Supplementary evidence for T violation

Robert G. Sachs
Enrico Fermi Institute and the Department of Physics, The University of Chicago, 5640 South Ellis Avenue, Chicago, Illinois 60637
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The only significant experimental evidence for T violation is based on the data on CP violation and other measured properties of the decay amplitudes of neutral K mesons and their combined use in an analysis based on the Bell-Steinberger unitarity relation or its equivalent. It has been pointed out by Kenny and Sachs that there is a technical possibility that this conclusion can be evaded because the argument depends on the assumption that the *effective* (phenomenological) weak-interaction Hamiltonian from which the decay amplitudes are derived is Hermitian. The purpose of this paper is to show that this assumption can be circumvented by making use of additional data on the unitarity of the decay amplitudes of both the neutral and charged K mesons to establish an upper limit on the magnitude of amplitudes associated with T-invariant anti-Hermitian interactions. It is assumed that the order of magnitude of such terms is in conformity with the values expected on the basis of the usual approximate isotopic spin selection rules for the weak interactions. Then it is found that if all the relevant weak interactions are T invariant, the data on unitarity would place an upper limit on the CP-violation parameter $|\eta_{+-}|$ of $|\eta_{+-}|<0.34\times10^{-3}$, which is an order of magnitude smaller than the observed value. It is therefore found that the possibility of including in the analysis a non-Hermitian effective weak Hamiltonian up to the limit permitted by the data on unitarity does not alter the conclusion that T invariance is violated. Confidence in this conclusion could be increased by experiments that improve the precision of some of the available data used for this analysis.

I. INTRODUCTION

The only well-established evidence for the violation of T invariance is based on the observed CP violation in the decay of the neutral K meson. The evidence is indirect; it makes use of the relationship with other decay amplitudes of the parameters η_{+-} and η_{00} measuring CP violation by decay of the K_L into two pions. This relationship is usually formulated in terms of the Bell-Steinberger unitarity conditions or the equivalent.[1]

The Bell-Steinberger conditions arise directly from the assumption that the phenomenological weak interactions are Hermitian.[2] That the *fundamental* interactions are Hermitian is generally taken to be a sacrosanct principle because it underlies the unitarity of all the dynamics. However it is the *effective* weak interaction that enters into the description of the $K^0,\bar{K}^0$ phenomena and is the basis for the analysis leading to tests of T invariance and CPT invariance. The assumption that effective interactions are Hermitian does not stand on the same firm ground as the corresponding assumption for the underlying fundamental interactions. In fact it is common practice to take account of unobserved absorptive channels in scattering and reaction processes by introducing non-Hermitian phenomenological terms into the interaction.

Although no specific theory leading to a non-Hermitian effective interaction for the $K^0,\bar{K}^0$ decay phenomena is suggested here, the possibility that the correct theory will turn out to have this property does exist. For example, one might imagine the participation of an undetected massless (or nearly massless) spin-zero particle that is odd under CP, like an axion, in weak decays. That could introduce absorptive effects and, possibly at the same time, account for CP violation in the observed decays.

On a more general level, the recent revival of the notion that our four-dimensional space-time is embedded in a higher-dimensional manifold allows the speculation that, while unitarity holds sway in the total manifold, there may be deviations from unitarity within the space-time subspace which is our realm of observation. That there should be room in physical theories for such a concept is suggested by the notion of the unification of quantum field theory with general relativity. A unification that permits the concept of the black hole must lead to the possibility of an *apparent* violation of unitarity to account for the swallowing by the black hole of such quantum numbers as the baryon number.[3]

These remarks are meant merely to suggest that it is possible to conceive of a situation in which the correct theory leads to non-Hermitian effective weak interactions. It is not my purpose here to propose, or to investigate, the general consequences of such a theory but, rather, to reevaluate the experimental evidence for T violation in the light of the possibility of such interactions. I will show that, if T invariance is assumed to be an overriding principle, the highly sensitive experiments on CP violation provide a measure of the anti-Hermitian terms in the strangeness-changing *effective* weak interactions while, at the same time, the anti-Hermitian interactions imply an *apparent* (i.e., observable) violation of unitarity, which is subject to independent experimental test. Therefore the loop can be closed; the consistency of the anti-Hermitian contribution required by the assumption of T invariance with the deviations from Hermiticity permitted by the data on unitarity becomes a test for T violation.

For this paper the loop will be closed by using the data on unitarity to determine a limit on the non-Hermiticity in the effective weak interactions under the assumption of T invariance and then to confront the CP violation that would be "predicted" on the basis of this T-invariant, non-Hermitian effective interaction with the observed CP violation. The direct data on CP violation will be found to strongly disagree with this prediction. Therefore it can be concluded that *T invariance must be violated even if the anti-Hermitian effective weak interactions responsible for K-meson decay occur up to the limit permitted by the data on unitarity.*

It will be found that the amplitude measuring the anti-Hermitian interaction depends on the CP-violation parameter α_0, which is defined by setting $I=0$ in

$$\alpha_I=(A_I-\overline{A}_I)/(A_I+\overline{A}_I)\ , \tag{1}$$

where A_I ($\overline{A}_I$) is the $I=0$ or 2, 2π amplitude for decay of the K^0 ($\overline{K}^0$).

In the conventional treatment of these phenomena, use is made of the CPT theorem and the freedom to choose the phase of A_0 to obtain $\alpha_0=0$. However the proof of the CPT theorem depends on the Hermiticity of the interactions so that the assumed non-Hermiticity of the phenomenological Hamiltonian implies a failure of the CPT theorem *for the phenomenological amplitudes A_I* and $\overline{A}_I$. Therefore $\alpha_0\neq0$ in the case under consideration.

The stated estimates of error in the data on CP violation that enter into the determination of α_0 are crucial to the final conclusion drawn here. Some of the estimated errors are large, leaving room for doubt as to whether the estimates are large enough. Therefore confidence in this conclusion would be increased by a repetition of some of these measurements with a higher degree of precision, if that is possible. Two experiments that would serve this purpose are the following.

(1) A precision measurement of $\phi_{00}-\phi_{+-}$, which is needed to provide an accurate determination of ϵ'', given by Eq. (23), a quantity that enters into the indirect determination of α_0 made here [see Eq. (18c)].

(2) A high-precision measurement of the CP violation in the 2π mode of the $K^{\pm}$, which could be used to obtain a more precise measure of α_2 than has been used here. This (α_2) is the other (and more important) quantity needed for the indirect determination of α_0 by means of Eq. (18c).

An alternative (or supplemental) experiment would be as follows.

(3) The direct measurement of α_0, which is possible in principle but has not been reported.

In contrast with the conventional tests based on unitarity which allow the observed CP violation to be separated into measurable T-violating and CPT-violating parts,[4] separate quantitative measures of the T-violating and anti-Hermitian contributions to the phenomenological interactions do not emerge from this analysis. In particular, once the assumption of T invariance is abandoned, the number of parameters proliferates to such an extent as to break down the simple relationship between the measure of unitarity and the anti-Hermitian terms. The resulting ambiguity in the interpretation leads to uncertainty in the connection between them. The possibility that *both* T invariance and Hermiticity are violated cannot be excluded.

