Global symmetry breaking in two-dimensional flat spacetime and in de Sitter spacetime

L. H. Ford and Alexander Vilenkin
Department of Physics and Astronomy, Tufts University, Medford, Massachusetts 02155
(Received 3 January 1986)

The breaking of global continuous symmetries in two-dimensional flat spacetime and in four-dimensional de Sitter spacetime is investigated. Infrared divergences require physically allowable quantum states in these spaces to break Lorentz or de Sitter invariance, resulting in two-point functions which are explicitly time dependent. Field expectation values which break the global symmetry must decay in time, but it is possible to have a state which exhibits broken symmetry for a finite time. It is also possible for field correlations and energy density produced by a broken-symmetry state to persist after the symmetry has been restored.

I. INTRODUCTION

It has long been recognized that the dimensionality of spacetime can have a significant effect upon the breaking of global continuous symmetries. In four-dimensional Minkowski spacetime, such symmetry breaking can give rise to a nonzero vacuum expectation value of the quantum field in a Lorentz-invariant vacuum state, a phenomenon first discussed by Goldstone.[1] However, in two-dimensional spacetime[2] and in three-dimensional spacetime at finite temperature[3] this is not possible. In both of these cases, the nonappearance of the Goldstone phenomena is linked to the infrared behavior of a massless scalar field.[4] In curved spacetimes the symmetry-breaking behavior can be quite different from that in flat space. For example, Inami and Ooguri[5] have recently shown that global symmetry breaking can occur in two-dimensional anti–de Sitter spacetime.

It has recently been recognized that a massless scalar field in four-dimensional de Sitter space also has unusual infrared properties which are similar to those in two-dimensional flat space.[6] The effects of this behavior have been considered by Ratra,[7] who argues that global continuous symmetries must be restored in de Sitter space. In this paper we will examine the breaking of such symmetries in both two-dimensional flat spacetime and in de Sitter space from a different viewpoint.

Our approach is motivated by inflationary cosmological scenarios in which the Universe goes through a de Sitter phase of exponential expansion in its early history. Before inflation, the Universe is described by a Robertson-Walker metric in which the massless scalar-field propagator has no infrared divergences and spontaneous breaking of global symmetries is allowed. As the Universe enters the de Sitter phase, the propagator remains finite, but it is explicitly time dependent, and thus the corresponding state is not de Sitter invariant. More generally, we consider a class of physical states in two-dimensional Minkowski and four-dimensional de Sitter space in which the massless scalar-field theory has no infrared divergences. Such states necessarily break Lorentz or de Sitter invariance. We will examine the behavior of the expectation value and correlations of a scalar field in these infrared-finite states. Our analysis indicates that the physical properties of such states can be similar to those of the broken-symmetry state.

II. TWO-DIMENSIONAL FLAT SPACETIME

We wish to consider a complex, self-coupled scalar field Φ for which the Lagrangian has a global U(1) symmetry and is of the form

$$L=\partial_\alpha\Phi^*\partial^\alpha\Phi-V(|\Phi|)\,, \tag{2.1}$$

where $V(|\Phi|)$ has a degenerate family of absolute minima with $|\Phi|\neq 0$. For example, if

$$V(|\Phi|)=-\tfrac{1}{2}m_0{}^2\Phi^*\Phi+\tfrac{1}{4}\lambda_0(\Phi^*\Phi)^2\,, \tag{2.2}$$

then the absolute minima occur at $|\Phi|=(m_0{}^2/\lambda_0)^{1/2}$. Let

$$\Phi=\chi e^{i\theta}\,, \tag{2.3}$$

where χ and θ are real fields. Then

$$L=\partial_\alpha\chi\,\partial^\alpha\chi-V(\chi)+\chi^2\partial_\alpha\theta\,\partial^\alpha\theta\,. \tag{2.4}$$

The expectation value of χ, $\langle\chi\rangle=\sigma$, is at the minimum of the potential $V(\chi)$, while the expectation value of θ is arbitrary. As usual, we can define $\chi=\sigma+\delta\chi$, where $\langle\delta\chi\rangle=0$. For energies much less than m_0, or for length scales $\gg m_0{}^{-1}$, the interaction between θ and $\delta\chi$ is weak, and we can approximately treat the Goldstone boson $\phi\equiv\sigma\theta$ as a free massless scalar field. In the same approximation, we can represent Φ as

$$\Phi=\sigma e^{i\phi/\sigma},\ \Box\,\phi=0\,. \tag{2.5}$$

Decompose the quantum operator ϕ into two parts, $\phi=\phi^++\phi^-$, where for a given quantum state $|\Psi\rangle$,

