Ambiguities of the chiral-anomaly graph in higher dimensions

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We compute vector-current and axial-vector-current divergence relations for the chiral-anomaly graph in $2n$ dimensions. We demonstrate that ambiguities arising from (i) arbitrariness in the routing of loop momenta in $2n$-dimensional Feynman integrals, (ii) arbitrariness associated with the location of γ^{2n+1} within γ-matrix traces in dimensional regularization, and (iii) arbitrariness associated with the ordering of matrices within traces in dimensional reduction all lead to equivalent relations between vector-current and axial-vector-current divergences in the absence of externally imposed constraints of vector-current conservation and Bose symmetry. The anomaly is shown to reside in an alternating sum of current divergences. We also show that further ambiguities, associated with the projection of less-than-$2n$-dimensional momenta onto $2n$-dimensional γ matrices, occur within dimensional reduction.

I. INTRODUCTION

The importance of the chiral anomaly in the development of particle physics in the last 16 years can hardly be underestimated.[1] Recently there has been a growing interest in the analysis of anomalies in higher spacetime dimensions. This is not only because of the constraints imposed on physical theories by the anomaly condition,[2] but also because of the deep mathematical significance of anomalies.[1] Indeed, it has been shown that chiral anomalies can be determined by differential-geometric methods that entirely avoid evaluation of Feynman diagrams.[1] Of course the chiral anomaly has been evaluated in higher dimensions using diagrammatic techniques.[2,3] However, such techniques invariably use regulating methods that impose vector-current conservation and Bose symmetry; the usual $2n$-dimensional result is only obtained after *a priori* imposition of these physical constraints.

Of crucial importance to the evaluation of the chiral anomaly in four dimensions is the well-known fact that naive shifts of the integration variable in more than logarithmically divergent integrals are not permitted.[4] (This result has recently been generalized to 2ω dimensions.[5,6]) As pointed out by Adler,[7] there would be no anomaly if such shifts were allowed; indeed, Frampton and Kephart's evaluation of the $2n$-dimensional chiral anomaly[2] makes use of finite surface terms which arise from such shifts. They obtain the anomaly after imposing vector-current conservation.

In this paper we demonstrate how regularization procedures in which naive shifts of integration variable *are* allowed [specifically, conventional dimensional regularization (CDR) and regularization by dimensional reduction (RDR)] reproduce results obtained explicitly in $2n$ dimensions (where n is an integer) from finite surface terms associated with shifts of integration variable in divergent Feynman integrals. In particular, we examine V^nA $(n+1)$-agon graph ambiguities which occur in all three procedures [i.e., $2n$-dimensional integration, $(2n+\epsilon)$-dimensional integration (CDR), and $(2n-\epsilon)$-dimensional integration (RDR)] in the absence of constraints imposed by Bose symmetry and vector-current conservation. It will be shown that equivalent relations arise in all three procedures graph by graph between vector-current and axial-vector-current divergences. The anomaly will be seen to reside in the sum of these divergences.

Such equivalence between these procedures for the VVA $(2+1)$-agon (triangle) graph has already been demonstrated.[8] Remarkably enough, this equivalence carries over to any even dimension, as we will show. We stress that this is true despite differing parametrizations of the ambiguities inherent in each procedure. Indeed, each procedure introduces arbitrary parameters associated with these ambiguities. Since the number of parameters is n for a given graph, and since there are $n!$ graphs in the amplitude in $2n$ dimensions, the number of these parameters grows as $n \times n!$. In $2n$ dimensions the parameters arise from the arbitrariness in the loop momentum routing; this leads to additional surface terms which contribute to vector-current and axial-vector-current divergences. In $2n+\epsilon$ dimensions (CDR), the obtained result depends upon whether or not one chooses to anticommute γ^{2n+1} through propagator traces prior to continuing to $2n+\epsilon$ dimensions.[3,9] In RDR one must (selectively) abandon the property of trace cyclicity of Dirac γ matrices; in this case the ambiguity is associated with the choice of vertex that may occur at the beginning of the $(n+1)$-agon trace. As expected, the usual $2n$-dimensional chiral anomaly is obtained for choices in each of the three procedures which are consistent with vector-current conservation. What we find remarkable is that all current divergences in each procedure are graph-by-graph equivalent. despite completely differing ambiguity parametrizations that have numbers of arbitrary parameters that grow faster than $n!$.

We shall use the following notation. We shall denote by $\Gamma_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$ the Green's function corresponding to $\langle 0 | T(J^{2n+1}_{\alpha_0}(0) J_{\alpha_1}(x_1) \cdots J_{\alpha_n}(x_n)) | 0 \rangle$ where J^{2n+1}_μ denotes the axial-vector current $\bar{\psi}\gamma_\mu \gamma^{2n+1}\psi$, and J_μ denotes the vector current $\bar{\psi}\gamma_\mu\psi$. All external momenta shall be taken to be incoming; for $J_{\alpha_i}(x_i)$ we shall use q_i. The basic $V^n A$ $(n+1)$-agon graph is shown in Fig. 1; by momentum conservation $\sum_{i=0}^{n} q_i^\mu = 0$. We shall denote this graph by $S_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$; the full amplitude is given by

$$\Gamma_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n) = \sum_{\text{perms }(\alpha_i, q_i)} S_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n) ; \tag{1.1}$$

