Hadronic transverse energy and QCD

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The mean transverse energy of the produced hadrons in $p\bar{p}$ interactions is calculated in the leading-logarithm approximation of QCD. Besides the computation of the contribution of large-p_T jets, we have shown that gluon bremsstrahlung, which dominates at small x, corresponds to events with a large transverse energy and a large multiplicity. Experimental results seem to support this QCD property up to hadronization effects which we have estimated from phenomenology.

I. INTRODUCTION

The dynamics of lepton-induced hadroproduction is explained as a factorizable mechanism. At high enough energy, partons are generated in a pointlike interaction described by perturbative QCD at short distances. At large distances, these partons fragment into hadrons, this last step being governed by confinement forces, the dynamics of which is not yet fully understood. The hadron-hadron interaction is more intricate since this factorization property is not obvious. Recent results in pp and $p\bar{p}$ collisions at the CERN collider shed some light on this problem. Hard collisions are indeed observed as two or more large-transverse-momentum jets of particles. But these events emerge from a background of nonjet configurations corresponding to small-transverse-momentum particles with a large multiplicity.[1] Quite a few of these events are characterized by a large transverse energy $E_T=\sum_n (E_T)_n$, where the sum runs over all the particles which deposit energy inside the calorimeters.

A priori it is not straightforward to relate the low-transverse-momentum events and QCD since there is no obvious large scale of momentum allowing perturbative calculations. However, we have shown in a previous work that gluon or quark radiation induced by "semihard" collisions may explain most of the low-p_T events at high energy. Indeed the rise of the total cross section at high energy is attributed to the low-x parton (mainly gluon) cascading. The relevance of perturbative QCD calculations at low x have been recently confirmed theoretically.[2] Roughly speaking, Q^2/x can be made large enough at moderate Q^2 to apply safely to perturbative QCD, which explains why QCD is relevant to describe small-x events (large multiplicity) and medium transverse momentum.

To study the mean total transverse energy $\langle E_T\rangle$ or $d\sigma/dE_T$ at the hadronic level, we have to calculate first the corresponding observables at the partonic level using perturbative QCD and take into account both the partonization of the incident hadrons and the fragmentation of the outgoing partons. On one hand, we use our formalism developed in Ref. 3 for the constituent quarks sharing the total hadron momentum. On the other hand, the final-state hadronization contribution to the transverse energy has to be considered. Following an analysis made in e^+e^- annihilation into hadrons,[4] the effects of hadronization are minimal when calculating $\langle E_T\rangle$. This is why we will focus in this paper on the calculation of the mean value of the total energy $\langle E_T\rangle$.

Our paper is organized as follows. In Sec. II, we recall the formalism previously used in Ref. 3 to describe the rise of the pp and $p\bar{p}$ total cross section. In Sec. III we perform the perturbative QCD calculation for $d\sigma/dE_T$ and $\langle E_T\rangle$ at the partonic level and discuss the origin of the large contribution to $\langle E_T\rangle$. Section IV is devoted to the estimate of hadronization effects, allowing the comparison of our results with the full-acceptance, minimal-bias events available at CERN ISR and CERN SPS collider energies.[5] In the Conclusion we emphasize the relevance of perturbative QCD for this problem. In Appendixes A, B, and C we discuss the validity of QCD at low x and derive $\langle E_T\rangle$ at the partonic level.

II. HADRONIC CROSS SECTIONS: THE FORMALISM

Let us recall briefly the formalism which has yielded the rise of the total hadronic cross section between the ISR and collider energies.[3] In a first stage, one singles out for each incoming hadron p or $\bar{p}$ one constituent quark of low-transverse momentum $m\simeq 300$–400 MeV$\simeq m_N/3$ where m_N is the nucleon mass. Its nonperturbative Q^2-independent distribution $\psi(Z)$, where Z is the fraction of longitudinal momentum, is constrained by both valence-number and energy-momentum conservation: namely,

$$\int_0^1 \psi(Z)dZ=3,\quad \int_0^1 Z\psi(Z)dZ=1\ .$$

The contribution to the $p\bar{p}$ cross section is given by

$$\sigma_{p\bar{p}}(s)=\int dZ_1 dZ_2\psi(Z_1)\psi(Z_2)\sigma^{\text{part}}(Z_1Z_2s)\ . \tag{1}$$

To describe σ^{part} at high energy, we retain as the dominant contribution the exchange of one hard gluon of virtual mass squared $|\hat{t}|\gg m^2$ (see Fig. 1). One gets

$$\sigma^{\text{part}}(Z_1Z_2s=\hat{s}) =\sum_{a,b}\int\int\int_\Delta \frac{d\sigma}{d\hat{t}}D_q^a(x_a,\hat{t})D_{\bar{q}}^b(x_b,\hat{t})d\hat{t}\,dx_a dx_b\ .$$

x_a and x_b are the Bjorken variables of the interacting partons a,b; $d\sigma/d\hat{t}$ is the one-gluon-exchange cross section; and the D's are the parton distributions inside the constit-

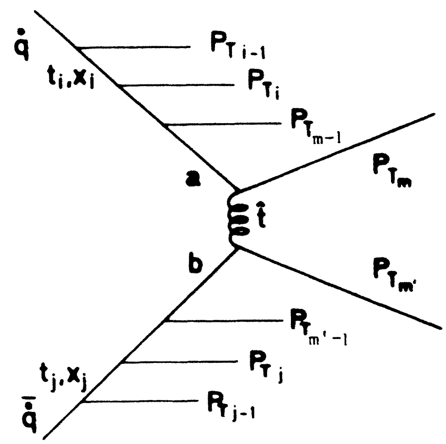

FIG. 1. Quark-gluon cascading. P_{Tm} and $P_{Tm'}$ are the outgoing transverse momenta in the parton-a—parton-b collision. The P_{Ti}'s are those of the cascading partons i or j ($i \leq m-1$, $j \leq m'-1$).

uent quarks. The Δ domain of integration specifies the phase space suitable for perturbative QCD calculations and corresponds to the following cutoff conditions.

