Transverse-polarization effects in e^+e^- collisions: The role of chiral symmetry

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Electrons and positrons in storage rings have natural polarization which is perpendicular to the beam direction. Effects of the transverse polarization to azimuthally integrated cross sections are studied. In the standard model, the polarization does not affect the cross section up to a correction proportional to the electron mass, which is negligible at high energies. This fact is closely related to the chiral symmetry of the standard model. It is not necessarily true if there are new interactions beyond the standard model. Polarization effects in composite and supersymmetric models are studied in detail. New particles such as scalar bosons and excited electrons in composite models can give nonzero polarization effects. In supersymmetric theories, the transverse polarization is useful to probe the mixing of the scalar-electron states. The nonvanishing effects are connected to the breaking of the chiral symmetry by the new interactions.

I. INTRODUCTION

Electron-positron collision experiments[1-3] have been one of the most important and fruitful fields in high-energy physics over the past ten years. The center-of-mass energy has increased from 3 GeV at Frascati's ADONE to 47 GeV at DESY's PETRA. Throughout these energies, the mass of the electron $m_e = 0.5\times10^{-3}$ GeV is practically negligible. It is an excellent approximation to treat electrons as massless particles. In this limit positive-helicity electrons (and negative-helicity positrons) completely decouple from negative-helicity electrons (and positive-helicity positrons). They behave as if they are unrelated two Weyl fermions.

It is well known that the guiding magnetic field of storage rings can produce nearly complete transverse polarization of the beams through its coupling to the magnetic moment of electrons (by the emission of spin-flip synchrotron radiation).[4-6] The maximum value of the possible polarization amounts to be $P_0 = 8/5\sqrt{3} = 92.4\%$.

Beam polarization has been observed in many storage rings.[7] It has provided a very accurate calibration of the beam energy by the resonance depolarization method, giving precise determination of particle masses: ϕ at Novosibirsk's VEPP-2M (Ref. 8), ψ and ψ' at VEPP-4 (Ref. 9), Υ, Υ', and Υ'' at VEPP-4 (Ref. 10), the Cornell Electron Storage Ring (CESR) (Ref. 11), and DESY's DORIS (Ref. 12). The beam energy was measured at PETRA (Ref. 13) with a fractional error of $\sim 10^{-5}$. At SLAC's SPEAR, azimuthal asymmetry due to the polarization[14] has been observed in the reactions $e^+e^- \to e^+e^-$ and $\mu^+\mu^-$ (Ref. 15), inclusive hadron production,[16] and jet-axis distribution.[17] Similar effects at PETRA were also reported.[18]

A transversely polarized electron state is a linear combination of two helicity states. Transverse polarization is thus a manifestation of the tiny electron mass which causes a transition between left-handed and right-handed electrons. This property of transverse polarization is quite unlike longitudinal polarization, which selects one of the two helicity states.

Effects of transverse polarization have been calculated for a great variety of processes ranging over pure QED (Refs. 19 and 20) and neutral-current[21-25] processes, exclusive[26] and inclusive[27] hadron production, jet cross section,[28-30] W pair production,[31] Higgs-boson production,[32] and so on.[33-35]

For instance, the cross section for $e^+e^- \to \mu^+\mu^-$ in the presence of transverse polarization to the lowest order in QED is[20]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s}(1+\cos^2\theta - P^2\sin^2\theta\cos2\phi) , \tag{1.1}$$

where θ is the polar angle of μ^- with respect to the e^- direction, ϕ is the azimuthal angle[36] of μ^- relative to the polarization direction, and P is the magnitude of the polarization (we take the polarization of electrons and positrons equal and opposite). The extra term proportional to P has a $\cos2\phi$ dependence. After integrating over the azimuthal angle, there remains no trace of polarization.

A remarkable fact is that this disappearance of the transverse-polarization effect—the special form of the transverse-polarization dependence—is a property which is common to (almost[37]) all calculated processes in e^+e^- collisions. Polarization-dependent parts of the cross section always have a factor $\cos2\phi$ (or $\sin2\phi$ in few cases), where ϕ is the overall azimuthal angle. There remains no effect of the polarization in the ϕ-integrated cross section. This fact makes it possible to use the calculation of unpolarized cross sections in comparing experiments with the theory even if the beams are polarized.

We should stress that this is not a general consequence of Lorentz invariance since the polarization vectors of electrons and positrons are correlated with each other. On the contrary, if the e^+ and e^- spin vector directions are averaged independently, we get the unpolarized cross section as we will see in Sec. II.

Besides numerous calculations of specific processes, we can find a few general studies of the transverse-polarization effect. Baǐer and Khoze[38] considered the total cross section of electromagnetic processes and conclud-

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ed that "the total cross section of an arbitrary process coincides, accurate to the terms of order m_e^2/E^2, with the cross section for unpolarized particles" if the final state contains one or more particles with mass $\gg m_e$. Kheifets and Khoze[39] wrote down the general form of the angular distribution for two-particle production via s-channel photon exchange and showed that the transverse-polarization-dependent term has a $\cos 2\phi$ dependence. They related this fact to the helicity conservation at high energies. Avram and Schiller[40] studied the exclusive process $e^+e^- \to$multihadron for arbitrary final state within the one-photon-exchange approximation and noted that any ϕ-averaged distribution does not depend at all on the transverse polarization of the beams. Tsai[41] also considered the effects of transversely polarized beams on inclusive hadron production and some exclusive reactions including the Bhabha scattering and $e^+e^- \to \gamma\gamma$. He gave an explanation of the appearance of the $\cos 2\phi$ factor in the polarization-dependent part by appealing to time-reversal and parity invariance and the neglect of radiative corrections.

These existing explanations are, however, limited to parity-conserving processes and only applicable to reactions involving electromagnetic and strong interactions. We are unable to tell if it continues to be true where the weak interactions play an important role. More speculative reactions with new particles predicted by supersymmetry or composite models are also out of these considerations. Moreover, it is not clear whether the conclusion is unaffected by the inclusion of radiative corrections.

In a recent paper[42] (hereafter called I) I proved that transverse polarization has no effect on the azimuthally integrated cross section for any process in e^+e^- collisions to all orders in the standard model if the electron mass can be neglected. (We refer to this statement as the null theorem in the following.) The proof presented in I was based on an analysis of the γ-matrix structure of the amplitude. Although the proof was rather simple, the physics behind the manipulation was not very clear.

In this paper I will give a new proof of the null theorem in terms of helicity amplitudes. It turns out that the theorem is the consequence of the approximate chiral symmetry of the standard model. In the limit of massless electrons, the global "electronic" chiral symmetry which rotates the left- and right-handed electrons differently guarantees that the theorem holds in all orders of perturbation theory. Violation of this theorem is solely due to the electron mass and the tiny Yukawa coupling of the electron, and, hence, is negligible at high energies.

So far almost all experimental facts can be explained by the standard model, a gauge theory with the SU(3) $\otimes$SU(2)$\otimes$(U(1) gauge group. However, there are theoretical expectations[43] that the standard model may not be the whole story—the weak mass scale $G_F^{-1/2} \sim 300$ GeV should be associated with some "new physics."

Two extensively studied directions are composite models and supersymmetry. In these models the null theorem does not hold in general. The transverse polarization can be used for probing the chiral structure of new interactions.

This paper is organized as follows. In Sec. II a general discussion of kinematics in electron-positron collisions with transverse polarization is presented and the azimuthal-angle dependence of the cross section is extracted. It is convenient to work with the "reduced" cross section which is the square of the amplitude. The relation of this quantity and the cross section is given in Appendix A. The cross-section formula for beams of arbitrary polarization is provided in Appendix B.

In Sec. III we define the electronic chiral symmetry in the standard model and present the proof of the null theorem in the standard model based on symmetry considerations. As explicit examples, the amplitudes and the cross sections for the processes $e^+e^- \to \mu^+\mu^-$, $e^+e^- \to e^+e^-$, and $e^+e^- \to \gamma\gamma$ are given in Appendixes C–E. The electron mass is retained so that we can see the violation of the theorem due to the explicit breaking of the symmetry. The process $e^+e^- \to \gamma\gamma$ is particularly interesting since the electron mass is essential to keep the forward and total cross sections finite.

Sections IV and V are devoted to the study of new physics models: compositeness and supersymmetry. Whether the theorem holds or not depends on the chiral structure of the model. If the chiral symmetry can be extended to the new particles and interactions, there remains no effect of transverse polarization. The formation of a spinless resonance gives a nice illustration how the theorem is violated. Excited electron interactions are also discussed in Sec. IV. In Sec. V the extension of chiral symmetry for supersymmetric models is discussed and the production of scalar-electron pairs and photino pairs is studied. The effect of transverse polarization depends on the structure of the scalar-electron mass matrix and the gaugino masses. It is shown that the mixing between scalar-electron states can be probed using the beam polarization. The detailed result for these processes is given in Appendixes F and G. Section VI includes discussions and a summary.

It is to be noted that the phase convention of the helicity amplitudes in this paper follows that of Jacob and Wick.[44]

II. GENERAL FRAMEWORK

Although the results in this section are rather well known, it is convenient to review them for use in later sections and to establish our notation.

A. Electron polarization states in quantum mechanics

The space of spin-$\frac{1}{2}$ particle polarization states is a two-dimensional complex vector space. Disregarding the irrelevant overall normalization and phase, the polarization states can be characterized by two real parameters. The direction of the expectation value of the spin vector $\langle \mathbf{S} \rangle$ in the rest frame completely determines a pure state. The magnitude of $\langle \mathbf{S} \rangle$ is fixed for spin-$\frac{1}{2}$ particles: $|\langle \mathbf{S} \rangle|^2 = \frac{1}{4}$. Any mixed state (or rather, mixed ensemble) can be described by allowing the magnitude of $\langle \mathbf{S} \rangle$ to vary. The polarization vector is

$$\mathbf{P} = 2\langle \mathbf{S} \rangle .$$

A state with $|\mathbf{P}|<1$ may be regarded as a mixture of two pure states with $\langle\mathbf{S}\rangle$ parallel and antiparallel to $\mathbf{P}$. Note that these properties are specific to spin-$\frac{1}{2}$ particles. For particles with spin≥ 1, expectation values of higher moments (quadrupole,. . .) are needed for complete labeling of the state. The magnitude of $\langle\mathbf{S}\rangle$ varies even for pure states.

B. Transversely polarized e^+e^- states

To describe the initial spin states in e^+e^- collisions, we choose the helicity states as the basis, which is natural and convenient at high energies. We work in the e^+e^- center-of-mass frame, and take the electron momentum direction as the z axis. As our discussion will be in terms of amplitudes, it is convenient to consider pure states only. The case of incomplete polarization can be treated simply by taking a weighted average after squaring the amplitudes.

A transversely polarized electron state is a superposition of the two helicity states with equal weight. In general, the state vector of an electron with the polarization vector in the direction (θ,χ) $(\theta,\chi$ being the polar and azimuthal angle in the rest frame, respectively) can be written

$$|\theta,\chi\rangle=\cos\frac{\theta}{2}\,|h=+\rangle+\sin\frac{\theta}{2}e^{i\chi}\,|h=-\rangle\ , \qquad (2.1)$$

where h denotes the electron helicity. Restricting ourselves to the purely transverse-polarization case, $\theta=\pi/2$, we have

$$|\chi\rangle=\frac{1}{\sqrt{2}}(\,|h=+\rangle+e^{i\chi}\,|h=-\rangle)\ . \qquad (2.2)$$

Similarly for the positron

$$|\bar{\chi}\rangle=\frac{1}{\sqrt{2}}(e^{i\bar{\chi}}\,|\bar{h}=+\rangle+|\bar{h}=-\rangle)\ , \qquad (2.3)$$

where $\bar{\chi}$ is the azimuthal angle of the positron polarization and $\bar{h}$ the helicity of the positron. (Note that the positron direction is in the negative z axis.)

The state vector of the electron-positron system is the direct product

$$|\chi,\bar{\chi}\rangle=\tfrac{1}{2}(\,|+-\rangle+e^{i\bar{\chi}}\,|++\rangle+e^{i\chi}\,|--\rangle +e^{i(\chi+\bar{\chi})}\,|-+\rangle)\ , \qquad (2.4)$$

where $|+-\rangle=|h=+\rangle\otimes|\bar{h}=-\rangle$, etc. These states in (2.4) span a two-dimensional subspace of the six-dimensional spin space of the e^+e^- system.