i.e., it is obtained by adding all graphs of the form of Fig. 1 with indices α_i and momenta q_i permuted $(i = 1, \ldots, n)$. Standard techniques for obtaining the anomaly[1–3,9] involve Bose symmetrization of the external momenta and imposition of vector-current conservation $(q_r^{\alpha_r}\Gamma_{\alpha_0 \cdots \alpha_r \cdots \alpha_n} = 0;\ r \neq 0)$; the anomaly is then obtained by computing $q_0^{\alpha_0}\Gamma_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$ in a procedure that respects the aforementioned constraints. In this paper we shall compute the quantities $q_r^{\alpha_r} S_{\alpha_0 \cdots \alpha_r \cdots \alpha_n}$ and $q_0^{\alpha_0} S_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$ in each of the three procedures previously mentioned without imposing Bose symmetry or vector-current conservation. These quantities will be shown to be equivalent in each procedure; from this the equivalence of $\Gamma_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$ in each procedure follows. Furthermore, we shall show that

$$\sum_{r=0}^{n} (-1)^r q_r^\mu S_{\alpha_1 \cdots \alpha_r \mu \alpha_{r+1} \cdots \alpha_n} = -\frac{2}{(2\pi)^n n!} \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} \tag{1.2}$$

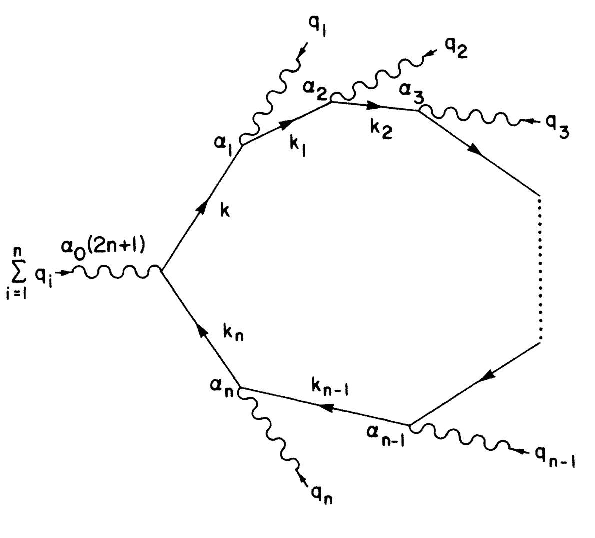

FIG. 1. The generic chiral-anomaly graph in $2n$ dimensions.

which leads to

$$\sum_{r=0}^{n} (-1)^r q_r^\mu \Gamma_{\alpha_1 \cdots \alpha_r \mu \alpha_{r+1} \cdots \alpha_n} = -\frac{2}{(2\pi)^n} \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} \tag{1.3}$$

showing that the anomaly resides in the (alternating) *sum* of the current divergences in each procedure. [Note that the right-hand sides of (1.2) and (1.3) are automatically Bose symmetric.] When vector-current conservation is imposed, we obtain, for (1.3),

$$q_0^{\alpha_0}\Gamma_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n) = \frac{-2}{(2\pi)^n} \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} \tag{1.4}$$

which is the usual expression for the anomaly. Finally, we shall also demonstrate that for $n = 2k+1$, the relation (1.2) "splits" to become

$$\sum_{r=0}^{n} q_{2r}^\mu S_{\alpha_1 \cdots \alpha_{2r}\mu\alpha_{2r+1} \cdots \alpha_n} = \frac{-1}{(2\pi)^n n!} \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} \tag{1.5}$$

and

$$\sum_{r=0}^{n} q_{2r+1}^\mu S_{\alpha_1 \cdots \alpha_{2r+1}\mu\alpha_{2r+2} \cdots \alpha_n} = \frac{1}{(2\pi)^n n!} \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} . \tag{1.6}$$

The outline of this paper is as follows. In Sec. II we compute the current divergence relations using $2n$-dimensional integration variable shift techniques.[5] Such techniques have been successfully employed elsewhere in perturbative field-theory computations;[10] such a procedure has been called preregularization.[11] In Secs. III and IV current divergence relations are computed in CDR and RDR, respectively. In Sec. V we demonstrate the equivalence of the differing ambiguities and show that the anomaly resides in the sum of the current divergences. Finally, we summarize our work in a concluding section.

II. $2n$ DIMENSIONS

The $V^n A$ $(n+1)$-agon graph $S_{\alpha_0 \cdots \alpha_n}$ of Fig. 1 is given by

$$S_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n) = \int \frac{d^{2n}k}{(2\pi)^{2n}} \mathrm{Tr}\left[\gamma^{2n+1} \prod_{j=0}^{n} \gamma_{\alpha_j} \not{k}_j^{-1}\right] , \tag{2.1}$$

where k_r^μ is defined to be

$$k_r^\mu \equiv (k+s)^\mu + \sum_{i=1}^{r} q_j^\mu \tag{2.2}$$

(which is easily seen from Fig. 1), and $k_0^\mu \equiv (k+s)^\mu$.

The rth vector-current divergence is

$$q_r^{\alpha_r} S_{\alpha_0 \cdots \alpha_r \cdots \alpha_n} = \int \frac{d^{2n}k}{(2\pi)^{2n}} \mathrm{Tr} \left[\gamma^{2n+1} \prod_{j=0}^{r-1} \gamma_{\alpha_j} \not{k}_j^{-1} \prod_{l=r+1}^{n} \gamma_{\alpha_l} \not{k}_l^{-1} - \gamma^{2n+1} \sum_{j=0}^{r-2} \gamma_{\alpha_j} \not{k}_j^{-1} \gamma_{\alpha_{r-1}} \not{k}_r^{-1} \prod_{l=r+1}^{n} \gamma_{\alpha_l} \not{k}_l^{-1} \right] \quad (2.3)$$