(i) Short-range interactions imply a transverse-momentum cutoff $Q_T{}^2 \gg \Lambda_{\rm QCD}{}^2$ where $\Lambda_{\rm QCD}$ is the QCD scale parameter of the coupling constant

$$\frac{\alpha_s(\hat{t})}{4\pi} = \frac{1}{\frac{11}{3}N_c - N_f}\,\frac{1}{\ln(\hat{t}/\Lambda^2)}$$

and $Q_T{}^2$ has to be chosen for consistency larger than m^2. $Q_T{}^2 \gtrsim 1$ GeV^2, for instance.

(ii) The kinematics of the a,b collision implies

$$\hat{t} < \hat{s} = x_a x_b \tilde{s} \ .$$

(iii) The required validity of the leading-logarithm approximation (LLA) implies a cutoff on the longitudinal momentum x_a, x_b. Indeed for a given Q^2, x cannot be too small (this point is discussed in Appendix A). In a model-dependent way,[3] we have fixed the longitudinal cutoff as

$$x_a, x_b \geq x_c = \frac{m^2}{\hat{t}} \ . \tag{2}$$

The integration on x_a, x_b, and the summation over parton species a, b reads, in a compact form,

$$\sigma^{\rm part}(\tilde{s}) = \frac{\pi}{2}\int_{Q_T{}^2}^{\tilde{s}} \alpha_s{}^2(\hat{t})\frac{d\hat{t}}{\hat{t}^2}(C_2\langle n\rangle_s + N_c\langle n\rangle_g)^2 \ , \tag{3}$$

where $\langle n\rangle_s$, $\langle n\rangle_g$ are defined as the truncated multiplicities of excited quarks and gluons at the scale $\hat{t}$:

$$\langle n\rangle_{g,s} = \int_{x_{\rm min}}^{1} D_q^{g,s}(x,\hat{t})dx \ ,$$

where $x_{\rm min}$ is specified by conditions (ii) and (iii). In a factorized approximation one gets[3]

$$x_{\rm min} = x_c = \frac{m^2}{\hat{t}} \quad \text{for } Q_T{}^2 < \hat{t} < m^2\left[\frac{\tilde{s}}{m^2}\right]^{1/3}$$

(domain I),

$$x_{\rm min} = \left[\frac{\hat{t}}{\tilde{s}}\right]^{1/2} \quad \text{for } m^2\left[\frac{\tilde{s}}{m^2}\right]^{1/3} < \hat{t} < \tilde{s}$$

(domain II). In a 2×2 matrix form, one may write, in an an elegant way,

$$C_2\langle n\rangle_s + N_c\langle n\rangle_g = (C_2, N_c)\left[\sum_{m=0}^{\infty}\int\prod_{i=1}^{m} dMi\right]\begin{bmatrix}1\\0\end{bmatrix} , \tag{4}$$

where i labels the step of parton cascading and

$$dMi = \frac{\alpha_s(t_i)}{4\pi}\frac{dt_i}{t_i}H\left[\frac{x_i+1}{x_i}\right]\frac{dx_i}{x_i} \ , \tag{5}$$

where $H(Z_i = x_{i+1}/x_i)$ is the usual singlet-gluon mixing matrix

$$\begin{bmatrix} P_{qq}(Z) & 2N_f P_{qg}(Z) \\ P_{gq}(Z) & P_{gg}(Z) \end{bmatrix} \tag{6}$$

and t_i, x_i labels the virtualness of parton i, and the longitudinal momentum fraction of parton i versus the constituent-quark momentum. Note that among the Altarelli-Parisi kernels[6] P_{gg} and P_{gq} behave as $1/Z$ near $Z=0$.

The t_i's (more precisely[7] the $S_i = t_i/x_i$) are strictly ordered in the LLA. This condition allows the resummation of all the ladder contribution (see Appendix C). The truncated multiplicity of the gluon and the singlet are simple functions of

$$\eta = \ln\frac{\hat{t}}{m^2}, \quad \eta_s = \ln\frac{\tilde{s}}{m^2} \ ,$$

and

$$\xi = \int_{\mu^2}^{\hat{t}} \frac{\alpha_s(t)}{4\pi}\frac{dt}{t} \ ,$$

where μ is a phenomenological parameter determined by Dokshitzer, Dyakonov, and Troyan[7] (DDT) to obtain the correct sharing of momentum between quarks and gluons in the nucleon at all scales.

With this formalism, we have obtained the correct rise of the pp total cross section between CERN ISR and collider energies.[3] This rise is explained by the excess of the increase of the parton multiplicities over the decrease of each individual cross section.

III. QCD CALCULATION OF THE PARTON TRANSVERSE ENERGY

Let us consider the differential transverse energy cross section for partons $d\sigma^{\rm part}/dE_T$. Following our formalism [see Eqs. (3) and (4)], one writes

$$\frac{d\sigma^{\text{part}}}{dE_T}=\frac{\pi}{2}\sum_{m,m'}\int_\Delta\left[(C_2,N_c)\prod_{i=1}^{m-1}dM_i\begin{bmatrix}1\\0\end{bmatrix}\right]\left[(C_2,N_c)\prod_{j=1}^{m'-1}dM_j\begin{bmatrix}1\\0\end{bmatrix}\right]\frac{\alpha_s{}^2(\hat t)}{\hat t^2}d\hat t\,\delta\left[E_T-\sum_{i=1}^{m}|p_{Ti}|-\sum_{j=1}^{m'}|p_{Tj}|\right], \tag{7}$$

where

$$|p_{T(i-1),(j-1)}|=t_{i,j}{}^{1/2}(1-Z_{i,j})^{1/2},\quad i=1,\ldots,m-1,\quad j=1,\ldots,m'-1$$

is the transverse momentum ($\approx$ transverse energy) of the parton of the upper or lower cascades (see Fig. 1). p_{Tm} and $p_{Tm'}$ are the transverse momenta of the interacting partons. E_T is the total transverse energy of the first produced partons. It is formally simple to express the moments of $d\sigma^{\text{part}}/dE_T$ thanks to the δ function. Indeed the Laplace transform $\phi(\tau)$ of $d\sigma/dE_T$ reads