For the natural polarization realized in storage rings, the polarization vectors of electrons and positrons are in the opposite direction because of the opposite sign of the magnetic moments. Setting $\bar{\chi}=\chi+\pi$ in Eq. (2.4), we get

$$|\chi,\chi+\pi\rangle=\tfrac{1}{2}(\,|+-\rangle-e^{i\chi}\,|++\rangle+e^{i\chi}\,|--\rangle -e^{2i\chi}\,|-+\rangle)\ . \qquad (2.5)$$

C. Azimuthal-angle dependence of the cross section

The scattering amplitude from the transversely polarized state in (2.4) to a final state f is

$$\langle f\,|\,T\,|\chi,\bar{\chi}\rangle=\tfrac{1}{2}(T_{+-}+e^{i\bar{\chi}}T_{++}+e^{i\chi}T_{--} +e^{i(\chi+\bar{\chi})}T_{-+})\ , \qquad (2.6)$$

where we used the notation

$$T_{+-}=\langle f\,|\,T\,|+-\rangle\ ,$$

etc. The cross section is proportional to

$$\begin{aligned}\Sigma(\chi,\bar{\chi})&\equiv|\langle f\,|\,T\,|\chi,\bar{\chi}\rangle|^2\\ &=\tfrac{1}{4}(\,|T_{+-}|^2+|T_{++}|^2+|T_{--}|^2+|T_{-+}|^2)\\ &\quad+\tfrac{1}{2}\mathrm{Re}[e^{i(\chi+\bar{\chi})}T^*_{+-}T_{-+}+e^{i\chi}(T^*_{+-}T_{--}+T^*_{++}T_{-+})+e^{i\bar{\chi}}(T^*_{+-}T_{++}+T^*_{--}T_{-+})+e^{i(\chi-\bar{\chi})}T^*_{++}T_{--}]\ .\end{aligned} \qquad (2.7)$$

The relation between Σ and the cross section is given in Appendix A.

It is possible to define the overall azimuthal angle ϕ of the final state f such that any angle between momenta of final or initial particles is independent of ϕ. When the final state is a two-particle state, we can choose ϕ as the azimuthal angle of one of the final particles.

We are interested in the ϕ dependence of the cross section. Instead of discussing the ϕ dependence directly, it is simpler to study the $\chi,\bar{\chi}$ dependence since the latter can be made explicit using (2.6). In fact, we can show that

$$|\langle\phi,\ldots|\,T\,|\chi,\bar{\chi}\rangle|^2=|\langle\phi=0,\ldots|\,T\,|\chi-\phi,\bar{\chi}-\phi\rangle|^2\ , \qquad (2.8)$$

from the rotational invariance with respect to the beam direction. In other words, the rotation of the final state by ϕ is equivalent to the rotation of the initial state by $-\phi$.

If we average (2.7) over χ and $\bar{\chi}$ independently, we get the unpolarized cross section

$$\int_0^{2\pi}\frac{d\chi}{2\pi}\int_0^{2\pi}\frac{d\bar{\chi}}{2\pi}\,|\langle f\,|\,T\,|\chi,\bar{\chi}\rangle|^2=\Sigma_{\rm unpol}\ , \qquad (2.9)$$

where

$$\Sigma_{\rm unpol}\equiv\tfrac{1}{4}(\,|T_{+-}|^2+|T_{++}|^2+|T_{--}|^2+|T_{-+}|^2)\ . \qquad (2.10)$$

Alternatively, this can be accomplished by averaging over four initial states $|\chi,\bar{\chi}\rangle$, $|-\chi,\bar{\chi}\rangle$, $|\chi,-\bar{\chi}\rangle$, and $|-\chi,-\bar{\chi}\rangle$ for fixed χ and $\bar{\chi}$

$$\frac{1}{4}\sum_{\pm\chi}\sum_{\pm\bar{\chi}}|\langle f\,|\,T\,|\,\pm\chi,\pm\bar{\chi}\rangle|^2=\Sigma_{\rm unpol}\,. \tag{2.11}$$

Note that the unpolarized cross section is independent of the final-state azimuthal angle ϕ thanks to (2.8).

For the naturally polarized initial state (2.5), the cross section is proportional to

$$\begin{aligned}\Sigma(\chi,\chi+\pi)=\Sigma_{\rm unpol}&-\tfrac{1}{2}{\rm Re}T^*_{++}T_{--}-\tfrac{1}{2}{\rm Re}e^{2i\chi}T^*_{+-}T_{-+}\\&-\tfrac{1}{2}{\rm Re}e^{i\chi}[T^*_{+-}(T_{++}-T_{--})\\&\qquad-(T^*_{++}-T^*_{--})T_{-+}]\,.\end{aligned} \tag{2.12}$$

Interference terms of various helicity amplitudes in (2.12) are the effects of transverse polarization which mixes the initial helicity states.

Now we fix the direction of the electron polarization to positive x axis, i.e., $\chi=0$. The ϕ dependence can be explicitly extracted from (2.8) and (2.12):

$$\begin{aligned}\Sigma_{\rm pol}&=\Sigma(-\phi,-\phi+\pi)\\&=\Sigma_{\rm unpol}-\tfrac{1}{2}{\rm Re}T^*_{++}T_{--}-\tfrac{1}{2}{\rm Re}e^{-2i\phi}T^*_{+-}T_{-+}\\&\quad-\tfrac{1}{2}{\rm Re}e^{-i\phi}[T^*_{+-}(T_{++}-T_{--})\\&\qquad-(T^*_{++}-T^*_{--})T_{-+}]\,.\end{aligned} \tag{2.13}$$

Here we should keep in mind that the amplitudes $T_{h\bar{h}}$ are to be evaluated at $\phi=0$.

When we average over ϕ we obtain

$$\bar{\Sigma}_{\rm pol}\equiv\int_0^{2\pi}\frac{d\phi}{2\pi}\Sigma_{\rm pol}=\Sigma_{\rm unpol}-\tfrac{1}{2}{\rm Re}T^*_{++}T_{--}\,, \tag{2.14}$$

which differs from the unpolarized cross section by the last term. This term reflects the correlation between the electron and positron spins [compare with (2.9)] and is the only effect of transverse polarization to the azimuthally integrated cross section.

Equations (2.10), (2.13), and (2.14) are our basic formulas which will be frequently used in the following sections.

When the polarization is not complete (as is always the case for storage rings), the cross section becomes

$$\begin{aligned}\Sigma=\Sigma_{\rm unpol}&-\tfrac{1}{2}P\bar{P}\,{\rm Re}T^*_{++}T_{--}-\tfrac{1}{2}P\bar{P}\,{\rm Re}e^{-2i\phi}T^*_{+-}T_{-+}\\&+\tfrac{1}{2}P\,{\rm Re}e^{-i\phi}(T^*_{+-}T_{--}+T^*_{--}T_{-+})\\&-\tfrac{1}{2}\bar{P}\,{\rm Re}e^{-i\phi}(T^*_{+-}T_{++}+T^*_{--}T_{-+})\,,\end{aligned} \tag{2.15}$$

where P and $\bar{P}$ are the magnitude of the electron and positron polarization, respectively. (Their directions are set to the natural directions in storage rings. The electron spin direction is taken as the x axis, i.e., $\phi=0$.) If $P=\bar{P}$, we have

$$\begin{aligned}\Sigma=\Sigma_{\rm unpol}&-\tfrac{1}{2}P^2{\rm Re}T^*_{++}T_{--}-\tfrac{1}{2}P^2{\rm Re}e^{-2i\phi}T^*_{+-}T_{-+}\\&-\tfrac{1}{2}P\,{\rm Re}e^{-i\phi}[T^*_{+-}(T_{++}-T_{--})\\&\qquad-(T^*_{++}-T^*_{--})T_{-+}]\,.\end{aligned} \tag{2.16}$$

III. THE STANDARD MODEL

A. Electronic chiral symmetry

The standard model has an (global) approximate electronic chiral symmetry:

$$l_L\to e^{-i\alpha}l_L\,,\quad e_R\to e^{i\alpha}e_R \tag{3.1}$$

(all other fields remain intact). Here

$$l_L=\begin{bmatrix}\nu_e\\ e\end{bmatrix}_L$$

and α is a real parameter. The kinetic terms and gauge interactions respect this symmetry. It is explicitly broken by the Yukawa interaction

$$\mathcal{L}=-h_e\bar{e}_R\varphi^\dagger l_L+{\rm H.c}\,. \tag{3.2}$$

and consequently, by the electron mass.[45] (Here φ stands for the Higgs field.) The magnitude of this coupling is extremely small,

$$h_e=2.83\times10^{-6}\,,$$

and the chiral symmetry is virtually exact at high energies whenever the electron mass can be neglected.

This symmetry allows us to assign an (approximately) conserved quantum number—electronic chiral charge X_e to all particles. We may choose

$$X_e=\begin{cases}+1 & \text{for } e_R^-,e_R^+,\bar{\nu}_{eR}\,,\\ -1 & \text{for } e_L^-,\nu_{eL},e_L^+\,,\\ 0 & \text{otherwise}\,.\end{cases} \tag{3.3}$$

In the symmetric limit, we can identify this chiral charge with the helicity quantum number.

B. The null theorem

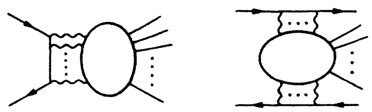
Before entering into the discussion of the transverse-polarization effects in the standard model, it is convenient to introduce a classification of amplitudes (or rather, diagrams).

Take any Feynman diagram for a process $e^+e^-\to f$. If you follow the fermion line starting from the initial electron, you either arrive at one of the final fermions (if any) or return to the initial positron. We call the former a *scattering-type* diagram and the latter an *annihilation-type* diagram. See Fig. 1. In the standard model, if the final state contains no electron or electron neutrino, all possible diagrams are of the annihilation type and we may call the process an annihilation process.

Let us discuss the annihilation process first. When there are no final electrons and electron neutrinos, the final state has $X_e=0$. The initial helicity states are eigenstates of X_e:

$$X_e=\begin{cases}0 & \text{for } |+-\rangle,|-+\rangle\,,\\ +2 & \text{for } |++\rangle\,,\\ -2 & \text{for } |--\rangle\,.\end{cases} \tag{3.4}$$

Since X_e is conserved, we have the result



(a) (b)

FIG. 1. (a) Annihilation-type diagram; (b) scattering-type diagram.

$$T_{++}=T_{--}=0\ . \tag{3.5}$$

In this case (2.13) reduces to

$$\Sigma_{\rm pol}=\tfrac{1}{4}(\,|T_{+-}|^2+|T_{-+}|^2) - \tfrac{1}{2}\cos2\phi\,{\rm Re}T^*_{+-}T_{-+}-\tfrac{1}{2}\sin2\phi\,{\rm Im}T^*_{+-}T_{-+}\ , \tag{3.6}$$

and the ϕ-averaged cross section

$$\overline{\Sigma}_{\rm pol}=\tfrac{1}{4}(\,|T_{+-}|^2+|T_{-+}|^2)\ . \tag{3.7}$$

This is just the unpolarized cross section. Thus we have shown that there is no effect of transverse polarization for the annihilation processes in the standard model once we integrate over the azimuthal angle ϕ. Moreover, ϕ dependence only appears as the form of $\cos2\phi$ or $\sin2\phi$. In most cases of interest, the amplitudes in the lowest order have a common phase at $\phi=0$ and there are $\cos2\phi$ terms only.[46] In fact, $\sin2\phi$ terms are T odd and there is no lowest-order contribution.[47]

For partial polarization the cross section becomes

$$\tfrac{1}{4}(\,|T_{+-}|^2+|T_{-+}|^2) - \tfrac{1}{2}P^2(\cos2\phi\,{\rm Re}T^*_{+-}T_{-+}+\sin2\phi\,{\rm Im}T^*_{+-}T_{-+})\ , \tag{3.8}$$

where P is the polarization of the electron and the positron (taken equal).

Now we turn to the case containing electrons (or electron neutrinos) in the final state. Typical reactions are the Bhabha scattering $e^+e^-\to e^+e^-$ and various two-photon processes. The amplitude is in general a sum of an annihilation-type and a scattering-type amplitude. The same argument as above is applied to the annihilation part. More consideration is necessary for the scattering part.

If we do not observe the spins of the final-state particles, or even if we measure the helicities of the final particles, we can work with states of definite X_e as a final state, so there is no interference between T_{++} and T_{--}, since X_e is conserved and the two initial states have different X_e values. In other words, if $\langle f\,|\,T\,|\,++\rangle\neq0$ for a final state f, the state has $X_e=+2$, so $\langle f\,|\,T\,|\,--\rangle=0$, and vice versa. Thus we see that the only possible interference is between the two $X_e=0$ states $|+-\rangle$ and $|-+\rangle$.

Going back to the cross section of (2.13), the above argument implies

$$\Sigma_{\rm pol}=\Sigma_{\rm unpol}-\tfrac{1}{2}{\rm Re}e^{-2i\phi}T^*_{+-}T_{-+}\ , \tag{3.9}$$

and for the ϕ-averaged cross section

$$\overline{\Sigma}_{\rm pol}=\Sigma_{\rm unpol}\ . \tag{3.10}$$

We conclude that the effect of the transverse polarization to the ϕ-averaged cross section in e^+e^- collisions is absent in the standard model provided (i) the electron mass can be neglected, and (ii) the transverse polarization of the final electrons (if any) is not measured. Both conditions are satisfied in practically all measurable processes in high-energy e^+e^- collisions. This completes the proof of the null theorem. Some examples including the violation of the theorem by the electron mass are discussed in Appendixes C–E.