which straightforwardly follows from (2.1) upon using

$$\not{q}_r = \not{k}_r - \not{k}_{r-1} \ . \quad (2.4)$$

Although the quantity in (2.3) is the difference between two divergent integrals (diverging as k^{n+1}), each term may be computed separately as follows. First we use

$$\mathrm{Tr}(\gamma^{2n+1}\gamma_{\mu_1} \cdots \gamma_{\mu_{2n}}) = 2^n i \epsilon_{\mu_1 \cdots \mu_{2n}} \quad (2.5)$$

to write the general form of such integrals as

$$\int \frac{d^{2n}k}{(2\pi)^{2n}} \mathrm{Tr} \left[\gamma^{2n+1} \prod_{j=0}^{n-1} \gamma_{\alpha_j} \not{k}_j^{-1} \right]$$

$$= (n-1)! \int \frac{d^{2n}k}{(2\pi)^{2n}} \int \prod_{l=0}^{n-1} dx_l \delta \left[\sum_{m=0}^{n-1} x_m - 1 \right] (2^n i \epsilon_{\mu_1 \cdots \mu_n}) \frac{(k+s)^{\rho_0} \cdots \left[k+1+\sum_{j=1}^{n-1} q_j \right]^{\rho^{n-1}}}{\left[\left[k+s+\sum_{i=1}^{n} \sum_{j=1}^{i} q_j x_i \right]^2 - D \right]^n} , \quad (2.6)$$

where D is some Feynman-parameter combination of external momenta. The expression in (2.6) is actually only linearly divergent, because the ϵ tensor cancels out all but one power of k in the numerator. Upon shifting $k^\mu \to k^\mu \to s^\mu - \sum_{i=1}^{n} \sum_{j=1}^{i} q_j x_i$, the results of Ref. 5 give

$$\int \frac{d^{2n}k}{(2\pi)^{2n}} \mathrm{Tr} \left[\gamma^{2n+1} \prod_{j=0}^{n-1} \gamma_{\alpha_j} \not{k}_j^{-1} \right]$$

$$= (n-1)!(2^n i \epsilon_{\mu_1 \cdots \mu_n}) \left[\int \frac{d^{2n}k}{(2\pi)^{2n}} \int \prod_{l=0}^{n-1} dx_l \delta \left[\sum_{m=0}^{n-1} x_m - 1 \right] \frac{\left[\sum_{i=1}^{n} \sum_{j=1}^{i} q_j x_i \right]^{\rho_0} q_1^{\rho_1} \cdots q_{n-1}^{\rho_{n-1}}}{(k^2-D)^n} \right.$$

$$\left. + \frac{1}{(2\pi)^n} \int \prod_{l=0}^{n-1} dx_l \delta \left[\sum_{m=0}^{n-1} x_m - 1 \right] \left[\frac{-i\pi}{n!} \right] \left[-s - \sum_{i=1}^{n} \sum_{j=1}^{i} q_j x_i \right]^{\rho_0} q_1^{\rho_1} \cdots q_{n-1}^{\rho_{n-1}} \right]$$

$$= -\frac{1}{(2\pi)^n n!} \epsilon_{\alpha_0 \rho_0 \cdots \alpha_{n-1} \rho_{n-1}} s^{\rho_0} q_1^{\rho_1} \cdots q_{n-1}^{\rho_{n-1}} \quad (2.7)$$

since all other terms vanish upon contraction with the ϵ tensor.

Using (2.7) in (2.3), we obtain

$$q_r^{\alpha_r} S_{\alpha_0 \cdots \alpha_r \cdots \alpha_n} = -\frac{1}{(2\pi)^n n!} \epsilon_{\alpha_0 \rho_0 \cdots \alpha_{r-1} \rho_{r-1} \alpha_{r+1} \rho_{r+1} \cdots \alpha_n \rho_n} [s^{\rho_0} q_1^{\rho_1} \cdots q_{r-2}^{\rho_{r-2}} (q_{r-1}^{\rho_{r-1}} q_r^{\rho_{r+1}} - q_r^{\rho_{r-1}} q_{r+1}^{\rho_{r+1}}) q_{r+2}^{\rho_{r+2}} \cdots q_n^{\rho_n}] \ . \quad (2.8)$$

We now parametrize the loop-momentum-routing ambiguity s^μ in terms of the external momenta:

$$s^\mu = \sum_{r=1}^{n} A_r q_r^\mu \ . \quad (2.9)$$

This is an n-fold ambiguity for the graph $S_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$; there are $n!$ such graphs, yielding a full $n \times n!$-fold ambiguity in $\Gamma_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n)$ in this procedure. Insertion of (2.9) into (2.8) gives

$$q_r^{\alpha_r} S_{\alpha_0 \cdots \alpha_r \cdots \alpha_n} = (-1)^{r+1} \frac{1}{(2\pi)^n n!} \epsilon_{\alpha_0 \rho_1 \cdots \alpha_{r-1} \rho_r \alpha_{r+1} \rho_{r+1} \cdots \alpha_n \rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} (A_{r+1} - A_{r-1}) , \quad (2.10)$$

where $A_0 \equiv (-1)$ and $A_{n-1} \equiv 0$.

For the axial-vector-current divergence we obtain

$$q_0^{\alpha_0} S_{\alpha_0 \cdots \alpha_n} = \int \frac{d^{2n}k}{(2\pi)^{2n}} \mathrm{Tr} \left[\gamma^{2n+1} \prod_{j=0}^{n-1} \not{k}_j^{-1} \gamma_{\alpha_{j+1}} + \gamma^{2n+1} \prod_{j=1}^{n} \gamma_{\alpha_j} \not{k}_j^{-1} \right] = \frac{-1}{(2\pi)^n n!} \epsilon_{\alpha_1 \rho_1 \cdots \alpha_n \rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} (A_1 - A_n + 1) \quad (2.11)$$

which follows from (2.7).