Although a clear-cut measure of the degree of T violation is not forthcoming, the result obtained here, when combined with the usual arguments based on Hermitian interactions, would seem to establish the qualitative existence of T violation. However, since all of these arguments are indirect, it would still be most desirable to have the conclusion confirmed by direct measurements on parameters that change sign under *motion* reversal.[5] Since the effect observed in $K^0,\overline{K}^0$ phenomena may be due to a large CP violation in the weak interactions of heavy quarks, measurements of such motion reversal parameters at high energies (corresponding to masses of mesons composed of heavy quarks) would appear to be the most promising.[6]

Because the standard treatment of the $K^0,\overline{K}^0$ decay phenomena assumes Hermiticity, some reconsideration of that treatment is needed here and will be provided in the following three sections. Section II introduces the necessary modification of the parametrization of the decay amplitudes, Sec. III develops the corresponding phenomenological relationships for the 2π decay mode, and Sec. IV the modifications in the form of the dispersive and absorptive terms in the mass matrix.

In Sec. V the connection is established between the CP-violation parameter η_{+-} and the parameters characterizing the foregoing analysis. Since the objective is to estimate an upper limit on $|\eta_{+-}|$, order-of-magnitude estimates of the parameters are adequate and they lead to simplifying relationships among the parameters. The orders of magnitude are fixed by the assumption that anti-Hermitian amplitudes are of the same order or smaller than the CP-violating amplitudes. It is also assumed that anti-Hermitian amplitudes of the $K^{\pm}$ bear the usual relationships to corresponding $K^0,\overline{K}^0$ amplitudes that are given by approximate isotopic spin selection rules. Some of the approximations are justified in the Appendix.

An estimate of the parameter α_0 is made in Sec. VI by making use of similar assumptions and, in Sec. VII, the experimental information on unitarity is used to place limits on the magnitude of T-invariant anti-Hermitian amplitudes, and thereby to establish the limit on $|\eta_{+-}|$ that is imposed by T invariance of these anti-Hermitian terms. This limit is an order of magnitude smaller than the observed value of $|\eta_{+-}|$. Therefore it is concluded that the introduction of these non-Hermitian terms does not alter the conclusion that T invariance is violated.

II. DECAY AMPLITUDES

The states of the $K^0,\overline{K}^0$ system will be denoted by $|j\rangle$, where $j=1,2$ and $|1\rangle\equiv|K^0\rangle$, $|2\rangle\equiv|\overline{K}^0\rangle$. The convention for the relative phase of $|2\rangle$ is taken to be

$$|\overline{K}^0\rangle=CP\,|K^0\rangle\ . \tag{2}$$

The dynamical basis for the analysis of the decay and interference properties of these states is the generic time-dependent Schrödinger equation:

$$i\frac{\partial}{\partial t}\mid\psi\rangle = H\mid\psi\rangle\ , \tag{3}$$

where the units have been chosen so that $\hbar = 1$ (and will be chosen so that $c=1$). Here, H is the Hamiltonian including the effective weak interactions which will be denoted by W, and W may include the non-Hermitian terms.

The relevant terms in W are the $\Delta S = \pm 1$ term W_1 and the $\Delta S = \pm 2$ term W_2. The $K^0, \bar{K}^0$ decay amplitudes into the actual or virtual channel $\mid c\rangle$ of energy E, which arise from W_1, are denoted by $A_{cj}(E)$. Then, by consequence of Eq. (3),

$$A_{cj}(E) = (2\pi\rho_c)^{1/2}\langle c,\text{out}\mid W_1\mid j\rangle\ , \tag{4}$$

where $\rho_c(E)$ is the density of states $\mid c\rangle$ and $\mid c,\text{out}\rangle$ is the state having the form of a free wave for $t\to\infty$ in the standard Lehmann-Symanzik-Zimmermann (LSZ) notation. The channels accessible in the decay process (i.e., those having a total mass $< m_K$) will be denoted by $\mid f\rangle$ and the amplitude is normalized in such a way that partial decay rates are given by

$$\Gamma_{fj} = \mid A_{fj}(m_K)\mid^2\ . \tag{5}$$

For convenience, in the following the on-the-mass-shell amplitude will be written simply as A_{fj}:

$$A_{fj} \equiv A_{fj}(E = m_K)\ . \tag{6}$$

It will be necessary to introduce the "transposed amplitude," $\tilde{A}_{cj}(E)$ defined by

$$\tilde{A}_{cj}(E) = (2\pi\rho_c)^{1/2}\langle j\mid W_1\mid c,\text{out}\rangle\ . \tag{7}$$

Clearly, if W_1 is Hermitian $\tilde{A}_{cj}(E) = A^*_{cj}(E)$, the conjugate complex, but, otherwise there is no direct relationship corresponding to Eq. (5) between $\tilde{A}_{fj} \equiv \tilde{A}_{fj}(m_K)$ and the decay process. Nevertheless it will be shown that $\tilde{A}_{fj}$ does play a role in determining the overall decay properties of the system.

The assumption of T invariance implies that

$$TW_\gamma T^{-1} = W_\gamma\ , \tag{8}$$

for both $\gamma = 1$ and 2, where T is the antiunitary time-reversal operator. When the states $\mid c,\text{out}\rangle$ are chosen to be the eigenstates of the strong-interaction S matrix, whose elements are given in general by $\langle c,\text{out}\mid c',\text{in}\rangle$, it follows from Eqs. (4) and (8) and the antiunitary property of T that

$$A^*_{cj}(E) = e^{-2i\delta_c(E)}A_{c'j}(E)\ , \tag{9a}$$

where $2\delta_c(E)$ is the eigenphase of the S matrix associated with the eigenstate $\mid c,\text{in}\rangle$ and the state $\mid c',\text{in}\rangle$ is obtained from $\mid c,\text{in}\rangle$ by motion reversal[5] of all quantum numbers. Similarly, it follows from Eq. (7) that

$$\tilde{A}^*_{cj}(E) = e^{2i\delta_c(E)}\tilde{A}_{c'j}(E)\ . \tag{9b}$$

Equations (9) suggest the introduction of the "reduced amplitudes"

$$a_{cj}(E) = A_{cj}(E)e^{-i\delta_c(E)} \tag{10a}$$

and

$$\tilde{a}_{cj}(E) = \tilde{A}_{cj}(E)e^{i\delta_c(E)} \tag{10b}$$

for which the condition of T invariance becomes

$$a^*_{cj}(E) = a_{c'j}(E) \tag{11a}$$

and

$$\tilde{a}^*_{cj}(E) = \tilde{a}_{c'j}(E)\ . \tag{11b}$$

Therefore reduced amplitudes that are even under motion reversal are real numbers and those that are odd are imaginary. But the 2π and 3π amplitudes are necessarily even and the only odd contributions to other amplitudes are those arising from electromagnetic (e.g., Coulomb) final-state-interaction effects, which are not included in the eigenphase δ_c. Since the electromagnetic effects are small, these odd terms may be neglected. Hence, in this approximation, all of the amplitudes $\alpha_{cj}(E)$ and $\tilde{\alpha}_{cj}(E)$ are real.[7]

The effective interactions W_γ, $\gamma = 1$ or 2, may be written as the sum of Hermitian and anti-Hermitian parts:

$$W_\gamma = W_\gamma^{(h)} + W_\gamma^{(a)}\ , \tag{12a}$$

where

$$W_\gamma^{(h)\dagger} = W_\gamma^{(h)},\quad W_\gamma^{(a)\dagger} = -W_\gamma^{(a)}\ . \tag{12b}$$

