Chiral symmetry, nonleptonic hyperon decay, and the Feinberg-Kabir-Weinberg theorem

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We demonstrate that additional (kaon-pole) diagrams must be appended to the traditional commutator-plus-baryon-pole approach to nonleptonic hyperon decay. The modified analysis satisfies the constraints of the Feinberg-Kabir-Weinberg theorem. The new terms are not numerically significant in the case of the usual current-current Hamiltonian, in the continuum limit, but can be important for work using lattice techniques or for other Hamiltonians.

The problem of understanding nonleptonic hyperon decay is an old one.[1] The traditional current-algebra–PCAC (partial conservation of axial-vector current) (chiral) approach analyzes parity-violating (s-wave) decay in the soft-pion limit, yielding the familiar commutator term

$$\langle \pi^a B'|H_w^{\rm pv}|B\rangle = -\frac{i}{F_\pi}\langle B'|[F_a^5,H_w^{\rm pv}]|B\rangle \ , \qquad (1)$$

while parity-conserving (p-wave) transitions are predicted using the simple baryon-pole model

$$\langle \pi^a B'|H_w^{\rm pc}|B\rangle \approx \sum_{B''}\left(\frac{\langle \pi^a B'|B''\rangle\langle B''|H_w^{\rm pc}|B\rangle}{m_B-m_{B''}} + \frac{\langle B'|H_w^{\rm pc}|B''\rangle\langle \pi^a B''|B\rangle}{m_{B'}-m_{B''}}\right) \ . \qquad (2)$$

This analysis has well-known defects—fitting s-wave commutators to experiment yields p-wave amplitudes which are a factor of 2 too small.[2] Nevertheless, recent work involving coupling to $\frac{1}{2}^-$ excitations (which vanish in the $q_a \to 0$ limit) appears promising in being able to resolve this long-standing problem.[3]

Our purpose in this Brief Report is not, however, to review this well traveled road, but rather to ask whether this chiral-symmetry-based approach to the problem satisfies the constraints implied by the Feinberg-Kabir-Weinberg (FKW) theorem,[4] which asserts that if we utilize a combination of scalar and pseudoscalar densities [which transform as $(3,\bar{3})\otimes(\bar{3},3)$ under $SU(3)_L\otimes SU(3)_R$] as our "weak Hamiltonian,"

$$``H_w" \sim u_6 + i v_7 + \text{H.c.} \ , \qquad (3)$$

instead of the usual current-current form [which transforms as $(8,1)\otimes(27,1)$], then all transitions mediated by the "H_w" must vanish. This is because the pseudoweak Hamiltonian "H_w" can be absorbed as a part of a generalized quark-mass matrix and then rotated away by a simple redefinition of the quark fields.

However, if we simply substitute v_7 and u_6 into Eq. (1) and (2), respectively, the resulting transition amplitudes do *not* vanish:

$$\langle \pi^a B'|v_7|B\rangle \cong -\frac{i}{F_\pi}\langle B'|[F_a^5,v_7]|B\rangle \neq 0 \ , \qquad (4a)$$

$$\langle \pi^a B'|u_6|B\rangle \cong \sum_{B''}\left(\langle \pi^a B'|B''\rangle\frac{\langle B''|u_6|B\rangle}{m_B-m_{B''}} + \langle \pi^a B''|B\rangle\frac{\langle B'|u_6|B''\rangle}{m_{B'}-m_{B''}}\right)\neq 0, \qquad (4b)$$

which gives one pause, to say the least, about the validity of the traditional chiral approach. Nevertheless, we shall demonstrate below how this apparent paradox can be resolved. The resolution has important implications for other calculations of hyperon decay.