III. $2n+\epsilon$ DIMENSIONS

In CDR, V^nA $(n+1)$-agon graph integrals are computed in $2\omega > 2n$ dimensions; we write $\omega = n+\epsilon/2$. No shift-of-integration variable surface terms occur;[5,9] instead, analogous to the $(4+\epsilon)$-dimensional case, we require that γ^{2n+1} commutes with the γ_d's corresponding to (unphysical) dimensions $2n < d \le 2\omega$, and anticommutes with the remaining γ_d's $(d \le 2n)$. Hence for physical momenta q_α $(q_\alpha = 0$ for $\alpha > 2n)$ and loop momenta k defined in $2n+\epsilon$ dimensions it is easy to show that

$$(\not k+\not q)^{-1}\not q\gamma^{2n+1}\not k^{-1}=\gamma^{2n+1}\not k^{-1}+(\not k+\not q)^{-1}\gamma^{2n+1}+(\not k+\not q)^{-1}(-2\gamma^{2n+1}\not e)\not k^{-1}\,, \tag{3.1}$$

where $e_\alpha \equiv [0(\alpha \le 2n); k_\alpha(2n < \alpha \le 2\omega)]$. The last term in (3.1) is absent for $\omega = n$. If such a term were absent, then the freedom to naively shift variables of integration in 2ω dimensions for $\omega \ne n$ would render *all* current divergences vanishing. Consequently, all anomalous contributions to current divergences must arise from the last term in (3.1).

The presence of such a term implies that the basic V^nA graph of Fig. 1 has an n-fold ambiguity associated with arbitrary anticommutations of γ^{2n+1} prior to continuation to 2ω dimensions.[3,9] This arbitrariness may be parametrized as[8]

$$\begin{aligned}S_{\alpha_0\cdots\alpha_n}(q_1,\ldots,q_n)=\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}[&B_0\mathrm{Tr}(\gamma_{\alpha_0}\gamma^{2n+1}\not k^{-1}\gamma_{\alpha_1}\not k_1{}^{-1}\cdots\gamma_{\alpha_n}\not k_n{}^{-1})+\cdots\\&+B_r\mathrm{Tr}(\gamma_{\alpha_0}\not k^{-1}\gamma_{\alpha_1}\not k_1{}^{-1}\cdots\gamma_{\alpha_r}\gamma^{2n+1}\not k_r{}^{-1}\cdots\gamma_{\alpha_n}\not k_n{}^{-1})+\cdots\\&+B_n\mathrm{Tr}(\gamma_{\alpha_0}\not k^{-1}\gamma_{\alpha_1}\not k_1{}^{-1}\cdots\gamma_{\alpha_n}\gamma^{2n+1}\not k_n{}^{-1})]\\=\sum_{r=0}^{n}\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}&B_r\mathrm{Tr}\left[\prod_{j=0}^{r}\gamma_{\alpha_{j-1}}\not k_{j-1}{}^{-1}(\gamma_{\alpha_r}\gamma^{2n+1}\not k_r{}^{-1})\prod_{l=r+1}^{n}\gamma_{\alpha_l}\not k_l{}^{-1}\right].\end{aligned} \tag{3.2}$$

Although superficially it appears that there are $(n+1)$ parameters B_i, these parameters must obey the constraint

$$\sum_{i=0}^{n}B_i=1 \tag{3.3}$$

since the normalization of $S_{\alpha_0\cdots\alpha_n}(q_1,\ldots,q_n)$ must not change upon continuation from $2n$ to 2ω dimensions. This yields an n-fold ambiguity for $S_{\alpha_0\cdots\alpha_n}(q_1,\ldots,q_n)$, and hence an $n\times n!$-fold ambiguity for $\Gamma_{\alpha_0\cdots\alpha_n}(q_1,\ldots,q_n)$ in CDR.

Computation of current divergences is simple because (a) anomalous terms can occur only when γ^{2n+1} is adjacent to the γ matrix which is contracted into the external momenta (all other terms vanish upon shifting the variable of integration), (b) all terms vanish unless numerator traces are quadratic in $\not e$ (this will be understood momentarily), and (c) the only other factors of e^μ present are in the loop momenta k^μ. From this we obtain

$$\begin{aligned}q_r^{\alpha_r}S_{\alpha_0\cdots\alpha_r\cdots\alpha_n}&=B_r\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\mathrm{Tr}\left[\prod_{j=0}^{r-1}\gamma_{\alpha_j}\not k_j{}^{-1}(-2\gamma^{2n+1}\not e)\not k_r{}^{-1}\prod_{l=r+1}^{n}\gamma_{\alpha_l}\not k_l{}^{-1}\right]\\&=-2B_r\left[\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\frac{1}{P}\right]\left[\sum_{t=0}^{r-1}\mathrm{Tr}\left[\prod_{j=0}^{t}\gamma_{\alpha_{j-1}}\not k_{j-1}\gamma_{\alpha_t}\not e\sum_{m=t+1}^{r-1}\gamma_{\alpha_m}\not k_m\gamma^{2n+1}\not e\not k_r\prod_{l=r+1}^{n}\gamma_{\alpha_l}\not k_l\right]\right.\\&\qquad+\sum_{t=r+2}^{n}\mathrm{Tr}\left[\prod_{j=0}^{r-1}\gamma_{\alpha_j}\not k_j\gamma^{2n+1}\not e\not k_r\prod_{l=r+1}^{t}\gamma_{\alpha_{l-1}}\not k_{l-1}\gamma_{\alpha_t}\not e\prod_{m=t+1}^{n}\gamma_{\alpha_m}\not k_m\right]\\&\qquad\left.+\mathrm{Tr}\left[\prod_{j=0}^{r-1}\gamma_{\alpha_j}\not k_j\gamma^{2n+1}\not e^2\prod_{l=r+1}^{n}\gamma_{\alpha_l}\not k_l\right]\right],\end{aligned} \tag{3.4}$$