Then the reduced amplitudes a_{cj} may also be written as the linear combination of contributions b_{cj} and β_{cj} arising [as in Eq. (4)] from $W_1^{(h)}$ and $W_1^{(a)}$, respectively,

$$a_{cj} = b_{cj} - \beta_{cj}\ , \tag{13a}$$

where the choice of sign in the definition of β_{cj} is a matter of convenience. It follows from the definition of the transposed amplitude, Eq. (7), and from Eq. (12b) that

$$\tilde{a}_{cj}(E) = b_{cj}(E) + \beta_{cj}(E)\ . \tag{13b}$$

The two amplitudes $a_{cj}(E)$ and $\tilde{a}_{cj}(E)$ are thereby replaced by two other real parameters $a_{cj}(E)$ and $\beta_{cj}(E)$, where $\beta_{cj}(E)$ provides a measure of the degree of non-Hermiticity of the effective interaction. It will be most convenient to parametrize the analysis in terms of the total amplitudes $a_{cj}(E)$ and their anti-Hermitian parts, $\beta_{cj}(E)$, by writing, in place of Eq. (13b),

$$\tilde{a}_{cj}(E) = a_{cj}(E) + 2\beta_{cj}(E)\ . \tag{13c}$$

These two real parameters $a_{cj}(E)$ and $\beta_{cj}(E)$ now supplant the two that characterize the usual analysis: the real and imaginary parts of each amplitude. Interference experiments make possible the determination of real and imaginary parts of a decay amplitude A_{fj} (on the mass shell) but no corresponding method is available to measure β_{fj}.

III. PHENOMENOLOGY OF CP VIOLATION IN THE 2π MODE

The standard analysis[8] yielding an expression for the K_L to K_S ratio of 2π decay amplitudes $\eta_{\pi\pi}$ ($=\eta_{+-}$ or η_{00}) in terms of the mass matrix and other parameters must be modified to take into account the apparent CPT violation that must occur under the assumption of T invariance. However the eigenstates of the mass matrix take the usual form

$$|K_S\rangle = 2^{-1/2}[(1+\epsilon-\bar{\epsilon})|K^0\rangle + (1-\epsilon+\bar{\epsilon})|\bar{K}^0\rangle] , \tag{14a}$$

$$|K_L\rangle = 2^{-1/2}[(1+\epsilon+\bar{\epsilon})|K^0\rangle - (1-\epsilon-\bar{\epsilon})|\bar{K}^0\rangle] . \tag{14b}$$

Here ϵ is directly related to *CP* violation and $\bar{\epsilon}$ to *CPT* violation[3] and both are of order 10^{-3} or less so that terms of second and higher order have been, and will be neglected.

The amplitudes for decay of the K_S and K_L into any final state $|f\rangle$ are related to the amplitudes A_f and $\bar{A}_f$ for decay of the K^0 and $\bar{K}^0$ into the same final state by the same linear relationships:

$$A_{fS} = 2^{-1/2}[(1+\epsilon-\bar{\epsilon})A_f + (1-\epsilon+\bar{\epsilon})\bar{A}_f] , \tag{15a}$$

$$A_{fL} = 2^{-1/2}[(1+\epsilon-\bar{\epsilon})A_f - (1-\epsilon-\bar{\epsilon})\bar{A}_f] , \tag{15b}$$

and the ratio $\eta_{\pi\pi} = A_{2\pi,L}/A_{2\pi,S}$ of the amplitudes into a 2π state is, therefore, to first order in $\epsilon,\bar{\epsilon}$,

$$\eta_{\pi\pi} = \epsilon + \bar{\epsilon} + \epsilon_{\pi\pi} , \tag{16a}$$

where $\epsilon_{\pi\pi}$ represents the *direct CP* violation in the 2π mode:

$$\epsilon_{\pi\pi} = (A_{2\pi} - \bar{A}_{2\pi})/(A_{2\pi} + \bar{A}_{2\pi}) . \tag{16b}$$

It is at this point that a departure from standard forms becomes evident: when $\epsilon_{\pi\pi}$ is written in terms of the reduced amplitudes a_{Ij} for the $I=0$ and $I=2$ eigenstates of the 2π system. First, because of the assumed *T* invariance, a_{0j} and a_{2j} are real numbers and the quantity $\epsilon' \sim \text{Im}\, a_2$, in terms of which $\epsilon_{\pi\pi}$ is normally expressed, vanishes. Second, the direct *CP* violation, which is in fact a phenomenological *CPT* violation, may be expressed in terms of a real number α_I given by Eq. (1) or

$$\alpha_I = (a_I - \bar{a}_I)/(a_I + \bar{a}_I) . \tag{17a}$$

Since α_I is small and need be considered only to first order, Eq. (17a) can be written as

$$\bar{a}_I = a_I(1 - 2\alpha_I) . \tag{17b}$$

When these definitions are used and the difference $\delta = \delta_2 - \delta_0$ between the $I=2$ and $I=0$ phase shifts is introduced the outcome for $\eta_{\pi\pi}$ is

$$\eta_{+-} = \epsilon + \bar{\epsilon} + \alpha_0 + \epsilon'' , \tag{18a}$$

$$\eta_{00} = \epsilon + \bar{\epsilon} + \alpha_0 - 2\epsilon'' , \tag{18b}$$

with

$$\epsilon'' = 2^{-1/2} e^{i\delta}(a_2/a_0)(\alpha_2 - \alpha_0) . \tag{18c}$$

Use has been made here of the well-known approximation $a_2/a_0 \ll 1$ [see Eq. (44b) below]. There are two important differences between Eqs. (18) and the more usual expressions for $\eta_{\pi\pi}$. One is the appearance of the common real term α_0 measuring the contribution of direct (phenomenological) *CPT* violation in the $I=0$, 2π mode. The other is that ϵ'' is 90° out of phase with

$$\epsilon' = 2^{-1/2} i e^{i\delta}\, \text{Im}(a_2/a_0) \tag{19}$$

which ordinarily appears in Eqs. (18) in place of ϵ''.

The significance of this phase difference lies in the methods available for measurement of the direct *CP* or *CPT* violation. The quantities measured are η_{+-} and η_{00}. Then the difference $\eta_{+-} - \eta_{00}$ yields a value for $3\epsilon'$ in the conventional case and for $3\epsilon''$ in the present case. In general a determination of this difference requires precise information about the phases of both η_{+-} and η_{00}. The measurement of ϕ_{00}, the phase of η_{00}, is particularly difficult and large estimated errors are given for the most recent measurements.[9] However, Cronin has remarked on the fortuitous circumstance that the measurements of the π-π phase shifts lead to a phase for ϵ' that is almost equal to the observed value of ϕ_{+-}. Thus, in the conventional case, ϵ', η_{+-}, and η_{00} all have nearly the same phase and the value of $|\epsilon'|/|\eta_{+-}|$ can be determined with precision from a measurement of $|\eta_{00}|/|\eta_{+-}|$. Measurement of ϕ_{00} is not required. The result[9] is that $|\epsilon'|$ is very small compared to $|\eta_{+-}|$.

