Discriminating horizontal symmetries from operator analysis and proton-decay experiments

Krishnanath Bandyopadhyay and Asim K. Ray
Department of Physics, Visva-Bharati University, Santiniketan 731235, West Bengal, India

Utpal Sarkar*
Center for Particle Theory, The University of Texas at Austin, Austin, Texas 78712
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The impact of different horizontal symmetries on nucleon decay is discussed within the framework of the standard and the left-right-symmetric theory. Under very general assumptions of operator analysis, it is demonstrated that $U(1)_H$, $SU(2)_H$, $SU(3)_H^{VL}$, and $SU(3)_H^{V}$ symmetries predict different nucleon decay modes and, hence, the on-going proton-decay experiments can be used to discriminate among the different horizontal symmetries. Furthermore, it is shown that, under favorable situations, the standard and the left-right-symmetric theory can also be distinguished.

I. INTRODUCTION

Experiments at currently available energies fit the standard $SU(3)_C\times SU(2)_L\times U(1)_Y$ theory[1] although an interesting and viable left-right-symmetric[2] alternative to the standard theory based on the gauge group $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ is not ruled out. However, the several fermion generations cannot be explained within the framework of these theories. Several horizontal symmetries[3–8] have been devised to understand the problem of different generations of fermions including their masses, mixing angles, and *CP* violation, but they usually make very few testable predictions. So any phenomenology which can discriminate among different horizontal groups is very much welcome. In this paper we focus on possible experimental tests of different horizontal symmetries. From very general arguments of operator analysis,[9–14] we demonstrate that, within the framework of the standard and left-right-symmetric theories, different predictions follow for baryon-number-nonconserving nucleon decay for different horizontal symmetries.

Following grand unification, several experiments[15–18] have been designed to measure the proton lifetime τ, and the present experimental limit $\tau(p\to e^+\pi^0)>1.5\times 10^{32}$ year is in clear conflict with the minimal SU(5) prediction of $\tau(p\to e^+\pi^0)<10^{31}$ year. Thus, the minimal SU(5) model[19] is ruled out and the proton instability has become an empirical question. Hence operator analysis, which does not rely on any specific grand unified model, offers a useful method of studying various aspects of baryon- and lepton-number-nonconserving processes, which are assumed to be mediated by some superheavy particles with a characteristic grand unification mass $M\approx 10^{15}$ GeV. Historically, operator analysis[9–11] has been used to discuss the baryon- and lepton-number-nonconserving operator structure of the effective Hamiltonian that is invariant under the group $SU(3)_C\times SU(2)_L\times U(1)_Y$. Thus, processes described by an effective Lagrangian of dimensionality M^d are associated with an effective coupling constant which is suppressed by a factor roughly of order M^{4-d}. It is further assumed that, after integrating out all the superheavy degrees of freedom an $SU(3)_C\times SU(2)_L\times U(1)_Y$-invariant field theory describes the physical processes at low energies. $SU(3)_C$ invariance requires at least three quark fields to enter into an operator with nonzero baryon number, and Lorentz invariance requires an even number of fermion fields only. Thus any baryon-number-violating operator must contain at least four fermions and the dimensionality of the operator is at least 6. Higher-dimensional operators are suppressed by successively higher powers of $(M)^{-1}$.

In the present work, we consider the phenomenological predictions for nucleon decay from the operator analysis of the effective Hamiltonian that is invariant under the standard and the left-right-symmetric theory including a horizontal symmetry G_H. We have considered $G_H\equiv U(1)_H$, $SU(2)_H$, $SU(3)_H^{V}$, and $SU(3)_H^{VL}$. With the inclusion of the horizontal symmetry, the operator leading to a baryon- and lepton-number-nonconserving process should not only be singlet under the groups describing the standard $[SU(3)_C\times SU(2)_L\times U(1)_Y]$ or the left-right-symmetric theory $[SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}]$ but also be singlet under G_H. The decay modes resulting from an operator can be predicted from the all possible Feynman diagrams involving the quarks and the leptons entering the operator. Here we demonstrate that, within the framework of the standard and the left-right-symmetric theory including G_H, the different horizontal symmetries can be distinguished from a study of the nucleon decay modes. Furthermore, we show that, for certain horizontal symmetries, the standard theory can be distinguished from the left-right-symmetric theory.

The plan of the paper is as follows. The different horizontal symmetries and the representation of the fermions and Higgs bosons under these symmetries are discussed in Sec. II. Notations and classifications are given in Sec. III. Section IV catalogues dimension-6 operators leading to $\Delta B=\Delta L$ nucleon decay modes. Dimension-7 operators leading to $\Delta B=\Delta L$ as well as $\Delta B=-\Delta L$ decay modes are presented in Sec. V. Results are summarized in Sec. VI. Section VII contains our conclusions.

II. HORIZONTAL SYMMETRIES

Horizontal symmetries have been proposed by many authors in different contexts. Although the main motivation for horizontal symmetries seems to be the existence of the generation puzzle, the other motivations are to calculate the electron-muon mass ratio[20] and the weak mixing angles[21] including *CP* violation,[22] to avoid the strong *CP* problem,[23] and to achieve dynamically broken theories.[24] A horizontal gauge model is a model in which a gauge symmetry between different generations of quarks and leptons is introduced. This horizontal symmetry may be spontaneously broken, leading to mass differences between different generations as required by experiments. In such models there exist horizontal gauge bosons capable of mediating flavor-changing transitions and Higgs particles depending on the horizontal gauge groups.

The simplest horizontal gauge symmetry is the $U(1)_H$ symmetry,[3] which is proposed to distinguish between two fermionic generations. With the extension of the notion of generation structure to the Higgs system, the requirements of anomaly-free conditions, lepton-number conservation, and proper Cabibbo structure determine the horizontal quantum number Y_H uniquely in the standard theory. In the two-generation scheme $Y_H = \mp\frac{1}{3}[Y+2(B-L)]$ for fermions, where the $(-)$ and $(+)$ signs correspond to the first and the second generation, respectively. B, L, and Y are the baryon number, lepton number, and Weinberg-Salam hypercharge, respectively. The corresponding quantum numbers are summarized in Table I. In the left-right-symmetric theory incorporating $U(1)_H$ symmetry we have assumed[25] that the fermions belonging to the left-handed and right-handed doublets have the same Y_H quantum numbers. The model can be generalized to 2^n $(n=1,2,\ldots)$ generations. However, the assignment of Y_H quantum number to the fermions is no longer unique.[3] For the present purpose, we restrict ourselves to the decay modes of the nucleon and, hence, consider the two-generation $U(1)_H$ model for its uniqueness. There are two Higgs fields $h(1,2,1,+\frac{1}{3})$ and $h'(1,2,1,-\frac{1}{3})$ in the $SU(3)_C\times SU(2)_L\times U(1)_Y\times U(1)_H$ theory and three Higgs fields $g(1,2,2,2,0)$, $g'(1,2,2,2,\frac{2}{3})$, and $g''(1,2,2,2,-\frac{2}{3})$ in the $SU(3)_C\times SU(2)_L\times SU(2)_R \times U(1)_{B-L}\times U(1)_H$ theory. These Higgs fields have been used to construct dimension-7 operators. It may be noted that these Higgs fields contribute to the fermion masses.

Some authors have introduced a horizontal $SU(2)_H$ symmetry[4] in addition to the usual $SU(2)_L\times U(1)_Y$ of the Weinberg-Salam model for achieving *CP* violation with two generations[21,22] of quarks and calculating weak mixing angles.[21] In the present work we have considered an $SU(2)_H$ model with three generations of quarks and leptons.[26] Besides their usual $SU(2)_L\times U(1)_Y$ representation, the left- and right-handed fermions, under $SU(2)_H$, transform as triplets for the three-generation model. To construct dimension-7 operators in the standard and left-right-symmetric theories including $SU(2)_H$ symmetry, we have used the following Higgs particles: (i) $\phi(1,2,1,1)$ and $\omega(1,2,1,3)$ in the $SU(3)_C\times SU(2)_L\times U(1)_Y\times SU(2)_H$ theory and (ii) $\phi'(1,2,2,2,1)$ and $\omega'(1,2,2,2,3)$ in the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(2)_H$ theory. It is to be noted that the above Higgs fields also contribute to the fermion masses.

TABLE I. Fermion classification within the two-generation scheme.

	T_3	Y	Y_H
ν_L^e,ν_L^μ	$\frac{1}{2}$	-1	$+1,-1$
e_L,μ_L	$-\frac{1}{2}$	-1	$+1,-1$
e_R,μ_R	0	-2	$+\frac{4}{3},-\frac{4}{3}$
u_L,c_L	$\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3},+\frac{1}{3}$
d_L,s_L	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3},+\frac{1}{3}$
d_R,s_R	0	$-\frac{2}{3}$	0,0
u_R,c_R	0	$\frac{4}{3}$	$-\frac{2}{3},+\frac{2}{3}$

Another natural horizontal symmetry is the $SU(3)_H$ symmetry[5] since there are three generations of fermions. This symmetry has been considered by several authors[6,7] to study the fermion masses, *CP* violation, and flavor-changing neutral currents (FCNC). We have incorporated the $SU(3)_H$ symmetry in the standard and left-right-symmetric theories to construct dimension-6 and dimension-7 baryon- and lepton-number-violating operators. The left-handed and right-handed fermions transform as triplets under $SU(3)_H$ and their charge conjugates as antitriplets. To avoid anomalies a right-handed neutrino triplet is also necessary. In the present work we refer to this version of $SU(3)_H$ symmetry as $SU(3)_H^V$ symmetry. The following Higgs particles are used for construction of dimension-7 operators: (i) $\chi(1,1,0,3)$, $\xi(1,1,0,\bar{6})$, $\rho(1,2,1,\bar{3})$, and $t(1,2,1,6)$ in the $SU(3)_C \times SU(2)_L\times U(1)_Y\times SU(3)_H^V$ theory; (ii) $\chi'(1,1,1,0,3)$, $\xi'(1,1,1,0,\bar{6})$, $\rho'(1,2,2,2,\bar{3})$, and $t'(1,2,2,2,6)$ in the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_H^V$ theory. These Higgs fields do not contribute to the fermion masses.

Another version of the $SU(3)_H$ symmetry, which is referred to as $SU(3)_H^{VL}$ in the present paper, has also been used to construct the baryon- and lepton-number-violating operators. This symmetry[8] has been used by several authors in the context of a maximal grand unification like SU(16), low-energy supersymmetric models, and light Dirac neutrinos. We assume that, while all the left-handed fermions and antifermions and right-handed mirror fermions and mirror antifermions transform as triplets under $SU(3)_H^{VL}$, the rest of the fermions transform as antitriplets under $SU(3)_H^{VL}$. The resulting theory is chiral in both fermionic and mirror-fermionic context but, as a whole, it is a vectorlike theory and, hence, it is anomaly-free. The mirror fermions couple to charged gauge particles $W_{L,R}^{\pm}$, generating familiar low-energy weak interactions through $V+A$ rather than $V-A$ projections, and mix little with the basic fermions. For construction of dimension-7 operators, the following Higgs particles are used: (i) $\eta(1,2,1,\bar{3})$, $H(1,2,1,6)$, $\zeta(1,2,1,1)$, and $\Delta(1,2,1,8)$ in the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ theory; (ii) $\eta'(1,2,2,2,\bar{3})$, $H'(1,2,2,2,6)$, $\zeta'(1,2,2,2,1)$, and $\Delta'(1,2,2,2,8)$ in the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$ theory. It is to be noted that while $\eta(1,2,1,\bar{3})$ and $H(1,2,1,6)$ contribute to fermion masses in the standard theory, $\eta'(1,2,2,2,\bar{3})$ and $H'(1,2,2,2,6)$ generate fermion masses in the left-right-symmetric theory.