We conclude this section with a few comments. It is evident that the theorem is exact in all orders of perturbation theory (for $m_e\to0$) since the whole standard-model Lagrangian is invariant under the chiral symmetry. In particular, unphysical scalar particles which appear in renormalizable gauges decouple from electrons in the massless limit. Ghosts do not couple to electrons either.

The theorem applies even to reactions with the Higgs particle in the final state. For instance, the reaction $e^+e^-\to ZH$ (Refs. 48 and 32) proceeds via s-channel Z exchange and t-channel electron exchange at the tree level. The latter contribution, which violates the theorem, is extremely smaller than the former. Another reaction $e^+e^-\to\gamma H$ (Ref. 49) receives a contribution only proportional to the Yukawa coupling at the tree level, for which the theorem does not hold. However, the one-loop contribution via s-channel photon and Z exchange, etc., which respects the theorem, dominates the cross section.

IV. BEYOND THE STANDARD MODEL: COMPOSITENESS

In the preceding section we saw that the null theorem is the consequence of the electronic chiral symmetry of the standard model. The theorem is not necessarily true in general because of the anticorrelation between the e^+ and e^- spins. Breakdown of this theorem would indicate the existence of new interactions. In this and the next sections we will examine several cases in which the theorem is violated. In the following we discuss new particles and interactions[50] expected in composite models.[51,52] Supersymmetry will be treated in Sec. V.

A. Spin-0 resonance

The simplest and most illustrative example[42] is the formation of spin-0 (scalar or pseudoscalar) resonance. Motivated by the "radiative Z decay" events at the CERN $p\bar{p}$ collider,[53,54] experiments[55–61] at PETRA looked for spinless bosons[62–64] which couple to electrons. Spin-0 boson are also expected in composite models. As pointed out in I, nonzero transverse polarization affects the total resonance cross section.

Scalar (0^+) case. Suppose the scalar coupling is of the form

$$\mathscr{L}_S=f_S\bar{e}eS\ . \tag{4.1}$$

Helicity amplitudes for $e^+e^-\to S$ are given by

$$T_{++}=T_{--}=2f_S p\ ,\qquad T_{+-}=T_{-+}=0\ , \tag{4.2}$$

where p is the electron momentum in the c.m. system. [The latter of (4.2) is due to angular momentum conservation.] Using (2.10) and (2.13), we have

$$\Sigma_{\text{unpol}}=2f_S{}^2p^2=\tfrac{1}{2}f_S{}^2(s-4m_e{}^2) \tag{4.3}$$

and

$$\Sigma_{\text{pol}}=0\ . \tag{4.4}$$

Here s is the e^+e^- c.m. energy. Thus the cross section is *exactly zero* for complete polarization. This is in contrast with the standard-model cases. If the polarization is finite the cross section is proportional to

$$2f_S{}^2p^2(1-P^2)\ . \tag{4.5}$$

Pseudoscalar (0^-) *case.* The interaction is

$$\mathscr{L}_P=if_P\bar{e}\gamma_5 eP\ . \tag{4.6}$$

Helicity amplitudes are

$$T_{++}=2if_PE,\quad T_{--}=-2if_PE\ ,\qquad T_{+-}=T_{-+}=0\ , \tag{4.7}$$

where E is the electron energy. The unpolarized and polarized cross sections are

$$\Sigma_{\text{unpol}}=2f_P{}^2E^2=\tfrac{1}{2}f_P{}^2s\ , \tag{4.8}$$

$$\Sigma_{\text{pol}}=2\Sigma_{\text{unpol}}\ . \tag{4.9}$$

Unlike the scalar case, the polarization makes the cross section twice as large as the unpolarized one. For partial polarization we have $(1+P^2)$ times the unpolarized cross section. The breakdown of the null theorem may be understood in terms of symmetry. The scalar coupling (4.1) or (4.6) inevitably violates the electronic chiral symmetry. There is no choice of X_e values for the spinless particle to preserve the symmetry. (In any case we cannot assign a nonzero X_e charge to a real scalar field.)

These results show that the PETRA experiments should be reanalyzed if there is finite transverse polarization. The polarization would make the bound for spin-0 resonance weaker for scalars, stronger for pseudoscalars.[65]

Inspection of (4.4) and (4.9) shows that if we have both a scalar and a pseudoscalar of the same masses and couplings, the polarization effect is canceled for $m_e\to 0$:

$$\Sigma_{\text{pol}}=\Sigma_{\text{unpol}}=f^2s\ , \tag{4.10}$$

where $f\equiv f_S=f_P$. The scalar sector in this model in fact respects a chiral symmetry. If we define the complex scalar field

$$\Phi=\frac{1}{\sqrt{2}}(S+iP)\ , \tag{4.11}$$

the interaction can be written

$$\begin{aligned}\mathscr{L}&=\mathscr{L}_S+\mathscr{L}_P\\ &=f\left[\bar{e}\frac{1+\gamma_5}{\sqrt{2}}e\Phi+\text{H.c.}\right]\\ &=\sqrt{2}f(\bar{e}_Le_R\Phi+\bar{e}_Re_L\Phi^\dagger)\ .\end{aligned} \tag{4.12}$$

It is obvious that the chiral symmetry is preserved (for $m_e\to 0$) if we assign $X_e=-2$ to Φ. This symmetry assures the disappearance of the transverse-polarization effect as in the standard model. (See Fig. 2.)

If this scalar couples not only to the electron but to all quarks and leptons, we should extend the chiral symmetry to include all fermions. The symmetry is broken by the fermion masses, particularly by the t-quark mass. It invalidates the null theorem by introducing interference between $X_e=+2$ and -2 states. For example, the cross sections for $e^+e^-\to t\bar{t}$ via s-channel Φ exchange (assuming universal coupling) are

$$\Sigma_{\text{unpol}}=6f^2\mathscr{P}^2(1-2m_t{}^2/s)\ , \tag{4.13}$$

$$\begin{aligned}\Sigma_{\text{pol}}&=\Sigma_{\text{unpol}}+12f^2\mathscr{P}^2m_t{}^2/s\\ &=6f^2\mathscr{P}^2\ ,\end{aligned} \tag{4.14}$$

where

$$\mathscr{P}^2=\frac{s^2}{(s-M^2)^2+M^2\Gamma^2}\ , \tag{4.15}$$

and M and Γ are the mass and the total width of the scalars. The polarization effect disappears for $m_t\to 0$ as the chiral symmetry is restored.

Since m_t is not small,[66,67] the chiral-symmetry-breaking effect influences the scalar spectrum. The two spinless states are no longer degenerate[63] and split into a scalar and a pseudoscalar mass eigenstate. In fact, if $M>2m_t$ and $M \not\gg 2m_t$, the widths of the two particles become different [compare (4.2) and (4.8)]. The real part of the self-mass is affected differently even if $M<2m_t$. If the mass difference of the two states are not much smaller than the widths, it may be possible to observe the transverse-polarization effect.

Another interaction of the scalars generally expected in composite models is the coupling with two photons. In the case of a single spinless particle, the polarization effect is the same as in the resonance formation cross section. An interesting case we will analyze below is the chiral-symmetric case. The interaction Lagrangian is given by

$$\mathscr{L}=\frac{e^2}{4\mu}SF_{\mu\nu}F^{\mu\nu}+\frac{e^2}{4\mu'}PF_{\mu\nu}\tilde{F}^{\mu\nu}\ , \tag{4.16}$$

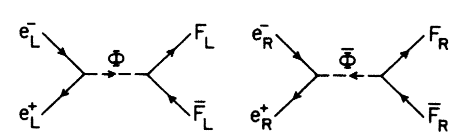


FIG. 2. Diagrams for $e^+e^-\to$a fermion pair via "chirality-conserving" scalar exchange.

where μ and μ' are parameters with a dimension of mass, $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$, and $\tilde{F}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$.[68]

The helicity amplitudes for the reaction $e^+e^-\to\gamma\gamma$ coming from s-channel scalar exchange (Fig. 3) are

$$\begin{aligned} T^{++}_{++}=T^{--}_{--}&=\frac{e^2 f s^{3/2}}{2(s-M^2)}\left[\frac{1}{\mu}+\frac{1}{\mu'}\right],\\ T^{--}_{++}=T^{++}_{--}&=\frac{e^2 f s^{3/2}}{2(s-M^2)}\left[\frac{1}{\mu}-\frac{1}{\mu'}\right]. \end{aligned} \tag{4.17}$$

Here we used the notation $T^{\lambda_1\lambda_2}_{h\bar{h}}$, with λ_i the helicity of the photons. In addition to these, we have QED contributions to the amplitudes T_{+-} and T_{-+} (see Appendix E). There is no interference between the two. The finite-width correction must be included in (4.17) near the resonance.

It can be seen from (4.17) that the transverse-polarization effect vanishes if $\mu=\mu'$ (then $T^{--}_{++}=T^{++}_{--}=0$) or $\mu=-\mu'$ ($T^{++}_{++}=T^{--}_{--}=0$).

Is there any symmetry for the case $|\mu|=|\mu'|$ to assure the null result? At first sight there seems to be no "chiral" symmetry since the electromagnetic field is self-conjugate. Nevertheless, it turns out that the Lagrangian actually possesses a continuous symmetry for the above two cases. For $\mu=\mu'$, the Lagrangian (4.16) can be cast in the following form:

$$\mathcal{L}=\frac{e^2}{4\sqrt{2}\mu}\Phi\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}+\text{H.c}\ . \tag{4.18}$$

Here we have introduced a complex tensor

$$\mathcal{F}_{\mu\nu}=\frac{1}{\sqrt{2}}(F_{\mu\nu}-i\tilde{F}_{\mu\nu})\ . \tag{4.19}$$

The interaction (4.18) is invariant under the transformation

$$\Phi\to e^{-2i\alpha}\Phi,\quad \mathcal{F}_{\mu\nu}\to e^{i\alpha}\mathcal{F}_{\mu\nu}\ . \tag{4.20}$$

Thus we are able to extend the chiral symmetry to include the interaction with photons. It is clear that the symmetry allows us to assign $X_e=+1$ for $\mathcal{F}_{\mu\nu}$, thereby guaranteeing the null result. Physically, $\mathcal{F}_{\mu\nu}$ ($\mathcal{F}^\dagger_{\mu\nu}$) corresponds to helicity $+1$ (-1) photons. (The helicity of the photon is completely Lorentz invariant unlike that of massive particles.) In the case $\mu=-\mu'$, $\mathcal{F}_{\mu\nu}$ in (4.18) should be changed to $\mathcal{F}^\dagger_{\mu\nu}$ and the second equation in (4.20) becomes $\mathcal{F}_{\mu\nu}\to e^{-i\alpha}\mathcal{F}_{\mu\nu}$.

It should be noted that the symmetry of (4.20) is not a symmetry of the total Lagrangian[69] because the kinetic term is proportional to $\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}+\mathcal{F}^\dagger_{\mu\nu}\mathcal{F}^{\mu\nu\dagger}$ and is not invariant under the transformation in (4.20). The photon propagator violates the symmetry. Therefore the symmetry is effective for on-shell photons only. This is in accord with the fact that the concept of helicity does not make sense for off-shell photons and the Coulomb interaction enters into the game. For instance, the symmetry is violated in the process $\gamma\gamma\to\Phi\Phi$ via (t- and u-channel) photon exchange. We obtain a nonzero result, although the symmetry would forbid the reaction. Also it is impossible to define a corresponding transformation law for the vector potential A_μ and "minimal" gauge interactions do not respect the symmetry.

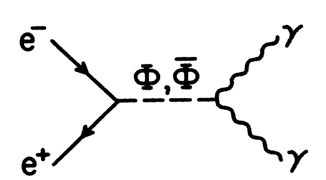


FIG. 3. The diagram for the scalar-exchange contribution to $e^+e^-\to\gamma\gamma$.

Similar couplings of the scalar to other gauge bosons (W^+W^-,ZZ,$Z\gamma$) are also expected [particularly because of the SU(2)⊗U(1) invariance]. The effect of scalar bosons to the reactions $e^+e^-\to W^+W^-$,ZZ with unpolarized beams was studied by Tanimoto.[70] Since the weak bosons are massive, the extension of a chiral symmetry such as (4.20) is not possible. The violation of the null theorem is expected for these processes.

At higher energies, pair production of the scalars becomes possible (via t-channel electron exchange). There is an essential difference between the single scalar formation and the pair production. In the lowest order we have $T_{++}=T_{--}=0$ [compare with (4.2) and (4.7)]. The polarization effects vanish even if the chiral symmetry does not exist. (However, if there are three-scalar interactions, the above conclusion does not follow because the reaction proceeds with s-channel scalar exchange.) In general we have maximal effects for an odd number of scalars, and no effects for an even number. The scalar pair production and the scalar-Z associated production (the latter via s-channel vector exchange) for polarized beams were studied by Narison and Wallet.[71] They obtained only asymmetries (terms proportional to $\cos 2\phi$) because of the above reason.