For the determination of ϵ'', the situation is just the opposite from this happy circumstance: because of the 90° difference in phase of ϵ'' and η_{+-} the determination of ϵ'' is very sensitive to the measurement of ϕ_{00} and there is a high degree of uncertainty in its value. This indeterminacy will result in a corresponding uncertainty in the estimate of the "predicted" value of $|\eta_{+-}|$ based on the assumption of *T* invariance.

The equation

$$3\epsilon'' = \eta_{+-} - \eta_{00} , \tag{20}$$

which follows from Eqs. (18), along with the fact[9] that the phase of ϵ'',

$$\delta \approx \phi_{+-} - \frac{\pi}{2} , \tag{21}$$

implies that the real part of

$$\frac{3\epsilon''}{\eta_{+-}} = 1 - \frac{\eta_{00}}{\eta_{+-}} \tag{22}$$

must vanish. Hence

$$\frac{3i\epsilon''}{\eta_{+-}} = \left|\frac{\eta_{00}}{\eta_{+-}}\right| \sin(\phi_{00} - \phi_{+-}) . \tag{23}$$

And if the values[9]

$$|\eta_{00}|/|\eta_{+-}| = 1.0138 \pm 0.0174 \tag{24a}$$

and

$$\phi_{00} - \phi_{+-} = 10° \pm 6° \tag{24b}$$

are inserted the result is

$$i\epsilon''/\eta_{+-} = 0.059 \pm 0.036 , \tag{25}$$

where the large estimated error is due to the estimated error in the measurement of $\phi_{00} - \phi_{+-}$.

IV. THE MASS MATRIX

The mixing parameters ϵ and $\bar{\epsilon}$ are directly related to the mass matrix responsible for the mixing through the equations

$$\epsilon = i(2\Lambda)^{-1}e^{i\phi_0}(M_{12}-M_{21}) \tag{26a}$$

and

$$\bar{\epsilon} = i(2\Lambda)^{-1}e^{i\phi_0}(M_{22}-M_{11}) , \tag{26b}$$

where

$$\Lambda = [(\Delta m)^2 + \tfrac{1}{4}(\Gamma_S - \Gamma_L)^2]^{1/2} , \tag{27a}$$

$$\phi_0 = \arctan[2\Delta m/(\Gamma_S - \Gamma_L)] , \tag{27b}$$

$$\Delta m = m_L - m_S . \tag{27c}$$

Equations (26) are valid to first order in the small quantities ϵ and $\bar{\epsilon}$.

Although these equations have the standard form in terms of the elements of the mass matrix, the expressions for these elements are modified significantly by the introduction of anti-Hermitian contributions to the effective interactions. If M_{jk} is written as

$$M_{jk} = m_{jk} - \frac{i}{2}\Gamma_{jk} \tag{28}$$

then, for non-Hermitian interactions[2]

$$\Gamma_{jk} = \sum_f \tilde{A}_{fj} A_{fk} \tag{29a}$$

and

$$m_{jk} = m_K \delta_{jk} + \langle j \mid W_2 \mid k \rangle - \frac{1}{2\pi} \mathrm{P} \int \frac{dE}{E - m_K} \sum_c \tilde{A}_{cj}(E) A_{ck}(E) . \tag{29b}$$

As a consequence of the conditions Eqs. (8) and (9) for T invariance, Γ_{jk} and M_{jk} are real numbers.[7] When they are expressed in terms of the reduced amplitudes, they may be written as the sum of real symmetric (Hermitian) and real antisymmetric (anti-Hermitian) matrices

$$\Gamma_{jk} = \Gamma^s_{jk} + \Gamma^a_{jk} \tag{30a}$$

and

$$m_{jk} = m^s_{jk} + m^a_{jk} \tag{30b}$$

with

$$\Gamma^s_{jk} = \sum_f (a_{fj}a_{fk} + a_{fj}\beta_{fk} + \beta_{fj}a_{fk}) , \tag{31a}$$

$$\Gamma^a_{jk} = \sum_f (\beta_{fj}a_{fk} - a_{fj}\beta_{fk}) , \tag{31b}$$

and so forth. The symmetric and antisymmetric parts of W_2 are simply its (real) Hermitian and anti-Hermitian parts.

V. *CP* VIOLATION RESULTING FROM ANTI-HERMITIAN AMPLITUDES

Equations (18), (26a), and (30) may now be combined to write

$$\eta_{+-} = \alpha_0 + (2\Lambda)^{-1}e^{i\phi_0}[\Gamma^a_{12} + \tfrac{1}{2}(\Gamma^s_{22} - \Gamma^s_{11}) + i(2m^a_{12} + m^s_{22} - m^s_{11})] + \epsilon'' . \tag{32}$$

Since the observed value[9] of $\phi_{+-} \approx \phi_0$, the phase of the third term in the square brackets is nearly the same as that of ϵ''. Therefore they can be combined and Eq. (32) may be rewritten as

$$\eta_{+-} = \alpha_0 + (2\Lambda)^{-1}e^{i\phi_0}[\Gamma^a_{12} + \tfrac{1}{2}(\Gamma^s_{22} - \Gamma^s_{11})](1 + i\xi) , \tag{33}$$

where ξ is a real number. Since ξ is the ratio of dispersive to absorptive terms in the mass matrix (except for the small contribution of ϵ'') which is of the order of $\Delta m/\Gamma_S$, it is expected that $|\xi| \approx 1$.

Equation (33) will now be used to express η_{+-} in terms of other measurable parameters such as the decay amplitudes appearing in the absorptive terms. No such expressions for the dispersive terms are available but ξ will be treated as a parameter to be determined by the condition that the phase of η_{+-} given by Eq. (33) be equal to the observed phase. That is possible because, from T invariance, α_0, Γ^a_{12}, Γ^s_{jj}, and ξ are real numbers. The situation is strikingly at odds with that associated with *CPT* invariance because, then, $\alpha_0 = 0$, the dispersive term is intrinsically imaginary, and it is only necessary to confirm that the absorptive terms (which are *not* real) are small in order to demonstrate that $\phi_{+-} \approx \phi_0$. Thus the phase of η_{+-} is obtained in a more "natural" way under the assumption of *CPT* invariance than in the present case. Nevertheless it is important to provide substantive evidence that the "unnatural" way is excluded.