where P is given by

$$P=\prod_{i=0}^{n}k_i{}^2\,. \tag{3.5}$$

Using (2.5), we get

$$q_r^{\alpha_r} S_{\alpha_0 \cdots \alpha_r \cdots \alpha_n} = -2B_r \left[\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{e^2}{P} \right] (2^n i \epsilon_{\alpha_0\rho_0 \cdots \alpha_{r-1}\rho_{r-1}\alpha_{r+1}\rho_{r+1} \cdots \alpha_n\rho_n}) \times \left[\sum_{j=0}^{n} (-1)^{j+1} k_0^{\rho_0} \cdots k_{j-1}^{\rho_{j-1}} k_{j+1}^{\rho_{j+1}} \cdots k_n^{\rho_n} (-1)^r \right]$$

$$= 2(-1)^{r+1} B_r \left[\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{e^2}{P} \right] (2^n i \epsilon_{\alpha_0\rho_1 \cdots \alpha_{r-1}\rho_r\alpha_{r+1}\rho_{r+1} \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n}) \tag{3.6}$$

due to the antisymmetry of the ϵ tensor.

We can simplify (3.6) further in the $\omega \to n$ limit using the 2ω-dimensional result[3,8,9]

$$\lim_{\omega \to n} \left[\int \frac{d^{2\omega}k}{(2\pi)^{2\omega}} \frac{e^2}{(k^2 - D)^{n+1}} \right] = -\frac{i\pi^n}{(2\pi)^{2n} n!} . \tag{3.7}$$

Only e^2 terms can give a nonzero result in (3.7); higher powers of e vanish for $\omega \to n$. Hence we obtain

$$q_r^{\alpha_r} S_{\alpha_0 \cdots \alpha_r \cdots \alpha_n} = (-1)^{r+1} \frac{2B_r}{(2\pi)^n n!} \epsilon_{\alpha_0\rho_1 \cdots \alpha_{r-1}\rho_r\alpha_{r+1}\rho_{r+1} \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots q_n^{\rho_n} \tag{3.8}$$

for the vector-current divergence on the rth index.

The axial-vector-current divergence is obtained in a completely analogous manner:

$$q_0^{\alpha_0} S_{\alpha_0 \cdots \alpha_n}(q_1, \ldots, q_n) = -\frac{2B_0}{(2\pi)^n n!} \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} q_1^{\rho_1} \cdots \rho_n^{\rho_n} . \tag{3.9}$$

IV. $2n - \epsilon$ DIMENSIONS

We now consider RDR, a procedure in which momentum vectors and loop integrals are computed in $2\omega < 2n$ dimensions.[12] As a result, γ^{2n+1} is fully anticommuting and naive shifts of integration variable are also allowed. In $4-\epsilon$ dimensions, the usual chiral anomaly is obtained by abandoning trace cyclicity of γ matrices; departures from trace cyclicity are proportional to ϵ (Ref. 13). We will assume this ansatz in $2n - \epsilon$ dimensions; specifically, consider

$$\mathrm{Tr}[\gamma^{2n+1}(\not{a} - \not{b})\not{a}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}\not{b}] . \tag{4.1}$$

If a, b, and c are $2n$-dimensional momenta, the first $\not{b}$ could be anticommuted through γ^{2n+1} and then trace cyclicity could be used to attach it to the other end of the trace. However, in $2\omega < 2n$ dimensions this is not permitted; instead we must anticommute the first $\not{b}$ to the end of the trace:[13]

$$\begin{aligned} \mathrm{Tr}[\gamma^{2n+1}(\not{a} - \not{b})\not{a}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}\not{b}] &= a^2 \mathrm{Tr}(\gamma^{2n+1}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}\not{b}) - 2a\cdot b\, \mathrm{Tr}(\gamma^{2n+1}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}\not{b}) \\ &\quad + 2b_{\alpha_1} \mathrm{Tr}(\gamma^{2n+1}\not{a}\not{c}_1 \cdots \not{b}) - 2c_1\cdot b\, \mathrm{Tr}(\gamma^{2n+1}\not{a}\gamma_{\alpha_1}\gamma_{\alpha_2} \cdots \not{b}) \\ &\quad + \cdots - b^2 \mathrm{Tr}(\gamma^{2n+1}\not{a}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}) \\ &= a^2 \mathrm{Tr}(\gamma^{2n+1}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}\not{b}) + b^2 \mathrm{Tr}(\gamma^{2n+1}\not{a}\gamma_{\alpha_1}\not{c}_1 \cdots \not{c}_{n-1}\gamma_{\alpha_n}) \\ &\quad - 2 \times 2^n i c_1^{\rho_1} \cdots c_n^{\rho_n} b^{\rho_{n+1}} b^\theta A_{\alpha_1\rho_1 \cdots \alpha_n\rho_n\rho_{n+1},\theta} , \end{aligned} \tag{4.2}$$

where

$$A_{\alpha_1\rho_1 \cdots \alpha_n\rho_n\rho_{n+1},\theta} \equiv \epsilon_{\rho_1 \cdots \alpha_n\rho_n\rho_{n+1}} g_{\alpha_1\theta} - \epsilon_{\alpha_1\alpha_2 \cdots \alpha_n\rho_n\rho_{n+1}} g_{\rho_1\theta} + \cdots + (-1)^n \epsilon_{\alpha_1\rho_1 \cdots \alpha_n\rho_n} g_{\rho_{n+1}\theta} . \tag{4.3}$$