B. Excited electron

Excited states of fermions[72–78] are typical particles expected in composite models. Excited electrons are of interest here because they have a direct coupling to electrons (which could be the definition of excited electrons). Electromagnetic gauge invariance requires the lowest-dimensional interaction of the excited electron e^* with the electron to be a magnetic-transition type. If we require the parity invariance the interaction has the form[79]

$$\mathcal{L}=\frac{e}{2\Lambda}(\bar{E}\sigma^{\mu\nu}e+\bar{e}\sigma^{\mu\nu}E)F_{\mu\nu}\ , \tag{4.21}$$

where E denotes the excited electron field. This interaction violates the chiral symmetry if the e^* mass is finite. [If it were negligible, it would be possible to assign the opposite chirality for $e^*_{R(L)}$ as $e_{R(L)}$. However, we know that the excited electron, if it exists, is substantially heavy.[80] At $s\gg m_{e^*}{}^2$ the symmetry becomes nearly exact, but we also expect that higher-dimensional interactions than (4.21) becomes important.] It is expected that the null theorem is violated in reactions where e^* is involved.

On the other hand, it has been argued[81,74] that the success of the QED prediction for the electron and muon anomalous magnetic moments puts a stringent limit on the coupling unless the excited lepton interaction is maximally parity violating. The interaction

$$\mathscr{L}=\frac{e}{2\sqrt{2}\Lambda}[\bar{E}\sigma^{\mu\nu}(1-\gamma_5)e+\bar{e}\sigma^{\mu\nu}(1+\gamma_5)E]F_{\mu\nu} \tag{4.22}$$

respects a chiral symmetry because the excited electron interacts only with e_L^- and e_R^+. For the rest of the section we discuss a few examples of the e^* effect.

Our first example is again the reaction $e^+e^-\to\gamma\gamma$, where the exchange of virtual e^* contributes. The standard-model prediction for this process is displayed in Appendix E. The interaction of (4.16) or (4.17) gives an extra contribution by t- and u-channel e^* exchange.

The relevant diagrams are shown in Fig. 4. Nonzero helicity amplitudes $T_{h\bar{h}}^{\lambda_1\lambda_2}$ at $\phi=0$ are the following.[82] For the parity-conserving coupling of (4.21),

$$\begin{aligned}
T^{+-}_{+-}=-T^{-+}_{-+}&=\frac{e^2}{2\Lambda^2}\frac{s^2}{u-M^{*2}}(1+\cos\theta)\sin\theta\ ,\\
T^{+-}_{-+}=-T^{-+}_{+-}&=\frac{e^2}{2\Lambda^2}\frac{s^2}{t-M^{*2}}(1-\cos\theta)\sin\theta\ ,\\
T^{--}_{++}=T^{++}_{--}&=\frac{e^2s^{3/2}M^*}{\Lambda^2}\left[\frac{1-\cos\theta}{t-M^{*2}}+\frac{1+\cos\theta}{u-M^{*2}}\right],
\end{aligned} \tag{4.23}$$

where M^* is the excited electron mass. For the chiral coupling in (4.22),

$$\begin{aligned}
T^{+-}_{-+}&=\frac{e^2}{\Lambda^2}\frac{s^2}{t-M^{*2}}(1-\cos\theta)\sin\theta\ ,\\
T^{-+}_{-+}&=-\frac{e^2}{\Lambda^2}\frac{s^2}{u-M^{*2}}(1+\cos\theta)\sin\theta\ .
\end{aligned} \tag{4.24}$$

The cross sections for longitudinally polarized beams obtained from (4.24) and (E3) are consistent with the results of Ref. 77. The latter has no chirality-flip amplitude ($T_{++}=T_{--}=0$) and the null theorem holds as expected. There exist chirality-flip amplitudes for the parity-conserving case of (4.23). However, according to the fact that $T^{++}_{++}=T^{--}_{--}=0$, there is no polarization effect for the ϕ-averaged cross section.

Again, this is understood in terms of symmetry. The interaction in (4.21) can be rewritten as

$$\mathscr{L}=\frac{e}{2\sqrt{2}\Lambda}(\bar{E}\sigma^{\mu\nu}e_R\mathscr{F}_{\mu\nu}+\bar{E}\sigma^{\mu\nu}e_L\mathscr{F}^{\dagger}_{\mu\nu})+\text{H.c}\ . \tag{4.25}$$

[$\mathscr{F}_{\mu\nu}$ has been defined by (4.19)], which is invariant under

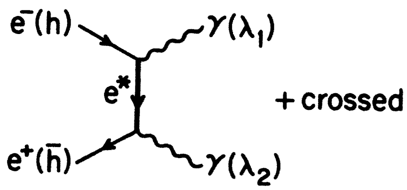


FIG. 4. Diagrams for the excited-electron contribution to $e^+e^-\to\gamma\gamma$. Helicities of the particles are indicated in parentheses.

the transformation

$$e_L\to e^{-i\alpha}e_L,\quad e_R\to e^{i\alpha}e_R,\quad \mathscr{F}_{\mu\nu}\to e^{-i\alpha}\mathscr{F}_{\mu\nu}\ . \tag{4.26}$$

This chiral symmetry prevents the appearance of transverse-polarization effects. In fact, this symmetry can be defined for general tensor interaction

$$\begin{aligned}
\mathscr{L}&=\frac{e}{2\Lambda}[\bar{E}\sigma^{\mu\nu}(c-d\gamma_5)eF_{\mu\nu}+\text{H.c.}]\\
&=\frac{e}{2\Lambda}[(c-d)\bar{E}\sigma^{\mu\nu}e_R\mathscr{F}_{\mu\nu}+(c+d)\bar{E}\sigma^{\mu\nu}e_L\mathscr{F}^{\dagger}_{\mu\nu}+\text{H.c.}]\ .
\end{aligned} \tag{4.27}$$

It is to be repeated that the symmetry in (4.26) is valid only for on-shell photons. Note that the transformation in (4.26) for $\mathscr{F}_{\mu\nu}$ is *different* from (4.20). If there are both interactions (4.16) (with $\mu=\mu'$) and (4.21), we have nonvanishing polarization effects.

In the large-M^* limit, the leading term of the chiral amplitudes [Eq. (4.24)] and the chirality-nonflip amplitudes in the parity-conserving case [Eq. (4.23)] is proportional to M^{*-2}. On the other hand, the chirality-flip amplitudes in (4.23) have a M^{*-1} dependence because they are proportional to the mass term of the e^* propagator. In the cross section, however, the dominant term comes from the interference of the chirality-nonflip amplitudes and the standard-model contribution. The unpolarized cross section for the parity-conserving interaction is

$$\Sigma_{\text{unpol}}=4e^4\left[\frac{1+\cos^2\theta}{1-\cos^2\theta}+\frac{s^2}{2\Lambda^2M^{*2}}(1+\cos^2\theta)+\frac{s^3}{2\Lambda^4M^{*2}}\right]. \tag{4.28}$$

The last term is absent for the chiral interaction. This term is unfortunately suppressed by a factor s/Λ^2 compared to the leading term. Thus the type of the interaction cannot be distinguished from the unpolarized cross section. Transverse polarization does not improve the situation because we have

$$\Sigma_{\text{pol}}-\Sigma_{\text{unpol}}=-4e^4\cos2\phi\left[1+\frac{s^2}{2\Lambda^2M^{*2}}\sin^2\theta\right] \tag{4.29}$$

for both cases. Of course, longitudinal polarization can be used to separate each helicity component and to determine the tensor and pseudotensor coupling separately (see Appendix B).

Excited electrons can also have couplings with W and Z bosons of the same type as (4.21) or (4.22). The effects of these couplings to the processes $e^+e^-\to W^+W^-,ZZ$ for the chiral couplings for unpolarized beams were considered by Tanimoto.[70] If the coupling is not chiral, nonvanishing transverse-polarization effects should be observed since an extended symmetry such as (4.26) is broken by the gauge-boson mass.

Now we turn to the real production of e^*. An experimentally favored process is the single excitation $e^+e^-\to e^*e$ (Fig. 5). In this reaction it is easy to see that the chirality of the nonexcited electron is conserved and no

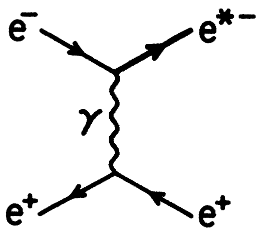


FIG. 5. The diagram for single production of the excited electron.

interesting effect arises from transverse polarization.

Pair production of e^* receives contribution from two sources (Fig. 6). One is the QED production via s-channel γ exchange, whose amplitudes are the same as $e^+e^-\to\mu^+\mu^-$ discussed in Appendix C (up to possible form-factor and anomalous moment[78] effects). The other contribution is the double excitation via t-channel γ exchange. The latter amplitudes contain various helicity components and transverse-polarization effects remain after azimuthal integration. The results are not reproduced here since the expressions are quite complicated.

C. Others

Other particles such as excited quarks and leptons (except e^*) do not directly couple to the electron (as far as there is no flavor-changing interactions). Their interactions do not affect the proof of the null theorem in Sec. III.

Leptoquarks, bosons with nonzero lepton and baryon numbers, are expected in composite models[83] and technicolor models,[84] as well as in some extended gauge models.[85] Electrons can become quarks by emitting a leptoquark boson. In the reaction $e^+e^-\to q\bar{q}$, which occurs through s-channel γ and Z exchange, leptoquarks give an additional contribution through u-channel exchange. If the interactions of leptoquark mass eigenstates are not chiral, the breakdown of the null theorem occurs in the following cases: (i) $e^+e^-\to t\bar{t}$ by scalar leptoquark exchange; (ii) $e^+e^-\to q\bar{q}$ by scalar and vector leptoquark exchange. Other reactions such as single leptoquark production (via photon-leptoquark fusion) and leptoquark pair production are examples where the chiral-symmetry-breaking effect is not seen in the lowest order.

A color-octet "electron,"[86] which can exist if leptons are made of colored constituents, has a magnetic-transition-type interaction with the electron by the emission of a gluon. This interaction can contribute to the total multihadron cross section[87] through the reaction $e^+e^-\to gg$ (via t-channel exchange). At higher energies, pair production of the octet electron by gluon exchange becomes possible. The effect of transverse polarization to these processes is exactly the same as the case of the excited electron. (Single production is forbidden in this order. It proceeds through the electron dissociation into an octet electron and a gluon with photon exchange, for example. No polarization effect is expected.)

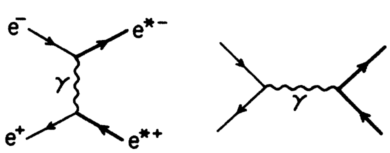


FIG. 6. Diagrams for the excited-electron pair production.

Even if no new particles exist at reachable energies, there is a possibility that some contact interactions of the four-fermion type[88] may be induced by preon dynamics. They modify the Bhabha and fermion-pair-production cross section at high energies. Experiments[89] show that the mass scale of this new interaction must be larger than several TeV assuming the coupling constant is of the order of unity. If those interactions are mediated by new gauge bosons as is originally supposed, there is no transverse-polarization effect because gauge couplings respect chiral symmetry. If, however, there also exist interaction induced by scalar exchange, the null theorem breaks down in general.

In this section we saw that there are several examples in composite models where the null theorem does not hold. The origin of the violation is the scalar or tensor coupling of the electron. In the special case of chiral coupling, however, the chiral symmetry can be extended to the new sector such that the transverse-polarization effect does not appear. In fact, this chiral symmetry is required[90] if we demand the predictions of the standard model as the low-energy effective theory should not be disturbed by the new interactions. In this case the only remaining effect of transverse polarization is via the t-quark mass, for instance, in the reaction $e^+e^-\to t\bar{t}$ by scalar exchange.

V. BEYOND THE STANDARD MODEL: SUPERSYMMETRY

Phenomenology of supersymmetric extensions[91] of the standard model has been extensively studied in the literature. Electron-positron collisions[92-121] is a good place to look for new particles predicted by supersymmetry. In this section we discuss the effect of transverse polarization on reactions expected by supersymmetric models.

A. Chiral symmetry in supersymmetric models

In the standard model, the only interactions of the electron (for $m_e\to 0$) are the gauge interactions with the photon, W, and Z. In supersymmetric theories, we have other interactions of the electron which are obtained from the gauge interactions by supersymmetry transformation, namely, the interactions with gauginos (photino $\tilde{\gamma}$ and W and Z gauginos $\tilde{w}$ and $\tilde{z}$) and the scalar partner of the electron (scalar electron $\tilde{e}$) or the neutrino (scalar neutrino $\tilde{\nu}$) (Ref. 122). These are shown in Fig. 7.

These new interactions respect the electronic chiral symmetry if the supersymmetry is not broken because we can extend the global chiral symmetry so that it commutes with supersymmetry. This symmetry is just the phase rotation of the electron superfields.