$$\phi(\tau)=\int_0^{\sqrt{s}}\frac{d\sigma}{dE_T}e^{-\tau E_T}dE_T,\quad \text{Re}\tau>0$$

$$=\frac{\pi}{2}\int_\Delta\left[(C_2,N_c)\prod_{i=1}^{m-1}dM_ie^{-\tau|p_{Ti}|}\begin{bmatrix}1\\0\end{bmatrix}\right]\left[(C_2,N_c)\prod_{j=1}^{m'-1}dM_je^{-\tau|p_{Tj}|}\begin{bmatrix}1\\0\end{bmatrix}\right]\frac{\alpha_s{}^2(\hat t)}{\hat t^2}e^{-\tau(|p_{Tm}|+|p_{Tm'}|)}d\hat t \tag{8}$$

and

$$\left\langle\sum_i E_{Ti}{}^n\right\rangle\sigma_{\text{in}}=(-1)^n\left.\frac{d^{(n)}\phi(\tau)}{d\tau^n}\right|_{\tau=0}. \tag{9}$$

Note that for this specific problem, the usual exponentiation methods for moments do not apply. For instance, the x_i integrations defined by the domain Δ require some care (see Appendix C). The mean transverse energy is obtained for $n=1$. To perform the calculation we note that each term $|p_{Ti}|$in $\partial\phi/\partial\tau$ cancels the $1/t_i$ singularity of the matrix element dM_i [see Eq. (5)]. Indeed one can write this integral as

$$\int^{t_{i+1}}\frac{\alpha_s(t_i)}{4\pi}\frac{dt_i}{t_i}t_i{}^{1/2}\cdots\simeq\frac{\alpha_s(t_{i+1})}{4\pi}2t_{i+1}{}^{1/2}\cdots,$$

if we neglect the logarithmic dependence of the running coupling constant on t_i. As a consequence the term $t_{i+1}{}^{1/2}$ cancels the singularity of the next matrix element dM_{i+1}, which in turn cancels the next singularity with the appearance of a power of $\alpha_s(t)$ at each step. This calculation is performed in Appendix C by using the assumption of strong ordering of the t_i's inside each chain $m^2\ll\cdots\ll t_i\ll t_{m-1}$. We get an order-$\alpha_s$ correction to the main contribution. This order-α_s contribution is nothing but a sort of soft-gluon bremsstrahlung and corresponds to the sum of the transverse energy arising from the cascading partons inside each chain. Under the strong ordering assumption the main contribution to $\langle E_T\rangle$ comes from $|p_{Ti,j}|$, $i=m-1,m$, $j=m'-1,m'$. The exact calculation is difficult to handle since from energy-momentum conservation in the parton-parton collision $\mathbf{p}_{Tm}-\mathbf{p}_{T(m-1)}=\mathbf{p}_{Tm'}-\mathbf{p}_{T(m'-1)}$ and $|\mathbf{p}_{Tm}-\mathbf{p}_{T(m-1)}|^2\approx\hat t$. However, we show from kinematics that, as in deep-inelastic scattering, the maximum value of $(p_{T(m-1)})^2$ $[(p_{T(m'-1)})^2]$ is equal to $\hat t(1-Z_a)/4Z_a$ $[\hat t(1-Z_b)/4Z_b]$. For $Z_{a,b}$ of order 1 this maximum value is smaller than $\hat t$ whereas it is larger for small $Z_{a,b}$.

In the first case, the balance of transverse momentum is realized between the two outgoing partons a and b: $\mathbf{p}_{Tm}\sim\mathbf{p}_{Tm'}$, and $|\mathbf{p}_{Tm}|\sim\hat t^{1/2}$. E_T is thus nothing but $2\hat t^{1/2}$ and corresponds to the usual hard-scattering contribution to large-p_T events.

In the second case, for $Z_a\ll1$ say, the energy-momentum conservation is realized between the outgoing parton a and parton $m-1$, $|\mathbf{p}_{Tm}|\sim|\mathbf{p}_{T(m-1)}|\gg\hat t$. This is nothing but the configuration of a deep-inelastic process where it is $W^2=Q^2(1-Z)/4Z$ rather than Q^2 which sets the scale $\langle p_T{}^2\rangle$ (Ref. 8). The first configuration $[Z_{a,b}\approx O(1)]$ is very easy to handle for the main contribution. Indeed $E_T\simeq2\hat t^{1/2}$ at this order and we get

$$\delta_1=\langle E_T\rangle_1\sigma_{\text{in}}$$

$$=\pi\int_{Q_T{}^2}^{s}\frac{\alpha_s(\hat t)}{\hat t^2}(C_2\langle n\rangle_s+N_c\langle n\rangle_g)^2\hat t^{1/2}d\hat t\,.$$

The order-α_s correction arising from the contribution of the remaining p_{Ti}'s of the cascading partons is calculated in Appendix C.

The final result reads

$$\delta_1=\langle E_T\rangle_1\sigma_{\text{in}}=\pi\int_{Q_T{}^2}^{s}\frac{\alpha_s(\hat t)}{\hat t^2}(C_2\langle n\rangle_s+N_c\langle n\rangle_g)^2\hat t^{1/2}[1+X(\hat t)]\,, \tag{10}$$

where

$$X(\hat{t})=\frac{\alpha_s(\hat{t})}{\pi}\left[\frac{N_c\ln\frac{1}{x_{\min}}}{\xi(\hat{t})}\right]^{1/2}\frac{I_1\left[16N_c\xi\ln\frac{1}{x_{\min}}\right]^{1/2}}{I_0\left[16N_c\xi\ln\frac{1}{x_{\min}}\right]^{1/2}}$$

$$\simeq\frac{\alpha_s(\hat{t})}{\pi}\left[\frac{N_c\ln\frac{1}{x_{\min}}}{\xi(\hat{t})}\right]^{1/2}\quad\text{for large } 16N_c\xi\ln\frac{1}{x_{\min}}\ .$$