Equations (31) may now be used in Eq. (33) to express η_{+-} in terms of amplitudes since

$$\Gamma^a_{12} + \tfrac{1}{2}(\Gamma^s_{22} - \Gamma^s_{11}) = \sum_f [(\beta_f \bar{a}_f - a_f \bar{\beta}_f) + \tfrac{1}{2}(\bar{a}_f^{\,2} - a_f^{\,2}) + (\bar{a}_f \bar{\beta}_f - a_f \beta_f)] . \tag{34}$$

Because the $I = 0$, 2π amplitudes are much larger than all others, it is a good approximation to include only those terms ($f \equiv 0$) in Eq. (34). The validity of this approximation is confirmed in the Appendix by making use of available data on the other amplitudes. Furthermore, an upper limit on the *CPT* violation of the anti-Hermitian amplitude may be estimated by assuming it to be of the same order as that of the total amplitude given by Eq. (17a) and terms of order higher than the first are neglected. Then,

$$\beta_I - \bar{\beta}_I = 2\gamma_I \alpha_I \beta_I \tag{35a}$$

with

$$|\gamma_I| \lesssim 1 , \tag{35b}$$

and Eq. (34) becomes

$$\Gamma^a_{12} + \tfrac{1}{2}(\Gamma^s_{22} - \Gamma^s_{11}) \approx -2\alpha_0 a_0^{\,2}(1 + 2\beta_0/a_0) . \tag{36}$$

Now from Eq. (27a)

$$\Lambda \approx 2^{-1/2}\Gamma_S \approx 2^{1/2}a_0{}^2 \tag{37}$$

so that Eq. (33) may be replaced by

$$\eta_{+-} \approx \alpha_0[1-2^{-1/2}e^{i\phi_0}(1+2\beta_0/a_0)(1+i\xi)] \ . \tag{38}$$

Since $\phi_{+-} \approx \phi_0$,

$$\eta_{+-}e^{-i\phi_0} = \alpha_0[e^{-i\phi_0} - 2^{-1/2}(1+2\beta_0/a_0)(1+i\xi)] \tag{39}$$

must be a real number and ξ is determined by the condition

$$\sin\phi_0 + 2^{-1/2}\xi(1+2\beta_0/a_0) \approx 0 \ . \tag{40}$$

Then

$$\eta_{+-}e^{-i\phi_{+-}} = 2^{-1/2}\alpha_0\left[\sin\left[\frac{\pi}{4}-\phi_0\right] - \frac{2\beta_0}{a_0}\right] . \tag{41}$$

Since Eq. (27b) gives[9] $\phi_0 = 43.7°$, the relationship between the parameter measuring *CP* violation and that measuring the anti-Hermitian amplitude in the $I=0$, 2π state is

$$\eta_{+-}e^{-i\phi_{+-}} = 2^{-1/2}\alpha_0(2.27\times 10^{-2} - 2\beta_0/a_0) \ . \tag{42}$$

A value of α_0 is now needed to determine the connection between *CP* violation and the anti-Hermitian amplitude. Although α_0 can, in principle, be measured directly, no result has been reported and it is necessary to fall back on an indirect measurement at this time.

The procedure to be followed here is to make use of the rather poorly determined value of $|\epsilon''|$, given by Eq. (25), along with its definition, Eq. (18c), and an estimate of α_2 from the measurement of *CP* violation in the 2π mode of the $K^\pm$ to obtain an estimate of α_0. This will then yield an estimate of the range of values of $|\eta_{+-}|$ that are consistent with acceptable deviations from unitarity under the assumed *T* invariance of the effective Hamiltonian.

VI. DETERMINATION OF α_0

The information on a_2, and on other parameters that will be needed later, is to be obtained from data on $K^\pm$ decay, especially data on *CP* and *CPT* violation in $K^\pm$ decay which are given in Table I. The connection between the $K^\pm$ decay amplitudes a_{f+}, a_{f-} and the amplitudes $a_f, \bar{a}_f$, respectively, for K^0 decay into analogous modes (the same label f refers to the analogue mode of appropriate charge and $j=1$ or 2 correspond to the subscript + or −, respectively) is given by

$$a_{fj} = \zeta_f a_{f\pm} \ , \tag{43}$$

where ζ_f is (because of the assumed *T* invariance) a real number.

It is an adequate approximation for the present purpose to assume that the ratio ζ_f in Eq. (43) is of the order of unity. That is justified on the basis of the usual assumption that the amplitudes for these analogue decays are simply different isotopic spin state manifestations of the same interactions. For example, the common assumption that $K^\pm \to \pi^\pm\pi^0$ decay is due to the same $\Delta I = \frac{3}{2}$ interaction that is responsible for the $I=2$, 2π mode of K_S^0 leads to[10]

$$\zeta_2 = \tfrac{3}{2} \ , \tag{44a}$$

giving

$$|a_2/a_0| \approx 0.045 \tag{44b}$$

by comparison of the K_{0S} decay rate (which is dominated by the $I=0$, 2π mode) with the 2π decay rate of $K^\pm$. This result is also consistent with the observed ratio $\Gamma_S(\pi^+\pi^-)/\Gamma_S(\pi^0\pi^0)$.

The ratios ζ_f for the other important analogue modes, $f=3\pi$ and $f=\pi l\nu$, may be obtained by direct comparison of the K_L^0 and $K^\pm$ lifetimes and are found to be

$$\zeta_f \approx 1.5 \ , \tag{45}$$

where, for the 3π state, an average over $\pi^+\pi^-\pi^0$ and $\pi^0\pi^0\pi^0$ states is used. There is, of course, no $K^\pm$ analogue to the amplitude a_0, nor is there a $K^0, \bar{K}^0$ analogue to the dominant $K^\pm \to \mu^\pm\nu$ mode.

Since only an estimate of their order of magnitude is required the anti-Hermitian parts of the amplitudes are obtained by using the same relationship:

$$\beta_{fj} = \zeta_f \beta_{f\pm} \ . \tag{46}$$

Furthermore, ζ_f is taken to be independent of j since any dependence must be small, of the order of the *CP* violation.

TABLE I. Tests of *CP* and *CPT* invariance from decay modes of $K^\pm$. $B_{f,\pm}$ is the branching ratio into mode f. The definition of $\alpha_{f,\pm}$ is $\alpha_{f,\pm} = (a_{f+} - a_{f-})/2a_{f-}$.

Test of	Mode	Measurement	Reference
CP	$K^\pm \to \pi^\pm\pi^0$	$\alpha_{2\pm} = (B_{2\pi,+} - B_{2\pi,-})/4B_{2\pi,-} = (2.0\pm 3.0)\times 10^{-3}$	a
CP	$K^\pm \to \mu^\pm\nu$	$\alpha_{\mu 2\pm} = (B_{\mu 2,+} - B_{\mu 2,-})/4B_{\mu 2,-} = (-1.4\pm 1.0)\times 10^{-3}$	b
CP	$K^\pm \to \pi^\pm\pi^\pm\pi^\mp$	$\alpha_{\tau\pm} = (B_{\tau+} - B_{\tau-})/4B_{\tau-} = (0.2\pm 0.3)\times 10^{-3}$	c
CP	$K^\pm \to \pi^\pm\pi^0\pi^0$	$\alpha_{\tau'\pm} = (B_{\tau'+} - B_{\tau'-})/4B_{\tau'} = (0.2\pm 1.5)\times 10^{-3}$	d
CPT	$K^\pm$ lifetime	$(\Gamma_+ - \Gamma_-)/\Gamma_+ = (1.1\pm 0.9)\times 10^{-3}$	e

[a]D. Herzo *et al.*, Phys. Rev. **186**, 1403 (1969).
[b]W. T. Ford *et al.*, Phys. Rev. Lett. **18**, 1214 (1967).
[c]W. T. Ford *et al.*, Phys. Rev. Lett. **25**, 1370 (1970).
[d]K. M. Smith *et al.*, Nucl. Phys. **B60**, 411 (1973).
[e]F. Lobkowicz *et al.*, Phys. Rev. **185**, 1676 (1969).