III. NOTATIONS AND CLASSIFICATIONS

The operators can be classified conveniently in terms of a multiplicative quantum number known as the F parity,[11] which is assumed to be conserved in baryon- and lepton-nonconserving interactions. A field is characterized by a pair of integers and/or half-integers (A,B), which specify their transformation under the homogeneous Lorentz group. T_L and T_R refer to the left-handed and right-handed weak isospin of the field, respectively. The F parity of a field is defined as $(-1)^{2A+2(T_L+T_R)}$. The F parity is even $(+1)$ for quarks and leptons and odd (-1) for the known gauge fields, space-time derivatives, and the Higgs fields. The F parity is of opposite sign for the corresponding fermions and antifermions but of the same sign for bosons and antibosons. The conservation of F parity requires that the operators with even parity are allowed but those with odd F parity are forbidden. Thus, the dimension-6 operator $QQQL$ $(\Delta B=\Delta L)$ with $F=+1$ is allowed but the dimension-6 operator $QQQ\bar{L}$ $(\Delta B=-\Delta L)$ with $F=-1$ is forbidden, and, to allow a $\Delta B=-\Delta L$ process, one has to combine a $\Delta B=-\Delta L$ operator with a Higgs field to construct a dimension-7 operator with $F=+1$.

We establish the following notations for our purpose. α, β, and γ are $SU(3)_C$ indices; i, j, k, and l are $SU(2)_L$ indices; i', j', k', and l' are $SU(2)_R$ indices; L_{iL} and $Q_{i\alpha L}$ are left-handed lepton and quark $SU(2)_L$ doublets; E_R, $U_{\alpha R}$, and $D_{\alpha R}$ are right-handed lepton, up- and down-quark $SU(2)_L$ singlets; $L_{i'R}$ and $Q_{i'\alpha R}$ are right-handed lepton and quark $SU(2)_R$ doublets; p, q, r, s, m, a, and n are horizontal indices for the groups $SU(2)_H$ and $SU(3)_H$ and these indices can assume values 1, 2, and 3 for three generations of fermions; superscript c denotes the Lorentz-invariant complex conjugate; ϵ_{ij}, $\epsilon_{i'j'}$, $\epsilon_{\alpha\beta\gamma}$, and ϵ_{pqr} are the totally antisymmetric $SU(2)_L$, $SU(2)_R$, $SU(3)_C$, and $SU(3)_H$ tensors, respectively, with $\epsilon_{12}\equiv 1$ and $\epsilon_{123}\equiv +1$ and δ is the usual Kronecker delta symbol. In the horizontal representation the asterisk represents the antitriplet representation. An operator O in the standard theory is represented as $\tilde{O}$ in the left-right-symmetric theory.

It is to be noted that we do not write those dimension-6 and dimension-7 operators, which lead to nucleon decay modes involving heavier mesons (with c, t, or b flavors) and the heavy lepton (τ) violating the conservation of energy.

IV. DIMENSION-6 OPERATORS

We are now in a position to write the $\Delta B=\Delta L$ dimension-6 operators leading to the nucleon decay in the $SU(3)_C \times SU(2)_L \times U(1)_Y \times G_H$ and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times G_H$ theories. We consider different horizontal symmetries like $G_H \equiv U(1)_H$, $SU(2)_H$, $SU(3)_H^{VL}$, and $SU(3)_H^{V}$, respectively, and demonstrate that these symmetries lead to different nucleon decay modes.

A. $U(1)_H$ symmetry

There are six dimension-6 operators in the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_H$ theory which are

$$O^1=(\bar{D}^c_{1\alpha R} U_{1\beta R})(\bar{Q}^c_{1i\gamma L} L_{1jL})\epsilon_{ij}\epsilon_{\alpha\beta\gamma} , \tag{1}$$

$$O^2=(\bar{D}^c_{2\alpha R} U_{1\beta R})(\bar{Q}^c_{1i\gamma L} L_{1jL})\epsilon_{ij}\epsilon_{\alpha\beta\gamma} , \tag{2}$$

$$O^3=(\bar{Q}^c_{1i\alpha L} Q_{1j\beta L})(\bar{U}^c_{1\gamma R} E_{1R})\epsilon_{ij}\epsilon_{\alpha\beta\gamma} , \tag{3}$$

$$O^4=(\bar{Q}^c_{1i\alpha L} Q_{1j\beta L})(\bar{Q}^c_{1k\gamma L} L_{1lL})\epsilon_{ij}\epsilon_{kl}\epsilon_{\alpha\beta\gamma} , \tag{4}$$

$$O^5=(\bar{D}^c_{1\alpha R} U_{1\beta R})(\bar{U}^c_{1\gamma R} E_{1R})\epsilon_{\alpha\beta\gamma} , \tag{5}$$

$$O^6=(\bar{D}^c_{2\alpha R} U_{1\beta R})(\bar{U}^c_{1\gamma R} E_{1R})\epsilon_{\alpha\beta\gamma} . \tag{6}$$

These operators lead to the following decay modes of the nucleon:

$$p \to \pi^+\bar{\nu}_e, \pi^+\pi^0\bar{\nu}_e, \pi^+\pi^-e^+, \pi^0e^+, K^+\bar{\nu}_e, K^+\pi^0\bar{\nu}_e ,$$

$$K^0\pi^+\bar{\nu}_e, K^0e^+, K^+\pi^-e^+, K^0\pi^0e^+, \pi^0\pi^0e^+$$

and

$$n \to \pi^0\bar{\nu}_e, \pi^0\pi^0\bar{\nu}_e, \pi^0\pi^-e^+, K^0\bar{\nu}_e, K^0\pi^0\bar{\nu}_e, K^0\pi^-e^+ ,$$

$$\pi^-e^+, \pi^+\pi^-\bar{\nu}_e .$$

It has already been pointed out[13] that the $U(1)_H$ symmetry forbids the nucleon decay into the leptons of the second generation and, hence, the decays like $p \to K^0\mu^+$, $K^+\bar{\nu}_\mu$ and $n \to K^0\bar{\nu}_\mu$ are ruled out.

In the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_H$ theory the following four dimension-6 operators are possible:

$$\tilde{O}^1=(\bar{Q}^c_{1i'\alpha R} Q_{1j'\beta R})(\bar{Q}^c_{1i\gamma L} L_{1jL})\epsilon_{ij}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma} , \tag{7}$$

$$\tilde{O}^2=(\bar{Q}^c_{1i\alpha L} Q_{1j\beta L})(\bar{Q}^c_{1i'\gamma R} L_{1j'R})\epsilon_{ij}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma} , \tag{8}$$

$$\tilde{O}^3=(\bar{Q}^c_{1i\alpha L} Q_{1j\beta L})(\bar{Q}^c_{1k\gamma L} L_{1lL})\epsilon_{ij}\epsilon_{kl}\epsilon_{\alpha\beta\gamma} , \tag{9}$$

$$\tilde{O}^4=(\bar{Q}^c_{1i'\alpha R} Q_{1j'\beta R})(\bar{Q}^c_{1k'\gamma R} L_{1l'R})\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma} . \tag{10}$$

These give rise to the following decays of nucleon:

$$p \to \pi^+\bar{\nu}_e, \pi^0e^+, \pi^+\pi^0\bar{\nu}_e, \pi^0\pi^0e^+, \pi^+\pi^-e^+$$

and

$$n \to \pi^0\bar{\nu}_e, \pi^-e^+, \pi^+\pi^-\bar{\nu}_e, \pi^-\pi^0e^+, \pi^0\pi^0\bar{\nu}_e .$$

Like the standard theory, the left-right-symmetric theory including $U(1)_H$ forbids the nucleon decay into the leptons of the second generation but, unlike the standard theory, the latter also forbids the decay of nucleon into strange particles.

B. $SU(2)_H$ symmetry

We consider an $SU(2)_H$ model with three generations of fermions.[26] We have constructed the following operators in the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(2)_H$ theory:

$$\mathscr{P}^1 = (\bar{D}^{cp}_{\alpha R} U^q_{\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (11)$$

$$\mathscr{P}^2 = (\bar{D}^{cp}_{\alpha R} U^q_{\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (12)$$

$$\mathscr{P}^3 = (\bar{D}^{cp}_{\alpha R} U^q_{\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}(\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (13)$$

$$\mathscr{P}^4 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{U}^{cr}_{\gamma R} E^s_R)\epsilon_{ij}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (14)$$

$$\mathscr{P}^5 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{U}^{cr}_{\gamma R} E^s_R)\epsilon_{ij}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (15)$$

$$\mathscr{P}^6 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{k\gamma L} L^s_{lL})\epsilon_{ij}\epsilon_{kl}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (16)$$

$$\mathscr{P}^7 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{k\gamma L} L^s_{lL})\epsilon_{ij}\epsilon_{kl}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (17)$$

$$\mathscr{P}^8 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{k\gamma L} L^s_{lL}) \times (\tau\epsilon)_{ij} \cdot (\tau\epsilon)_{kl}(\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (18)$$

$$\mathscr{P}^9 = (\bar{D}^{cp}_{\alpha R} U^q_{\beta R})(\bar{U}^{cr}_{\gamma R} E^s_R)\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (19)$$

$$\mathscr{P}^{10} = (\bar{D}^{cp}_{\alpha R} U^q_{\beta R})(\bar{U}^{cr}_{\gamma R} E^s_R)(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (20)$$

$$\mathscr{P}^{11} = (\bar{D}^{cp}_{\alpha R} U^q_{\beta R})(\bar{U}^{cr}_{\gamma R} E^s_R)(\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (21)$$

$$\mathscr{P}^{12} = (\bar{U}^{cp}_{\alpha R} U^q_{\beta R})(\bar{D}^{cr}_{\gamma R} E^s_R)(\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} . \quad (22)$$

The following nucleon decay modes are predicted:[26]

$$p \to \pi^+\bar{\nu}_e, \pi^+\pi^0\bar{\nu}_e, \pi^0 e^+, \pi^+\pi^- e^+, \pi^0\pi^0 e^+, K^+\bar{\nu}_\mu , \; K^+\pi^0\bar{\nu}_\mu, K^0\pi^+\bar{\nu}_\mu, K^0\mu^+, K^+\pi^-\mu^+, K^0\pi^0\mu^+ , \; K^+K^0\bar{\nu}_e, \pi^0\mu^+, \pi^0\pi^0\mu^+, \pi^+\pi^-\mu^+$$

and

$$n \to \pi^0\bar{\nu}_e, \pi^0\pi^0\bar{\nu}_e, \pi^0\pi^- e^+, K^0\bar{\nu}_\mu, K^0\pi^0\bar{\nu}_\mu, K^0\pi^-\mu^+ , \; K^0K^0\bar{\nu}_e, \pi^- e^+, \pi^+\pi^-\bar{\nu}_e, K^+\pi^-\bar{\nu}_\mu, \pi^-\mu^+, \pi^0\pi^-\mu^+ .$$

The model allows $\Delta S = 0$ nucleon decay modes involving the leptons of the first generation and only the charged lepton of the second generation whereas $\Delta S = 1$ decays involve leptons of the second generation only. Furthermore, the model predicts $\Delta S = 2$ nucleon decays involving the neutral lepton of the first generation.