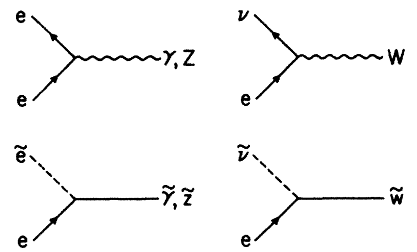


FIG. 7. Gauge interactions of the electron (above) and their supersymmetric counterpart (below).

Supersymmetry must be broken if it is to be realized in nature as the known particles do not form supermultiplets. Whether the above chiral symmetry continues to hold after supersymmetry breaking depends on the mechanism of the supersymmetry breakdown. The structure of the scalar-electron mass matrix is crucial for the chiral symmetry. Thus transverse polarization gives a tool to analyze the mechanism of supersymmetry breaking.

Let us examine each of the new couplings. The coupling with $\tilde{w}$ always respects the chiral symmetry because the $\tilde{w}$ couples to left-handed electrons only.

There are two ways to assign the chiral charge to the other two couplings. If the gauginos are light, we can make the following assignment to these fermions:

$$X_e=\begin{cases}+1 & \text{for } \tilde{\gamma}_L,\tilde{z}_L\ ,\\ -1 & \text{for } \tilde{\gamma}_R,\tilde{z}_R\ .\end{cases} \tag{5.1}$$

Since the gaugino couplings are of Yukawa type, the above chiral charges are opposite to those for electrons. The gaugino mass terms break the symmetry. Although the photino may be light, it is unlikely that the $\tilde{z}$ mass is rather small. Thus this assignment of the chiral charge is not very useful except at very high energies where the gaugino masses are negligible.

Another possible assignment is to rotate not the fermions (gauginos) but the scalars (scalar electrons). This is the natural assignment in the supersymmetric limit. In this limit the two scalar-electron states $\tilde{e}_R$ and $\tilde{e}_L$ are distinct particles with no transition between them for $m_e \to 0$. (Our notation is such that $\tilde{e}_R^{\pm}$ is associated with e_R^- and e_L^+, and so on.) The conserved chiral charges are

$$X_e=\begin{cases}+1 & \text{for } \tilde{e}_R^-,\tilde{e}_L^+\ ,\\ -1 & \text{for } \tilde{e}_L^-,\tilde{e}_R^+\ .\end{cases} \tag{5.2}$$

Supersymmetry-breaking effects do not necessarily respect the symmetry. The general mass matrix of the scalar-electron sector is (assuming CP invariance)

$$\mathcal{M}^2=\begin{array}{c c} & \begin{array}{cc}\tilde{e}_R & \tilde{e}_L\end{array}\\ \begin{array}{c}\tilde{e}_R^\dagger\\ \tilde{e}_L^\dagger\end{array} & \begin{pmatrix} m_R{}^2 & \mu^2\\ \mu^2 & m_L{}^2\end{pmatrix}\end{array} . \tag{5.3}$$

The mass term is invariant under the transformation induced by (5.2)

$$\tilde{e}_R \to e^{i\alpha}\tilde{e}_R,\quad \tilde{e}_L \to e^{-i\alpha}\tilde{e}_L \tag{5.4}$$

only when $\mu=0$.

In the parity-conserving case $|m_R|=|m_L|$, the two mass eigenstates are equal mixtures of $\tilde{e}_R$ and $\tilde{e}_L$, one with scalar, the other with pseudoscalar coupling to the electron. (We will call these states $\tilde{e}_S$ and $\tilde{e}_P$ in the following.) The violation of the chiral symmetry is in a sense maximal and we expect to see the effect of transverse polarization, which will be discussed in the following subsections.

On the other hand, if $|m_R{}^2-m_L{}^2| \gg \mu^2$, the mass eigenstates are approximately $\tilde{e}_R$ and $\tilde{e}_L$ and the chiral-symmetry breaking is small. In minimal models with minimal particle content and soft-supersymmetry-breaking terms induced by $N=1$ supergravity, $\mu^2/m_{R,L}{}^2$ is proportional to m_e. The mixing angle is thus very small. This pattern is usually assumed in phenomenological studies so far.

Incidentally, the anomalous magnetic moment of the muon gives a constraint[123] on the superpartner spectrum. The constraint is stricter for the nonchiral mass matrix of scalar muons. However, it depends on the gaugino masses and the mass splitting of two scalar-muon states as well as the mixing angle. The electron $g-2$ gives much weaker bounds.

B. Scalar-electron pair production

Scalar electrons,[124] as charged particles, can be produced via the coupling with photons. While other scalar leptons and scalar quarks can also be produced in this way, there is another mechanism for scalar-electron production: t-channel gaugino exchange (see Fig. 8). Actually there are four diagrams, with $\gamma, Z, \tilde{\gamma}, \tilde{z}$ exchange.

Various aspects of this reaction have been studied.[92,103,107,113,115,116,118,119] In particular, transverse-polarization effects were considered by Chiappetta, Soffer, Taxil, and Renard and Sorba.[116] However, all studies assume the scalar-electron mass eigenstates are $\tilde{e}_R$ and $\tilde{e}_L$. Although Schiller and Wähner[117] treat the general mixing case in polarized e^+e^- annihilation, they limit themselves to the production of other scalar leptons and scalar quarks only.

In this section we study the reaction $e^+e^- \to \tilde{e}^+\tilde{e}^-$ for the chiral and parity-conserving cases. As our interest lies on the transverse-polarization effect, we make some simplification. Diagrams with Z and $\tilde{z}$ exchange are neglected because their structure is rather similar to the diagrams with γ and $\tilde{\gamma}$ exchange. We compare two cases $e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$ and $e^+e^- \to \tilde{e}_S^+\tilde{e}_S^-$. Although our results are thus only applicable to the case $2m_{\tilde{e}_{R(S)}}$

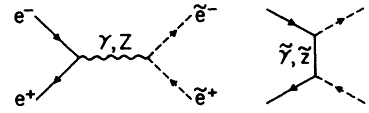


FIG. 8. Diagrams for scalar-electron pair production.

$<\sqrt{s}<2m_{\tilde{e}_{L(P)}}$ and $\sqrt{s} \ll M_Z, m_{\tilde{z}}$, the qualitative behavior is expected to be generally applicable. The amplitudes for other final-state combinations are presented in Appendix F.

We assume the following interactions (those are prescribed by supersymmetry up to the phase convention). For the chiral case,

$$\mathscr{L}=\sqrt{2}e\left[\bar{\tilde{\gamma}}\frac{1+\gamma_5}{2}e\tilde{e}_R^* + \bar{e}\frac{1-\gamma_5}{2}\tilde{\gamma}\,\tilde{e}_R - \bar{\tilde{\gamma}}\frac{1-\gamma_5}{2}e\tilde{e}_L^* - \bar{e}\frac{1+\gamma_5}{2}\tilde{\gamma}\,\tilde{e}_L\right] . \tag{5.5}$$

For the parity-conserving case,

$$\mathscr{L}=e(\bar{e}\tilde{\gamma}\,\tilde{e}_S+\bar{\tilde{\gamma}}e\tilde{e}_S^* - \bar{e}\gamma_5\tilde{\gamma}\,\tilde{e}_P + \bar{\tilde{\gamma}}\gamma_5 e\tilde{e}_P^*) . \tag{5.6}$$

The helicity amplitudes for $e^+e^-\to\tilde{e}_R^+\tilde{e}_R^-$ are [for $\phi=0$ (Ref. 125)]

$$\begin{aligned} T^{RR}_{+-}&=-e^2\beta_R\left[1+\frac{s}{t'}\right]\sin\theta ,\\ T^{RR}_{-+}&=e^2\beta_R\sin\theta ,\\ T^{RR}_{++}&=T^{RR}_{--}=0 , \end{aligned} \tag{5.7}$$

where $\beta_R{}^2=1-4m_{\tilde{e}_R}{}^2/s$, $t'=t-m_{\tilde{\gamma}}{}^2$. The cross section is

$$\frac{d\sigma^{RR}}{d\Omega}=\frac{\alpha^2\beta_R{}^2}{16s}\sin^2\theta\left[1+\left[1+\frac{s}{t'}\right]^2 + 2P^2\left[1+\frac{s}{t'}\right]\cos2\phi\right] . \tag{5.8}$$

The polarization-dependent term vanishes after ϕ integration as expected.

The amplitudes for $e^+e^-\to\tilde{e}_S^+\tilde{e}_S^-$ are

$$\begin{aligned} T^{SS}_{+-}&=-T^{SS}_{-+}=-e^2\beta_S\left[1+\frac{s}{t'}\right]\sin\theta ,\\ T^{SS}_{++}&=T^{SS}_{--}=-e^2\frac{\sqrt{s}\,m_{\tilde{\gamma}}}{t'} , \end{aligned} \tag{5.9}$$

with $\beta_S{}^2=1-4m_{\tilde{e}_S}{}^2/s$. Note the existence of the chirality-flip terms. The cross section is

$$\frac{d\sigma^{SS}}{d\Omega}=\frac{\alpha^2\beta_S}{8s}\left[\beta_S{}^2\sin^2\theta\left[1+\frac{s}{2t'}\right]^2(1+P^2\cos2\phi) + \frac{sm_{\tilde{\gamma}}{}^2}{t'^2}(1-P^2)\right] . \tag{5.10}$$

The last term survives after ϕ integration and directly indicates the violation of the chiral symmetry.

The two cross sections have a different behavior even in the absence of the transverse polarization:

$$\frac{d\sigma^{RR}}{d\Omega}=\frac{\alpha^2\beta_R}{16s}\beta_R{}^2\sin^2\theta\left[2+\frac{2s}{t'}+\frac{s^2}{t'^2}\right] , \tag{5.8'}$$

$$\frac{d\sigma^{SS}}{d\Omega}=\frac{\alpha^2\beta_S}{16s}\left[\beta_S{}^2\sin^2\theta\left[2+\frac{2s}{t'}+\frac{s^2}{2t'^2}\right]+\frac{sm_{\tilde{\gamma}}{}^2}{t'^2}\right] . \tag{5.10'}$$

The last term of (5.10′) has an S-wave threshold behavior and is isotropic. This rapid raise at the threshold is another signal of the $\tilde{e}_R$-$\tilde{e}_L$ mixing.

Glück and Reya[103] found similar behavior in the reaction $e^+e^-\to\tilde{e}_R\tilde{e}_L$ as also may be seen in Appendix F. However, as noted by Kobayashi and Kuroda,[107] that rapid rise is obscured by the lower-threshold reaction $e^+e^-\to\tilde{e}_R^+\tilde{e}_R^-$ (or $\tilde{e}_L^+\tilde{e}_L^-$), and, hence, rather difficult to observe (if $m_{\tilde{e}_R}\neq m_{\tilde{e}_L}$). On the contrary, in our case the rise is at the lowest threshold (the same is true for $\tilde{e}_P^+\tilde{e}_P^-$ if $m_{\tilde{e}_P}<m_{\tilde{e}_S}$) and the S-wave behavior is significant.

C. Photino pair production

The next process we examine is $e^+e^-\to\tilde{\gamma}\,\tilde{\gamma}$ (Refs. 98 and 95). Although this reaction is experimentally relevant for the case of massive decaying photino[126] only, it can serve as a simple illustration of the scalar-electron mass-mixing effect.

The reaction occurs through t-channel $\tilde{e}$ exchange (see Fig. 9). In this subsection we give the results for the limiting cases: (i) $s\ll m_{\tilde{e}_R}{}^2\ll m_{\tilde{e}_L}{}^2$ (chiral case) and (ii) $s\ll m_{\tilde{e}_S}{}^2\ll m_{\tilde{e}_P}{}^2$ (parity-conserving case). More complete treatment is done in Appendix G.

The helicity amplitudes for case (i) are

$$\begin{aligned} T^{+-}_{+-}&=e^2\frac{s\tilde{\beta}}{m_{\tilde{e}_R}{}^2}(1+\cos\theta) ,\\ T^{-+}_{+-}&=-e^2\frac{s\tilde{\beta}}{m_{\tilde{e}_R}{}^2}(1-\cos\theta) . \end{aligned} \tag{5.11}$$

Here $\tilde{\beta}^2=1-4m_{\tilde{\gamma}}{}^2/s$. Other amplitudes vanish. The null theorem holds obviously. Since only one initial-state contributes, there is no $\cos2\phi$ term either. The cross section is

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2 s\tilde{\beta}^3}{8m_{\tilde{e}_R}{}^4}(1+\cos^2\theta) . \tag{5.12}$$

The threshold rise is rather slow.