In the second configuration, both the first- and second-order soft-gluon bremsstrahlung contributions need some more care. The complete calculation is performed in Appendix C yielding the result

$$\delta_2=\langle E_T\rangle_2\sigma_{\rm in}=\pi\int_{Q_T{}^2}^{\tilde{s}}\frac{\alpha_s(\hat{t})}{\hat{t}^2}(C_2\langle n\rangle_s+N_c\langle n\rangle_g)Y(\hat{t})d\hat{t}\ , \tag{11}$$

where

$$Y(\hat{t})=4N_c\frac{\alpha_s(\hat{t})}{\pi}\left[\frac{\hat{t}}{x_{\min}}\right]^{1/2}\left[1+2N_c\frac{\alpha_s(\hat{t})}{\pi}\right](C_2\langle y^{1/2}\rangle_s+N_c\langle y^{1/2}\rangle_g)$$

and

$$\langle y^{1/2}\rangle_{s,g}=\int_{x_{\min}}^{1}y^{1/2}D_q^{s,g}(y,\hat{t})dy$$

is the $\frac{3}{2}$ truncated moment of the singlet and the gluon structure function, respectively.

Comparing formulas (11) and (10) we notice that δ_2 is of order $\alpha_s(\hat{t})$ compared to the large-p_T contribution δ_1. But the energy scale is $(\hat{t}/x_{\min})^{1/2}$ rather than $\hat{t}^{\,1/2}$ and is quite large for small $x_{\min}$. This explains why δ_2 gives an appreciable contribution to the transverse partonic energy. This result has already been obtained in deep-inelastic scattering[8] where the scale of $\langle p_T{}^2\rangle$ is $W^2\sim Q^2/4x$ rather than Q^2. Note that it is not the truncated multiplicity but the $\frac{3}{2}$ truncated moment which enters in the definition of $Y(\hat{t})$. Indeed the maximum value of $p_{T(m-1)}$ is $(\hat{t}x_{m-1}/4x_a)^{1/2}$ and not $(\hat{t}/4x_a)^{1/2}$. As shown in Appendix C, this extra $x_{m-1}{}^{1/2}$ arising from the integration over t_{m-1} changes the first moment into the $\frac{3}{2}$ moment of the gluon and singlet distributions.

A general comment is in order. Our calculation relies heavily upon the non-Abelian character of the theory, namely, the existence of the three-gluon vertex yielding the $1/Z$ singularity of the P_{gg} and P_{gq} kernels. We thus think that this large-E_T background observed in hadron-hadron collisions at high energy may be connected to the non-Abelian character of QCD.

A rough quantitative estimate can be made when comparing δ_1 and δ_2. Indeed in the model of Ref. 3 for the rise of the total cross sections, one considers $x_{\min}=m^2/\hat{t}$ $[(\tilde{s}\,\hat{t})^{1/2}]$ in domain I [domain II]. In this model, the minimum value of $x_{\min}$ is $(m^2/s)^{1/3}=7\times10^{-3}$ at $\sqrt{s}=540$ GeV and $\langle y^{1/2}\rangle_g/(x_{\min})^{1/2}\simeq2.4\langle n\rangle_g$. Thus if we consider only the main contribution for the gluon we get $(\delta_2/\delta_1)_g\simeq10\alpha_s(\hat{t})$. In this model,[3] $\langle\hat{t}\rangle$ is of order 3–5 which means $\alpha_s(\langle\hat{t}\rangle)$ of the order 0.25. Thus $\delta_2/\delta_1\approx4$. As far as the soft-gluon contribution is concerned, we get a correction

$$\frac{2N_c}{\pi}\alpha_s(\hat{t})\simeq2\alpha_s(\hat{t})\simeq0.5$$

for δ_2 and

$$\left[\frac{N_c\ln\frac{1}{x_{\min}}}{\xi(\hat{t})}\right]^{1/2}\frac{\alpha_s(\hat{t})}{\pi}$$

or $2.5\alpha_s(\hat{t})\simeq0.6$ for δ_1.

This rough quantitative estimate tells us that, at the partonic level, the direct hard-scattering contribution to E_T is four times smaller than the bremsstrahlung ones (the gluon gives the main contribution to E_T).

In Sec. IV a more quantitative estimate taking into account the hadronization is given. Note that at the partonic level the s dependence of E_T may be parametrized as $\tilde{s}^{\alpha}$. In our model we have found $\alpha\approx0.25$ which is intermediate between the $\alpha\simeq0$ expected theoretically from soft interaction and $\alpha\approx0.5$ characteristic of scale-invariant hard interaction.

IV. HADRONIZATION EFFECTS AND RESULTS

In order to estimate the hadronization we first have to discuss the nonperturbative distribution $\psi(Z)$ [see Eq. (1)]. In this equation the partonic contribution $\sigma^{\rm part}(\tilde{s})$ depends on Z_1 and Z_2 only through $\tilde{s}=Z_1Z_2s$. The energy dependence of $\langle E_T\rangle_{\rm part}$ is rougly $A\tilde{s}^{\alpha}$ with $\alpha\approx0.25$ for model of Ref. 3. The effect of the initial hadrons on the determination of $\langle E_T\rangle_{\rm had}$ is to give an overall factor

$$\frac{\sigma^{\rm had}(s)}{\sigma^{\rm part}(s)}=\left[\int_0^1Z^{\alpha}\psi(Z)dZ\right]^2\ .$$

Although the distribution $\psi(Z)$ is not well known, its first and second moments are determined to be 3 and 1, respectively. A phenomenological form

$$3\frac{\Gamma(a+b+2)}{\Gamma(a+1)\Gamma(b+1)}Z^a(1-Z)^b$$

with $b=2a+1$ and $-1/2\leq a\leq 0$ yields an overall factor of $\frac{9}{2}$ rather than 9 for an independent quark model. It is interesting to note that the order of magnitude is not far from the expectation of the additive quark model $\psi(Z)=3\delta(\frac{1}{3}-Z)$ which yields

$$9\sigma^{\text{part}}\left[\frac{s}{9}\right]\approx 5.4As^{\alpha}=5.4\sigma^{\text{part}}(s)\ .$$

Indeed the Z_1,Z_2 components of the distribution functions near 1 are depleted by energy sharing among the constituent quark.