In the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(2)_H$ theory the following ten dimension-6 operators are possible:

$$\tilde{\mathscr{P}}^1 = (\bar{Q}^{cp}_{i'\alpha R} Q^q_{j'\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}\epsilon_{i'j'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (23)$$

$$\tilde{\mathscr{P}}^2 = (\bar{Q}^{cp}_{i'\alpha R} Q^q_{j'\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}\epsilon_{i'j'}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (24)$$

$$\tilde{\mathscr{P}}^3 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{k\gamma L} L^s_{lL})\epsilon_{ij}\epsilon_{kl}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (25)$$

$$\tilde{\mathscr{P}}^4 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{k\gamma L} L^s_{lL})\epsilon_{ij}\epsilon_{kl}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (26)$$

$$\tilde{\mathscr{P}}^5 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{k\gamma L} L^s_{lL}) \times (\tau\epsilon)_{ij} \cdot (\tau\epsilon)_{kl}(\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (27)$$

$$\tilde{\mathscr{P}}^6 = (\bar{Q}^{cp}_{i'\alpha R} Q^q_{j'\beta R})(\bar{Q}^{cr}_{k'\gamma R} L^s_{l'R})\epsilon_{i'j'}\epsilon_{k'l'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (28)$$

$$\tilde{\mathscr{P}}^7 = (\bar{Q}^{cp}_{i'\alpha R} Q^q_{j'\beta R})(\bar{Q}^{cr}_{k'\gamma R} L^s_{l'R}) \times \epsilon_{i'j'}\epsilon_{k'l'}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (29)$$

$$\tilde{\mathscr{P}}^8 = (\bar{Q}^{cp}_{i'\alpha R} Q^q_{j'\beta R})(\bar{Q}^{cr}_{k'\gamma R} L^s_{l'R}) \times (\tau\epsilon)_{i'j'} \cdot (\tau\epsilon)_{k'l'}(\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} , \quad (30)$$

$$\tilde{\mathscr{P}}^9 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{i'\gamma R} L^s_{j'R})\epsilon_{ij}\epsilon_{i'j'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma} , \quad (31)$$

$$\tilde{\mathscr{P}}^{10} = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr}_{i'\gamma R} L^s_{j'R})\epsilon_{ij}\epsilon_{i'j'}(\delta_{pr}\delta_{qs} + \delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma} . \quad (32)$$

These lead to nucleon decay modes as follows:

$$p \to \pi^0 e^+, \pi^+\pi^0\bar{\nu}_e, \pi^0\pi^0 e^+, \pi^+\bar{\nu}_e, \pi^+\pi^- e^+, K^+\bar{\nu}_\mu , \; K^+\pi^0\bar{\nu}_\mu, K^0\pi^+\bar{\nu}_\mu, K^0\mu^+, K^+\pi^-\mu^+, K^+K^0\bar{\nu}_e, K^0\pi^0\mu^+$$

and

$$n \to \pi^0\bar{\nu}_e, \pi^0\pi^0\bar{\nu}_e, \pi^+\pi^-\bar{\nu}_e, \pi^- e^+, \pi^0\pi^- e^+, K^0\bar{\nu}_\mu , \; K^0\pi^0\bar{\nu}_\mu, K^0\pi^-\mu^+, K^0K^0\bar{\nu}_e, K^+\pi^-\bar{\nu}_\mu .$$

Interestingly, unlike the standard theory, the left-right-symmetric theory including $SU(2)_H$ forbids the $\Delta S = 0$ nucleon decays involving the charged lepton of the second generation.

C. $SU(3)_H^{VL}$ symmetry

There are only two dimension-6 operators in the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H^{VL}$ theory, which are

$$R^1 = (\bar{D}^{cp^*}_{\alpha R} U^{q^*}_{\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}\delta_{p^*r}\delta_{q^*s}\epsilon_{\alpha\beta\gamma} , \quad (33)$$

$$R^2 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{U}^{cr^*}_{\gamma R} E^{s^*}_R)\epsilon_{ij}\delta_{pr^*}\delta_{qs^*}\epsilon_{\alpha\beta\gamma} . \quad (34)$$

These operators predict the following decay modes of the nucleon:

$$p \to \pi^+\bar{\nu}_e, \pi^+\pi^0\bar{\nu}_e, \pi^0 e^+, \pi^+\pi^- e^+, \pi^0\pi^0 e^+, K^+\bar{\nu}_\mu , \; K^+\pi^0\bar{\nu}_\mu, K^0\pi^+\bar{\nu}_\mu, K^0\mu^+, K^+\pi^-\mu^+, K^0\pi^0\mu^+$$

and

$$n \to \pi^0\bar{\nu}_e, \pi^0\pi^0\bar{\nu}_e, \pi^0\pi^- e^+, K^0\bar{\nu}_\mu, K^0\pi^0\bar{\nu}_\mu, K^0\pi^-\mu^+ , \; \pi^- e^+, \pi^+\pi^-\bar{\nu}_e .$$

It may be noted that this model, like the three-generation $SU(2)_H$ model, allows $\Delta S = 1$ decays involving only the leptons of the second generation but unlike the three-generation $SU(2)_H$ model, the $SU(3)_H^{VL}$ symmetry allows the $\Delta S = 0$ decays involving the leptons of the first generation only.

In the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^{VL}$ theory, there are also the following dimension-6 operators:

$$\tilde{R}^1 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{Q}^{cr}_{i\gamma L} L^s_{jL})\epsilon_{ij}\epsilon_{i'j'}\delta_{p^*r}\delta_{q^*s}\epsilon_{\alpha\beta\gamma} , \quad (35)$$

$$\tilde{R}^2 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{Q}^{cr^*}_{i'\gamma R} L^{s^*}_{j'R})\epsilon_{ij}\epsilon_{i'j'}\delta_{pr^*}\delta_{qs^*}\epsilon_{\alpha\beta\gamma} . \quad (36)$$

It is to be noted that the operators in the left-right-symmetric theory incorporating $SU(3)_H^{VL}$ lead to the same nucleon decay modes as those predicted in the standard theory including the $SU(3)_H^{VL}$ symmetry although the former allows left-handed as well as right-handed neutrinos whereas the latter yields left-handed neutrinos only.

D. $SU(3)_H^V$ symmetry

All fermions of the three families transform as triplets and all antifermions transform as antitriplets under $SU(3)_H^V$ symmetry. Therefore, the operators containing a multiple of three fermions or a fermion-antifermion pair will be invariant under $SU(3)_H^V$ symmetry. But Lorentz invariance permits only an even number of fermionic fields. Thus, in the $SU(3)_C\times SU(2)_L\times U(1)_Y\times SU(3)_H^V$ theory, baryon- and lepton-number-violating operators containing four fermion fields do not preserve $SU(3)_H^V$ invariance and, hence, the construction of dimension-6 operators is not possible.[13] The same is also true for the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(3)_H^V$ theory.

V. DIMENSION-7 OPERATORS

We now write the dimension-7 operators. It is worthwhile to point out that a dimension-7 operator contains four fermion fields and one Higgs field (H) and the effective coupling constant of a nucleon decay mode described by a dimension-7 operator will be suppressed by a factor $M^{-3}\langle H\rangle$. If we assume the unification mass scale $M\approx 10^{15}$ GeV, $\langle H\rangle = m_W\approx 10^2$ GeV, we get the proton lifetime $\tau\approx 10^{44}$ year. However, the value of τ is reduced with the choice of a lower value of M or an intermediate mass scale ($\tilde{m}$) responsible for horizontal symmetry breaking. In particular, if proton decay is mediated by some exotic particles with mass $M\approx 10^{13}$ GeV, and the horizontal symmetry is broken at a mass scale $\langle H\rangle = m\approx 10^5$ GeV (as determined from the constraints on flavor-changing neutral currents, we obtain $\tau\approx 10^{31}$ year.

A. $U(1)_H$ symmetry

In the $SU(2)_C\times SU(2)_L\times U(1)_Y\times U(1)_H$ theory there are two possible Higgs fields, $h(1,2,1,+\frac{1}{3})$ and $h'(1,2,1,-\frac{1}{3})$, which have been used to construct dimension-7 operators. We do not write the operators associated with $h'(1,2,1,-\frac{1}{3})$ as these operators lead to nucleon decay modes involving heavy-quark fields. The relevant dimension-7 operators with $h(1,2,1,+\frac{1}{3})$ are

$$L^1=(\bar{Q}^c_{1i\alpha L}Q_{1j\beta L})(\bar{L}_{2kL}D_{1\gamma R})h_l^\dagger\epsilon_{ij}\epsilon_{kl}\epsilon_{\alpha\beta\gamma}\ , \qquad (37)$$

$$L^2=(\bar{Q}^c_{1i\alpha L}Q_{1j\beta L})(\bar{L}_{2kL}D_{2\gamma R})h_l^\dagger\epsilon_{ij}\epsilon_{kl}\epsilon_{\alpha\beta\gamma}\ , \qquad (38)$$

$$L^3=(\bar{D}^c_{2\alpha R}D_{1\beta R})(\bar{L}_{2iL}U_{1\gamma R})h_j^\dagger\epsilon_{ij}\epsilon_{\alpha\beta\gamma}\ . \qquad (39)$$

The decay modes resulting from these operators are

$$p\to\pi^+\nu_\mu,\pi^+\pi^0\nu_\mu,\pi^+K^0\nu_\mu,\pi^0K^+\nu_\mu,K^+\nu_\mu$$

and

$$n\to\pi^0\nu_\mu,\pi^0\pi^0\nu_\mu,\pi^+\pi^-\nu_\mu,\pi^0K^0\nu_\mu,\pi^-K^+\nu_\mu,K^0\nu_\mu\ .$$

Here dimension-7 operators allow nucleon decay modes into the neutral lepton of the second generation only.

In the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times U(1)_H$ theory, there are three possible Higgs fields $g(1,2,2,2,0)$, $g'(1,2,2,2,+\frac{2}{3})$, and $g''(1,2,2,2,-\frac{2}{3})$, which are necessary to generate the quark mass matrices. The dimension-7 operators associated with the Higgs field $g(1,2,2,2,0)$ are

$$G^1=(\bar{Q}^c_{1i\alpha L}Q_{1j\beta L})(\bar{L}_{2kL}Q_{1i'\gamma R})g_{lj'}^\dagger\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (40)$$

$$G^2=(\bar{Q}^c_{1i\alpha L}Q_{1j\beta L})(\bar{L}_{2i'R}Q_{1k\gamma L})g_{lj'}^\dagger\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (41)$$

$$G^3=(\bar{Q}^c_{1i'\alpha R}Q_{1j'\beta R})(\bar{L}_{2iL}Q_{1k'\gamma R})g_{jl'}^\dagger\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \qquad (42)$$

$$G^4=(\bar{Q}^c_{1i'\alpha R}Q_{1j'\beta R})(\bar{L}_{2k'R}Q_{1i\gamma L})g_{jl'}^\dagger\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ . \qquad (43)$$

These operators allow the following nucleon decays:

$$p\to\pi^+\nu_\mu,\pi^+\pi^0\nu_\mu$$

and

$$n\to\pi^0\nu_\mu,\pi^0\pi^0\nu_\mu,\pi^+\pi^-\nu_\mu\ .$$

With $g'(1,2,2,2,+\frac{2}{3})$, the operators are

$$H^1=(\bar{Q}^c_{1i\alpha L}Q_{1j\beta L})(\bar{L}_{2kL}Q_{2i'\gamma R})g'^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (44)$$

$$H^2=(\bar{Q}^c_{1i\alpha L}Q_{1j\beta L})(\bar{L}_{2i'R}Q_{2k\gamma L})g'^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (45)$$

$$H^3=(\bar{Q}^c_{1i'\alpha R}Q_{1j'\beta R})(\bar{L}_{2iL}Q_{2k'\gamma R})g'^\dagger_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \qquad (46)$$

$$H^4=(\bar{Q}^c_{1i'\alpha R}Q_{1j'\beta R})(\bar{L}_{2k'R}Q_{2i\gamma L})g'^\dagger_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \qquad (47)$$

$$H^5=(\bar{Q}^c_{1i\alpha L}Q_{2j\beta L})(\bar{L}_{2kL}Q_{1i'\gamma R})g'^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (48)$$

$$H^6=(\bar{Q}^c_{1i\alpha L}Q_{2j\beta L})(\bar{L}_{2kL}Q_{1i'\gamma R})g'^\dagger_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (49)$$

$$H^7=(\bar{Q}^c_{1i\alpha L}Q_{2j\beta L})(\bar{L}_{2i'R}Q_{1k\gamma L})g'^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (50)$$

$$H^8=(\bar{Q}^c_{1i\alpha L}Q_{2j\beta L})(\bar{L}_{2i'R}Q_{1k\gamma L})g'^\dagger_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (51)$$

$$H^9=(\bar{Q}^c_{1i'\alpha R}Q_{2j'\beta R})(\bar{L}_{2iL}Q_{1k'\gamma R})g'^\dagger_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \qquad (52)$$

$$H^{10}=(\bar{Q}^c_{1i'\alpha R}Q_{2j'\beta R})(\bar{L}_{2iL}Q_{1k'\gamma R})\times g'^\dagger_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \qquad (53)$$

$$H^{11}=(\bar{Q}^c_{1i'\alpha R}Q_{2j'\beta R})(\bar{L}_{2k'R}Q_{1i\gamma L})g'^\dagger_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \qquad (54)$$

$$H^{12}=(\bar{Q}^c_{1i'\alpha R}Q_{2j'\beta R})(\bar{L}_{2k'R}Q_{1i\gamma L})g'^\dagger_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}\epsilon_{\alpha\beta\gamma}\ . \qquad (55)$$

These predict

$$p\to K^+\nu_\mu,K^+\pi^0\nu_\mu,K^0\pi^+\nu_\mu,K^+\pi^+\mu^-$$

and

$$n\to K^0\nu_\mu,K^+\mu^-,K^0\pi^0\nu_\mu,K^+\pi^-\nu_\mu,K^0\pi^+\mu^-,K^+\pi^0\mu^-\ .$$