$$\phi^-|\Psi\rangle=0 \tag{2.6a}$$

and

$$\langle\Psi|\phi^+=0\,. \tag{2.6b}$$

For example, let

$$\phi^+(x)=\sum_j a_j^\dagger f_j^*(x) \tag{2.7a}$$

and

$$\phi^-(x)=\sum_j a_j f_j(x), \tag{2.7b}$$

where $\{f_j\}$ are an orthonormal set of positive-norm solutions of the massless Klein-Gordon equation, $\Box f_j=0$. In the particular case that $|\Psi\rangle$ is the Lorentz-invariant vacuum, $\phi^\pm$ are the usual positive- and negative-frequency parts of ϕ. However, here we are not requiring that the f_j be purely positive-frequency mode functions and hence $|\Psi\rangle$ could be a more general state. Because the $a_j^\dagger$ and a_j satisfy

$$[a_j,a_l^\dagger]=\delta_{jl}, \tag{2.8}$$

we have that

$$[\phi^-(x),\phi^+(y)]=\langle\phi(x)\phi(y)\rangle, \tag{2.9}$$

where the expectation value is taken in the state $|\Psi\rangle$. The Campbell-Baker-Hausdorf formula states that

$$e^{A+B}=e^A e^{-[A,B]/2} e^B \tag{2.10}$$

for any pair of operators A and B which each commute with $[A,B]$. Using this relation we find that

$$\langle\Phi(x)\rangle=\sigma e^{\langle\phi^2(x)\rangle/2\sigma^2}. \tag{2.11}$$

Similarly, the correlation functions are

$$\langle\Phi(x)\Phi(y)\rangle=\sigma^2\exp\{-[\langle\phi(x)\phi(y)\rangle+\tfrac{1}{2}\langle\phi^2(x)\rangle+\tfrac{1}{2}\langle\phi^2(y)\rangle]/\sigma^2\} \tag{2.12}$$

and

$$\langle\Phi(x)\Phi^*(y)\rangle=\sigma^2\exp\{[\langle\phi(x)\phi(y)\rangle-\tfrac{1}{2}\langle\phi^2(y)\rangle-\tfrac{1}{2}\langle\phi^2(y)\rangle]/\sigma^2\}. \tag{2.13}$$

Although $\langle\phi^2(x)\rangle$ is formally infinite due to ultraviolet divergences, we are primarily concerned with the long-wavelength behavior of the theory for the purposes of symmetry breaking. In flat spacetime these ultraviolet divergences may be removed by a wave-function renormalization. Let $\Phi_R=Z\Phi_B$, where Φ_B and Φ_R are the bare and renormalized quantities, respectively. If we let $\langle\phi^2\rangle_{\rm reg}=\langle\phi^2\rangle_{\rm fin}+D$, where $\langle\phi^2\rangle_{\rm reg}$ and D are regulator-dependent quantities and $\langle\phi^2\rangle_{\rm fin}$ is finite in the absence of a regulator, then we need $Z=e^{D/2}$. Henceforth we will assume that $\Phi=\Phi_R$ and $\langle\phi^2\rangle=\langle\phi^2\rangle_{\rm fin}$. Even after the ultraviolet divergences in $\langle\phi^2\rangle$ have been removed, $\langle\phi(x)\phi(y)\rangle$ is still singular as $x\to y$. This causes $\langle\Phi(x)\Phi(y)\rangle$ and $\langle\Phi(x)\Phi^*(y)\rangle$ either to diverge or vanish as in this limit. However, the above expressions for the correlation functions are only valid if $|(x-y)^2|\gtrsim m_0^{-2}$. At smaller distances, the approximation in which the dynamics of the χ field are neglected would fail.

To this point our discussion applies to flat spacetimes of any dimensionality. Let us now consider two-dimensional spacetime. Here $\langle\phi^2\rangle$ and $\langle\phi(x)\phi(y)\rangle$ contain an infrared divergence in the Lorentz-invariant vacuum. If we were to insert an infrared cutoff, such as a small mass, these quantities would diverge ($\to+\infty$) in the limit that the cutoff is removed. Hence both $\langle\Phi\rangle$ and $\langle\Phi(x)\Phi(y)\rangle$, although not $\langle\Phi(x)\Phi^*(y)\rangle$, vanish in this limit. This is the sense in which spontaneously broken continuous symmetries do not arise in two dimensions. The thermal averages of ϕ^2 and $\phi(x)\phi(y)$ also contain an infrared divergence in three-dimensional spacetime, so $\langle\Phi\rangle=0$ at finite temperature in three dimensions.

In this paper we wish to approach this problem from a somewhat different viewpoint in which we do not insist upon the Lorentz invariance of the vacuum state. A similar approach was taken in the earlier work on infrared divergences in curved spacetime.[6,8] We wish to choose a "vacuum" state $|\Psi\rangle$ which is free of infrared divergences. Let the mode functions be of the form

$$f_k=(4\pi\omega)^{-1/2}e^{ikx}[c_1(\omega)e^{i\omega t}+c_2(\omega)e^{-i\omega t}], \tag{2.14}$$

where $\omega=|k|$ and

$$|c_2(\omega)|^2-|c_1(\omega)|^2=1. \tag{2.15}$$

The field operator is

$$\phi=\int_{-\infty}^{\infty}dk\,(a_k f_k+a_k^\dagger f_k^*). \tag{2.16}$$

The two-point function in the state $|\Psi\rangle$ is

$$\langle\phi(x,t)\phi(x',t')\rangle=(2\pi)^{-1}\int_0^\infty d\omega\,\omega^{-1}\{\cos\omega(x-x')[c_1(\omega)e^{i\omega t}+c_2(\omega)e^{-i\omega t}][c_1^*(\omega)e^{-i\omega t'}+c_2^*(\omega)e^{i\omega t'}]\}. \tag{2.17}$$

This integral converges at the lower limit provided that

$$|c_1+c_2|\to 0 \quad\text{as}\quad \omega\to 0. \tag{2.18}$$

Thus the infrared divergences may be avoided by a suitable choice of the quantum state.