For case (ii) we have

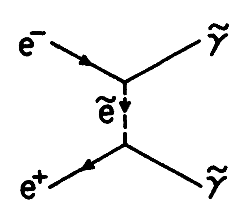


FIG. 9. Diagrams for photino pair production.

$$
\begin{aligned}
T^{+-}_{+-} &= T^{-+}_{-+} = e^2 \frac{s\tilde{\beta}}{2m_{\tilde{e}_S}{}^2}(1+\cos\theta) , \\
T^{-+}_{+-} &= T^{+-}_{-+} = -e^2 \frac{s\tilde{\beta}}{2m_{\tilde{e}_S}{}^2}(1-\cos\theta) , \\
T^{++}_{++} &= T^{--}_{--} = -e^2 \frac{s(1-\tilde{\beta})}{2m_{\tilde{e}_S}{}^2} , \\
T^{--}_{++} &= T^{--}_{++} = e^2 \frac{s(1+\tilde{\beta})}{2m_{\tilde{e}_S}{}^2} ,
\end{aligned}
\tag{5.13}
$$

and

$$
\frac{d\sigma}{d\Omega} = \frac{\alpha^2 s\tilde{\beta}}{16m_{\tilde{e}_S}{}^4}\left[3 - \frac{2}{\tilde{\gamma}^2} + \tilde{\beta}^2\cos^2\theta + P^2\left[\frac{1}{\tilde{\gamma}^2} + \tilde{\beta}^2\sin^2\theta\cos 2\phi\right]\right] . \tag{5.14}
$$

with $\tilde{\gamma} = \sqrt{s}/2m_{\tilde{\gamma}}$. This is different from (5.12) in several aspects. The null theorem is violated here since the parity-conserving Lagrangian does not possess the chiral symmetry. Note, however, the violating term vanishes for $s \gg m_{\tilde{\gamma}}{}^2$. This is due to the other possible chiral symmetry for massless photinos discussed in Sec. V A. Also, the threshold rise is more rapid than (5.12). At threshold we have the maximal polarization effect

$$
\frac{d\sigma}{d\Omega} \simeq \frac{\alpha^2 s\tilde{\beta}}{16m_{\tilde{e}_S}{}^4}(1+P^2) . \tag{5.15}
$$

Similar results are expected for other gaugino pair-production processes: $e^+e^- \to \tilde{z}\tilde{\gamma}$ (Refs. 106, 104, and 127) or $\tilde{z}\tilde{z}$ (Ref. 105). The chiral nature of the $\tilde{w}$ coupling precludes any violation of the null theorem for $e^+e^- \to \tilde{w}^+\tilde{w}^-$ (Refs. 102, 105, 114, and 128). Chiappetta *et al.*[120] studied the polarization effects for these reactions. Their results obey the null theorem for they employ the chiral scalar-electron mass matrix.

Another important reaction of this type is the radiative photino pair production $e^+e^- \to \gamma\tilde{\gamma}\tilde{\gamma}$. It is a favorable process to look for photinos and scalar electrons and has been discussed by several groups.[95,98,109–111,121] Experimental limits have also been obtained.[129] Since the emission of a photon does not change the chiral structure, our qualitative results for $e^+e^- \to \tilde{\gamma}\tilde{\gamma}$ is also applicable to this process.

VI. CONCLUSION

Transverse polarization has a unique position in high-energy e^+e^- interactions, which is quite different from longitudinal polarization. Longitudinal beam polarization selects some of the initial helicity states. It enables us to measure the magnitudes of each helicity amplitude and is a powerful tool to study the weak interactions. On the contrary, transverse polarization prepares a state which is a linear combination of some helicity states. Its effect is thus quantum mechanical in a sense and it gives us an opportunity to decide the relative phases of the helicity amplitudes.

Among the "elementary" particles we know, the electron is the lightest massive particle. Its mass is less than 10^{-6} times the W-boson mass which essentially sets the scale of the masses of quarks and leptons. The near masslessness of the electron is related to the nearly exact chiral symmetry of the standard model. Any theory which supercedes the standard model should guarantee the smallness of the electron mass. The problem is more important in those theories since they usually involve larger mass scale(s) than the weak scale. One possible protection mechanism is chiral symmetry.[130]

Chiral symmetry has an important consequence for the transverse-polarization effect in electron-positron collisions. Azimuthally integrated cross sections do not depend on the polarization if the theory has chiral symmetry which acts on the electron. In the standard model, the degree of the violation of this null theorem is proportional to the electron mass and negligible at experimentally relevant energies.

Although chiral symmetry is respected by all the gauge interactions, it is not obvious whether the other interactions, if they exist at all, obey the symmetry. The standard model has a Yukawa interaction which breaks the symmetry, but the effect is extremely small. New particles and/or interactions predicted in extensions of the standard model can break the symmetry, as we saw in previous sections for composite models and supersymmetric models. Thus the transverse polarization supplies a useful tool to probe the chiral structure of new interactions.

The null theorem is exact in the standard model for $m_e \to 0$. Whether the theorem holds for an extended model depends on whether it is possible to define chiral symmetry for the model. If chiral symmetry can be extended to the whole Lagrangian of the model, the null theorem applies to that model in all orders of perturbation theory. However, even if the symmetry does not exist, the lowest-order amplitudes sometimes obey the theorem due to some accidental reason. In these cases, we expect that the theorem is violated by higher-order corrections.

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APPENDIX A: THE CROSS SECTION

Our discussion in the text is made in terms of the reduced cross section Σ for convenience. In this appendix the relation between Σ and the true cross section is given.

For a reaction $e^+e^- \to f$ with f any n-body final state the cross section is

$$
d\sigma = \frac{1}{2s\beta_e}\Sigma\, d\Phi_n , \tag{A1}
$$

where s is the center-of-mass energy squared, β_e the electron velocity $\beta_e=(1-4m_e^2/s)^{1/2}$, which can practically be set to unity, and $d\Phi_n$ is the phase-space factor

$$d\Phi_n=(2\pi)^{4-3n}\delta^4\left(p+\bar{p}-\sum_{i=1}^{n}p_i\right)\prod_{i=1}^{n}\frac{d^3p_i}{2E_i} . \qquad \text{(A2)}$$

Here p and $\bar{p}$ are the initial electron and positron momentum, respectively, and p_i is the momentum of the ith final particle.

I give some explicit expressions of $d\Phi_n$ for small n. For $n=1$

$$d\Phi_1=2\pi\delta(s-M^2) , \qquad \text{(A3)}$$

where M is the final-particle mass, so that

$$\sigma=\frac{\pi}{s}\Sigma\delta(s-M^2) . \qquad \text{(A4)}$$

The finite-width correction due to the decay of the resonance can be written

$$d\Phi_1=\frac{2M\Gamma}{(s-M^2)^2+M^2\Gamma^2} \qquad \text{(A5)}$$

for $s\sim M^2$. Here Γ is the total decay width of the particle.

For $n=2$ we have

$$d\Phi_2=\frac{\bar{\beta}}{32\pi^2}d\Omega , \qquad \text{(A6)}$$

where $\bar{\beta}=2p/\sqrt{s}$, with p the momentum of each final particle:

$$\bar{\beta}=[(1+r_1+r_2)(1+r_1-r_2)\times(1-r_1+r_2)(1-r_1-r_2)]^{1/2} , \qquad \text{(A7)}$$

where $r_{1,2}=m_{1,2}/\sqrt{s}$ and m_1,m_2 are the masses of final particles. In the case $m_1=m_2$, $\bar{\beta}$ is equal to the velocity of each particle. The cross section is given by

$$\frac{d\sigma}{d\cos\theta\, d\phi}=\frac{\bar{\beta}}{64\pi^2 s}\Sigma . \qquad \text{(A8)}$$

APPENDIX B: CROSS SECTION FOR ARBITRARY POLARIZATION

In Sec. II we gave the cross-section formula for transversely polarized beams. In this appendix the expression for arbitrary polarized beams is presented.

The state vector of the electron polarized in the direction (θ,χ) can be written

$$|\theta,\chi\rangle=\cos\frac{\theta}{2}|h=+\rangle+\sin\frac{\theta}{2}e^{i\chi}|h=-\rangle . \qquad \text{(2.1)}$$

Similarly for the positron we have

$$|\bar{\theta},\bar{\chi}\rangle=\sin\frac{\bar{\theta}}{2}e^{i\bar{\chi}}|\bar{h}=+\rangle+\cos\frac{\bar{\theta}}{2}|\bar{h}=-\rangle . \qquad \text{(B1)}$$

The (reduced) cross section is obtained from (2.1) and (B1):

$$\begin{aligned}\Sigma&=|\langle f|T|\theta,\chi;\bar{\theta},\bar{\chi}\rangle|^2\\&=\tfrac{1}{4}[(1+\cos\theta)(1+\cos\bar{\theta})|T_{+-}|^2+(1+\cos\theta)(1-\cos\bar{\theta})|T_{++}|^2\\&\quad+(1-\cos\theta)(1+\cos\bar{\theta})|T_{--}|^2+(1-\cos\theta)(1-\cos\bar{\theta})|T_{-+}|^2\\&\quad+2(1+\cos\theta)\sin\bar{\theta}\,\mathrm{Re}e^{i\bar{\chi}}T^*_{+-}T_{++}+2(1-\cos\theta)\sin\bar{\theta}\,\mathrm{Re}e^{i\bar{\chi}}T^*_{--}T_{-+}\\&\quad+2\sin\theta(1+\cos\bar{\theta})\mathrm{Re}e^{i\chi}T^*_{+-}T_{--}+2\sin\theta(1-\cos\bar{\theta})\mathrm{Re}e^{i\chi}T^*_{++}T_{-+}\\&\quad+2\sin\theta\sin\bar{\theta}\,\mathrm{Re}e^{i(\chi+\bar{\chi})}T^*_{+-}T_{-+}+2\sin\theta\sin\bar{\theta}\,\mathrm{Re}e^{i(\chi-\bar{\chi})}T^*_{++}T_{--}] .\end{aligned} \qquad \text{(B2)}$$

This can be rewritten in terms of the polarization vectors $\mathbf{P}=(\sin\theta\cos\chi,\sin\theta\sin\chi,\cos\theta)$ and $\bar{\mathbf{P}}=(\sin\bar{\theta}\cos\bar{\chi},\sin\bar{\theta}\sin\bar{\chi},\cos\bar{\theta})$:

$$\begin{aligned}\Sigma=\tfrac{1}{4}[&(|T_{+-}|^2+|T_{++}|^2+|T_{--}|^2+|T_{-+}|^2)+P_z(|T_{+-}|^2+|T_{++}|^2-|T_{--}|^2-|T_{-+}|^2)\\&+\bar{P}_z(|T_{+-}|^2-|T_{++}|^2+|T_{--}|^2-|T_{-+}|^2)+P_z\bar{P}_z(|T_{+-}|^2-|T_{++}|^2-|T_{--}|^2+|T_{-+}|^2)\\&+2P_x\mathrm{Re}(T^*_{+-}T_{--}+T^*_{++}T_{-+})-2P_y\mathrm{Im}(T^*_{+-}T_{--}+T^*_{++}T_{-+})+2\bar{P}_x\mathrm{Re}(T^*_{+-}T_{++}+T^*_{--}T_{-+})\\&-2\bar{P}_y\mathrm{Im}(T^*_{+-}T_{++}+T^*_{--}T_{-+})+2P_x\bar{P}_x\mathrm{Re}(T^*_{+-}T_{-+}+T^*_{++}T_{--})-2P_x\bar{P}_y\mathrm{Im}(T^*_{+-}T_{-+}-T^*_{++}T_{--})\\&-2P_y\bar{P}_x\mathrm{Im}(T^*_{+-}T_{-+}+T^*_{++}T_{--})-2P_y\bar{P}_y\mathrm{Re}(T^*_{+-}T_{-+}-T^*_{++}T_{--})+2P_z\bar{P}_x\mathrm{Re}(T^*_{+-}T_{++}-T^*_{--}T_{-+})\\&-2P_z\bar{P}_y\mathrm{Im}(T^*_{+-}T_{++}-T^*_{--}T_{-+})+2P_x\bar{P}_z\mathrm{Re}(T^*_{+-}T_{--}-T^*_{++}T_{-+})-2P_y\bar{P}_z\mathrm{Im}(T^*_{+-}T_{--}-T^*_{++}T_{-+})] .\end{aligned} \qquad \text{(B3)}$$

Equation (B3) is actually valid for partially polarized beams although it is derived for the complete polarization case. This is due to the fact that the cross section is a linear function of the polarization vector $\mathbf{P}$ (and $\bar{\mathbf{P}}$).

For annihilation processes (or processes with $T_{++}=T_{--}=0$) expression (B3) is considerably simplified:

$$\begin{aligned}\Sigma=\tfrac{1}{4}[&(1+P_z\bar{P}_z)(|T_{+-}|^2+|T_{-+}|^2)\\&+(P_z+\bar{P}_z)(|T_{+-}|^2-|T_{-+}|^2)\\&+2(P_x\bar{P}_x-P_y\bar{P}_y)\mathrm{Re}T^*_{+-}T_{-+}\\&-2(P_x\bar{P}_y+P_y\bar{P}_x)\mathrm{Im}T^*_{+-}T_{-+}] .\end{aligned} \qquad \text{(B4)}$$

It is easy to extract the ϕ dependence of the cross section from (B3) or (B4) using the fact that $T_{h\bar{h}}$ is proportional to $e^{i(h-\bar{h})\phi}$. (Note that $h,\bar{h}=\pm\frac{1}{2}$.)