Furthermore, dimension-7 operators associated with $g''(1,2,2,2,-\frac{2}{3})$ are given by

$$m^1=(\bar{Q}^c_{2i\alpha L}Q_{1j\beta L})(\bar{L}_{1kL}Q_{2i'\gamma R})g''^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (56)$$

$$m^2=(\bar{Q}^c_{2i\alpha L}Q_{1j\beta L})(\bar{L}_{1kL}Q_{2i'\gamma R})g''^\dagger_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (57)$$

$$m^3=(\bar{Q}^c_{2i\alpha L}Q_{1j\beta L})(\bar{L}_{1i'R}Q_{2k\gamma L})g''^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \qquad (58)$$

$$m^4=(\bar{Q}^{\,c}_{2i\alpha L}Q_{1j\beta L})(\bar{L}_{1i'R}Q_{2k\gamma L})g''^{\dagger}_{ij'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{i'j'}\epsilon_{\alpha\beta\gamma}\ , \tag{59}$$

$$m^5=(\bar{Q}^{\,c}_{2i'\alpha R}Q_{1j'\beta R})(\bar{L}_{1iL}Q_{2k'\gamma R})g''^{\dagger}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \tag{60}$$

$$m^6=(\bar{Q}^{\,c}_{2i'\alpha R}Q_{1j'\beta R})(\bar{L}_{1iL}Q_{2k'\gamma R})g''^{\dagger}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \tag{61}$$

$$m^7=(\bar{Q}^{\,c}_{2i'\alpha R}Q_{1j'\beta R})(\bar{L}_{1k'R}Q_{2i\gamma L})g''^{\dagger}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\epsilon_{\alpha\beta\gamma}\ , \tag{62}$$

$$m^8=(\bar{Q}^{\,c}_{2i'\alpha R}Q_{1j'\beta R})(\bar{L}_{1k'R}Q_{2i\gamma L})g''^{\dagger}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}\epsilon_{\alpha\beta\gamma}\ . \tag{63}$$

These lead to

$$p\to K^+K^0\nu_e, K^+K^+e^-$$

and

$$n\to K^0K^0\nu_e, K^+K^0e^-\ .$$

Interestingly, dimension-7 operators with the Higgs field $g(1,2,2,2,0)$ allow only $\Delta S=0$ nucleon decays; those with $g'(1,2,2,2,+\frac{2}{3})$ lead to only $\Delta S=1$ nucleon decays but the operators with $g''(1,2,2,2,-\frac{2}{3})$ give rise to nucleon decay modes with $\Delta S=2$ only.

B. $SU(2)_H$ symmetry

There are eight dimension-7 operators with the Higgs field $\phi(1,2,1,1)$ in the $SU(3)_C\times SU(2)_L\times U(1)_Y\times SU(2)_H$ theory with three generations of fermions. These are

$$F^1=(\bar{D}^{\,cp}_{\alpha R}U^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\phi^{\dagger}_j\epsilon_{ij}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{64}$$

$$F^2=(\bar{D}^{\,cp}_{\alpha R}U^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\phi^{\dagger}_j\epsilon_{ij}(\delta_{pr}\delta_{qs}+\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{65}$$

$$F^3=(\bar{D}^{\,cp}_{\alpha R}U^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\phi^{\dagger}_j\epsilon_{ij}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{66}$$

$$F^4=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kl}D^s_{\gamma R})\phi^{\dagger}_l\epsilon_{ij}\epsilon_{kl}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{67}$$

$$F^5=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kL}D^s_{\gamma R})\phi^{\dagger}_l\epsilon_{ij}\epsilon_{kl}(\delta_{pr}\delta_{qs}+\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{68}$$

$$F^6=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kL}D^s_{\gamma R})\times\phi^{\dagger}_l(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{69}$$

$$F^7=(\bar{D}^{\,cp}_{\alpha R}D^q_{\beta R})(\bar{L}^{\,r}_{iL}U^s_{\gamma R})\phi^{\dagger}_j\epsilon_{ij}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{70}$$

$$F^8=(\bar{D}^{\,cp}_{\alpha R}D^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\phi_i(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ . \tag{71}$$

These give rise to the following decay modes:

$$p\to\pi^+\nu_e,\pi^+\pi^0\nu_e,K^+\nu_\mu,K^+\pi^0\nu_\mu,K^0\pi^+\nu_\mu,K^+K^0\nu_e\ ,\quad K^+\pi^+e^-,K^+\pi^+\mu^-$$

and

$$n\to\pi^0\nu_e,\pi^0\pi^0\nu_e,\pi^0K^0\nu_\mu,K^0\nu_\mu,K^0K^0\nu_e,\pi^+\pi^-\nu_e\ ,\quad K^+\pi^-\nu_\mu,K^0\pi^+e^-,K^+\pi^0e^-,K^+\mu^-,K^0\pi^+\mu^-\ ,\quad K^+\pi^0\mu^-,K^+e^-\ .$$

Furthermore, there is another set of eight dimension-7 operators with the Higgs field $\omega(1,2,1,3)$. These are given by

$$N^1=(\bar{D}^{\,cp}_{\alpha R}U^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\omega^{\dagger m}_j\epsilon_{ij}\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{72}$$

$$N^2=(\bar{D}^{\,cp}_{\alpha R}U^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\omega^{\dagger m}_j\epsilon_{ij}\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ , \tag{73}$$

$$N^3=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kL}D^s_{\gamma R})\omega^{\dagger m}_l(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{74}$$

$$N^4=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kL}D^s_{\gamma R})\omega^{\dagger m}_l(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ , \tag{75}$$

$$N^5=(\bar{D}^{\,cp}_{\alpha R}D^q_{\beta R})(\bar{L}^{\,r}_{iL}U^s_{\gamma R})\omega^{\dagger m}_j\epsilon_{ij}\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{76}$$

$$N^6=(\bar{D}^{\,cp}_{\alpha R}D^q_{\beta R})(\bar{L}^{\,r}_{iL}U^s_{\gamma R})\omega^{\dagger m}_j\epsilon_{ij}\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ , \tag{77}$$

$$N^7=(\bar{D}^{\,cp}_{\alpha R}D^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\omega^m_i\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{78}$$

$$N^8=(\bar{D}^{\,cp}_{\alpha R}D^q_{\beta R})(\bar{L}^{\,r}_{iL}D^s_{\gamma R})\omega^m_i\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ . \tag{79}$$

The decay modes resulting from these operators are

$$p\to K^+\nu_e,K^+\pi^0\nu_e,K^0\pi^+\nu_e,K^+K^0\nu_\mu,K^+\pi^+e^-\ ,\quad K^+K^+\mu^-,K^+\nu_\tau,K^+\pi^0\nu_\tau,K^0\pi^+\nu_\tau,K^+K^0\nu_\tau$$

and

$$n\to K^0\nu_e,K^0\pi^0\nu_e,K^0K^0\nu_\mu,K^+e^-,K^0\pi^+e^-,K^+K^0\mu^-\ ,\quad K^0\nu_\tau,K^0\pi^0\nu_\tau,K^+\pi^-\nu_\tau,K^0K^0\nu_\tau,K^+\pi^0e^-,K^+\pi^-\nu_e\ .$$

Obviously, the decay modes resulting from dimension-7 operators with the Higgs particles $\phi(1,2,1,1)$ and $\omega(1,2,1,3)$ are different; the latter leads to $\Delta S=1$ and $\Delta S=2$ nucleon decays, whereas the former allows $\Delta S=0$, $\Delta S=1$, and $\Delta S=2$ nucleon decays.

In the $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\times SU(2)_H$ theory, twelve operators with the Higgs field $\phi'(1,2,2,2,1)$ are possible:

$$\tilde{F}^1=(\bar{Q}^{\,cp}_{i'\alpha R}Q^q_{j'\beta R})(\bar{L}^{\,r}_{iL}Q^s_{k'\gamma R})\phi'^{\dagger}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{80}$$

$$\tilde{F}^2=(\bar{Q}^{\,cp}_{i'\alpha R}Q^q_{j'\beta R})(\bar{L}^{\,r}_{il}Q^s_{k'\gamma R})\times\phi'^{\dagger}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}(\delta_{pr}\delta_{qs}+\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{81}$$

$$\tilde{F}^3=(\bar{Q}^{\,cp}_{i'\alpha R}Q^q_{j'\beta R})(\bar{L}^{\,r}_{iL}Q^s_{k'\gamma R})\times\phi'^{\dagger}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{82}$$

$$\tilde{F}^4=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kL}Q^s_{i'\gamma R})\phi'^{\dagger}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{83}$$

$$\tilde{F}^5=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kL}Q^s_{i'\gamma R})\times\phi'^{\dagger}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}(\delta_{pr}\delta_{qs}+\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{84}$$

$$\tilde{F}^6=(\bar{Q}^{\,cp}_{i\alpha L}Q^q_{j\beta L})(\bar{L}^{\,r}_{kl}Q^s_{i'\gamma R})\times\phi'^{\dagger}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{i'j'}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{85}$$

$$\tilde{F}^7=(\bar{Q}^{\,cp}_{i'\alpha R}Q^q_{j'\beta R})(\bar{L}^{\,r}_{k'R}Q^s_{i\gamma L})\phi'^{\dagger}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \tag{86}$$

$$\tilde{F}^8=(\bar{Q}^{\,cp}_{i'\alpha R}Q^q_{j'\beta R})(\bar{L}^{\,r}_{k'R}Q^s_{i\gamma L})\times\phi'^{\dagger}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}(\delta_{pr}\delta_{qs}+\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \tag{87}$$

$$\tilde{F}^{9}=(\bar{Q}\,^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}\,^{r}_{k'R}Q^{s}_{i\gamma L})$$
$$\times\phi'^{\dagger}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}(\tau\epsilon)_{k'l'}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \qquad (88)$$

$$\tilde{F}^{10}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{i'R}Q^{s}_{k\gamma L})\phi'^{\dagger}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\delta_{pq}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \qquad (89)$$

$$\tilde{F}^{11}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{i'R}Q^{s}_{k\gamma L})$$
$$\times\phi'^{\dagger}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}(\delta_{pr}\delta_{qs}+\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ , \qquad (90)$$

$$\tilde{F}^{12}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{i'R}Q^{s}_{k\gamma L})$$
$$\times\phi'^{\dagger}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}\epsilon_{i'j'}(\delta_{pr}\delta_{qs}-\delta_{ps}\delta_{qr})\epsilon_{\alpha\beta\gamma}\ . \qquad (91)$$

These predict the following nucleon decays:

$$p\to\pi^+\nu_e,\pi^+\pi^0\nu_e,\pi^0K^+\nu_\mu,\pi^+K^0\nu_\mu,\pi^+K^+\mu^-,K^+\nu_\mu\ ,$$
$$K^+K^0\nu_e,K^+K^+e^-\ .$$

and

$$n\to\pi^0\nu_e,\pi^0\pi^0\nu_e,\pi^+\pi^-\nu_e,K^0\nu_\mu,K^0\pi^0\nu_\mu,K^0K^0\nu_e\ ,$$
$$K^+\pi^-\nu_\mu,K^+K^0e^-,K^+\mu^-,K^0\pi^+\mu^-,K^+\pi^0\mu^-\ .$$

Another eight operators with the Higgs field $\omega'(1,2,2,2,3)$ can be constructed as follows:

$$\tilde{N}^{1}=(\bar{Q}\,^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}\,^{r}_{iL}Q^{s}_{k'\gamma R})\omega'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \qquad (92)$$

$$\tilde{N}^{2}=(\bar{Q}\,^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}\,^{r}_{iL}Q^{s}_{k'\gamma R})\omega'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ , \qquad (93)$$

$$\tilde{N}^{3}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{kL}Q^{s}_{i'\gamma R})\omega'^{\dagger m}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}$$
$$\times\epsilon_{i'j'}\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \qquad (94)$$

$$\tilde{N}^{4}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{kL}Q^{s}_{i'\gamma R})\omega'^{\dagger m}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}$$
$$\times\epsilon_{i'j'}\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ , \qquad (95)$$

$$\tilde{N}^{5}=(\bar{Q}\,^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}\,^{r}_{k'R}Q^{s}_{i\gamma L})\omega'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \qquad (96)$$

$$\tilde{N}^{6}=(\bar{Q}\,^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}\,^{r}_{k'R}Q^{s}_{i\gamma L})\omega'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ , \qquad (97)$$

$$\tilde{N}^{7}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{i'R}Q^{s}_{k\gamma L})\omega'^{\dagger m}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}$$
$$\times\epsilon_{i'j'}\epsilon_{pqm}\delta_{rs}\epsilon_{\alpha\beta\gamma}\ , \qquad (98)$$

$$\tilde{N}^{8}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r}_{i'R}Q^{s}_{k\gamma L})\omega'^{\dagger m}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}$$
$$\times\epsilon_{i'j'}\epsilon_{pqr}\delta_{sm}\epsilon_{\alpha\beta\gamma}\ . \qquad (99)$$