The energy-momentum tensor for a massless scalar field is

$$T_{\mu\nu}=\phi_{,\mu}\phi_{,\nu}-\tfrac{1}{2}g_{\mu\nu}\phi_{,\rho}\phi^{,\rho} \tag{2.19}$$

and hence,

$$\langle T_{00}\rangle=(4\pi)^{-1}\int_0^\infty d\omega\,\omega(|c_1e^{i\omega t}-c_2e^{-i\omega t}|^2+|c_1e^{i\omega t}+c_2e^{-i\omega t}|^2). \tag{2.20}$$

This quantity should also be free of infrared divergences, so we require that

$$\omega^2|c_1-c_2|^2\to 0 \quad\text{as}\quad \omega\to 0. \tag{2.21}$$

Thus physically allowable choices for the vacuum state are those associated with functions c_1 and c_2 which satisfy Eqs. (2.15), (2.18), and (2.21). For simplicity, let us also restrict our attention to the case in which c_1 and c_2 are real functions. Then

$$c_2{}^2-c_1{}^2=1\,,\quad c_1+c_2\to 0,\ \ \omega(c_1-c_2)\to 0\ \text{ as } \omega\to 0\,. \tag{2.22}$$

The quantity $c_2-c_1=(c_1+c_2)^{-1}$ becomes infinite as $\omega\to 0$, but less rapidly than ω^{-1}. Let

$$f(\omega)=c_1+c_2\,, \tag{2.23}$$

so

$$c_1=(f^2-1)/(2f) \tag{2.24a}$$

and

$$c_2=(f^2+1)/(2f)\,. \tag{2.24b}$$

If f varies as a power of ω near $\omega=0$, then

$$f(\omega)\sim a\omega^{\lambda},\ \ \omega\to 0\,, \tag{2.25}$$

where $0<\lambda<1$.

Consider the time derivative of the two-point function at equal times:

$$\frac{d}{dt}\langle\phi(x,t)\phi(x',t)\rangle = -\pi^{-1}\int_0^\infty d\omega\, c_1(\omega)c_2(\omega)[\sin\omega(\Delta x+2t) - \sin\omega(\Delta x-2t)]\,, \tag{2.26}$$

where $\Delta x=x-x'$. Let us first examine the $t\to\infty$ limit of $\langle\phi^2\rangle$. Set $\Delta x=0$ and use the small ω forms of c_1 and c_2 to find

$$\frac{d}{dt}\langle\phi^2\rangle=-2\pi^{-1}\int_0^\infty d\omega\, c_1c_2\sin 2t\omega \sim(2\pi a^2)^{-1}\int_0^\infty d\omega\,\omega^{-2\lambda}\sin 2t\omega = \frac{\sec[\pi(1-2\lambda)/2]}{2^{3-2\lambda}\Gamma(2\lambda)a^2}t^{2\lambda-1}\,, \tag{2.27}$$

and hence

$$\langle\phi^2\rangle\sim\frac{\sec[\pi(1-2\lambda)/2]}{2^{4-2\lambda}\lambda\Gamma(2\lambda)a^2}t^{2\lambda},\ \ t\to\infty\,. \tag{2.28}$$

This growth of $\langle\phi^2\rangle$ is very similar to the behavior which occurs in de Sitter space (see Sec. III). Quantum states which are free of infrared divergences necessarily break Lorentz invariance in such a way as to force $\langle\phi^2\rangle$ to be a growing function of time.

A particular example of an allowable state is constructed by matching the massless mode functions to those for a massive field. Assume that for $t<0$, f_k has the form

$$f_k=(4\pi\Omega)^{-1/2}e^{i(kx-\Omega t)}\,, \tag{2.29}$$

where $\Omega=(k+m^2)^{1/2}$, and the form of Eq. (2.14) for $t>0$. This state has a physical interpretation. Suppose that in the past the ϕ field had interactions (such as those at finite temperature) which caused it to acquire an effective mass; after these interactions have been switched off and the field becomes massless, the ϕ field will be described by an infrared-divergence-free state equivalent to that obtained by matching. Requiring that f_k and its first time derivative be continuous at $t=0$ yields

$$c_1=\frac{\omega-\Omega}{2\sqrt{\omega\Omega}},\ \ c_2=\frac{\omega+\Omega}{2\sqrt{\omega\Omega}}\,. \tag{2.30}$$

This matching procedure is accurate for $\omega\Delta t\ll 1$, where Δt is the duration of the transition from $m_\phi=m$ to $m_\phi=0$. As long as we are not interested in the ultraviolet behavior, we can use Eq. (2.30) assuming that Δt is sufficiently small. The forms of c_1 and c_2 in (2.30) satisfy Eq. (2.22).