A is an object antisymmetric in all subscripts which precede θ; it vanishes in $2n$ dimensions. For the reduced metric $g^{\mu\nu}_{(2\omega)}$ it is easy to show that[8,13]

$$g^{\mu_i\theta}_{(2\omega)} A_{\mu_1\mu_2 \cdots \mu_{2n+1},\theta} = \epsilon_{\mu_1\mu_2 \cdots \mu_{i-1}\mu_{i+1} \cdots \mu_{2n+1}} (2\omega - 2n) . \tag{4.4}$$

Parametrizing the choice as to which vertex should begin the trace yields, for Fig. 1 (Ref. 8),

$$S_{\alpha_0\cdots\alpha_n}(q_1,\ldots,q_n)=\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}[C_0\mathrm{Tr}(\gamma^{2n+1}\gamma_{\alpha_0}\not{k}_0{}^{-1}\gamma_{\alpha_1}\not{k}_1{}^{-1}\cdots\gamma_{\alpha_n}\not{k}_n{}^{-1})+\cdots$$
$$+C_1\mathrm{Tr}(\gamma^{2n+1}\gamma_{\alpha_1}\not{k}_1{}^{-1}\cdots\gamma_{\alpha_n}\not{k}_n{}^{-1}\gamma_{\alpha_0}\not{k}_0{}^{-1})+\cdots$$
$$+C_n\mathrm{Tr}(\gamma^{2n+1}\gamma_{\alpha_n}\not{k}_n{}^{-1}\gamma_{\alpha_0}\not{k}_0{}^{-1}\gamma_{\alpha_1}\not{k}_1{}^{-1}\cdots\gamma_{\alpha_{n-1}}\not{k}_{n-1}{}^{-1})]$$
$$=\sum_{j=0}^{n}\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}C_j\mathrm{Tr}[\gamma^{2n+1}(\gamma_{\alpha_j}\not{k}_j{}^{-1}\cdots\gamma_{\alpha_n}\not{k}_n{}^{-1})(\gamma_{\alpha_0}\not{k}_0{}^{-1}\cdots\gamma_{\alpha_{j-1}}\not{k}_{j-1}{}^{-1})]\ . \tag{4.5}$$

As in CDR, the normalization of $S_{\alpha_0\cdots\alpha_n}(q_1,\ldots,q_n)$ must not change upon reduction from $2n$ to 2ω dimensions, so

$$\sum_{r=0}^{n}C_r=1\ . \tag{4.6}$$

The vector-current divergence at the rth index is given by

$$q_r^{\alpha_r}S_{\alpha_0\cdots\alpha_r\cdots\alpha_n}=\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\frac{1}{P}C_r\mathrm{Tr}[\gamma^{2n+1}(\not{k}_r-\not{k}_{r-1})\not{k}_r\cdots\gamma_{\alpha_n}\not{k}_n\gamma_{\alpha_0}\not{k}_0\cdots\gamma_{\alpha_{r-1}}\not{k}_{r-1}] \tag{4.7}$$

since all other terms vanish upon shifting the variable of integration.[5,8] Here P is defined as in (3.5). If we employ (4.2) in (4.7) we get

$$q_r^{\alpha_r}S_{\alpha_0\cdots\alpha_r\cdots\alpha_n}=\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\frac{1}{P}C_r(-2^{n+1}ik_r^{\rho_r}k_{r+1}^{\rho_{r+1}}\cdots k_n^{\rho_n}k_0^{\rho_0}k_{r-1}^{\rho_{r-1}}k_{r-1}^{\theta}A_{\rho_r\alpha_{r+1}\cdots\rho_n\alpha_0\cdots\rho_{r-1},\theta})\ . \tag{4.8}$$

The antisymmetry in A means that the only nonvanishing contributions to (4.8) are for terms in the integrand numerator that are bilinear in k and contain k^{θ}:

$$\int\frac{d^{2\omega}k}{(2\pi)^{2\omega}}\frac{k^{\rho}k^{\theta}}{P}=\frac{-i\pi^{\omega}}{(2\pi)^{2\omega}\Gamma(\omega+1)(2n-2\omega)}g_{(n)}^{\rho\theta}[1+O(n-\omega)]\ . \tag{4.9}$$

Using this, (4.7) becomes

$$q_r^{\alpha_r}S_{\alpha_0\cdots\alpha_r\cdots\alpha_n}=(-1)^{r+1}\frac{2C_r}{(2\pi)^n n!}\epsilon_{\alpha_0\rho_1\cdots\alpha_{r-1}\rho_r\alpha_{r+1}\rho_{r+1}\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ . \tag{4.10}$$

The axial-vector-current divergence is obtained by completely identical manipulations; it is

$$q_0^{\alpha_0}S_{\alpha_0\cdots\alpha_r\cdots\alpha_n}=(-1)\frac{2C_0}{(2\pi)^n n!}\epsilon_{\alpha_0\rho_1\cdots\alpha_{r-1}\rho_r\alpha_{r+1}\rho_{r+1}\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ . \tag{4.11}$$

It is worthwhile at this point to mention a disturbing feature of the RDR procedure. Consider the quantity

$$\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdot\cdot\gamma_{\alpha_{2n}}\not{b})\ . \tag{4.12}$$