APPENDIX C: $e^+e^- \to \mu^+\mu^-$

The null theorem is violated by the electron mass. The lowest-order QED amplitudes and the cross section for the reaction $e^+e^- \to \mu^+\mu^-$ are presented here. Both electron and muon masses are kept finite. We can see the nonvanishing effect of transverse polarization on the ϕ-averaged cross section for $m_e \neq 0$. The Z-exchange contribution is also discussed. The absorptive part of the propagator induces a $\sin 2\phi$ dependence which may be used to measure the total width or the couplings of Z.

The diagram for the process is shown in Fig. 10. The helicity amplitudes $T_{h\bar{h}}^{h'\bar{h}'}$ [at $\phi=0$ (Ref. 131)] are given by

$$
\begin{aligned}
T_{+-}^{+-} &= T_{-+}^{-+} = -e^2(1+\cos\theta)\,,\\
T_{+-}^{-+} &= T_{-+}^{+-} = -e^2(1-\cos\theta)\,,\\
T_{+-}^{\pm\pm} &= -T_{-+}^{\pm\pm} = e^2\frac{1}{\gamma_\mu}\sin\theta\,,\\
T_{\pm\pm}^{\pm\mp} &= -T_{\pm\pm}^{\mp\pm} = -e^2\frac{1}{\gamma_e}\sin\theta\,,\\
T_{\pm\pm}^{\pm\pm} &= T_{\pm\pm}^{\mp\mp} = -e^2\frac{1}{\gamma_\mu\gamma_e}\cos\theta\,,
\end{aligned}
\tag{C1}
$$

where $\gamma_\mu=\sqrt{s}\,/2m_\mu$ and $\gamma_e=\sqrt{s}\,/2m_e$.

The cross section for finite polarization is

$$
\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\frac{\beta_\mu}{\beta_e}\left[1+\cos^2\theta+\frac{1}{\gamma_\mu^{\,2}}\sin^2\theta + (1-P^2)\frac{1}{\gamma_e^{\,2}}\left[\sin^2\theta+\frac{1}{\gamma_\mu^{\,2}}\cos^2\theta\right] - P^2\beta_\mu^{\,2}\sin^2\theta\cos 2\phi\right]\,,
\tag{C2}
$$

where $\beta_\mu^{\,2}=1-4m_\mu^{\,2}/s$ and $\beta_e^{\,2}=1-4m_e^{\,2}/s$. This result is consistent with Godine and Hankey.[22] The total cross section is

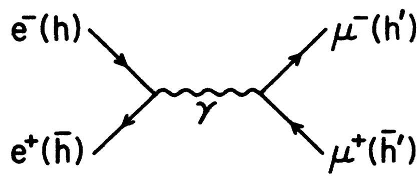

FIG. 10. The QED diagram for $e^+e^- \to \mu^+\mu^-$. Helicities of the leptons are indicated in parentheses.

$$
\sigma=\frac{4\pi\alpha^2}{3s}\frac{\beta_\mu}{\beta_e}\left[1+\frac{1}{2\gamma_\mu}\right]\left[1+\frac{1}{2\gamma_e}(1-P^2)\right]\,.
\tag{C3}
$$

If we take the limit $m_e \to 0$, the cross section does not depend on P even if we keep the muon mass finite.

The inverse reaction $\mu^+\mu^- \to e^+e^-$ at threshold is sensitive to the muon polarization (we take $m_e \to 0$)

$$
\frac{d\sigma}{d\Omega}\simeq\frac{\alpha^2}{4s\beta_\mu}[2-P^2\sin^2\theta(1+\cos 2\phi)]\,.
\tag{C4}
$$

At high energies, Z-exchange contribution to $e^+e^- \to \mu^+\mu^-$ becomes important. For $m_e \to 0$, nonzero amplitudes are the following:[132]

$$
\begin{aligned}
T_{+-}^{+-} &= -g_Z^{\,2}(v_e-a_e)(v_\mu-\beta_\mu a_\mu)\mathscr{P}(1+\cos\theta)\,,\\
T_{+-}^{\pm\pm} &= +g_Z^{\,2}(v_e-a_e)v_\mu\frac{1}{\gamma_\mu}\mathscr{P}\sin\theta\,,\\
T_{+-}^{-+} &= -g_Z^{\,2}(v_e-a_e)(v_\mu+\beta_\mu a_\mu)\mathscr{P}(1-\cos\theta)\,,\\
T_{-+}^{+-} &= -g_Z^{\,2}(v_e+a_e)(v_\mu-\beta_\mu a_\mu)\mathscr{P}(1-\cos\theta)\,,\\
T_{-+}^{\pm\pm} &= -g_Z^{\,2}(v_e+a_e)v_\mu\frac{1}{\gamma_\mu}\mathscr{P}\sin\theta\,,\\
T_{-+}^{-+} &= -g_Z^{\,2}(v_e+a_e)(v_\mu+\beta_\mu a_\mu)\mathscr{P}(1+\cos\theta)\,.
\end{aligned}
\tag{C5}
$$

Here

$$
\mathscr{P}=\frac{s}{s-M_Z^{\,2}+iM_Z\Gamma_Z}\,,
$$

and $g_Z=e/\sin\theta_W\cos\theta_W$, $v_e=v_\mu=-\frac{1}{4}+\sin^2\theta_W$, and $a_e=a_\mu=-\frac{1}{4}$.

The polarized cross section including both γ and Z exchanges for the limit m_e, $m_\mu \to 0$ is

$$
\begin{aligned}
\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\Bigg[&\left[1+\frac{2v_ev_\mu}{\sin^2\theta_W\cos^2\theta_W}\frac{s(s-M_Z^{\,2})}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}+\frac{(v_e^{\,2}+a_e^{\,2})(v_\mu^{\,2}+a_\mu^{\,2})}{\sin^4\theta_W\cos^4\theta_W}\frac{s^2}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}\right](1+\cos^2\theta)\\
&+\left[\frac{4a_ea_\mu}{\sin^2\theta_W\cos^2\theta_W}\frac{s(s-M_Z^{\,2})}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}+\frac{8v_ea_ev_\mu a_\mu}{\sin^4\theta_W\cos^4\theta_W}\frac{s^2}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}\right]\cos\theta\\
&-P^2\left[1+\frac{2v_ev_\mu}{\sin^2\theta_W\cos^2\theta_W}\frac{s(s-M_Z^{\,2})}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}+\frac{(v_e^{\,2}-a_e^{\,2})(v_\mu^{\,2}-a_\mu^{\,2})}{\sin^4\theta_W\cos^4\theta_W}\frac{s^2}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}\right]\sin^2\theta\cos 2\phi\\
&+P^2\frac{2a_ev_\mu}{\sin^2\theta_W\cos^2\theta_W}\frac{sM_Z\Gamma_Z}{(s-M_Z^{\,2})^2+M_Z^{\,2}\Gamma_Z^{\,2}}\sin^2\theta\,\sin 2\phi\Bigg]\,.
\end{aligned}
\tag{C6}
$$

This is a standard formula, but an interesting point lies in the last term proportional to $\sin 2\phi$, which comes from the absorptive part of the propagator. It provides a new method to measure the Z total width by the use of transverse polarization. At the top of the resonance, the angular distribution becomes

$$\frac{d\sigma}{d\Omega} \propto 1+\cos^2\theta+0.11\cos\theta-0.94P^2\sin^2\theta\cos2\phi+0.64P^2\frac{\Gamma_Z}{M_Z}\sin^2\theta\sin2\phi\ , \tag{C7}$$

where we take $\sin^2\theta_W=0.22$.

APPENDIX D: BHABHA SCATTERING

The amplitude for the reaction $e^+e^-\to e^+e^-$ has both annihilation and scattering contribution. The lowest-order QED graphs are shown in Fig. 11. The helicity amplitudes calculated from these diagrams with the electron mass kept finite are (at $\phi=0$; notations are the same as in Appendix C)

$$\begin{aligned}
T^{+-}_{+-}&=T^{-+}_{-+}=e^2\left[\frac{1+\beta^2}{\beta^2}\frac{1}{1-\cos\theta}-1\right](1+\cos\theta)\ ,\\
T^{-+}_{+-}&=T^{+-}_{-+}=e^2\left[\frac{1}{\gamma^2\beta^2}-(1-\cos\theta)\right]\ ,\\
T^{\pm\mp}_{\pm\pm}&=-T^{\mp\pm}_{\pm\pm}=-T^{\pm\pm}_{+-}=T^{\pm\pm}_{-+}=e^2\frac{1}{\gamma}\left[\frac{1}{\beta^2(1-\cos\theta)}-1\right]\sin\theta\ ,\\
T^{++}_{++}&=T^{--}_{--}=e^2\left[\frac{1+\beta^2}{\beta^2}\frac{2}{1-\cos\theta}-\frac{1+\beta^2}{\gamma^2\beta^2}+\frac{1}{\gamma^2}(1-\cos\theta)\right]\ ,\\
T^{--}_{++}&=T^{++}_{--}=e^2\left[-\frac{1+\beta^2}{\gamma^2\beta^2}+\frac{1}{\gamma^2}(1-\cos\theta)\right]\ ,
\end{aligned} \tag{D1}$$

where $\gamma=\sqrt{s}/2m_e$ and $\beta^2=1-4m_e{}^2/s$. These amplitudes are consistent with the results of Stehle.[133] It can be seen that the chiral symmetry becomes exact at high energies. The amplitudes with initial and final states having different chiral charges are suppressed by $\gamma^{-|\Delta X_e|}$ for $\gamma\to\infty$ [the third and last equations in (D1)].

At high energies (D1) becomes

$$\begin{aligned}
T^{+-}_{+-}&=T^{-+}_{-+}=e^2\left[\frac{2}{1-\cos\theta}-1\right](1+\cos\theta)\ ,\\
T^{-+}_{+-}&=T^{+-}_{-+}=-e^2(1-\cos\theta)\ ,\\
T^{++}_{++}&=T^{--}_{--}=\frac{4e^2}{1-\cos\theta}\ .
\end{aligned} \tag{D2}$$

The cross section for finite polarization (for $m_e\neq0$) is

$$\begin{aligned}
\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\Bigg\{&\frac{4(1+\beta^2)^2}{\beta^4}\frac{1}{(1-\cos\theta)^2}+\frac{2}{\beta^2}\left[-8+\frac{1}{\gamma^4}\right]\frac{1}{1-\cos\theta}+4\left[2+\frac{1}{\gamma^2}\right]+(1+\beta^2\cos\theta)^2\\
&+P^2\frac{1}{\gamma^2}\left[\frac{2(2+\beta^2)}{\beta^2}\frac{1}{1-\cos\theta}-3+\left[\frac{1}{\beta}+\beta\cos\theta\right]^2\right]\\
&+P^2\left[-\frac{2}{\gamma^2\beta^2}\frac{1}{1-\cos\theta}+\frac{1}{\gamma^2}+\left[\frac{1}{\beta}+\beta\cos\theta\right]^2\right]\cos2\phi\Bigg\}\ .
\end{aligned} \tag{D3}$$

The result for $P=0$ agrees with Bhabha.[134]

At high energies $s\gg m_e{}^2$, we have a very simple expression:

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\left[\frac{(3+\cos^2\theta)^2}{(1-\cos\theta)^2}+P^2(1+\cos\theta)^2\cos2\phi\right]\ . \tag{D4}$$

This fact makes the process favorable for probing new effects. They have been discussed in Secs. IV and V. In this appendix the orthodox QED results are shown for convenience. The effect of the electron mass is also stud-

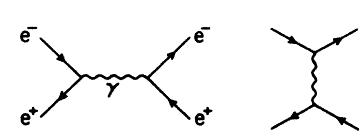

FIG. 11. QED diagrams for the Bhabha scattering.

APPENDIX E: $e^+e^-\to\gamma\gamma$ IN THE STANDARD MODEL

The lowest-order amplitude for the process $e^+e^-\to\gamma\gamma$ comes solely from the QED diagrams shown in Fig. 12.

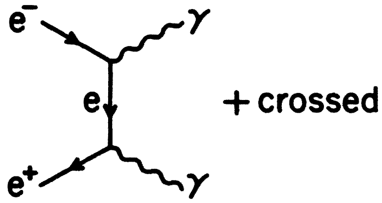

FIG. 12. The standard-model diagrams for $e^+e^- \to \gamma\gamma$.

ied.

The helicity amplitudes $T_{h\bar{h}}^{\lambda_1\lambda_2}$ [at $\phi=0$ (Ref. 82)] with $m_e \neq 0$ are

$$T_{h\bar{h}}^{\lambda_1\lambda_2} = \frac{-2e^2}{1-\beta^2\cos^2\theta} t_{h\bar{h}}^{\lambda_1\lambda_2} , \tag{E1}$$

with

$$\begin{aligned}
t_{+-}^{+-} &= -t_{-+}^{-+} = \beta(1+\cos\theta)\sin\theta , \\
t_{-+}^{+-} &= -t_{+-}^{-+} = \beta(1-\cos\theta)\sin\theta , \\
t_{+-}^{\pm\pm} &= t_{-+}^{\pm\pm} = 0 , \\
t_{\pm\pm}^{\pm\mp} &= t_{\pm\pm}^{\mp\pm} = \frac{\beta}{\gamma}\sin^2\theta , \\
t_{++}^{++} &= t_{--}^{--} = -\frac{1+\beta}{\gamma} , \\
t_{++}^{--} &= t_{--}^{++} = \frac{1-\beta}{\gamma} = \frac{1}{\gamma^3(1+\beta)} .
\end{aligned} \tag{E2}$$

The definition of β and γ is the same as in Appendix D. Note that $h(\bar{h}) = \pm$ refers to $\pm\frac{1}{2}$, whereas $\lambda_i = \pm$ means ± 1.