Here we have $a=m^{-1/2}$ and $\lambda=\frac{1}{2}$, so

$$\langle\phi^2\rangle\sim mt/4,\ \ t\to\infty\,. \tag{2.31}$$

Let us examine the behavior of the two-point function $\langle\phi(x,t)\phi(x',t)\rangle$ as either $t\to\infty$ or $\rho\to\infty$, where $\rho=|\Delta x|=|x-x'|$. First consider the case in which Δx is fixed and $t\to\infty$. From Eq. (2.26) we find that

$$\langle\phi(x,t)\phi(x',t)\rangle\sim\langle\phi^2\rangle\left[1+\lambda(2\lambda-1)\left[\frac{\rho}{2t}\right]^2+\cdots\right],\quad t\to\infty,\ \rho\text{ fixed}\,. \tag{2.32}$$

Similarly,

$$\langle\phi(x,t)\phi(x',t)\rangle\sim\frac{\sec[\pi(1-2\lambda)/2]}{2^{2-2\lambda}\lambda\Gamma(2\lambda)a^2}(2\lambda-1)\rho^{2\lambda-3}t^2\to 0\,,\quad \rho\to\infty,\ t\text{ fixed}\,. \tag{2.33}$$

We can now relate the above results to the discussion of symmetry breaking in two dimensions. From Eq. (2.11) we see that $\langle\Phi\rangle\to 0$ as $t\to\infty$. However, the rate of growth of $\langle\phi^2\rangle$ depends upon the choice of state and may be arbitrarily slow. Hence it is possible to have states in which $\langle\Phi\rangle$ is approximately constant over any finite-time interval and hence exhibits broken-symmetry behavior during that interval. From Eqs. (2.12), (2.13), and (2.33), we see that both $\langle\Phi(x,t)\Phi(x',t)\rangle$ and $\langle\Phi(x,t)\Phi^*(x',t)\rangle$ approach nonzero values as $\rho\to\infty$ at fixed t if $\sigma\neq 0$. Thus there exist correlations in the Φ field over arbitrarily large spatial separations. As $t\to\infty$ with ρ fixed, $\langle\Phi(x,t)\Phi(x',t)\rangle\to 0$, but $\langle\Phi(x,t)\Phi^*(x',t)\rangle$ approaches a nonzero value. If by a broken-symmetry state, one means a state in which $\langle\Phi\rangle\neq 0$, then it is true that such a configuration cannot be sustained for an infinite time in two dimensions. However, it can be sustained for any finite period of time. Furthermore, even after the symmetry has been restored in the sense that $\langle\Phi\rangle$ and $\langle\Phi(x,t)\Phi(x',t)\rangle$ have decayed to effectively zero values, there still remain long-range correlations exhibited by the nonvanishing of $\langle\Phi(x,t)\Phi^*(x',t)\rangle$ over large but finite distances which do not decay in time. Thus the theorem[2] that "there are no Goldstone bosons in two dimensions," although formally correct, is somewhat weaker than it might appear to be at first sight and does not prevent the realization of certain aspects of broken symmetry in two dimensions.

We can understand the behavior of $\langle\phi(x,t)\phi(x',t)\rangle$ and hence of $\langle\Phi(x,t)\Phi^*(x',t)\rangle$ in more detail by calculating these quantities in a particular quantum state. Let the state be that defined by Eq. (2.30), the state obtained by matching the massless field to a field of mass m at $t=0$.

From Eqs. (2.17) and (2.30) we find that, at $t=0$,

$$\langle \phi(x,0)\phi(x',0)\rangle = \frac{1}{2\pi}K_0(m\rho) \sim \begin{cases} (2\pi)^{-1}e^{-m\rho}, & m\rho \gg 1, \\ -(2\pi)^{-1}\ln(m\rho), & m\rho \ll 1 . \end{cases} \quad (2.34)$$

For $t>0$,

$$\langle \phi(x,t)\phi(x't)\rangle = \frac{1}{2\pi}K_0(m\rho)+F(\rho,t), \quad (2.35)$$

where

$$F(\rho,t)=\frac{m^2}{2\pi}\int_0^\infty \frac{d\omega}{\omega^2(\omega^2+m^2)^{1/2}}\cos\omega\rho\sin^2\omega t . \quad (2.36)$$