The main point is that both Eqs. (4a) and (4b) are incomplete. The problem with the p-wave pole amplitude has been emphasized previously[5]—we must also append a kaon-pole term. The full amplitude then becomes

$$\langle \pi^a B'|u_6|B\rangle \cong \sum_{B''}\left(\langle \pi^a B'|B''\rangle\frac{\langle B''|u_6|B\rangle}{m_B-m_{B''}} + \langle \pi^a B''|B\rangle\frac{\langle B'|u_6|B''\rangle}{m_{B'}-m_{B''}}\right) - \langle K^a B'|B\rangle\frac{\langle \pi^a|u_6|K^a\rangle}{m_K^2-m_\pi^2} \ . \qquad (5)$$

However, the u_6 matrix elements can be related to baryon and meson mass differences, since

$$m_B - m_{B'} = -\left(\tfrac{2}{3}\right)^{1/2}(m_s-m_u)\left(\langle B|u_8|B\rangle - \langle B'|u_8|B'\rangle\right),$$
$$m_K^2 - m_\pi^2 = -\left(\tfrac{2}{3}\right)^{1/2}(m_s-m_u)\left(\langle K|u_8|K\rangle - \langle \pi|u_8|\pi\rangle\right). \qquad (6)$$

Thus, for example, consider $\Sigma^+ \to p\pi^0$. We find

$$m_{\Sigma^+} - m_p = \left(\tfrac{1}{2}\right)^{1/2}(m_s-m_u)\langle B|u|B\rangle = \sqrt{2}(m_s-m_u)\langle p|u_6|\Sigma^+\rangle \ ,$$
$$m_K^2 - m_\pi^2 = \left(\tfrac{1}{2}\right)^{1/2}(m_s-m_u)\langle P|u|P\rangle = -2(m_s-m_u)\langle \pi^0|u_6|K^0\rangle \ . \qquad (7)$$

Then, using the standard $P\bar{B}B$ couplings

$$\langle \pi^0 p|p\rangle \sim \tfrac{1}{2}(d+f) \ ,$$
$$\langle \pi^0\Sigma^+|\Sigma^+\rangle \sim f \ , \quad (8)$$
$$\langle \bar{K}^0 p|\Sigma^+\rangle \sim \tfrac{1}{2}(d-f) \ ,$$

we determine

$$\langle \pi^0 p|u_6|\Sigma^+\rangle = \bar{u}(p')\gamma_5 u(p)\left[\frac{\frac{1}{2}(d+f)-f-\frac{1}{2}(d-f)}{\sqrt{2}(m_s-m_u)}\right]=0 \quad (9)$$

and similarly for the other amplitudes, in agreement with the FKW restriction.

The resolution of the problem for the s waves is more subtle and has not been previously noted. Here the point is that in addition to the usual commutator term, which in this case has the form

$$\langle \pi^a B'|v_7|B\rangle^{\text{commutator}} = -\frac{i}{F_\pi}\langle B'|[F_a^5, v_7]|B\rangle \ , \quad (10)$$

there exists also a contribution coming from the strong vertex $B \to B'\pi^a K^0$ followed by the annihilation of the kaon by the weak "Hamiltonian" v_7. This "pole" contribution can be written as

$$\langle \pi^a B'|v_7|B\rangle^{\text{pole}} = \langle 0|v_7|K^0\rangle \frac{1}{m_K{}^2}\langle K^0\pi^a B'|B\rangle \ . \quad (11)$$

If we now allow the pion to become soft (as well as the kaon), the strong vertex can be approximated in terms of the so-called σ term:

$$\langle K^0\pi^a B'|B\rangle \underset{q_\pi\to 0}{\longrightarrow} -\frac{1}{F_\pi F_K}\langle B'|[F_a^5, \partial^\mu A_\mu^7]|B\rangle \ . \quad (12)$$

However, since v^7 and $\partial^\mu A_\mu^7$ are directly proportional to one another, using the quark equations of motion

$$i\partial^\mu A_\mu^7 = (m_s+m_d)v^7 \ , \quad (13)$$

we may interchange their role in Eqs. (11) and (12), yielding

$$\langle \pi^a B'|v_7|B\rangle^{\text{pole}} \underset{q_\pi\to 0}{\longrightarrow} \langle 0|\partial^\mu A_\mu^7|K^0\rangle \frac{1}{F_\pi F_K m_K{}^2}\langle B'|[F_a^5, v_7]|B\rangle = -\frac{1}{F_\pi}\langle B'|[F_a^5, v_7]|B\rangle \ . \quad (14)$$

Hence,

$$\langle \pi^a B'|v_7|B\rangle \underset{q_\pi\to 0}{\longrightarrow} 0 \quad (15)$$

in accord with the FKW theorem.