From Eqs. (43) and (17a) it is then found that

$$\alpha_2=(a_{2+}-a_{2-})/2a_{2\pm} \tag{47}$$

so that $\alpha_2=\alpha_{2\pm}$ given in Table I. Then Eqs. (25) and (18c) give

$$\alpha_0=(2.0\pm3.0)\times10^{-3}-(1.85\pm1.13)\eta_{+-}e^{-i\phi_{+-}} \tag{48}$$

which may be applied to Eq. (42) to yield

$$|\eta_{+-}|=|(1.41\pm2.12)(2.27\times10^{-2}-2\beta_0/a_0)|\,|1+(1.308\pm0.799)(2.27\times10^{-2}-2\beta_0/a_0)|^{-1}\times10^{-3} . \tag{49}$$

VII. TEST OF UNITARITY

The eigenstates $|K_\alpha\rangle$, with $\alpha\equiv S$ or L, of the mass matrix M satisfy

$$M|K_\alpha\rangle=\left[M_\alpha-\frac{i}{2}\Gamma_\alpha\right]|K_\alpha\rangle \tag{50}$$

from which the "unitarity" condition

$$2\langle\beta|m^a|\alpha\rangle-i\langle\beta|\Gamma^s|\alpha\rangle=\left[(m_\alpha-m_\beta)-\frac{i}{2}(\Gamma_\alpha+\Gamma_\beta)\right]\langle K_\beta|K_\alpha\rangle \tag{51}$$

follows directly.[2] Here, the states $|K_\alpha\rangle\equiv|\alpha\rangle$ and $\langle K_\beta|\equiv\langle\beta|=|\beta\rangle^*$ are given by Eqs. (14) and the matrices m^a_{jk}, Γ^s_{jk} are defined by Eqs. (30).

Of particular interest is the case $\alpha=\beta$ for which Eq. (51) becomes

$$\Gamma_\alpha=\sum_f[\,|a_{f\alpha}|^2+2\,\mathrm{Re}(a^*_{f\alpha}\beta_{f\alpha})]+2i\langle\alpha|m^a|\alpha\rangle \tag{52}$$

when Γ^s is expressed in terms of amplitudes by means of Eq. (31a). Although m^a is an antisymmetric matrix (in j,k) it has diagonal elements in the $|\alpha\rangle$ representation but these elements are proportional to $\mathrm{Im}(\epsilon+\bar\epsilon)m_{12}$. Furthermore m^a_{12} itself is antisymmetric for exchange of K^0 and $\bar K^0$; hence, it is CP violating and can also be expected to be of order $(|\epsilon|+|\bar\epsilon|)$. Therefore the term $\langle\alpha|m^a|\alpha\rangle$ in Eq. (52) is of second order in $(|\epsilon|+|\bar\epsilon|)$ and is negligible.

The amplitudes $a_{f\alpha},\beta_{f\alpha}$ are obtained from the real quantities a_{fj},β_{fj} by means of the transformation Eqs. (15) and are complex because ϵ and $\bar\epsilon$ are complex. Since the branching ratios of the K_α into the final state $|f\rangle$ are given by[11]

$$B_{f\alpha}=|a_{f\alpha}|^2/\Gamma_\alpha , \tag{53}$$

Eq. (52) may be written as

$$\sum_f B_{f\alpha}[1+2\,\mathrm{Re}(\beta_{f\alpha}/a_{f\alpha})]=1 . \tag{54}$$

This statement replaces the usual statement of unitarity:

$$\sum_f B_{f\alpha}=1 \tag{55}$$

and may be used as a basis for determining the deviation from unitarity due to the presence of the anti-Hermitian terms. In the case $\alpha=S$, $B_{0S}\approx1$ for the $I=0$, 2π state, all other contributions to the sum in Eq. (54) are small and it becomes

$$\sum_f B_{fS}+2\,\mathrm{Re}(\beta_{0S}/a_{0S})=1 . \tag{56}$$

Now to zeroth order in $(|\epsilon|+|\bar\epsilon|)$, a_{0S} and β_{0S} are real and

$$\beta_{0S}/a_{0S}=\beta_0/a_0 . \tag{57}$$

Therefore

$$2\beta_0/a_0=1-\sum_f B_{fS} \tag{58}$$

gives the remaining parameter appearing in Eq. (49).

The sum can be determined from the measured K_S branching ratios for the $\pi^+\pi^-$, $\pi^0\pi^0$, and $\pi^+\pi^-\gamma$ modes, which are tabulated,[12] and the semileptonic modes, which are not. However, the last can be obtained on the basis of evidence for the $\Delta S=\Delta Q$ rule which involves a direct measurement of the partial semileptonic decay rates of the K_S and K_L. The implication is $\Gamma_S(\pi^\pm l^\mp\nu)=\Gamma_L(\pi^\pm l^\mp\nu)$, a result that is consistent (to zero order in $|\epsilon|+|\bar\epsilon|$) with the observed charge asymmetry in the semileptonic decay of the K_L. It follows from the data[13] on $\Gamma_L(\pi^\pm l^\mp\nu)$ that

$$\sum_l B_S(\pi l\nu)=(1.03\pm0.06)\times10^{-3} \tag{59}$$

and, from those on the other branching ratios that

$$\sum_f B_{fS}=0.982\pm0.016 , \tag{60}$$

whence

$$\beta_0/a_0=0.0090\pm0.008 . \tag{61}$$

An upper limit on the value of $|\eta_{+-}|$ permitted by Eq. (49) is obtained by using the upper limits on the two factors in the numerator. Since the estimated errors in both factors are large, a conservative limit will be obtained by taking the extreme values associated with three standard deviations. Thus $2\beta_0/a_0<6.60\times10^{-2}$ and

$$|\eta_{+-}|<0.34\times10^{-3} . \tag{62}$$

The small corrections, of order 1%, to the denominator in Eq. (49) have been neglected. The upper limit Eq. (62) is smaller than the observed value[9]

$$|\eta_{+-}|=(2.27\pm0.02)\times10^{-3} \tag{63}$$

by an order of magnitude.

It should be recognized that the result in Eq. (62) is dominated by estimated experimental errors. The largest effect is that due to the uncertainty of the *CP* violation in the 2π decay mode of the $K^{\pm}$, given in Table I. The other is associated with the data on branching ratios. Any departure from unitarity is lost in the noise. The fact that the precision of these measurements is high enough that even when the estimated errors are multiplied by three they are small compared to the very small *CP* violation is the significant result obtained here.

Taken along with the usual argument[1] based on the assumption of *Hermitian* effective interactions, this result leads to the conclusion that *T invariance is violated even if anti-Hermitian interactions occur up to the limit permitted by the data on unitarity.* It should be remarked that the weakest link in the overall evidence for *T* violation appears to be the indirect measurement of the π-π phase shifts δ_0 and δ_2 whose difference δ enters all of these arguments in a crucial way. Improved precision of these data and experiments providing direct evidence of the violation of motion reversal symmetry are needed to substantiate the existence of *T* violation.

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APPENDIX: CONTRIBUTIONS OF OTHER DECAY MODES

The foregoing discussion has been limited to the treatment of the role of the anti-Hermitian term in the dominant $I=0$, 2π mode of the $K^0,\bar{K}^0$ system. The purpose of this appendix is to place limits on the contributions of anti-Hermitian terms to other modes and to justify the neglect of these modes in arriving at the crucial approximation Eq. (36). Since the only purpose here is to show that these terms may be neglected, order-of-magnitude estimates of some of the parameters will be used.