If either ρ or t are large compared to m^{-1}, the dominant contribution to $F(\rho,t)$ comes from values of $\omega \ll m$. Thus,

$$\begin{aligned} F(\rho,t) &\approx \frac{m}{2\pi}\int_0^\infty \frac{d\omega}{\omega^2}\cos\omega\rho\sin^2\omega t \\ &= \frac{m^2}{4\pi}\lim_{\beta\to 0}\int_0^\infty \frac{d\omega}{\omega^2+\beta^2}\cos\omega\rho(1-\cos 2\omega t) \\ &= \frac{m}{16}(\,|2t-\rho|+2t-\rho), \end{aligned} \quad (2.37)$$

so

$$\langle \phi(x,t)\phi(x',t)\rangle \sim \begin{cases} \frac{1}{8}m(2t-\rho), & 2t>\rho \gg m^{-1}, \\ \frac{1}{4}mt-\frac{1}{2\pi}\ln(m\rho), & t>m^{-1}>\rho, \\ O(e^{-m\rho}), & \rho>2t\gg m^{-1} . \end{cases} \quad (2.38)$$

Combining Eqs. (2.13), (2.34), and (2.38) we find that the correlation function has the following behavior:

$$\langle \Phi(x,t)\Phi^*(x',t)\rangle \sim \begin{cases} \sigma^2(m\rho)^{-(1/2\pi\sigma^2)}, & m_0^{-1}<\rho<m^{-1}, \\ \sigma^2 e^{-m\rho/8\sigma^2}, & 2t>\rho\gg m^{-1}, \\ \sigma^2 e^{-mt/4\sigma^2}, & \rho>2t\gg m^{-1} . \end{cases} \quad (2.39)$$

This correlation function is illustrated in Fig. 1.

This function illustrates the general features discussed above: as $t\to\infty$ with ρ fixed, the correlation function approaches nonzero limits. Note, however, that if both ρ and t become large, $\langle \Phi(x,t)\Phi^*(x',t)\rangle \to 0$. Thus at very late times the value of the correlation function becomes very small as $\rho\to\infty$. The characteristic time scale here is m^{-1} which can be chosen to be arbitrarily large.

III. de SITTER SPACETIME

A massless, minimally coupled scalar field in four-dimensional de Sitter space has infrared behavior which is similar to that in two-dimensional flat spacetime. Take the metric on de Sitter space to be

$$ds^2=dt^2-e^{2Ht}d\mathbf{x}^2=(H\eta)^{-2}(d\eta^2-d\mathbf{x}^2), \quad (3.1)$$

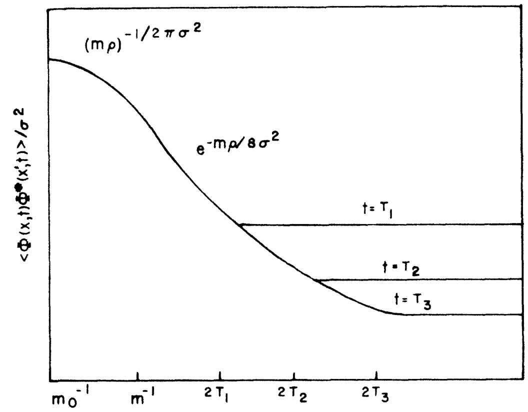

FIG. 1. The correlation function $\langle \Phi(x,t)\Phi^*(x',t)\rangle$ in two-dimensional spacetime in the state obtained by matching a massless scalar field to a field of mass m is shown. It is plotted as a function of $\rho=|x-x'|$ for various values of the time t. For large ρ, $\langle \Phi(x,t)\Phi^*(x',t)\rangle$ approaches the constant value $\sigma^2 e^{-mt/4\sigma^2}$.

where $\eta=-H^{-1}e^{-Ht}$. A free massless field which satisfies

$$\Box\phi=0 \quad (3.2)$$

has the property that $\langle\phi^2\rangle$ in a spatially homogeneous state in the above coordinates must be a growing function of time; for $t\gg H^{-1}$, it is given by[6,9,10]

$$\langle\phi^2\rangle=\frac{H^3t}{4\pi^2} . \quad (3.3)$$

This is the analog of Eq. (2.28) in two dimensions. This linear growth of $\langle\phi^2\rangle$ is again due to the infrared behavior of the theory. For the free field theory there is no accompanying growth in the energy density; however, in interacting theories $\langle T_{\mu\nu}\rangle$ can also become time dependent.[11]

The time dependence of $\langle\phi^2\rangle$ necessarily breaks de Sitter invariance. This is to be expected because no meaningful de Sitter-invariant vacuum exists for the massless field due to infrared divergences (for a recent detailed discussion of this point see Allen,[12] where a rigorous proof of the nonexistence of a de Sitter-invariant two-point function is given). This is completely analogous to the need to give up Lorentz invariance in order to have a well-defined massless scalar quantum field theory in two-dimensional flat spacetime. The physically allowed states are those in which the two-point function is free of infrared divergences. States which are free of infrared divergences at one time will remain so at later times.[8,13] Ratra[7] has attempted to give a counterexample to this result; however, his example requires taking the limit as time goes to infinity before examining the zero frequency limit, whereas these limits should be taken in the reverse order.