The new pieces described above will be present for other forms of the Hamiltonian also. The PCAC analysis will, in general, have Eqs. (1) and (2) modified to read

$$\langle \pi^a B'|H_w^{\text{PV}}|B\rangle = -\frac{i}{F_\pi}\langle B'|[F_a^5, H_w^{\text{PV}}]|B\rangle + \langle 0|H_w^{\text{PV}}|K^0\rangle \frac{1}{m_K{}^2}\langle K^0\pi^a B'|B\rangle \quad (16)$$

and

$$\langle \pi^a B'|H_w^{\text{PC}}|B\rangle = \sum_{B''}\left[\frac{\langle \pi^a B'|B''\rangle\langle B''|H_w^{\text{PC}}|B\rangle}{m_B-m_{B''}} + \frac{\langle B'|H_w^{\text{PC}}|B''\rangle\langle \pi^a B''|B\rangle}{m_{B'}-m_{B''}}\right] + \frac{\langle \pi^a|H_w^{\text{PC}}|K^a\rangle\langle K^a B'|B\rangle}{m_\pi{}^2-m_K{}^2} \ . \quad (17)$$

As indicated in Eq. (12), it is important to realize that the new term for the parity-violating Hamiltonian does not vanish in the soft-pion limit, due to the σ term. These modifications should be taken into account in analysis of hyperon decay.

Does this imply that all past treatments of the usual current-current Hamiltonian are incorrect? Fortunately we can show that the new terms are small or vanishing, to leading order in chiral symmetry, in the case of the usual $\Delta S=1$ Hamiltonian. The significant feature is that the usual weak interaction transforms as $(8_L,1_R)$ and $(27_L,1_R)$ under chiral $SU(3)_L\times SU(3)_R$. We give the analysis below for the (8,1) portion; that for (27,1) is similar. The simplest way to discuss the weak interactions of kaons and pions is in terms of effective chiral Lagrangians.[6,7] The (8,1) representation which arises from the usual current-current interaction has the effective Lagrangian[6]

$$L_{\text{eff}}^{(8,1)} \sim \text{Tr}\lambda_i \partial_\mu\Sigma\,\partial^\mu\Sigma^\dagger \ , \quad (18)$$

where

$$\Sigma = \exp\left(i\frac{1}{F_\pi}\lambda\cdot\phi\right) \quad (19)$$

is the usual nonlinear representation, $F_\pi = 94$ MeV being the pion-decay constant and λ_i being the usual SU(3) matrices normalized via

$$\text{Tr}\lambda_i\lambda_j = 2\delta_{ij} \ . \quad (20)$$

Expansion of this Lagrangian in terms of the meson fields ϕ^a gives the relative sizes of the various $K\to 0$, $K\to\pi$, $K\to 2\pi$, etc., vertices and summarizes the constraints of a PCAC analysis. We see that the K-vacuum matrix element for the usual weak Hamiltonian *vanishes.* (In the valence-quark model this follows from the feature that the kaon is a $q\bar{q}$ state, while H_w is a *four*-quark operator.) Thus, the s-wave transition amplitude is given in terms of the commutator term alone, in agreement with Eq. (1). In the case of the P-wave decay, we see that the K-π weak amplitude is nonvanishing. However, because of the momentum dependence of the effective Lagrangian, Eq. (18), the kaon-pole term is suppressed by factors of $m_\pi{}^2/m_K{}^2$ with respect to the usual baryon-pole terms of Eq. (2), and is numerically insignificant.

Although in the standard model these new terms are small, there may be situations where they are important. For example, in the theory of *CP* violation, one considers other forms of weak Hamiltonians generated by the *CP*-violating part of the theory. For these, one should in general include the kaon-pole terms. In addition, our results may be important when lattice-field-theoretic techniques are applied to hyperon decay. As discussed in Ref. 7, lattice calculations may not completely remove self-energy contributions, and these could in practice require the inclusion of the $K \rightarrow$ vacuum term in order to have an analysis consistent with PCAC.

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