The estimate of the hadronization effect of the first produced partons is less reliable. The broadening of parton jets modifies the value of $\langle E_T\rangle_{\text{had}}$. Here, we rely upon similar calculation done in e^+e^- annihilation. Indeed comparing a QCD calculation at the LLA with recent experimental results, it is possible[4] to compare $\langle E_T\rangle_{\text{part}}$ and $\langle E_T\rangle_{\text{had}}$ for the jets seen in e^+e^- annihilation at different energies. $\langle E_T\rangle_{\text{had}}$ can be described by the same perturbative QCD formula as $\langle E_T\rangle_{\text{part}}$ but with a rescaled larger Λ ($\Lambda_{\text{had}}\approx 600$ MeV rather than $\Lambda_{\text{QCD}}\approx 100$ MeV). We have tentatively applied the same recipe in our calculation of $\langle E_T\rangle_{\text{had}}$, namely, by replacing Λ_{QCD} by $\Lambda_{\text{had}}\approx 600$ MeV both in $Y(\hat{t})$ and $X(\hat{t})$. To get a rough quantitative estimate of the broadening of the jets, we note that $\langle\hat{t}\rangle\sim m^2(s/9m^2)^{1/3}$ (see Ref. 3) $\simeq 3-6$ GeV^2 at $\sqrt{s}=540$ GeV. Thus

$$\frac{\alpha_s(\Lambda_{\text{had}},\langle\hat{t}\rangle)}{\alpha_s(\Lambda_{\text{QCD}},\langle\hat{t}\rangle)}\simeq 2-2.5\ .$$

In this scheme, the final hadronization increase of the amount of transverse energy overcompensates the decreasing effect of the partonization of the initial hadrons.

A direct phenomenological comparison with experimental data at each energy is not reliable due to our lack of knowledge of the cut-off Q_T^2 and our nonperturbative hadronization uncertainty. However as in the model of Ref. 3 we calculate the rise of $\langle E_T\rangle\sigma_{\text{in}}$ between the ISR and the SPS energy which does not depend on Q_T^2. With the same set of parameters as in Ref. 3 we get

$$\Delta(E_T\sigma_{\text{in}})=E_T\sigma_{\text{in}}\,|_{\sqrt{s}=540\text{ GeV}}-E_T\sigma_{\text{in}}\,|_{\sqrt{s}=50\text{ GeV}}$$
$$\simeq 750\text{ GeV mb}\ .$$

to be compared to (850 ± 115) GeV mb quoted by the experimentalists.[5]

The correct order of magnitude of the effect is reproduced by our calculation. This seems to indicate that the mechanism we have imagined is not too far from the reality.

Note that other authors[9] have treated the same problem by using different methods such as, for instance, Monte Carlo calculations. They have obtained a fair agreement with the data. However, our QCD-inspired approach gives a consistent description of the rise of both the hadronic cross section and the hadronic mean transverse energy.

V. CONCLUSION AND OUTLOOK

We have shown that, in addition to the large-P_T jets corresponding to hard parton-parton interaction, quite a few hadron-hadron events at very high energy can be described by gluon bremsstrahlung related to the low-x singularities of perturbative QCD. These events, characterized by a large total transverse energy and a high multiplicity are produced for intermediate values of the interaction scale $\hat{t}$. In this domain, the smallness of the asymptotically free coupling constant is offset by the large multiplicity of partons at low x. In our specific case the dominant contribution to the total transverse energy is the second term of the perturbative expansion

$$\delta_2\sim\left\langle\alpha_s(\hat{t})\left[\frac{\hat{t}}{x_{\min}}\right]^{1/2}\right\rangle$$

rather than the first one $\delta_1\sim\langle t^{1/2}\rangle$.

The reasonable order of magnitude obtained for the average E_T is linked to the non-Abelian character of the underlying field theory. An essential role is indeed played in our mechanism by the low-x singularities of perturbative QCD for medium size scale. This specific regime has been investigated by numerous authors.[2] These studies have shown the relevance of perturbative QCD for x values larger than $x_{\min}(Q^2)$ which can be very small. [The higher the Q^2, the smaller the lower bound $x_{\min}(Q^2)$.]

The success of the semihard interactions to explain both the rise of the total $p\bar{p}$ cross section[3] and the mean total transverse energy in hadronic collisions at high energy is an incentive to get a better understanding of this lower bound below which perturbative QCD does not apply any more. On a phenomenological ground, we hope that our scheme can explain the so-called "minimum-bias" events observed recently at the SPS collider and provides a good tool to study the future hadronic experiments of the very-high-energy machines (Fermilab Tevatron Large Hadron Collider and Superconducting Super Collider). Because of the expected dominance of semihard interactions, the main prediction of our scheme could be the presence of many minijets or middle momentum jets among these "minimum-bias" events. More quantitative study of this expected phenomena is required.

APPENDIX A: LONGITUDINAL-MOMENTUM CUTOFFS IN PERTURBATIVE QCD

The calculations of E_T and other related observables rely upon the validity of perturbative QCD at small x and the knowledge of $x_{\min}(\hat{t})$. In a previous work[3] we have shown that the choice $x_{a,b}\geq m^2/\hat{t}$ allowed to describe the rise of the total cross section for $p\bar{p}$ at high energy. The purpose of this appendix is to argue this choice and to discuss the validity of perturbative QCD in the vicinity of the lower bound.

Known results[10] in the Regge limit of Yang-Mills theories yield naturally a perturbative cutoff x_c such that $\ln 1/x_c \sim \ln(\hat{t}/t_0)$. Indeed, one has to consider three different domains when either $\ln 1/x$ or $\ln \hat{t}/\Lambda^2$ $[\simeq 1/\alpha_s(\hat{t})]$ or both of them are large.