For the discussion of symmetry breaking in de Sitter space, we need to examine the two-point function. Express the field operator as

$$\phi=(2\pi)^{-3/2}\int d^3k[a_{\mathbf{k}}\psi_k(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}+\text{H.c.}], \quad (3.4)$$

where

$$\psi_k(\eta)=\tfrac{1}{2}\pi^{1/2}H\,|\eta|^{3/2}[c_1H^{(1)}_{3/2}(k\eta)+c_2H^{(2)}_{3/2}(k\eta)]\,. \quad (3.5)$$

The coefficients c_1 and c_2 are functions of k/H and satisfy

$$|c_2|^2-|c_1|^2=1\,. \quad (3.6)$$

If the vacuum is defined by $a_{\mathbf{k}}|0\rangle=0$, then the two-point function is

$$\langle\phi(x)\phi(x')\rangle=(2\pi)^{-3}\int d^3k\,\psi_k(\eta)\psi_k^*(\eta')e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\,. \quad (3.7)$$

This quantity is free of infrared divergences provided that

$$F(k/H)=|c_1(k/H)+c_2(k/H)|^2\to 0 \quad \text{as } k\to 0\,. \quad (3.8)$$

Let us first consider the limit of the two-point function as $t\to\infty$ ($|\eta|\to 0$) with $\Delta\mathbf{x}$ fixed. Set $\eta=\eta'$ and change the integration variable in Eq. (3.7) to $\mathbf{z}=\mathbf{k}/H$. For small, negative arguments the Hankel functions have the limiting forms

$$H^{(1)}_{3/2}(-\xi)\sim H^{(2)}_{3/2}(-\xi)\sim\sqrt{2/\pi}\,\xi^{-3/2}\,. \quad (3.9)$$

Thus at late times the two-point function approaches a nonzero constant:

$$\langle\phi(x),\phi(x')\rangle_{\eta=\eta'}\to\frac{H^3}{16\pi^3}\int d^3z\,z^{-3}F(z)\times e^{i\mathbf{z}\cdot\Delta\mathbf{x}H},\quad \eta\to 0\,. \quad (3.10)$$

This form applies when the coordinate separation $|\Delta\mathbf{x}|$ is held fixed. It is also of interest to consider the limit $\eta\to 0$ with the proper separation $\boldsymbol{\rho}=\Delta\mathbf{x}(H|\eta|)^{-1}$ held fixed. Let $\boldsymbol{\xi}=-\eta\mathbf{k}$. Then

$$\langle\phi(x)\phi(x')\rangle_{\eta=\eta'}=\frac{H^2}{32\pi^2}\int d^3\xi\,|c_1(\xi H^{-1}|\eta|^{-1})H^{(1)}_{3/2}(-\xi)+c_2(\xi H^{-1}|\eta|^{-1})H^{(2)}_{3/2}(-\xi)|^2e^{i\boldsymbol{\xi}\cdot\boldsymbol{\rho}H}\,. \quad (3.11)$$

Take the derivative of this equation with respect to η with $\boldsymbol{\rho}$ held constant and use the asymptotic forms given in Eq. (3.9) to find

$$\frac{d}{d\eta}\langle\phi(x)\phi(x')\rangle_{\eta=\eta'}\sim\frac{H}{16\pi^3\eta^2}\int\frac{d^3\xi}{\xi^2}e^{i\boldsymbol{\xi}\cdot\boldsymbol{\rho}H}\times F'(\xi H^{-1}|\eta|^{-1})\,. \quad (3.12)$$

Let $\mathbf{z}=\boldsymbol{\xi}H^{-1}|\eta|^{-1}$ and take $e^{i\mathbf{z}\cdot\boldsymbol{\rho}|\eta|H}\approx 1$. Then because $F(0)=0$ and $F(\infty)=1$, we find

$$\frac{d}{d\eta}\langle\phi(x)\phi(x')\rangle_{\eta=\eta'}\sim\frac{H^2}{4\pi|\eta|} \quad (3.13)$$

and hence

$$\langle\phi(x),\phi(x')\rangle_{\eta=\eta'}\sim\langle\phi^2\rangle\sim\frac{H^3t}{4\pi^2},\quad t\to\infty,\ \rho\ \text{constant}\,. \quad (3.14)$$

This is the analog of Eq. (2.32).