By anticommuting γ_{α_i} and $\gamma_{\alpha_{i+1}}$ we obtain

$$\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{2n}}\not{b})=g_{\gamma_{\alpha_i}\gamma_{\alpha_{i+1}}}\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{i-1}}\gamma_{\alpha_{i+2}}\gamma_{\alpha_{2n}}\not{b})-\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{i+1}}\gamma_{\alpha_i}\gamma_{\alpha_{2n}}\not{b})$$
$$=g_{\gamma_{\alpha_i}\gamma_{\alpha_{i+1}}}2^n ib^{\rho}b^{\sigma}\epsilon_{\rho_{\alpha_1}\cdots\alpha_{i-1}\rho_{i+2}\cdots\alpha_{2n}\sigma}-\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{i+1}}\gamma_{\alpha_i}\gamma_{\alpha_{2n}}\not{b})$$
$$=-\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{i+1}}\gamma_{\alpha_i}\gamma_{\alpha_{2n}}\not{b})\ , \tag{4.13}$$

where (2.5) has been used, an identity taken to be valid in the RDR procedure.[13] From (4.13) we see that the quantity in (4.12) is skew symmetric in any adjacent pair of indices $\alpha_1,\ldots,\alpha_{2n}$. Hence it is fully skew symmetric in *all* of these indices, and so

$$\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{2n}}\not{b})=K\epsilon_{\alpha_1\cdots\alpha_{2n}}\ . \tag{4.14}$$

By contracting both sides of (4.13) with the $2n$-dimensional ϵ tensor, it is easy to show that $K=-2^n ib^2$, and so

$$\mathrm{Tr}(\gamma^{2n+1}\not{b}\gamma_{\alpha_1}\cdots\gamma_{\alpha_{2n}}\not{b})=-2^n ib^2\epsilon_{\alpha_1\cdots\alpha_{2n}}\ , \tag{4.15}$$

a result identical to that which would have been obtained had trace cyclicity been assumed.

This result is inconsistent with the manipulations used to obtain Eqs. (4.10) and (4.11). (Such an inconsistency has previously been pointed out for the *VVA* triangle graph in Ref. 8.) This inconsistency has the consequence of rendering *all* current divergences zero within the RDR procedure for the V^nA $(n+1)$-agon graph. As in the four-dimensional case, the problem essentially is that the quantity

$$A_{\alpha_1\cdots\alpha_{2n}\rho,\theta}=\frac{1}{2^n i}[\mathrm{Tr}(\gamma^{2n+1}\gamma_\rho\gamma_{\alpha_i}\cdots\gamma_{\alpha_{2n}}\gamma_\theta)+\mathrm{Tr}(\gamma^{2n+1}\gamma_\theta\gamma_{\alpha_1}\cdots\gamma_{\alpha_{2n}}\gamma_\rho)] \tag{4.16}$$

which can be nonzero (consistent with the chiral anomaly) if trace cyclicity is abandoned, vanishes if $2n$-dimensional γ-matrix identities are employed in such a way as to restore trace cyclicity.

In the following, we shall take Eqs. (4.10) and (4.11) to be the equations which give the current divergences as calculated in RDR, avoiding the aforementioned ambiguity.

V. EQUIVALENCE OF THE THREE PROCEDURES

From Eqs. (2.10) and (2.11), (3.8) and (3.9), and (4.10) and (4.11), we see that the current divergences in all three procedures are equivalent, and that the one-to-one mapping between them is

$$A_{r+1}-A_{r-1}=2B_r=2C_r\ , \tag{5.1}$$

$$A_1-A_n+1=2B_0=2C_0\ . \tag{5.2}$$

Clearly one of the equations in this set is redundant; this follows from $\sum_{r=0}^{n}B_r=\sum_{r=0}^{n}C_r=1$. Consequently, we have the general result that

$$q_0^\mu S_{\mu\alpha_1\cdots\alpha_n}-q_1^\mu S_{\alpha_1\mu\alpha_2\cdots\alpha_n}+\cdots+(-1)^r q_r^\mu S_{\alpha_1\cdots\alpha_r\mu\alpha_{r+1}\cdots\alpha_n}+\cdots\equiv\sum_{r=0}^{n}(-1)^r q_r^\mu S_{\alpha_1\cdots\alpha_r\mu\alpha_{r+1}\cdots\alpha_n}$$
$$=-\frac{2}{(2\pi)^n n!}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ . \tag{5.3}$$

The right-hand side of (5.3) is $1/n!$ times the usual topological invariant related to the chiral anomaly.[1,2] It is automatically Bose symmetric, despite the fact that Bose symmetry was *not* imposed in any of the three procedures. Hence the sum of *all* diagrams obeys

$$\sum_{r=0}^{n}(-1)^r q_r^\mu \Gamma_{\alpha_1\cdots\alpha_r\mu\alpha_{r+1}\cdots\alpha_n}=-\frac{2}{(2\pi)^n}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ . \tag{5.4}$$

Imposing vector-current conservation yields

$$q_0^\mu\Gamma_{\mu\alpha_1\cdots\alpha_n}=-\frac{2}{(2\pi)^n}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n} \tag{5.5}$$

which is the momentum-space form of the chiral-anomaly equation[1]

$$\partial_\mu J_5^\mu(x)=-\frac{2^{1-n}}{(2\pi)^n n!}\epsilon^{\alpha_1\cdots\alpha_{2n}}F_{\alpha_1\alpha_2}\cdots F_{\alpha_{2n-1}\alpha_{2n}} \tag{5.6}$$

with $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ in $2n$ dimensions.