(i) The domain $\ln 1/x \ll \ln \hat{t}/\Lambda^2$ where the LLA is valid.

(ii) The Reggeization domain $\ln 1/x \gg \ln \hat{t}/\Lambda^2$.

(iii) The intermediate domain $\ln 1/x \sim \ln \hat{t}/\Lambda^2$.

Actually, it can be shown[10] that the results obtained for the LLA in domain (i) still apply in the intermediate domain. In particular, the nonplanar diagrams which contribute in this domain restore the strong ordering of virtualities which characterize domain (i).

In a previous work[3] we tried a form $x_c = m^2/\hat{t}$, and got a reasonable description of the rise of the total $p\bar{p}$ cross section between the ISR and SPS colliders for m identified as the intrinsic transverse momentum of the constituent quark $\simeq 330$ MeV. Note that for a timelike cascade this bound must be obeyed from kinematics.

It is now of paramount importance to check that perturbative QCD calculations, based upon the factorization property of one-gluon exchange and the Altarelli-Parisi evolution equations, are valid for x value bounded both by x_c and the kinematical conditions $x_a x_b \tilde{s} \gg \hat{t}$ which yields

$$x_{a,b} \ge x_c = \frac{m^2}{\hat{t}} \quad \text{in domain I} ,$$

$$x_{a,b} \ge \left[\frac{\hat{t}}{\tilde{s}} \right]^{1/2} \quad \text{in domain II} .$$

Note that the absolute minimum of x is $(m^2/\hat{t})^{1/3} = 7\times 10^{-3}$ at collider energy.

Recent theoretical investigations[2] have shown that there exists a simple criterion to test the validity of perturbative QCD at small x; namely, the packing fractions of partons W has to be much smaller than one:

$$W(x,\hat{t}) = x f(x,\hat{t}) = \frac{m_\pi^2}{\hat{t}} \ll 1 ,$$

where $f(x,\hat{t})$ is the structure function of the involved partons. For the gluon sector we get after integration $x_{\min} < x < 1$

$$\langle n \rangle_g = \int_{x_{\min}}^{1} f(x,\hat{t}) dx \ll \frac{\hat{t}}{m_\pi^2} \int_{x_{\min}}^{1} \frac{dx}{x} = \frac{\hat{t}}{m_\pi^2} \ln \frac{1}{x_{\min}} .$$

For large $\hat{t}$ this yields

$$16 N_c \xi \ln \frac{1}{x_{\min}} \ll \left[\ln \frac{\hat{t}}{m_\pi^2} \right]^2 ,$$

which is obviously satisfied if $\ln 1/x_{\min} \sim \ln \hat{t}/m^2$.

It is worthwhile to note that in the model of Ref. 3, $\langle n \rangle_g$ is always 100 times smaller than $(\hat{t}/m_\pi^2)\ln 1/x_{\min}$ which justifies the use of perturbative QCD at small x.

APPENDIX B: THE TRUNCATED MULTIPLICITIES (REF. 3)

For valence quarks we have approximated the truncated multiplicity by the full multiplicity which is finite: $n^v(\eta) = 1$. For gluon and sea quarks the truncated multiplicities are obtained from Eq. (4) and the normalization is fixed by means of the second moments. This yields

$$\langle n \rangle_g = \lambda_g \{ I_0[(16 N_c \xi \bar{\eta})^{1/2}] - 1 \} ,$$

$$\langle n \rangle_s = \lambda_s \left[\frac{I_1[(16 N_c \xi \bar{\eta})^{1/2}]}{(16 N_c \xi \bar{\eta})^{1/2}} - \frac{1}{2} \right] + 1 ,$$

where

$$\bar{\eta} = \begin{cases} \eta, \quad \eta \le \dfrac{\eta_s}{3}, & \text{domain I} , \\[2ex] \dfrac{\eta_s - \eta}{2}, \quad \eta > \dfrac{\eta_s}{3}, & \text{domain II} , \end{cases}$$

and

$$\eta = \ln \frac{\hat{t}}{m^2}, \quad \eta_s = \ln \frac{\tilde{s}}{m^2}, \quad \xi = \int_{\mu^2}^{\hat{t}} \frac{\alpha_s(t)}{4\pi} \frac{dt}{t} ,$$

$$\lambda_g = \frac{4C_2 \exp(-4N_c\xi)}{4C_2 + N_f} \, \frac{1 - \exp[-\frac{2}{3}(4C_2 + N_f)\xi]}{1 - \exp(-4N_c\xi)} ,$$

$$\lambda_s = \frac{8 N_c \xi \left[\dfrac{N_f}{4C_2 + N_f} - \exp(-\frac{8}{3}C_2\xi) + \dfrac{4C_2}{4C_2 + N_f} \exp[-\frac{2}{3}\xi(4C_2 + N_f)] \right]}{\exp(4N_c\xi) - 1 - 4N_c\xi} ,$$

I_0 and I_1 are modified Bessel functions.

APPENDIX C: QCD, RESUMMATION FORMULAS FOR TRANSVERSE-ENERGY DISTRIBUTIONS

From formulas (8) and (9), $\langle E_T \rangle \sigma_{\rm in}$ reads

$$\langle E_T \rangle \sigma_{\rm in} = 2\times\frac{\pi}{2}\int_{Q_T^2}^{\bar s}\frac{\alpha_s^2(\hat t)}{\hat t^2}(C_2\langle n\rangle_s + N_c\langle n\rangle_g)\times\left.\frac{\partial\psi(\tau,\hat t)}{\partial\tau}\right|_{\tau=0},$$

$$\psi(\tau,\hat t) = -\sum_m\int_{x_{\min}} dx(C_2,N_c)\prod_{i=1}^{m-1} dM_i\, e^{-\tau|p_{T_i}|}\begin{bmatrix}1\\0\end{bmatrix}e^{-\tau p_{T_m}} = C_2\psi_s(\tau,\hat t)+N_c\psi_g(\tau,\hat t),$$

where $\psi_{s,g}(\tau,\hat t)$ is one ladder contribution for the singlet and gluon sector, respectively. The factor 2 in front takes into account the two ladder contributions.