Finally we need the limiting form as either $|\Delta\mathbf{x}|$ or $\rho=|\boldsymbol{\rho}|\to\infty$ with η fixed. From Eq. (3.11) we can see that in this limit the exponential factor forces the dominant contribution to the integral to come from small values of ξ. Again using Eq. (3.9) and changing the integration variable to $\mathbf{z}$ yields

$$\langle\phi(x)\phi(x')\rangle_{\eta=\eta'}\sim\frac{H^3}{16\pi^3}\int\frac{d^3z}{z^3}F(z)e^{i\mathbf{z}\cdot\boldsymbol{\rho}|\eta|}\,. \quad (3.15)$$

Let $F(z)$ have the limiting form

$$F(z)\sim az^{\beta},\quad \beta>0,\ z\to 0\,. \quad (3.16)$$

If we substitute this form in Eq. (3.15), we find

$$\langle\phi(x)\phi(x')\rangle_{\eta=\eta'}\sim\frac{H^2a}{4\pi}\Gamma(\beta-1)\sin[\tfrac{1}{2}(\beta-1)\pi]\times(H^2\rho|\eta|)^{-\beta},\quad \rho\to\infty,\eta\ \text{fixed}\,. \quad (3.17)$$

Thus $\langle\phi(x)\phi(x')\rangle_{\eta=\eta'}$ vanishes as $\rho\to\infty$ with η (or t) fixed;[14] this is the analog of Eq. (2.33). Thus the two-point function in de Sitter space as a function of proper separation has qualitatively the same behavior as does the two-point function in two-dimensional flat spacetime.

Equations (2.11)–(2.13) which relate expectation values involving Φ to those involving ϕ are still valid in de Sitter space. Thus

$$\langle\Phi\rangle\sim\sigma e^{-H^3t/4\pi^2\sigma^2}\,. \quad (3.18)$$

We can also give the behavior of the equal-time correlation functions in various limits.

(1) Late time and fixed coordinate separation:

$$\langle\Phi(x)\Phi(x')\rangle\sim\langle\Phi(x)\Phi^*(x')\rangle\sim\sigma^2e^{-H^3t/2\pi^2\sigma^2}\to 0,\quad t\to\infty,\ \Delta\mathbf{x}\ \text{fixed}\,. \quad (3.19)$$

(2) Late time and fixed proper separation:

$$\langle\Phi(x)\Phi(x')\rangle\sim\sigma^2e^{-H^3t/2\pi^2\sigma^2}\to 0,\qquad \langle\Phi(x)\Phi^*(x')\rangle\to\text{const},\quad t\to\infty,\ \rho\ \text{fixed}\,. \quad (3.20)$$

(3) Fixed time and large coordinate or proper separation:

$$\langle\Phi(x)\Phi(x')\rangle\sim\langle\Phi^*(x)\Phi(x')\rangle\sim\sigma^2e^{-H^3t/2\pi^2\sigma^2}=\text{const},\quad \rho\to\infty,\ t\ \text{fixed}\,. \quad (3.21)$$

Thus if we were to graph $\langle \Phi(x)\Phi^*(x') \rangle$ as a function of ρ at various times, the result would look qualitatively like Fig. 1. For any fixed time, $\langle \Phi(x)\Phi^*(x') \rangle$ decays until it reaches a limiting value as ρ increases. However, this limiting value is proportional to $e^{-H^3 t/2\pi^2 \sigma^2}$ and hence at late times the correlation function decays to very small values at large proper separations.

A few remarks concerning ultraviolet divergences are in order here. We have assumed that the ultraviolet divergent part of $\langle \phi^2 \rangle$ has been removed in writing such relations as Eq. (3.3). In a general curved spacetime $\langle \phi^2 \rangle$ contains divergent pieces which are of the form of a constant and of a constant multiplied by R, the scalar curvature, which is constant in de Sitter space. In this particular case the resulting divergences in quantities such as $\langle \Phi \rangle$ could be removed by a wave-function renormalization, as was described in Sec. II. This procedure is not completely satisfactory because it would fail in more general spacetimes. In general $\langle \Phi \rangle$ will contain a factor of $e^{\alpha R}$ where α is a regulator-dependent constant and R is a function of position; such a divergence is apparently not renormalizable. This difficulty is probably due to the breakdown of the approximation used in writing Eq. (2.5). In a more careful treatment of the ultraviolet behavior of the theory the quantum fluctuations of the χ field cannot be ignored. We are here primarily concerned with symmetry breaking and the infrared behavior of the theory for which this is an inessential complication.

IV. DISCUSSION AND CONCLUSIONS

From Eqs. (3.18)–(3.20) we see that $\langle \Phi \rangle$ and the correlation functions have very similar behavior in de Sitter space to that found in two-dimensional flat spacetime. In particular $\langle \Phi \rangle$ and $\langle \Phi(x)\Phi(x') \rangle$ must decay to zero on a time scale of the order of H^{-1}. The correlation function $\langle \Phi(x)\Phi^*(x') \rangle$ does not decay if the two points are at fixed proper separation, but does decay if the two points are at fixed coordinate separation. These properties may be given an operational interpretation. If we establish a state in which correlations between Φ and Φ^* exist at one time, then any subsequent measurements made over the same physical length scale will reveal the presence of the same correlations. On the other hand, if we design an experiment in which the correlations are measured by a pair of comoving observers (who remain at fixed coordinate separation but exponentially increasing proper separation), these observers will see the correlations decay exponentially in time. At any fixed time, both $\langle \Phi(x)\Phi(x') \rangle$ and $\langle \Phi(x)\Phi^*(x') \rangle$ can reveal the presence of correlations over infinitely large distances.