The application of Eq. (52) to the case of $a=L$ provides the basis for estimating the magnitude of β_f/a_f for $f\neq 0$ because K_L has many decay modes having comparable branching ratios. As before, the contribution of m^a is negligible so that

$$\Gamma_L=\sum_f |a_{fL}|^2[1+2\,\mathrm{Re}(\beta_{fL}/a_{fL})] \ . \tag{A1}$$

Therefore, if an average value of (β_{fj}/a_{fj}) is defined by

$$\langle \beta_L/a_L\rangle=\sum_f B_{fL}\,\mathrm{Re}(\beta_{fL}/a_{fL})\Big/\sum_f B_{fl} \ , \tag{A2}$$

where B_{fL} is defined by Eq. (53), then

$$2\langle \beta_L/a_L\rangle=\left[\sum_f B_{fL}\right]^{-1}-1 \ . \tag{A3}$$

Similar relationships may be obtained for the amplitudes $a_{f\pm}$ and $\beta_{f\pm}$ defined for the $K^{\pm}$ decay in the same manner as a_{fj} and β_{fj} were defined for $K^0,\bar{K}^0$, Eqs. (13). Since there is no mixing in this case, the total decay rate $\Gamma_{\pm}$ is given directly by the equivalent of Γ_{jj}, Eq. (29a):

$$\Gamma_{\pm}=\sum_f a_{f\pm}{}^2[1+2(\beta_{f\pm}/a_{f\pm})] \ . \tag{A4}$$

When the branching ratios

$$B_{f\pm}=a_{f\pm}{}^2/\Gamma_{\pm} \tag{A5}$$

are introduced it is possible to obtain a measure of the anti-Hermitian term for charged decays analogous to Eq. (A3):

$$2\langle \beta_{\pm}/a_{\pm}\rangle=\left[\sum_f B_{f\pm}\right]^{-1}-1 \ . \tag{A6}$$

From the tabulated data[14] on the branching ratios it is found that

$$\sum_f B_{fL}=0.933\pm 0.038 \tag{A7}$$

and

$$\sum_f B_{f\pm}=1.005\pm 0.007 \ , \tag{A8}$$

whence it follows that

$$\langle \beta_L/a_L\rangle=0.033\pm 0.019 \tag{A9}$$

and

$$\langle \beta_{\pm}/a_{\pm}\rangle=-0.003\pm 0.004 \ . \tag{A10}$$

These averages serve as a basis for estimating the order of magnitude of the anti-Hermitian term for specific modes.

The remaining question concerns the contributions to Eq. (34) of the modes other than $f=0$, i.e., the $I=0$, 2π mode. If the contributions of these modes to Γ^a_{12} and Γ^s_{jj} are denoted by $\Delta\Gamma^a_{12}$ and $\Delta\Gamma^s_{12}$, the effect on η_{+-}, given by Eq. (33), is determined by a comparison of

$$(2\Lambda)^{-1}[\Delta\Gamma^a_{12}+\tfrac{1}{2}(\Delta\Gamma^s_{22}-\Delta\Gamma^s_{11})] \tag{A11}$$

with $|\eta_{+-}|$ given by Eq. (62).

The evaluation of

$$(4\Lambda)^{-1}(\Delta\Gamma^s_{22}-\Delta\Gamma^s_{11})=\tfrac{1}{2}\sum_{f\neq 0}\left[a_{\bar{f}}\frac{\beta_{\bar{f}}}{a_{\bar{f}}}-a_f{}^2\left[1+2\frac{\beta_f}{a_f}\right]\right] \tag{A12}$$

can be made by making use of Eq. (A4) to write

$$\Gamma_+-\Gamma_-=\sum_f\left[a_{f+}{}^2\left[1+2\frac{\beta_{f+}}{a_{f+}}\right]-a_{f-}{}^2\left[1+2\frac{\beta_{f-}}{a_{f-}}\right]\right] \tag{A13}$$

and then relating the two similar expressions by means of Eqs. (43) and (46) for the analogue modes of $K^{\pm}$ and $K^0,\bar{K}^0$. Equation (A13) includes the $K^{\pm}_{\mu 2}$ mode for which there is no analogue of $K^0,\bar{K}^0$. Therefore Eq. (A12) becomes

$$(4\Lambda)^{-1}(\Delta\Gamma_{22}^{s}-\Delta\Gamma_{11}^{s})$$
$$=(4\Lambda)^{-1}\zeta^2\left\{\Gamma_+-\Gamma_- -\left[a_{\mu+}{}^2\left[1+2\frac{\beta_{\mu+}}{a_{\mu+}}\right]-a_{\mu-}{}^2\left[1+2\frac{\beta_{\mu-}}{a_{\mu-}}\right]\right]\right\}. \quad \text{(A14)}$$

where ζ^2 is an average of $\zeta_f{}^2$ and is expected to be of order unity.

An upper limit on $|\beta_{\mu\pm}/a_{\mu\pm}|$ may be obtained from Eq. (A10) by assuming that it is the *only* nonvanishing contribution to the average. Then, since $B_{\mu 2,\pm}\approx 63\%$,

$$2|\beta_{\mu\pm}/a_{\mu\pm}|\leq 0.008\pm 0.012\ . \quad \text{(A15)}$$

Also

$$|a_{\mu+}|^2-|a_{\mu-}|^2=(B_{\mu 2,+}+B_{\mu 2,-})\Gamma_+\ . \quad \text{(A16)}$$

Therefore

$$(4\Lambda)^{-1}(\Delta\Gamma_{22}^{s}-\Delta\Gamma_{11}^{s})$$
$$=(4\Lambda)^{-1}\Gamma_f\zeta^2\left[\frac{\Gamma_+-\Gamma_-}{\Gamma_+}-(B_{\mu 2,+}-B_{\mu 2,-})\right] \quad \text{(A17)}$$

if the corrections of (at most) 1% or 2% due to Eq. (A15) are neglected. From Eq. (37)

$$(4\Lambda)^{-1}\Gamma_+=2^{-3/2}(\Gamma_+/\Gamma_S)=2.55\times 10^{-3} \quad \text{(A18)}$$

and $(\Gamma_+-\Gamma_-)/\Gamma_+$ as well as $(B_{\mu 2,+}-B_{\mu 2,-})/B_{\mu 2,-}$ as given in Table I, this contribution to Eq. (A11) is found to be

$$(4\Lambda)^{-1}(\Delta\Gamma_{22}^{s}-\Delta\Gamma_{11}^{s})=(1.18\pm 0.68)\times 10^{-5}\zeta^2 \quad \text{(A19)}$$

which is 2 orders of magnitude smaller than η_{+-} and, therefore negligible.

The term

$$(2\Lambda)^{-1}\Delta\Gamma_{12}^{a}=(2\Lambda)^{-1}\sum_{f\neq 0}(\beta_f\bar{a}_f-a_f\bar{\beta}_f) \quad \text{(A20)}$$

may also be shown to be negligible compared to $|\eta_{+-}|$ by noting, first, that only those states f that are common modes of both K^0 and $\bar{K}^0$ contribute. Therefore the $\Delta S=\Delta Q$ rule excludes the semileptonic states and only the contributions of the $I=2$, 2π mode and the 3π modes are expected to be significant.