To proceed further, we use the following assumptions. (i) The t_i's (rather than the s_i's$=t_i/x_i$) are strongly ordered, namely, $m^2 \ll t_1 \ll \cdots \ll t_{m-1}$. (ii) The x_i are also strongly ordered which means that we keep only the small Z_i contribution to $\psi(\tau,\hat t)$. With this approximation the gluon contribution is dominant and $P_{gg}(Z_i) \simeq 4N_c/Z_i$, $Z_i = x_{i+1}/x_i$.

With these assumptions it is possible to get the contribution to $\langle E_T\rangle$ the cascading partons by performing the integrals over dt_i and dx_i, $i=1,\ldots,m-2$. Indeed, for the gluon contribution

$$dM_i\, e^{-\tau|p_{T_i}|} \simeq 4N_c\frac{dx_i}{x_i}\frac{\alpha_s(t_i)}{4\pi}\frac{dt_i}{t_i}e^{-\tau t_i^{1/2}}$$
$$\equiv f(x_i)g(t_i)dx_i dt_i\,.$$

Note that $p_{Ti}=t_i^{1/2}(1-Z_i)^{1/2}\simeq t_i^{1/2}$ at small Z_i. The calculation of the nested integrals in x_i and t_i is simple since

$$\int_y^1 h(y_n)dy_n\cdots\int_{y_1}^1 h(y_0)dy_0 = \frac{\left[\int_y^1 h(y')dy'\right]^{n+1}}{(n+1)!}\,.$$

The result for $\psi_g(\tau,\hat t)$ reads, up to a normalization,

$$\psi_g(\tau,\hat t)\simeq\int_{x_{\min}}4N_c\frac{dx_a}{x_a}\int_{x_a}4N_c\frac{dx_{m-1}}{x_{m-1}}\int^{(p_{T_{m-1}})^{\max}} e^{-\tau(p_{T_m}+p_{T_{m-1}})}$$
$$\times\frac{\alpha_s(t_{m-1})}{4\pi}\frac{dt_{m-1}}{t_{m-1}}\sum_m\frac{\left[4N_c\ln\frac{1}{x_{m-1}}\right]^{m-2}}{(m-2)!}\frac{[J(\tau,t_{m-1})]^{m-1}}{(m-1)!},$$

where

$$J(\tau,t)=\int^t\frac{\alpha_s(t')}{4\pi}\frac{dt'}{t'}e^{-\tau t'^{1/2}},\quad J(0,t)=\xi(t),\quad -\left.\frac{\partial J(\tau,t)}{\partial\tau}\right|_{\tau=0}=\int^t\frac{\alpha_s(t')}{4\pi}\frac{dt'}{(t')^{1/2}}\sim 2\frac{\alpha_s(t)}{4\pi}t^{1/2}$$

if we neglect in the last integral the small logarithmic dependence of $\alpha_s(t')$.

Note that without assumption *i*, $dM_i\, e^{-\tau(p_{Ti})}$ would not have been factorizable and the summation impossible.

In the case $Z_a\simeq O(1)[(p_{T(m-1)})^2]_{\max}=p_{Tm}{}^2\simeq\hat t$ and we can easily perform the last three integrations:

$$\psi_g(\tau,\hat t)=e^{-\tau\hat t^{1/2}}\sum_m\frac{\left[4N_c\ln\frac{1}{x_{\min}}\right]^m}{m!}\frac{[J(\tau,\hat t)]^m}{m!}$$

and

$$-\left.\frac{\partial\psi_g(\tau,\hat t)}{\partial\tau}\right|_{\tau=0}=\hat t^{1/2}\sum_m\frac{\left[4N_c\ln\frac{1}{x_{\min}}\xi(\hat t)\right]^m}{m!m!}+2\frac{\alpha_s(\hat t)}{4\pi}\hat t^{1/2}\sum_m\frac{\left[4N_c\ln\frac{1}{x_{\min}}\right]^m}{m!}\frac{[\xi(\hat t)]^{m-1}}{(m-1)!}\,.$$

Remembering that

$$\sum_m \frac{\left[4N_c \ln \frac{1}{x_{\min}} \xi(\hat{t})\right]^m}{m!m!} = I_0 \left[16N_c \xi \ln \frac{1}{x}\right]^{1/2} \simeq \int_x^1 D_q^g(x',\hat{t})dx' ,$$

$$4N_c \sum_m \frac{\left[4N_c \ln \frac{1}{x}\right]^m}{m!} \frac{\xi^{m+1}}{(m+1)!} = 4N_c \xi \frac{I_1 \left[16N_c \xi \ln \frac{1}{x}\right]^{1/2}}{\left[4N_c \xi \ln \frac{1}{x}\right]^{1/2}} \simeq x D_q^g(x,\hat{t}) ,$$

we get at once

$$\sum_m \frac{\left[4N_c \xi \ln \frac{1}{x_{\min}}\right]^m}{m!m!} \simeq \langle n \rangle_g$$

which is the truncated multiplicity for the gluon and

$$\sum_m \frac{\left[4N_c \ln \frac{1}{x_{\min}}\right]^m}{m!} \frac{[\xi(\hat{t})]^{m-1}}{(m-1)!} = \left[\frac{4N_c \ln \frac{1}{x_{\min}}}{\xi}\right]^{1/2} I_1 \left[16N_c \xi \ln \frac{1}{x_{\min}}\right]^{1/2} .$$

The total contribution reads

$$-\left.\frac{\partial \psi(\tau,\hat{t})}{\partial \tau}\right|_{\tau=0} = \hat{t}^{1/2} \langle n \rangle_g \left[1 + 2\frac{\alpha_s(\hat{t})}{4\pi} \left[\frac{4N_c \ln \frac{1}{x_{\min}}}{\xi(\hat{t})}\right]^{1/2} \frac{I_1 \left[16N_c \xi \ln \frac{1}{x_{\min}}\right]^{1/2}}{I_0 \left[16N_c \xi \ln \frac{1}{x_{\min}}\right]^{1/2}} \right] .$$

The second case (ii) is a little bit difficult to handle.