These operators permit the following nucleon decay modes:

$$p\to K^+\nu_e,K^+\pi^0\nu_e,K^0\pi^+\nu_e,K^+K^0\nu_\mu,K^+\pi^+e^-\ ,$$
$$K^+K^+\mu^-,K^+\nu_\tau,K^+\pi^0\nu_\tau,K^0\pi^+\nu_\tau,K^+K^0\nu_\tau$$

and

$$n\to K^0\nu_e,K^0\pi^0\nu_e,K^0K^0\nu_\mu,K^+\pi^-\nu_e,K^+e^-,K^+\pi^0e^-\ ,$$
$$K^0\pi^+e^-,K^+K^0\mu^-,K^0\nu_e,K^0\pi^0\nu_\tau,K^+\pi^-\nu_\tau,K^0K^0\nu_\tau\ .$$

Clearly, the decay modes arising from dimension-7 operators with $\phi'(1,2,2,2,1)$ and $\omega'(1,2,2,2,3)$ are different; the former leads to $\Delta S=0$, $\Delta S=1$, and $\Delta S=2$ nucleon decays while the latter allows $\Delta S=1$ and $\Delta S=2$ nucleon decays only.

Interestingly, with $SU(2)_H$-triplet (ω,ω') Higgs bosons, the standard and the left-right-symmetric theory cannot be distinguished although, with $SU(2)_H$ singlet (ϕ,ϕ'), they lead to different predictions.

C. $SU(3)_H^{VL}$ symmetry

In the $SU(3)_C\times SU(2)_L\times U(1)_Y\times SU(3)_H^{VL}$ theory, dimension-7 operators have been constructed using the Higgs fields $\eta(1,2,1,\bar{3})$, $H(1,2,1,6)$, $\Delta(1,2,1,8)$, and $\zeta(1,2,1,1)$. The following two operators with the Higgs field $\eta(1,2,1,\bar{3})$ are possible:

$$X^{1}=(\bar{D}\,^{cp*}_{\alpha R}U^{q*}_{\beta R})(\bar{L}\,^{r*}_{iL}D^{s*}_{\gamma R})\eta_j^{\dagger m}\epsilon_{ij}\epsilon_{p^*q^*r^*}\delta_{s^*m}\epsilon_{\alpha\beta\gamma}\ , \qquad (100)$$

$$X^{2}=(\bar{D}\,^{cp*}_{\alpha R}D^{q*}_{\beta R})(\bar{L}\,^{r*}_{iL}U^{s*}_{\gamma R})\eta_j^{\dagger m}\epsilon_{ij}\epsilon_{p^*q^*r^*}\delta_{s^*m}\epsilon_{\alpha\beta\gamma}\ . \qquad (101)$$

The decay modes are

$$p\to K^+\nu_\tau,K^+\pi^0\nu_\tau,K^0\pi^+\nu_\tau,K^+K^0\nu_\tau$$

and

$$n\to K^0\nu_\tau,K^0\pi^0\nu_\tau,K^+\pi^-\nu_\tau,K^0K^0\nu_\tau\ .$$

With the Higgs field $H(1,2,1,6)$, there are two operators which are

$$I^{1}=(\bar{D}\,^{cp*}_{\alpha R}U^{q*}_{\beta R})(\bar{L}\,^{r*}_{iL}D^{s*}_{\gamma R})H_j^{\dagger m^*n^*}$$
$$\times\epsilon_{ij}\epsilon_{p^*q^*m^*}\epsilon_{r^*s^*n^*}\epsilon_{\alpha\beta\gamma}\ , \qquad (102)$$

$$I^{2}=(\bar{D}\,^{cp*}_{\alpha R}U^{q*}_{\beta R})(\bar{L}\,^{r*}_{iL}D^{s*}_{\gamma R})H_j^{\dagger m^*n^*}$$
$$\times\epsilon_{ij}\epsilon_{p^*q^*r^*}\epsilon_{s^*m^*n^*}\epsilon_{\alpha\beta\gamma}\ . \qquad (103)$$

These operators lead to the following decay modes of the nucleon:

$$p\to K^+\nu_\tau,K^+\pi^0\nu_\tau,K^0\pi^+\nu_\tau,K^+K^0\nu_\tau,K^+\nu_\mu,K^+\pi^0\nu_\mu\ ,$$
$$K^0\pi^+\nu_\mu,K^+K^0\nu_e$$

and

$$n\to K^0\nu_\tau,K^0\pi^0\nu_\tau,K^+\pi^-\nu_\tau,K^0K^0\nu_\tau,K^0\nu_\mu,K^0\pi^0\nu_\mu\ ,$$
$$K^+\pi^-\nu_\mu,K^0K^0\nu_e\ .$$

There are only two operators with the Higgs field $\Delta(1,2,1,8)$:

$$J^{1}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r*}_{kL}D^{s*}_{\gamma R})\Delta_l^{\dagger m^*n}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}$$
$$\times\epsilon_{pqn}\epsilon_{r^*s^*m^*}\epsilon_{\alpha\beta\gamma}\ , \qquad (104)$$

$$J^{2}=(\bar{Q}\,^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}\,^{r*}_{kL}D^{s*}_{\gamma R})\Delta_l^{\dagger m^*n}\epsilon_{ij}\epsilon_{kl}$$
$$\times\delta_{pr^*}\delta_{qs^*}\delta_{m^*n}\epsilon_{\alpha\beta\gamma}\ . \qquad (105)$$

These operators allow the following decay modes:

$$p \to \pi^+\nu_e, K^+\nu_\mu, \pi^+\pi^0\nu_e, K^+\pi^0\nu_\mu, K^0\pi^+\nu_\mu, K^+K^0\nu_e\,,$$
$$K^+\nu_\tau, K^+\pi^0\nu_\tau, K^0\pi^+\nu_\tau, K^+K^0\nu_\tau, K^+K^+e^-\,,$$
$$K^+\pi^+\mu^-, K^+\pi^0\nu_e, K^0\pi^+\nu_e$$

and

$$n \to K^0\nu_\mu, K^0K^0\nu_e, K^0\pi^0\nu_\mu, K^0\nu_\tau, K^0\pi^0\nu_\tau, K^+\pi^-\nu_\mu\,,$$
$$K^+\pi^-\nu_\tau, K^0K^0\nu_\tau, \pi^0\nu_e, \pi^0\pi^0\nu_e, \pi^+\pi^-\nu_e, K^+K^0e^-\,,$$
$$K^+\mu^-, K^+\pi^0\mu^-, K^0\pi^+\mu^-, K^0\pi^0\nu_e, K^+\pi^-\nu_e\,.$$

The following operator is possible with the Higgs field $\zeta(1,2,1,1)$:

$$Z^1 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{L}^{r*}_{kL} D^{s*}_{\gamma R})\zeta^\dagger_l \epsilon_{ij}\epsilon_{kl}\delta_{pr^*}\delta_{qs^*}\epsilon_{\alpha\beta\gamma}\,. \tag{106}$$

The operator predicts the following decays modes:

$$p \to \pi^+\nu_e, \pi^+\pi^0\nu_e, K^+\nu_\mu, K^+\pi^0\nu_\mu, K^0\pi^+\nu_\mu, K^+K^0\nu_e\,,$$
$$K^+\pi^0\nu_e, K^0\pi^+\nu_e$$

and

$$n \to \pi^0\nu_e, \pi^+\pi^-\nu_e, \pi^0\pi^0\nu_e, K^0\nu_\mu, K^0\pi^0\nu_\mu, K^0K^0\nu_e\,,$$
$$K^0\pi^0\nu_e, K^+\pi^-\nu_e, K^+\pi^-\nu_\mu\,.$$

It is to be noted that the operators with the Higgs fields $\eta(1,2,1,\bar{3})$ and $H(1,2,1,6)$ lead to $\Delta S=1$ and $\Delta S=2$ nucleon decays. Furthermore, the operators with $\zeta(1,2,1,1)$ lead to $\Delta S=2$ nucleon decay involving ν_e, whereas those with $\Delta(1,2,1,8)$ lead to $\Delta S=2$ nucleon decays with ν_e, e^-, and ν_τ only. Interestingly, with $\eta(1,2,1,\bar{3})$, $H(1,2,1,6)$, and $\zeta(1,2,1,1)$, there are no operators leading to the nucleon decays into the charged leptons.

In the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)^{VL}_H$ theory, dimension-7 operators have been constructed using the Higgs fields $\eta'(1,2,2,2,\bar{3})$, $H'(1,2,2,2,6)$, $\Delta'(1,2,2,2,8)$, and $\zeta'(1,2,2,2,1)$. There is only one operator with $\eta'(1,2,2,2,\bar{3})$:

$$\tilde{X}^1 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{L}^{r^*}_{iL} Q^{s^*}_{k'\gamma R})\eta'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\delta_{s^*m}\epsilon_{p^*q^*r^*}\epsilon_{\alpha\beta\gamma}\,. \tag{107}$$

This predicts

$$p \to K^+\nu_\tau, K^+\pi^0\nu_\tau, K^0\pi^+\nu_\tau, K^+K^0\nu_\tau$$

and

$$n \to K^0\nu_\tau, K^0\pi^0\nu_\tau, K^+\pi^-\nu_\tau, K^0K^0\nu_\tau\,.$$

There are two possible operators with $H'(1,2,2,2,6)$:

$$\tilde{I}^1 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{L}^{r^*}_{iL} Q^{s^*}_{k'\gamma R})H'^{\dagger m^* n^*}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{p^*q^*r^*}\epsilon_{s^*m^*n^*}\epsilon_{\alpha\beta\gamma}\,, \tag{108}$$

$$\tilde{I}^2 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{L}^{r^*}_{iL} Q^{s^*}_{k'\gamma R})H'^{\dagger m^* n^*}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{p^*q^*m^*}\epsilon_{r^*s^*n^*}\epsilon_{\alpha\beta\gamma}\,. \tag{109}$$

They give rise to the following decays:

$$p \to K^+\nu_\tau, K^+\pi^0\nu_\tau, K^0\pi^+\nu_\tau, K^+K^0\nu_\tau, K^+K^0\nu_e, K^+\nu_\mu\,,$$
$$K^+\pi^0\nu_\mu, K^0\pi^+\nu_\mu, K^+K^+e^-, K^+\pi^+\mu^-$$

and

$$n \to K^0\nu_\tau, K^0\pi^0\nu_\tau, K^0K^0\nu_\tau, K^+\pi^-\nu_\tau, K^0K^0\nu_e, K^0\nu_\mu\,,$$
$$K^0\pi^0\nu_\mu, K^+\pi^-\nu_\mu, K^+K^0e^-, K^+\mu^-, K^+\pi^0\mu^-, K^0\pi^+\mu^-\,.$$

The following four operators are possible with $\Delta'(1,2,2,2,8)$:

$$\tilde{J}^1 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{L}^{r^*}_{kL} Q^{s^*}_{i'\gamma R})\Delta'^{\dagger m^* n}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}$$
$$\times\delta_{pr^*}\delta_{qs^*}\delta_{m^*n}\epsilon_{\alpha\beta\gamma}\,, \tag{110}$$

$$\tilde{J}^2 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{L}^{r^*}_{kL} Q^{s^*}_{i'\gamma R})\Delta'^{\dagger m^* n}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl}$$
$$\times\epsilon_{i'j'}\epsilon_{pqn}\epsilon_{r^*s^*m^*}\epsilon_{\alpha\beta\gamma}\,, \tag{111}$$

$$\tilde{J}^3 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{L}^{r}_{k'R} Q^{s}_{i\gamma L})\Delta'^{\dagger m^* n}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}$$
$$\times\delta_{p^*r}\delta_{q^*s}\delta_{m^*n}\epsilon_{\alpha\beta\gamma}\,, \tag{112}$$

$$\tilde{J}^4 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{L}^{r}_{k'R} Q^{s}_{i\gamma L})\Delta'^{\dagger m^* n}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'}$$
$$\times\epsilon_{p^*q^*m^*}\epsilon_{rsn}\epsilon_{\alpha\beta\gamma}\,. \tag{113}$$