As in the two-dimensional case, global symmetries are eventually restored in de Sitter space, but configurations which have some of the features usually associated with broken symmetry can continue to exist. It is true that a nonzero value of $\langle \Phi \rangle$ cannot persist forever; however, a state with correlations over a given physical length scale can persist. The Lagrangian and hence the energy-momentum tensor of the Φ field are functionals of $\Phi\Phi^*$ rather than $\langle \Phi \rangle$. Thus one expects that if one initially prepares a broken-symmetry state with a nonzero energy density, this energy density would not decay in time. An example of such a state is a string arising from global-symmetry breaking.[15] This is a spatially inhomogeneous state with a nonzero energy density. Because a string has a nonconstant $\langle \chi \rangle$ and in fact $\langle \chi \rangle = 0$ on the string itself, the analysis given in this paper, particularly Eqs. (2.11)–(2.13), does not apply directly to strings. Nonetheless, it seems more plausible that global strings can exist indefinitely in de Sitter space than that their energy must be dissipated on a time scale of H^{-1}. The quantities $\langle \Phi \rangle$ and $\langle \Phi(x)\Phi(x') \rangle$ are not U(1) invariant, whereas $\langle \Phi(x)\Phi^*(x') \rangle$ is U(1) invariant. Hence at late times the symmetry is restored in the formal sense even though the energy density may remain unchanged.

One difference between de Sitter space and the two-dimensional flat-space cases is that in the former there is a natural time scale (H^{-1}) set by the spacetime geometry on which $\langle \Phi \rangle$ decays, whereas in the latter case this time scale can be arbitrarily specified by the choice of quantum state. It is in principle possible to construct states in de Sitter space which carry other time scales and in which $\langle \Phi \rangle$ decays more rapidly. However, if the de Sitter space is regarded as arising in an inflationary model[16] with an earlier, non–de Sitter phase the natural time scale is necessarily H^{-1} (Ref. 6).

It is worthwhile noting that it would be completely consistent with all observations for our present Universe to be a de Sitter space with $H^{-1} \sim 10^{10}$ yr. It would be very surprising if this were to have any effect upon symmetry breaking on terrestrial or subatomic scales. From the above discussion we see that there is indeed no prohibition against global-symmetry-breaking phenomena on such small scales.

ACKNOWLEDGMENTS

We would like to thank B. Allen, A. Everett, and G. Smith for helpful discussions. This work was supported by the National Science Foundation under Grants Nos. PHY-8506593 and PHY-8351860 and by grants from the General Electric, Massachusetts Electric, and Dennison Manufacturing companies.

[1]J. Goldstone, Nuovo Cimento **19**, 154 (1961).
[2]S. Coleman, Commun. Math. Phys. **31**, 259 (1973).
[3]N. D. Mermin and H. Wagner, Phys. Rev. Lett. **17**, 1133 (1966); P. C. Hohenberg, Phys. Rev. **158**, 383 (1967).
[4]S.-k. Ma and R. Rajaraman, Phys. Rev. D **11**, 1701 (1975).
[5]T. Inami and H. Ooguri, Kyoto University Report No. RIFP-612, 1985 (unpublished).
[6]A. Vilenkin and L. H. Ford, Phys. Rev. D **26**, 1231 (1982).
[7]B. Ratra, Phys. Rev. D **31**, 1931 (1985).
[8]L. H. Ford and L. Parker, Phys. Rev. D **16**, 245 (1977).
[9]A. Linde, Phys. Lett. **116B**, 335 (1982).
[10]A. A. Starobinsky, Phys. Lett. **117B**, 175 (1982).

[11]L. H. Ford, Phys. Rev. D 31, 710 (1985).

[12]B. Allen, Phys. Rev. D 32, 3136 (1985).

[13]S. A. Fulling, M. Sweeny, and R. M. Wald, Commun. Math. Phys. 63, 257 (1978).

[14]This asymptotic limit appears to conflict with the Green's function discussed by Allen in Ref. 12, which has a logarithmic dependence upon ρ. However, this apparent conflict can be resolved by realizing that Allen's Green's function corresponds to the $\beta \rightarrow 0$ limit of a set of Green's functions defined by $F(z)$ having the form given in Eq. (3.16). If we let $\beta \rightarrow 0$ in Eq. (3.17) and ignore the $1/\beta$ term on the grounds that it is a constant independent of the coordinates, then we find $\langle \phi(x)\phi(x') \rangle \sim a(4\pi^2)^{-1}H^2(Ht - \ln H\rho)$. With $a=1$, this agrees with the Green's function given by Allen's Eq. (4.20) with $\alpha=0$ and $H\rho \gg 1$. For large values of t, the ρ dependence of $\langle \phi(x)\phi(x') \rangle$ is unimportant, so Allen's form of the two-point function reproduces the late-time behavior of the more general class of two-point functions.

[15]A. Vilenkin and A. E. Everett, Phys. Rev. Lett. 48, 1867 (1982).

[16]A. Guth, Phys. Rev. D 23, 347 (1981).