Indeed in this configuration ($Z_a \sim 0$), the maximum value $(p_{T_{(m-1)}}{}^2)_{\max} \simeq \hat{t}/4x_{m-1}/x_a$ does depend on x_a and x_{m-1} and $p_{Tm} \approx p_{T(m-1)}$ due to the balance of momentum inside the chain

$$\psi_g(\tau,\hat{t}) = \int_{x_{\min}} 4N_c \frac{dx_a}{x_a} \int_{x_a}^1 4N_c \frac{dx_{m-1}}{x_{m-1}} \int^{\hat{t}x_{m-1}/4x_a} \frac{\alpha_s(t_{m-1})}{4\pi} \frac{dt_{m-1}}{t_{m-1}} e^{-2\tau t_{m-1}{}^{1/2}}$$

$$\times \sum_m \frac{\left[4N_c \ln \frac{1}{x_{m-1}}\right]^{m-2}}{(m-2)!} \frac{[J(\tau,t_{m-1})]^{m-1}}{(m-1)!} ,$$

$$-\left.\frac{\partial \psi(\tau,\hat{t})}{\partial \tau}\right|_{\tau=0} \simeq \int_{x_{\min}}^1 4N_c \frac{dx_a}{x_a} \int_{x_a}^1 4N_c \frac{dx_{m-1}}{x_{m-1}} \int^{\hat{t}x_{m-1}/4x_a} \frac{\alpha_s(t_{m-1})}{4\pi} \frac{dt_{m-1}}{t_{m-1}} 2t_{m-1}{}^{1/2}$$

$$\times \left[\frac{1}{4N_c} \sum_m \frac{[4N_c \xi(t_{m-1})]^{m-1}}{(m-1)!} \frac{\left[\ln \frac{1}{x_{\min}}\right]^{m-2}}{(m-2)!} + \frac{\alpha_s(t_{m-1})}{4\pi} \sum_m \frac{[4N_c \xi(t_{m-1})]^{m-2}}{(m-2)!} \frac{\left[\ln \frac{1}{x_{\min}}\right]^{m-2}}{(m-2)!} \right] .$$

The integral on t_{m-1} can be readily expressed if here again we neglect the t_{m-1} dependence of $D_q^g(x,t_{m-1})$ and $\alpha_s(t_{m-1})$:

$$-\left.\frac{\partial\psi(\hat{t},\tau)}{\partial\tau}\right|_{\tau=0}\simeq 2\times\frac{4N_c\alpha_s(\hat{t})}{4\pi}\hat{t}^{1/2}\left[\int_{x_{\min}}\frac{dx_a}{x_a}\int_{x_a}\left[\frac{x_{m-1}}{x_a}\right]^{1/2}D_q^g(x_{m-1},\hat{t})dx_{m-1}\right.$$
$$\left.+\frac{4N_c\alpha_s(\hat{t})}{4\pi}\int_{x_{\min}}\frac{dx_a}{x_a}\int_{x_a}\left[\frac{x_{m-1}}{x_a}\right]^{1/2}\frac{dx_{m-1}}{x_{m-1}}\int_{x_{m-1}}D_q^g(x_{m-2},\hat{t})dx_{m-2}\right].$$

For the first contribution, we change the order of integration:

$$\int_{x_{\min}}\frac{dx_a}{(x_a)^{3/2}}\simeq\frac{2}{(x_{\min})^{1/2}}$$

and we get

$$\int_{x_{\min}}\frac{dx_a}{x_a}\int_{x_a}\left[\frac{x_{m-1}}{x_a}\right]^{1/2}D_q^g(x_{m-1},\hat{t})dx_{m-1}\simeq\frac{2}{(x_{\min})^{1/2}}\int_{x_{\min}}^{1}x_{m-1}{}^{1/2}D_q^g(x_{m-1},\hat{t})dx_{m-1}=\frac{2}{(x_{\min})^{1/2}}\langle y^{1/2}\rangle_g\ ,$$

where $\langle y^{1/2}\rangle_g$ is the $\frac{3}{2}$ truncated moment of the gluon structure function. By the same kind of trick, we get for the second integral

$$\frac{4}{(x_{\min})^{1/2}}\langle y^{1/2}\rangle_g\ .$$

The final result reads

$$-\left.\frac{\partial\psi(\hat{t},\tau)}{\partial\tau}\right|_{\tau=0}\simeq 4\frac{4N_c\alpha_s(\hat{t})}{4\pi}\left[\frac{\hat{t}}{x_{\min}}\right]^{1/2}N_c\langle y^{1/2}\rangle_g\left[1+2\frac{4N_c\alpha_s(\hat{t})}{4\pi}\right].$$

Following exactly the same method for the singlet sector we get the answer for the mean transverse energy $\langle E_T\rangle_2$ arising from the small-Z_a domain:

$$\langle E_T\rangle_2\sigma_{\text{in}}=\pi\int_{Q_T{}^2}^{\bar{s}}\frac{\alpha_s{}^2(\hat{t})}{\hat{t}}(C_2\langle n\rangle_s+N_c\langle n_g\rangle)Y(\hat{t})\ ,$$

where

$$Y(\hat{t})\simeq 4\frac{4N_c\alpha_s(\hat{t})}{4\pi}\left[\frac{\hat{t}}{x_{\min}}\right]^{1/2}\left[1+\frac{8N_c\alpha_s(\hat{t})}{4\pi}\right](C_2\langle y^{1/2}\rangle_s+N_c\langle y^{1/2}\rangle_g)\ ,$$

where $\langle y^{1/2}\rangle_s$ is the $\frac{3}{2}$ truncated moment of the singlet structure function.

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