The following decay modes are allowed:

$$p \to \pi^+\nu_e, \pi^+\pi^0\nu_e, K^+\nu_\mu, K^+\pi^0\nu_\mu, K^0\pi^+\nu_\mu, K^+K^0\nu_e, K^+\nu_\tau\,,$$
$$K^+\pi^0\nu_\tau, K^0\pi^+\nu_\tau, K^+K^0\nu_\tau, K^+K^+e^-, K^+\pi^+\mu^-$$

and

$$n \to K^0K^0\nu_e, K^0\nu_\mu, K^0\pi^0\nu_\mu, K^0\nu_\tau, K^0\pi^0\nu_\tau, K^0K^0\nu_\tau, \pi^0\nu_e\,,$$
$$\pi^0\pi^0\nu_e, \pi^+\pi^-\nu_e, K^+\pi^-\nu_\mu, K^+\pi^-\nu_\tau, K^+K^0e^-\,,$$
$$K^+\mu^-, K^+\pi^0\mu^-, K^0\pi^+\mu^-\,.$$

With the Higgs field $\zeta'(1,2,2,2,1)$, the following two operators can be constructed:

$$\tilde{Z}^1 = (\bar{Q}^{cp}_{i\alpha L} Q^q_{j\beta L})(\bar{L}^{r^*}_{kL} Q^{s^*}_{i'\gamma R})\zeta'^\dagger_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'}\delta_{pr^*}\delta_{qs^*}\epsilon_{\alpha\beta\gamma}\,, \tag{114}$$

$$\tilde{Z}^2 = (\bar{Q}^{cp^*}_{i'\alpha R} Q^{q^*}_{j'\beta R})(\bar{L}^{r}_{k'R} Q^{s}_{i\gamma L})\zeta'^\dagger_{jl'}$$
$$\times\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'}\delta_{p^*r}\delta_{q^*s}\epsilon_{\alpha\beta\gamma}\,. \tag{115}$$

These operators lead to the following decay modes:

$$p \to \pi^+\nu_e, K^+\nu_\mu, \pi^+\pi^0\nu_e, K^+\pi^0\nu_\mu, K^0\pi^+\nu_\mu, K^+K^0\nu_e$$

and

$$n \to \pi^0\nu_e, K^0\nu_\mu, \pi^0\pi^0\nu_e, \pi^+\pi^-\nu_e, K^0\pi^0\nu_\mu, K^+\pi^-\nu_\mu, K^0K^0\nu_e\,.$$

With η', dimension-7 operators lead to the same nucleon decay modes as those obtained in the standard theory. With H', the operators lead to nucleon decays involving e^- and μ^- which are forbidden in the standard theory with H. With Δ' and ζ', $\Delta S=1$ nucleon decays involving ν_e are forbidden although these decay modes are allowed in the standard theory.

D. $SU(3)_H^V$ symmetry

As mentioned in Sec. IV, it is not possible to construct dimension-6 operators in the $SU(3)_C \times SU(2)_L \times U(1)_Y \times SU(3)_H^V$ theory. With fermionic fields only, one needs at least six fermions to construct Lorentz-invariant dimension-9 operators which are $SU(3)_H^V$ singlets. However, in the presence of a Higgs field, it is possible to construct Lorentz-invariant dimension-7 operators, which are also invariant under $SU(3)_H^V$. We have considered four Higgs fields $\chi(1,1,0,3)$, $\xi(1,1,0,\overline{6})$, $\rho(1,2,1,\overline{3})$, and $t(1,2,1,6)$ and constructed the possible dimension-7 operators.

There are only two operators with $\chi(1,1,0,3)$ and these are

$$S^1 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{L}\,{}^{cr}_{iL} Q^s_{j\gamma L})\chi^{a*}\epsilon_{ij}\epsilon_{pqr}\delta_{a^*s}\epsilon_{\alpha\beta\gamma}\,, \quad (116)$$

$$S^2 = (\overline{Q}\,{}^{cp}_{i\alpha L} Q^q_{j\beta L})(\overline{L}\,{}^{r}_{kL} Q^s_{l\gamma L})\chi^{a*}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times \epsilon_{pqr}\delta_{a^*s}\epsilon_{\alpha\beta\gamma}\,. \quad (117)$$

These give rise to the following nucleon decay modes:

$$p \to K^+\overline{\nu}_\tau, K^+\pi^0\overline{\nu}_\tau, K^0\pi^+\overline{\nu}_\tau, K^+K^0\overline{\nu}_\tau$$

and

$$n \to K^0\overline{\nu}_\tau, K^0\pi^0\overline{\nu}_\tau, K^+\pi^-\overline{\nu}_\tau, K^0K^0\overline{\nu}_\tau\,.$$

With the Higgs field $\xi(1,1,0,\overline{6})$, the following six operators can be written:

$$T^1 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{L}\,{}^{cr}_{iL} Q^s_{j\gamma L})\xi^{\dagger mn}\epsilon_{ij}\epsilon_{pqm}\epsilon_{rsn}\epsilon_{\alpha\beta\gamma}\,, \quad (118)$$

$$T^2 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{L}\,{}^{cr}_{iL} Q^s_{j\gamma L})\xi^{\dagger mn}\epsilon_{ij}\epsilon_{pqr}\epsilon_{smn}\epsilon_{\alpha\beta\gamma}\,, \quad (119)$$

$$T^3 = (\overline{Q}\,{}^{cp}_{i\alpha L} Q^q_{j\beta L})(\overline{L}\,{}^{cr}_{kL} Q^s_{l\gamma L})\xi^{\dagger mn}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times \epsilon_{pqm}\epsilon_{rsn}\epsilon_{\alpha\beta\gamma}\,, \quad (120)$$

$$T^4 = (\overline{Q}\,{}^{cp}_{i\alpha L} Q^q_{j\beta L})(\overline{L}\,{}^{cr}_{kL} Q^s_{l\gamma L})\xi^{\dagger mn}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times \epsilon_{pqr}\epsilon_{smn}\epsilon_{\alpha\beta\gamma}\,, \quad (121)$$

$$T^5 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{E}\,{}^{cr}_{R} U^s_{\gamma R})\xi^{\dagger mn}\epsilon_{pqm}\epsilon_{rsn}\epsilon_{\alpha\beta\gamma}\,, \quad (122)$$

$$T^6 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{E}\,{}^{cr}_{R} U^s_{\gamma R})\xi^{\dagger mn}\epsilon_{pqr}\epsilon_{smn}\epsilon_{\alpha\beta\gamma}\,. \quad (123)$$

From these operators, the following decay modes are predicted:

$$p \to K^+\overline{\nu}_\mu, K^+\overline{\nu}_\tau, K^+K^0\overline{\nu}_e, K^+\pi^0\overline{\nu}_\mu, K^0\pi^+\overline{\nu}_\mu, K^+\pi^0\overline{\nu}_\tau\,, K^0\pi^+\overline{\nu}_\tau, K^0\mu^+, K^+\pi^-\mu^+, K^0\pi^0\mu^+, K^+K^0\overline{\nu}_\tau, K^+\overline{\nu}_e\,, K^+\pi^0\overline{\nu}_e, K^0\pi^+\overline{\nu}_e$$

and

$$n \to K^0\overline{\nu}_\mu, K^0\overline{\nu}_\tau, K^0\overline{\nu}_e, K^+\pi^-\overline{\nu}_\mu, K^0\pi^0\overline{\nu}_\mu, K^+\pi^-\overline{\nu}_\tau\,, K^0\pi^0\overline{\nu}_\tau, K^+\pi^-\overline{\nu}_e, K^0\pi^0\overline{\nu}_e, K^0K^0\overline{\nu}_e, K^0K^0\overline{\nu}_\tau, K^0\pi^-\mu^+\,.$$

The following three operators are obtained with the Higgs field $\rho(1,2,1,\overline{3})$:

$$V^1 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{L}\,{}^{r*}_{iL} D^s_{\gamma R})\rho^{\dagger m}_j\epsilon_{ij}\epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\,, \quad (124)$$

$$V^2 = (\overline{Q}\,{}^{cp}_{i\alpha L} Q^q_{j\beta L})(\overline{L}\,{}^{r*}_{kl} D^s_{\gamma R})\rho^{\dagger m}_l(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times \epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\,, \quad (125)$$

$$V^3 = (\overline{D}\,{}^{cp}_{\alpha R} D^q_{\beta R})(\overline{L}\,{}^{r*}_{iL} U^s_{\gamma R})\rho^{\dagger m}_j\epsilon_{ij}\epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\,. \quad (126)$$

The following nucleon decay modes are possible:

$$p \to K^+\nu_e, K^+\pi^0\nu_e, K^0\pi^+\nu_e, K^+K^0\nu_\mu, K^+\pi^+e^-, K^+K^+\mu^-$$

and

$$n \to K^0\nu_e, K^0\pi^0\nu_e, K^+\pi^-\nu_e, K^0K^0\nu_\mu, K^+e^-, K^+\pi^0e^-\,, K^0\pi^+e^-, K^+K^0\mu^-\,.$$

Furthermore, two operators can be constructed with the Higgs field $t(1,2,1,6)$:

$$W^1 = (\overline{D}\,{}^{cp}_{\alpha R} U^q_{\beta R})(\overline{L}\,{}^{r*}_{iL} D^s_{\gamma R})t^{\dagger m^*n^*}_j\epsilon_{ij} \times \delta_{pm^*}\delta_{qn^*}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\,, \quad (127)$$

$$W^2 = (\overline{Q}\,{}^{cp}_{i\alpha L} Q^q_{j\beta L})(\overline{L}\,{}^{r*}_{kl} D^s_{\gamma R})t^{\dagger m^*n^*}_l\epsilon_{ij}\epsilon_{kl} \times \delta_{pm^*}\delta_{qn^*}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\,. \quad (128)$$

These predict the following nucleon decays:

$$p \to \pi^+\nu_e, \pi^+K^0\nu_e, \pi^0K^+\nu_e, \pi^+\pi^0\nu_e, \pi^0K^+\nu_\mu\,, \pi^+K^0\nu_\mu, K^+\nu_e, K^+K^0\nu_\mu$$

and

$$n \to \pi^0\nu_e, \pi^0K^0\nu_\mu, \pi^0\pi^0\nu_e, \pi^+K^-\nu_\mu, K^0\pi^0\nu_e\,, \pi^+\pi^-\nu_e, K^0\nu_e, K^+\pi^-\nu_e, K^0K^0\nu_\mu\,.$$

It is to be noted that only strangeness-changing nucleon decay modes are allowed by dimension-7 operators with $\chi(1,1,0,3)$, $\xi(1,1,0,\overline{6})$, and $\rho(1,2,1,\overline{3})$ but strangeness-changing as well as strangeness-conserving decay modes arise from the operators with the Higgs field $t(1,2,1,6)$. Furthermore, the operators with $\chi(1,1,0,3)$ and $\xi(1,1,0,\overline{6})$ lead to $\Delta B = \Delta L$ decays while those with $\rho(1,2,1,\overline{3})$ and $t(1,2,1,6)$ give rise to $\Delta B = -\Delta L$ nucleon decay modes. Interestingly, $\Delta B = -\Delta L$ operators forbid nucleon decay into $\overline{\nu}_\tau$, which is allowed by $\Delta B = \Delta L$ operators. In addition, operators with the Higgs $\rho(1,2,1,\overline{3})$ allow $\Delta S = 2$ nucleon decay modes involving the leptons of the second generation, which are forbidden by $\Delta B = \Delta L$ operators. The operators with the Higgs $\chi(1,1,0,3)$ forbid nucleon decay into charged leptons but allow decay into $\overline{\nu}_\tau$ only. In contrast, $\Delta B = \Delta L$ operators associated with $\xi(1,1,0,\overline{6})$ allow nucleon decay into the neutral leptons of all generations and the charged lepton of the second generation only.