Generalizations of Eqs. (17b) and (35a) to include the 3π modes

$$a_f-\bar{a}_f=2\alpha_f a_f\ , \quad \text{(A21a)}$$

$$\beta_f-\bar{\beta}_f=2\gamma_f\alpha_f a_f\ , \quad \text{(A21b)}$$

for $f=2$ and $f=3\pi$, yields

$$(2\Lambda)^{-1}\Delta\Gamma_{12}^{a}=\Lambda^{-1}\sum_f a_f{}^2\alpha_f(\gamma_f-1)\beta_f/a_f\ , \quad \text{(A22)}$$

where the sum includes only the $I=2$, 2π state $(f=2)$ and the 3π states. An upper limit on $|\beta_2/a_2|\approx|\beta_{2\pm}/a_{2\pm}|$ may be obtained from Eq. (A10) by assuming that the only nonvanishing anti-Hermitian term is due to $I=2$ and, similarly, an upper limit on $|\beta_{3\pi}/a_{3\pi}|$ may be found from Eq. (A9). Together the results may be expressed in terms of the average for the two modes

$$\langle|\beta_f/a_f|\rangle\leq 0.083\pm 0.030\ . \quad \text{(A23)}$$

Furthermore, if, on the basis of Eq. (43), it is assumed that $\alpha_{fj}=\alpha_f^{\pm}$ for the analogue modes, Table I may be used to set the limit

$$|\alpha_f|\leq 2.3\times 10^{-2}\ . \quad \text{(A24)}$$

Finally, from Eq. (44b) and the relationship $a_{3\pi}{}^2/a_0{}^2=B_{3\pi,L}\Gamma_L/\Gamma_S$ it follows that

$$a_2{}^2/a_0{}^2<a_{3\pi}{}^2/a_0{}^2<6\times 10^{-2} \quad \text{(A25)}$$

so that from Eq. (37)

$$|(2\Lambda)^{-1}\Delta\Gamma_{12}^{a}|<(0.015\pm 0.005)\times 10^{-3}(\langle\gamma_f\rangle-1) \quad \text{(A26)}$$

which implies a correction of less than 10% to Eq. (62) for any reasonable value of the average $\langle\gamma_f\rangle$.

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[1]K. Winter, in *Proceedings of the Amsterdam International Conference on Elementary Particles, 1971*, edited by A. G. Tenner and M. Veltman (North-Holland, Amsterdam, 1972), p. 333, presents a summary of the evidence for T violation and *CPT* invariance based on the unitary conditions.

[2]B. G. Kenny and R. G. Sachs, Phys. Rev. D 8, 1605 (1973).

[3]For example, see R. M. Wald, Phys. Rev. D 21, 2742 (1980) where it is argued that particle creation by black holes is incompatible with *CPT* invariance. This argument can be rephrased in terms of an *apparent* violation of unitarity although unitarity is preserved when particle states within the black hole are included. The apparent deviation from unitarity can be ascribed to an apparent, or "effective," non-Hermitian interaction *among observable states*.

[4]R. G. Sachs, Prog. Theor. Phys. (Jpn.) 54, 809 (1975).

[5]"Motion reversal," in contrast with "time reversal" is defined as the transformation that changes the sign of all quantum numbers associated with observables that are odd under T such as momenta and angular momenta. It is well known that cross sections and decay rates may include terms that are odd under motion reversal even when the interactions are T invariant, and the magnitude of such terms is predictable in terms of final-state phase shifts. Evidence for T violation depends on a demonstration that such terms are different from what is expected on the basis of measured values of phase shifts.

[6]See, for example, R. G. Sachs, Phys. Rev. Lett. 36, 1014 (1976).

[7]The important result, to be used later, that the matrix elements m_{jk} and Γ_{jk} are real is a general consequence of T invariance and does not depend on this approximation although the explicit form of Eqs. (31) does depend on it.

[8]A summary is provided in K. Kleinknecht, Ann. Rev. Nucl.

Sci. **26**, 1 (1976).

[9]J. W. Cronin [Rev. Mod. Phys. **53**, 373 (1981)] provides a review of the analysis and a summary of the current position on world averages of the *CP*-violation parameters which will be used here unless otherwise specified. The value of $|\eta_{00}|/|\eta_{+-}|$ has been reconstructed from the result of the recent measurement by the Chicago-Saclay group: $\epsilon'/\epsilon = -0.0046 \pm 0.0053 \pm 0.0024$ [R. H. Bernstein *et al.*, Phys. Rev. Lett. **54**, 1631 (1985)].

[10]B. G. Kenny, Ann. Phys. (N.Y.) **45**, 25 (1967).

[11]Note that $\beta_{f\alpha}$ is the *absolute*, not the relative branching ratio. The absolute branching ratio is the ratio of the number of decay events to the number of decaying particles or $\Gamma_{f\alpha}/\Gamma_\alpha$, where $\Gamma_{f\alpha}$ is the directly measured partial decay rate. If the absolute branching ratio is known for *one* decay mode, $f=0$, and the relative branching ratios $B_f(\text{rel}) = \Gamma_f/\sum_f \Gamma_f$ are known for all modes, then the absolute B_f are given by $B_f = B_f(\text{rel}) B_0 / B_0(\text{rel})$.

[12]Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984). The absolute branching ratio for $K_S \to \pi^+\pi^-$ was measured by observing the decay of a tagged K_S beam in a hydrogen bubble chamber in several experiments and the value used here is the weighted average, p. S106. The method is presented by F. S. Crawford, Jr. *et al.*, Phys. Rev. Lett. **2**, 266 (1959). The tabulated realtive branching ratios and their estimated errors obtained from combined data of several experiments were then used to obtain the other absolute ratios by the method of Ref. 11. It will be shown that any possible deviation from unitarity is small compared to estimated experimental errors so that it is not the average value of B_f but is, instead, the estimate of error that is important for my purpose.

[13]The measured partial decay rates $\Gamma_L(\pi l \nu)$ (normalized to the absolute $\pi^+\pi^-$ rate of the K_S), are given in Ref. 12. The absolute branching ratio $B_S(\pi l \nu)$ is obtained from (Ref. 11) Eq. (53) by using the average $\Gamma_L(\pi l \nu)$ as $\Gamma_S(\pi l \nu)$ and dividing by the measured total decay rate Γ_S.

[14]$B_L(\pi l \nu) = \Gamma_L(\pi l \nu)/\Gamma_L$ where Γ_L is the measured total decay rate and $\Gamma_L(\pi l \nu)$ is obtained as in Ref. 13. The values of $\Gamma_L(\pi^+\pi^-\pi^0)$ were also normalized to the absolute $\pi^+\pi^-$ rate of the K_S. The small correction due to *CP* violation is ignored in making these estimates. $B_L(\pi^0\pi^0\pi^0)$ is obtained by comparison of the average $(\pi^0\pi^0\pi^0)$/(all charged) ratio with the average $(\pi^+\pi^-\pi^0)$/(all charged) ratio. The absolute value of $\Gamma_\pm(\pi^+\pi^-\pi^\pm)$ and the total decay rate of the charged *K*'s were used along with tabulated relative branching ratios to obtain Eq. (A8). For this absolute decay rate, see W. T. Ford *et al.*, Phys. Rev. Lett. **18**, 1214 (1967); **25**, 1370 (1970).