At high energies, we have much simpler expressions

$$\begin{aligned}
T_{+-}^{+-} &= -T_{-+}^{-+} = -2e^2 \frac{1+\cos\theta}{\sin\theta} , \\
T_{-+}^{+-} &= -T_{+-}^{-+} = -2e^2 \frac{1-\cos\theta}{\sin\theta} ,
\end{aligned} \tag{E3}$$

and other amplitudes vanish.

The cross section is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s\beta} \frac{1}{(1-\beta^2\cos^2\theta)^2} \left[1 - \beta^4\cos^4\theta + \frac{2\beta^2}{\gamma^2}\sin^2\theta - P^2 \left[\beta^2\sin^4\theta\cos 2\phi + \frac{\beta^2}{\gamma^2}\sin^4\theta - \frac{1}{\gamma^4} \right] \right] . \tag{E4}$$

This is consistent with Page.[19]

We study several limiting cases. At high energies, we have

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s} \left[\frac{1+\cos^2\theta}{\sin^2\theta} - P^2\cos 2\phi \right] . \tag{E5}$$

At threshold, the polarization effect is maximal

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{s\beta}(1+P^2) + O(\beta) . \tag{E6}$$

In the forward direction

$$\left.\frac{d\sigma}{d\Omega}\right|_{\theta=0} = \frac{\alpha^2}{s\beta}[\gamma^2(1+\beta^2) + P^2] . \tag{E7}$$

Here there is no contribution for chirality-conserving amplitudes T_{+-} and T_{-+}. However, the polarization effect is negligible at high energies. Despite the caution noted by Baier and Khoze,[38] the total cross section does not depend on the polarization up to $O(m_e{}^2/s)$ although the contribution of chirality-nonconserving amplitudes is suppressed only by a factor of $\ln\gamma$:

$$\sigma_{\text{tot}} = \frac{\pi\alpha^2}{s\beta} \left[(3-\beta^4)\frac{1}{\beta}\ln\frac{1+\beta}{1-\beta} - 2(2-\beta^2) - P\left[\frac{3-2\beta^2}{\gamma^2\beta^2} - \frac{3+2\beta^2}{2\gamma^4\beta^3}\ln\frac{1+\beta}{1-\beta} \right] \right] . \tag{E8}$$

APPENDIX F: SCALAR-ELECTRON PAIR PRODUCTION

In this appendix we list all amplitudes for various combinations of the reactions $e^+e^- \to \tilde{e}^+\tilde{e}^-$.

1. Chiral case

The mass eigenstates are $\tilde{e}_R$ and $\tilde{e}_L$. The interaction with the electron is given by (5.5).

For $e^+e^- \to \tilde{e}_L^+\tilde{e}_L^-$ [all helicity amplitudes in this appendix are for $\phi=0$ (Ref. 125)]

$$\begin{aligned}
T_{+-}^{LL} &= -e^2\beta_L\sin\theta , \\
T_{-+}^{LL} &= e^2\beta_L \left[1 + \frac{s}{t'} \right] \sin\theta , \\
T_{++}^{LL} &= T_{--}^{LL} = 0 ,
\end{aligned} \tag{F1}$$

$$\frac{d\sigma^{LL}}{d\Omega} = \frac{\alpha^2\beta_L{}^3}{16s}\sin^2\theta \left[1 + \left[1 + \frac{s}{t'} \right]^2 + 2P^2 \left[1 + \frac{s}{t'} \right] \cos 2\phi \right] . \tag{F2}$$

For $e^+e^- \to \tilde{e}_R^+\tilde{e}_L^-$ (there is no photon-exchange contribution since the electromagnetic coupling is diagonal; this is no longer true for $\tilde{z}$ exchange, however),

$$\begin{aligned}
T_{--}^{LR} &= 2e^2 \frac{\sqrt{s}\, m_{\tilde{\gamma}}}{t'} , \\
T_{++}^{LR} &= T_{+-}^{LR} = T_{-+}^{RL} = 0 ,
\end{aligned} \tag{F3}$$

$$\frac{d\sigma^{LR}}{d\Omega} = \frac{\alpha^2\beta_{LR} m_{\tilde{\gamma}}{}^2}{4t'^2} , \tag{F4}$$

where

$$\beta_{LR}{}^2 = [1-(m_{\tilde{e}_L}+m_{\tilde{e}_R})^2/s][1-(m_{\tilde{e}_L}-m_{\tilde{e}_R})^2/s] . \tag{F5}$$

For $e^+e^-\to\tilde{e}_L^+\tilde{e}_R^-$

$$T^{RL}_{++}=2e^2\frac{\sqrt{s}\,m_{\tilde{\gamma}}}{t'}\ ,\qquad T^{RL}_{+-}=T^{RL}_{-+}=T^{RL}_{--}=0\ , \tag{F6}$$

$$\frac{d\sigma^{RL}}{d\Omega}=\frac{d\sigma^{LR}}{d\Omega}\ . \tag{F7}$$

The results for $e^+e^-\to\tilde{e}_R^+\tilde{e}_L^-$ are given in Sec. V B. Note that the chiral charge [assigned as (3.3) and (5.2)] is conserved in each reaction. Thus the null theorem holds for this case.

2. Parity-conserving case

Mass eigenstates are $\tilde{e}_S$ and $\tilde{e}_P$. The relevant interaction Lagrangian is given by (5.6). Results for $e^+e^-\to\tilde{e}_S^+\tilde{e}_S^-$ are given by (5.9) and (5.10).

For $e^+e^-\to\tilde{e}_P^+\tilde{e}_P^-$

$$T^{PP}_{+-}=-T^{PP}_{-+}=-e^2\beta_P\left[1+\frac{s}{2t'}\right]\sin\theta\ ,\qquad T^{PP}_{++}=T^{PP}_{--}=e^2\frac{\sqrt{s}\,m_{\tilde{\gamma}}}{t'}\ , \tag{F8}$$

$$\frac{d\sigma^{PP}}{d\Omega}=\frac{d\sigma^{SS}}{d\Omega}\ . \tag{F9}$$

For $e^+e^-\to\tilde{e}_S^+\tilde{e}_P^-$

$$T^{PS}_{+-}=T^{PS}_{-+}=-e^2\beta_{SP}\frac{s}{2t'}\sin\theta\ ,\qquad T^{PS}_{++}=-T^{PS}_{--}=-e^2\frac{\sqrt{s}\,m_{\tilde{\gamma}}}{t'}\ , \tag{F10}$$

$$\frac{d\sigma^{PS}}{d\Omega}=\frac{\alpha^2\beta_{SP}}{8s}\left[\beta_{SP}{}^2\sin^2\theta\left[\frac{s}{2t'}\right]^2(1-P^2\cos2\phi)+(1+P^2)\frac{s m_{\tilde{\gamma}}{}^2}{t'^2}\right]\ , \tag{F11}$$

where β_{SP} is defined by (F5) with $L,R\to S,P$.

For $e^+e^-\to\tilde{e}_P^+\tilde{e}_S^-$

$$T^{SP}_{\pm\mp}=T^{PS}_{\pm\mp}\ ,\qquad T^{SP}_{\pm\pm}=-T^{PS}_{\pm\pm}\ , \tag{F12}$$

$$\frac{d\sigma^{SP}}{d\Omega}=\frac{d\sigma^{PS}}{d\Omega}\ . \tag{F13}$$

If the two scalar-electron states are degenerate, the sum of the cross sections for $e^+e^-\to\tilde{e}^+\tilde{e}^-$ becomes

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2\beta}{4s}\left\{\beta^2\sin^2\theta\left[\left[1+\frac{s}{t'}+\frac{s^2}{2t'^2}\right]+P^2\left[1+\frac{s}{t'}\right]\cos2\phi\right]+\frac{2sm_{\tilde{\gamma}}{}^2}{t'^2}\right\}\ . \tag{F14}$$

The two cases give the same results as they should because if the two masses are the same, the choice of the bases is arbitrary.

APPENDIX G: PHOTINO PAIR PRODUCTION

Here we give the results for the reaction $e^+e^-\to\tilde{\gamma}\tilde{\gamma}$ without the approximation $s\ll m_{\tilde{e}}{}^2$. The relevant diagram is shown in Fig. 9. Notations are the same as in Sec. V C.

1. Chiral case, $m_{\tilde{e}_R}\ll m_{\tilde{e}_L}$

The helicity amplitudes for $\phi=0$ are

$$T^{+-}_{+-}=\tfrac{1}{2}e^2s\left[\frac{1-\tilde{\beta}}{t'}-\frac{1+\tilde{\beta}}{u'}\right](1+\cos\theta)\ ,\qquad T^{\pm\pm}_{+-}=-\tfrac{1}{2}e^2s\cdot\frac{1}{\tilde{\gamma}}\left[\frac{1}{t'}-\frac{1}{u'}\right]\sin\theta\ ,\qquad T^{-+}_{+-}=\tfrac{1}{2}e^2s\left[\frac{1+\tilde{\beta}}{t'}-\frac{1-\tilde{\beta}}{u'}\right](1-\cos\theta)\ , \tag{G1}$$

where

$$t'=t-m_{\tilde{e}_R}{}^2=-\frac{s}{2}(1+\delta_R-\tilde{\beta}\cos\theta)\ ,\qquad u'=u-m_{\tilde{e}_R}{}^2=-\frac{s}{2}(1+\delta_R+\tilde{\beta}\cos\theta)\ , \tag{G2}$$

where $\delta_R=2(m_{\tilde{e}_R}{}^2-m_{\tilde{\gamma}}{}^2)/s$. The cross section is

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2\tilde{\beta}^3}{2s}\frac{(\delta_R+\sin^2\theta)^2+\delta_R{}^2\cos^2\theta+\tilde{\gamma}^{-2}\cos^2\theta\sin^2\theta}{[(1+\delta_R)^2-\tilde{\beta}^2\cos^2\theta]^2}\ . \tag{G3}$$

This is consistent with Ref. 98. There is no polarization dependence.

2. Parity-conserving case, $m_{\tilde{e}_S}\ll m_{\tilde{e}_P}$

The amplitudes are

$$T^{+-}_{+-}=T^{-+}_{-+}=\tfrac{1}{4}e^2s\left[\frac{1-\tilde{\beta}}{t'}-\frac{1+\tilde{\beta}}{u'}\right](1+\cos\theta)\ ,$$
$$T^{-+}_{+-}=T^{+-}_{-+}=\tfrac{1}{4}e^2s\left[\frac{1+\tilde{\beta}}{t'}-\frac{1-\tilde{\beta}}{u'}\right](1-\cos\theta)\ ,$$
$$T^{\pm\pm}_{+-}=-T^{+-}_{\pm\pm}=T^{-+}_{\pm\pm}=-T^{\pm\pm}_{-+}=-\tfrac{1}{4}e^2s\left[\frac{1}{t'}-\frac{1}{u'}\right]\cdot\frac{1}{\tilde{\gamma}}\sin\theta\ ,$$
$$T^{++}_{++}=T^{--}_{--}=\tfrac{1}{4}e^2s(1-\tilde{\beta})\left[\frac{1+\cos\theta}{t'}+\frac{1-\cos\theta}{u'}\right]\ ,$$
$$T^{--}_{++}=T^{++}_{--}=-\tfrac{1}{4}e^2s(1+\tilde{\beta})\left[\frac{1-\cos\theta}{t'}+\frac{1+\cos\theta}{u'}\right]\ , \tag{G4}$$

where $t'=t-m_{\tilde{e}_S}{}^2$, etc. The cross section is

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2\tilde{\beta}}{4s}\left[\frac{1}{[(1+\delta_S)^2-\tilde{\beta}^2\cos^2\theta]^2}\left[2\tilde{\beta}^2(\delta_S+\sin^2\theta)^2+(1+\delta_S-\tilde{\beta}^2\cos^2\theta)^2+\tilde{\beta}^2\delta_S{}^2\cos^2\theta+\frac{2\tilde{\beta}^2}{\tilde{\gamma}^2}\cos^2\theta\sin^2\theta\right]\right.$$
$$\left.+P^2\cdot\frac{1}{(1+\delta_S)^2-\tilde{\beta}^2\cos^2\theta}\left[\frac{1}{\tilde{\gamma}^2}+\tilde{\beta}^2\sin^2\theta\cos 2\phi\right]\right]. \tag{G5}$$

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