In the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_H^V$ theory we have also constructed dimension-7 operators with the Higgs fields $\chi'(1,1,1,0,3)$, $\xi'(1,1,1,0,\overline{6})$, $\rho'(1,2,2,2,\overline{3})$, and $t'(1,2,2,2,6)$, respectively. The following two operators can be written with the Higgs field $\chi'(1,1,1,0,3)$:

$$\tilde{S}^1 = (\overline{Q}\,{}^{cp}_{i\alpha L} Q^q_{j\beta L})(\overline{L}\,{}^{cr}_{kl} Q^s_{l\gamma L})\chi'^{a*}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times \epsilon_{pqr}\delta_{a^*s}\epsilon_{\alpha\beta\gamma}\,, \quad (129)$$

$$\tilde{S}^2=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{cr}_{k'R}Q^{s}_{l'\gamma R})\chi'^{a*}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'} \times\epsilon_{pqr}\delta_{a^*s}\epsilon_{\alpha\beta\gamma}\ . \tag{130}$$

These give rise to the following decays:

$$p\to K^+\bar{\nu}_\tau, K^+\pi^0\bar{\nu}_\tau, K^0\pi^+\bar{\nu}_\tau, K^+K^0\bar{\nu}_\tau$$

and

$$n\to K^0\bar{\nu}_\tau, K^0\pi^0\bar{\nu}_\tau, K^+\pi^-\bar{\nu}_\tau, K^0K^0\bar{\nu}_\tau\ .$$

With the Higgs field $\xi'(1,1,1,0,\bar{6})$ the following four operators are possible:

$$\tilde{T}^1=(\bar{Q}^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}^{cr}_{kL}Q^{s}_{l\gamma L})\xi'^{\dagger mn}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times\epsilon_{pqm}\epsilon_{rsn}\epsilon_{\alpha\beta\gamma}\ , \tag{131}$$

$$\tilde{T}^2=(\bar{Q}^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}^{cr}_{kl}Q^{s}_{l\gamma L})\xi'^{\dagger mn}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times\epsilon_{pqr}\epsilon_{smn}\epsilon_{\alpha\beta\gamma}\ , \tag{132}$$

$$\tilde{T}^3=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{cr}_{k'R}Q^{s}_{l'\gamma R})\xi'^{\dagger mn}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'} \times\epsilon_{pqm}\epsilon_{rsn}\epsilon_{\alpha\beta\gamma}\ , \tag{133}$$

$$\tilde{T}^4=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{cr}_{k'R}Q^{s}_{l'\gamma R})\xi'^{\dagger mn}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'} \times\epsilon_{pqr}\epsilon_{smn}\epsilon_{\alpha\beta\gamma}\ . \tag{134}$$

The following decays arise from the above operators:

$$p\to K^+K^0\bar{\nu}_e, K^+\pi^0\bar{\nu}_\mu, K^0\pi^+\bar{\nu}_\mu, K^+\bar{\nu}_\mu, K^0\mu^+, K^+\pi^-\mu^+\ ,$$
$$K^0\pi^0\mu^+, K^+\bar{\nu}_\tau, K^+\pi^0\bar{\nu}_\tau, K^0\pi^+\bar{\nu}_\tau, K^+K^0\bar{\nu}_\tau$$

and

$$n\to K^0K^0\bar{\nu}_e, K^0\pi^0\bar{\nu}_\mu, K^0\bar{\nu}_\mu, K^0\pi^-\mu^+, K^0\bar{\nu}_\tau, K^0\pi^0\bar{\nu}_\tau\ ,$$
$$K^0K^0\bar{\nu}_\tau, K^+\pi^-\bar{\nu}_\tau, K^+\pi^-\bar{\nu}_\mu\ .$$

We can construct the following four operators with the Higgs field $\rho'(1,2,2,2,\bar{3})$:

$$\tilde{V}^1=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{r^*}_{iL}Q^{s}_{k'\gamma R})\rho'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'} \times\epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ , \tag{135}$$

$$\tilde{V}^2=(\bar{Q}^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}^{r^*}_{kL}Q^{s}_{i'\gamma R})\rho'^{\dagger m}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times\epsilon_{i'j'}\epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ , \tag{136}$$

$$\tilde{V}^3=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{r^*}_{k'R}Q^{s}_{i\gamma L})\rho'^{\dagger m}_{jl'}\epsilon_{ij}(\tau\epsilon)_{i'j'}\cdot(\tau\epsilon)_{k'l'} \times\epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ , \tag{137}$$

$$\tilde{V}^4=(\bar{Q}^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}^{r^*}_{i'R}Q^{s}_{k\gamma L})\rho'^{\dagger m}_{lj'}(\tau\epsilon)_{ij}\cdot(\tau\epsilon)_{kl} \times\epsilon_{i'j'}\epsilon_{pqm}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ . \tag{138}$$

These operators predict the following decay modes of nucleon:

$$p\to K^+\nu_e, K^+\pi^0\nu_e, K^0\pi^+\nu_e, K^+K^0\nu_\mu, K^+\pi^+e^-, K^+K^+\mu^-$$

and

$$n\to K^0\nu_e, K^0\pi^0\nu_e, K^+\pi^-\nu_e, K^0K^0\nu_\mu, K^0\pi^+e^-\ ,$$
$$K^+\pi^0e^-, K^+K^0\mu^-\ .$$

Another set of four operators can be constructed with the Higgs field $t'(1,2,2,2,6)$:

$$\tilde{W}^1=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{r^*}_{iL}Q^{s}_{k'\gamma R})t'^{\dagger m^*n^*}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'} \times\delta_{pm^*}\delta_{qn^*}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ , \tag{139}$$

$$\tilde{W}^2=(\bar{Q}^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}^{r^*}_{kL}Q^{s}_{i'\gamma R})t'^{\dagger m^*n^*}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'} \times\delta_{pm^*}\delta_{qn^*}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ , \tag{140}$$

$$\tilde{W}^3=(\bar{Q}^{cp}_{i'\alpha R}Q^{q}_{j'\beta R})(\bar{L}^{r^*}_{k'R}Q^{s}_{i\gamma L})t'^{\dagger m^*n^*}_{jl'}\epsilon_{ij}\epsilon_{i'j'}\epsilon_{k'l'} \times\delta_{pm^*}\delta_{qn^*}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ , \tag{141}$$

$$\tilde{W}^4=(\bar{Q}^{cp}_{i\alpha L}Q^{q}_{j\beta L})(\bar{L}^{r^*}_{i'R}Q^{s}_{k\gamma L})t'^{\dagger m^*n^*}_{lj'}\epsilon_{ij}\epsilon_{kl}\epsilon_{i'j'} \times\delta_{pm^*}\delta_{qn^*}\delta_{r^*s}\epsilon_{\alpha\beta\gamma}\ . \tag{142}$$

These operators predict the following decay modes:

$$p\to\pi^+\nu_e, \pi^+\pi^0\nu_e, K^+\pi^0\nu_\mu, K^0\pi^+\nu_\mu, K^+\nu_e, K^+\pi^0\nu_e\ ,$$
$$K^0\pi^+\nu_e, K^+K^0\nu_\mu$$

and

$$n\to\pi^0\nu_e, \pi^0\pi^0\nu_e, \pi^+\pi^-\nu_e, \pi^0K^0\nu_\mu, \pi^-K^+\nu_\mu\ ,$$
$$K^0\nu_e, \pi^0K^0\nu_e, \pi^-K^+\nu_e, K^0K^0\nu_\mu\ .$$

Here also only the strangeness-changing $\Delta B=\Delta L$ nucleon decays are possible with Higgs fields $\chi'(1,1,1,0,3)$ and $\xi'(1,1,1,0,\bar{6})$ but $\Delta B=-\Delta L$ decays result from the operators with the Higgs field $\rho'(1,2,2,2,\bar{3})$. However, both the strangeness-changing and strangeness-conserving decays are allowed from the $\Delta B=-\Delta L$ operators with Higgs field $t'(1,2,2,2,6)$. Like the standard theory, nucleon decay modes involving $\bar{\nu}_\tau$ result only from $\Delta B=\Delta L$ operators. Moreover, $\Delta B=\Delta L$ operators forbid $\Delta S=2$ nucleon decay into the leptons of the second generation while $\Delta B=-\Delta L$ operators allow $\Delta S=2$ decays involving the leptons of the second generation only. Interestingly, no charged leptons arise from $\Delta B=\Delta L$ operators with $\chi'(1,1,1,0,3)$ or $\Delta B=-\Delta L$ operators with $t'(1,2,2,2,6)$ but the charged leptons result in the nucleon decay with the Higgs fields $\xi'(1,1,1,0,\bar{6})$ and $\rho'(1,2,2,2,\bar{3})$ associated with $\Delta B=\Delta L$ and $\Delta B=-\Delta L$ operators, respectively.

Decay modes such as $p\to K^+\bar{\nu}_e, K^+\pi^0\bar{\nu}_e, K^0\pi^+\bar{\nu}_e$ and $n\to K^0\bar{\nu}_e, K^+\pi^-\bar{\nu}_e, K^0\pi^0\bar{\nu}_e$ are possible in the standard theory with the Higgs field $\xi(1,1,0,\bar{6})$, but these are not allowed in the left-right-symmetric theory with the Higgs field $\xi'(1,1,1,0,\bar{6})$.

VI. RESULTS

We shall now summarize our results on the analysis of nucleon decay modes arising from dimension-6 and dimension-7 operators in the standard and the left-right-symmetric theory including G_H where $G_H\equiv U(1)_H$, $SU(2)_H$, $SU(3)_H^{VL}$, or $SU(3)_H^V$. We assume the grand unification mass scale $M\approx10^{15}$ GeV implying $\tau\approx10^{31}$ year. In the context of the standard theory our analysis on dimension-6 operators shows that $U(1)_H$ symmetry allows nucleon decay modes like

$$p \to K^0 e^+ K^+ \bar{\nu}_e, K^+ \pi^0 \bar{\nu}_e, K^0 \pi^+ \bar{\nu}_e, K^+ \pi^- e^+, K^0 \pi^0 e^+$$

and

$$n \to K^0 \bar{\nu}_e, K^0 \pi^0 \bar{\nu}_e, K^0 \pi^- e^+$$

but $SU(2)_H$ or $SU(3)_H^{VL}$ symmetry forbids these decay modes. Both $SU(2)_H$ and $SU(3)_H^{VL}$ symmetries allow decay modes like

$$p \to K^0 \mu^+, K^+ \bar{\nu}_\mu, K^+ \pi^0 \bar{\nu}_\mu, K^0 \pi^+ \bar{\nu}_\mu, K^+ \pi^- \mu^+, K^0 \pi^0 \mu^+$$

and

$$n \to K^0 \bar{\nu}_\mu, K^0 \pi^0 \bar{\nu}_\mu, K^+ \pi^- \bar{\nu}_\mu, K^0 \pi^- \mu^+$$

but $U(1)_H$ symmetry forbids nucleon decay into the leptons of the second generation. However, $SU(2)_H$ symmetry predicts

$$p \to \pi^0 \mu^+, K^+ K^0 \bar{\nu}_e, \pi^0 \pi^0 \mu^+, \pi^+ \pi^- \mu^+$$

and

$$n \to \pi^- \mu^+, K^0 K^0 \bar{\nu}_e, \pi^0 \pi^- \mu^+ ,$$

which are ruled out by $SU(3)_H^{VL}$ or $U(1)_H$ symmetry. Interestingly, the construction of dimension-6 operators is not possible in the standard and the left-right-symmetric theory incorporating $SU(3)_H^V$ symmetry. In the left-right-symmetric theory, $U(1)_H$ symmetry does not allow dimension-6 operators leading to the strangeness-changing nucleon decay; with $SU(2)_H$ symmetry, the operators allow decays such as $p \to K^+ K^0 \bar{\nu}_e$ and $n \to K^0 K^0 \bar{\nu}_e$ but $SU(3)_H^{VL}$ symmetry forbids these decays.