There is a further relation that exists between current divergences when n is odd. For $n=2m+1$ we have

$$\sum_{j=0}^{m}(A_{2j+2}-A_{2j})=1\ , \tag{5.7a}$$

$$\sum_{j=0}^{m-1}(A_{2j+3}-A_{2j+1})+(A_1-A_{2m+1}+1)=1\ . \tag{5.7b}$$

This implies that the sum of current divergences "splits" into

$$\sum_{r=0}^{n}q_{2r}^\mu S_{\alpha_1\cdots\alpha_{2r}\mu\alpha_{2r+1}\cdots\alpha_n}=-\frac{1}{(2\pi)^n n!}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ , \tag{5.8a}$$

$$\sum_{r=0}^{n}q_{2r+1}^\mu S_{\alpha_1\cdots\alpha_{2r+1}\mu\alpha_{2r+2}\cdots\alpha_n}=\frac{1}{(2\pi)^n n!}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ . \tag{5.8b}$$

Note that relations (5.6) are *not* derivable in either CDR or RDR. Without further assumptions, this splitting does not carry over to $\Gamma_{\alpha\cdots\alpha_n}$ because current divergences taken at odd-numbered vertices of Fig. 1 will mix with current divergences at even-numbered vertices in the sum over all diagrams (although for any individual diagram in $4m+2$ dimensions "splitting" does hold; i.e., the sum of the current divergences at the odd-numbered vertices is equal to an invariant, and the sum of the current divergences at the even-numbered vertices is equal to minus this same invariant for any *individual* diagram, vertices being numbered clockwise from the γ^{2n+1} vertex). However, if we impose Bose symmetry, then the q_rth current divergence is the same at any even- or odd-numbered vertex. Hence

$$\sum_{r=0}^{n}q_{2r}^\mu\Gamma_{\alpha_1\cdots\alpha_{2r}\mu\alpha_{2r+1}\cdots\alpha_n}=-\frac{1}{(2\pi)^n}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}\ , \tag{5.9a}$$

$$\sum_{r=0}^{n} q_{2r+1}^{\mu}\Gamma_{\alpha_1\cdots\alpha_{2r+1}\mu\alpha_{2r+2}\cdots\alpha_n} = \frac{1}{(2\pi)^n}\epsilon_{\alpha_1\rho_1\cdots\alpha_n\rho_n}q_1^{\rho_1}\cdots q_n^{\rho_n}, \quad (5.9b)$$

provided Bose symmetry is imposed.

Hence in $4m+2$ dimensions, one could require either vector-current conservation or axial-vector-current conservation. This is in contrast with the $4m$-dimensional case, in which vector-current conservation must be imposed because the A^{n+1} $(n+1)$-agon graph is proportional to the V^nA $(n+1)$-agon graph in $2n=4m$ dimensions. This is easily seen by anticommuting γ^{2n+1} through the propagators in $2n$-dimensions; there are an odd number of vertices $(2m+1)$ and so there is a net γ^{2n+1} left over. Hence if we require axial-vector-current conservation in the V^nA diagram, we will obtain an inconsistency when this constraint is imposed on the A^{n+1} diagram. In $2n=4m+2$ dimensions, this argument no longer applies; the A^{n+1} $(n+1)$-agon diagram has an even number of vertices, and so it is *not* proportional to the V^nA $(n+1)$-agon graph, but rather is proportional to the V^{n+1} $(n+1)$-agon graph. No inconsistency is obtained by demanding the conservation of the axial-vector-current (at the price of obtaining an anomalous vector-current divergence). Of course nothing prevents the imposition of vector-current conservation [yielding the usual anomalous axial-vector-current divergence relation (5.6)].

VI. CONCLUSION

We have demonstrated that ambiguities which arise in each of three calculational procedures [γ^{2n+1} location in $(2n+\epsilon)$-dimensional integration (CDR), matrix ordering in traces in $(2n-\epsilon)$-dimensional integration (RDR), and arbitrariness of loop-momentum routing in $2n$ dimensions (preregularization)] yield equivalent axial-vector-current and vector-current divergence relations in the absence of constraints imposed by vector-current conservation and Bose symmetry. This equivalence occurs graph by graph, reducing a potential $n\times n!$-fold ambiguity to an n-fold ambiguity in the full amplitude. The anomaly resides in the sum of the current divergences of the full amplitude, and is in fact "equally distributed" in each graph. In $4m+2$ dimensions the current divergence sums "split" in a given graph, as in Eqs. (1.5) and (1.6); if Bose symmetry is imposed, this splitting carries over to the full amplitude. Consequently, in $4m+2$ dimensions either vector-current *or* axial-vector-current conservation must be imposed, whereas in $4m$ dimensions *only* vector-current conservation may be imposed as is seen upon comparing the V^nA diagram with the A^{n+1} diagram. Upon imposing vector-current conservation, the usual anomaly relation is obtained in either case.

The equivalent between RDR and the other two procedures holds provided that certain calculational routes in the former [as outlined in Eqs. (4.12)—(4.16)] are avoided. Otherwise, the anomaly can be legislated to vanish in the RDR technique by a judicious choice of calculational procedures in this method. This would destroy the aforementioned equivalence and would be in conflict with previously established results.[1,2]

Finally, we close with a conjecture. The equivalence of each of the three procedures investigated leads us to conjecture that *all* calculational procedures used to evaluate the anomaly (perturbative and nonperturbative) contain ambiguities which are equivalent, and that current divergence relations are therefore also equivalent in all procedures (modulo inconsistencies in methods of the type pointed out in RDR). Indeed, it has already been demonstrated that Fujikawa's method[14] for computing the anomaly contains ambiguities.[15] Work on these issues is in progress.

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