Furthermore, the decay modes arising from dimension-7 operators can also discriminate among the different horizontal symmetries. In spite of the suppression of the effective coupling constant by the factor $(M)^{-1}$ for dimension-7 operators compared to dimension-6 operators, the former can lead to $\tau \approx 10^{31}$ year if we assume the unification mass scale $M \approx 10^{13}$ GeV, and the horizontal-symmetry-breaking mass scale $m = \langle H \rangle \approx 10^5$ GeV where $\langle H \rangle$ is the vacuum expectation value of the relevant Higgs field. Within the framework of the standard theory, dimension-7 operators lead to different nucleon decay modes for different horizontal symmetries. $U(1)_H$ symmetry allows

$$p \to \pi^+ \nu_\mu, \pi^+ \pi^0 \nu_\mu$$

and

$$n \to \pi^0 \nu_\mu, \pi^0 \pi^0 \nu_\mu ,$$

which are forbidden by $SU(2)_H$, $SU(3)_H^{VL}$, or $SU(3)_H^V$ symmetry. In addition, $U(1)_H$, $SU(2)_H$, and $SU(3)_H^{VL}$ symmetries predict $p \to K^+ \nu_\mu$ and $n \to K^0 \nu_\mu$ which are prohibited by $SU(3)_H^V$ symmetry. But $SU(2)_H$ and $SU(3)_H^{VL}$ symmetries allow

$$p \to K^+ K^0 \nu_e, K^+ \pi^+ \mu^-$$

and

$$n \to K^0 K^0 \nu_e, K^+ \pi^0 \mu^-, K^0 \pi^+ \mu^-, \pi^+, \pi^- \nu_\mu ,$$

which are ruled out by $SU(3)_H^V$ and $U(1)_H$ symmetries. In contrast, $SU(2)_H$ and $SU(3)_H^V$ symmetries allow

$$p \to K^+ \nu_e, K^+ \pi^+ e^-, K^+ K^0 \nu_\mu, K^+ K^+ \mu^-$$

and

$$n \to K^0 \nu_e, K^0 \pi^+ e^-, K^+ \pi^0 e^-, K^+ K^0 \mu^-, K^0 K^0 \nu_\mu ,$$

which are forbidden by $U(1)_H$ or $SU(3)_H^{VL}$ symmetry. Furthermore, $SU(3)_H^V$ symmetry gives rise to

$$p \to K^+ \bar{\nu}_\tau, K^+ \pi^0 \bar{\nu}_\tau, K^0 \pi^+ \bar{\nu}_\tau, K^+ K^0 \bar{\nu}_\tau$$

and

$$n \to K^0 \bar{\nu}_\tau, K^0 \pi^0 \bar{\nu}_\tau, K^+ \pi^- \bar{\nu}_\tau, K^0 K^0 \bar{\nu}_\tau$$

but $SU(2)_H$ and $SU(3)_H^{VL}$ symmetries lead to

$$p \to K^+ \nu_\tau, K^+ \pi^0 \nu_\tau, K^0 \pi^+ \nu_\tau, K^+ K^0 \nu_\tau$$

and

$$n \to K^0 \nu_\tau, K^0 \pi^0 \nu_\tau, K^+ \pi^- \nu_\tau, K^0 K^0 \nu_\tau$$

although the above decays are ruled out by $U(1)_H$ symmetry. In the left-right-symmetric theory $U(1)_H$, $SU(2)_H$, and $SU(3)_H^{VL}$ symmetries allow $p \to K^+ K^0 \nu_e, K^+ \pi^+ \mu^-$ and $n \to K^0 K^0 \nu_e, K^+ \pi^0 \mu^-, K^0 \pi^+ \mu^-$ which are prohibited by $SU(3)_H^V$ symmetry. However, $SU(2)_H$, $SU(3)_H^{VL}$, and $SU(3)_H^V$ symmetries allow $p \to \pi^+ \nu_e, \pi^+ \pi^0 \nu_e$ and $n \to \pi^0 \nu_e, \pi^0 \pi^0 \nu_e, \pi^+ \pi^- \nu_e$, which are forbidden by $U(1)_H$ symmetry. Interestingly, the decays like $p \to K^+ K^+ \mu^-$, $K^+ \pi^+ e^-$ and $n \to K^+ K^0 \mu^-, K^+ \pi^0 e^-, K^0 \pi^+ e^-$ are possible with $SU(2)_H$ and $SU(3)_H^V$ symmetries although these are ruled out by $SU(3)_H^{VL}$ and $U(1)_H$ symmetries. With $SU(2)_H$ and $SU(3)_H^{VL}$ symmetries, decays such as

$$p \to K^+ \nu_\tau, K^+ \nu_\mu, K^+ K^+ e^-, K^+ \pi^+ \mu^-$$

and

$$n \to K^0 \nu_\tau, K^0 \nu_\mu, K^+ K^0 e^-, K^+ \pi^0 \mu^-, K^0 \pi^+ \mu^-$$

are allowed, which are prohibited by $SU(3)_H^V$ symmetry.

The decay modes resulting from dimension-6 operators can distinguish between the standard and the left-right-symmetric theory for some horizontal symmetries. With $U(1)_H$ symmetry, the standard theory allows strangeness-changing nucleon decays such as

$$p \to K^+ \bar{\nu}_e K^0 e^+, K^+ \pi^0 \bar{\nu}_e, K^0 \pi^+ \bar{\nu}_e, K^+ \pi^- e^+, K^0 \pi^0 e^+$$

and

$$n \to K^0 \bar{\nu}_e, K^0 \pi^- e^+, K^0 \pi^0 \bar{\nu}_e ,$$

which are forbidden in the left-right-symmetric theory. With $SU(2)_H$ symmetry, nucleon decays such as $p \to \pi^0 \mu^+, \pi^0 \pi^0 \mu^+, \pi^+ \pi^- \mu^+$ and $n \to \pi^- \mu^+, \pi^0 \pi^- \mu^+$, are allowed in the standard theory but these decay modes are ruled out in the left-right-symmetric theory. However, with $SU(3)_H^V$ symmetry, both the standard and left-right-symmetric theories do not allow the construction of dimension-6 operators. With $SU(3)_H^{VL}$ symmetry, both the theories lead to similar nucleon decay modes from dimension-6 operators although the standard theory allows left-handed neutrinos only but the left-right-symmetric theory yields left-handed as well as right-handed neutrinos.

Dimension-7 operators also lead to decay modes which can distinguish between the standard and the left-right-symmetric theory for certain horizontal symmetries. With $U(1)_H$ symmetry, the left-right-symmetric theory allows

$$p \to K^+\pi^+\mu^-, K^+K^0\nu_e, K^+K^+e^-$$

and

$$n \to K^+\mu^-, K^0\pi^+\mu^-, K^+\pi^0\mu^-, K^0K^0\nu_e, K^+K^0e^- ,$$

which are ruled out in the standard theory. With the $SU(2)_H$ symmetry, the left-right-symmetric theory allows $p \to K^+K^+e^-$ and $n \to K^+K^0e^-$, which are forbidden in the standard theory. With $SU(3)_H^V$ symmetry, the standard theory predicts

$$p \to K^+\bar{\nu}_e, K^+\pi^0\bar{\nu}_e, K^0\pi^+\bar{\nu}_e$$

and

$$n \to K^0\bar{\nu}_e, K^+\pi^-\bar{\nu}_e, K^0\pi^0\bar{\nu}_e$$

which are not allowed in the left-right-symmetric theory. However, with the $SU(3)_H^{VL}$ symmetry, the standard theory allows

$$p \to K^+\pi^0\nu_e, K^0\pi^+\nu_e$$

and

$$n \to K^0\pi^0\nu_e, K^+\pi^-\nu_e ,$$

which are restricted in the left-right-symmetric theory.

VII. CONCLUSION

From the operator analysis of nucleon decay in the standard and the left-right-symmetric theory including G_H where $G_H \equiv U(1)_H$, $SU(2)_H$, $SU(3)_H^{VL}$, or $SU(3)_H^V$ we conclude that the ongoing proton decay experiments[27] can discriminate among the different horizontal symmetries. In particular, the determination of $\tau(p \to e^+\pi^0) \geq 10^{32}$ year will support $U(1)_H$, $SU(2)_H$, or $SU(3)_H^{VL}$ symmetry but will rule out the $SU(3)_H^V$ symmetry. On the other hand, the detection of the decay modes $p \to e^+\pi^0$, $\mu^+K^0, \bar{\nu}_\mu K^+$ will favor $SU(2)_H$ and $SU(3)_H^{VL}$ symmetries only. In contrast, if the decay mode such as $p \to K^+\nu_\mu$ is detected, $SU(3)_H^V$ is definitely ruled out and $SU(3)_H^{VL}$ symmetry is certainly favored although $U(1)_H$ and $SU(2)_H$ symmetries are not completely ruled out. Similarly, if the experiments measure $\tau(p \to \mu^+K^0, \bar{\nu}_\mu K^+) \geq 10^{31}$ year, $SU(3)_H^{VL}$ and $SU(2)_H$ symmetries are definitely favored although $SU(3)_H^V$ is not totally ruled out. It may be noted that, dimension-7 operators can lead to decay modes like $p \to \mu^+K^0, \bar{\nu}_\mu K^+$ for $SU(3)_H^V$ symmetry and $p \to K^+\nu_\mu$ for $U(1)_H$, $SU(2)_H$, and $SU(3)_H^{VL}$ symmetries with $\tau \geq 10^{31}$ year if $M \approx 10^{13}$ GeV and $\langle H \rangle \approx 10^5$ GeV. Furthermore, under a favorable situation, the standard and left-right-symmetric theories can also be distinguished in the nucleon decay. For example, the proton decay mode $p \to K^+\bar{\nu}_e$ can arise in the standard theory with $U(1)_H$ or $SU(3)_H^V$ symmetry but the mode is forbidden in the left-right-symmetric theory including $U(1)_H$, $SU(2)_H$, $SU(3)_H^{VL}$, or $SU(3)_H^V$ symmetry. Similarly, $p \to \pi^0\mu^+$ decay mode is possible in the standard theory with $SU(2)_H$ symmetry but the mode is ruled out in the left-right-symmetric theory including $U(1)_H$, $SU(2)_H$, $SU(3)_H^{VL}$, or $SU(3)_H^V$ symmetry.

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*Address after 1 September 1986: Physics Department, University of Toronto, Toronto, Ontario, Canada M5S 1A7.

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[25]There is an alternative version of this left-right-symmetric theory in which the Y_H quantum number of the left-handed doublet is opposite to that of the right-handed doublet. Within the framework of this version of the left-right-symmetric theory including $U(1)_H$, dimension-6 operators allow the strangeness-changing nucleon decays such as $p \to \pi^+ K^0 \bar{\nu}_\mu, \pi^0 K^+ \bar{\nu}_\mu$ and $n \to \pi^0 K^0 \bar{\nu}_\mu, \pi^- K^+ \bar{\nu}_\mu$, and dimension-7 operators lead to the decays like

$$p \to K^+\pi^0\nu_e, K^0\pi^+\nu_e, K^+\nu_e, K^+K^0\nu_\mu, \pi^+\pi^0\nu_e, \pi^+\nu_e ,$$
$$K^+\pi^+e^-, K^+K^+\mu^-$$

and

$$n \to K^0\pi^0\nu_e, K^+K^0\mu^-, K^+\pi^-\nu_e, K^0\nu_e, K^0K^0\nu_\mu, \pi^0\nu_e ,$$
$$\pi^+\pi^-\nu_e, \pi^0\pi^0\nu_e, K^+e^-, K^+\pi^0e^-, K^0\pi^+e^- ,$$

but all these decay modes are prohibited if the Y_H quantum number is the same for the left-handed as well as right-handed fermions.

[26]An alternative two-generation $SU(2)_H$ model as proposed by Shanker (Ref. 4) is also possible. Dimension-6 and dimension-7 operators can be constructed for the two-generation $SU(2)_H$ model within the framework of the standard and the left-right-symmetric theory. The nucleon decay modes resulting from these operators show interesting departures from those resulting from the three generation $SU(2)_H$ model. For example, in the context of the standard theory, the decay modes like $p \to \pi^+ \bar{\nu}_\mu$, $K^+ \bar{\nu}_e$, $K^0 e^+$, etc., which are forbidden by dimension-6 operators in the three-generation $SU(2)_H$ model, are allowed by the two-generation $SU(2)_H$ model. A detailed comparison of these two versions of the $SU(2)_H$ model for the baryon- and lepton-number-violating operators will be presented elsewhere.

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