Neutrino mixings and right-handed currents in M_{l2} decays

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With a view to ascertaining whether the present experimental values of muon polarization $P_{\mu^+}(M^+_{\mu 2})$ and the ratio $R_M = B(M^+ \to e^+\nu_e)/B(M^+ \to \mu^+\nu_\mu)$ in M_{l2} decays ($M = K, \pi$; $l = e, \mu$) could admit a finite neutrino mass, neutrino mixings, and right-handed currents (RHC's), we have reexamined the decays with the inclusion of these factors. The polarization value $P_{\mu^+}(K_{\mu 2})$, admits a varied range of neutrino mass along with RHC contributions. The present values of R_K and R_π do not favor Kobayashi-Maskawa mixings in the neutrino sector. Electron polarization $P_{e^+}(M_{e2})$, if measured, could be decisive in ascertaining the nature of neutrino mixings.

Recent measurements on muon polarization[1,2] in $K^+_{\mu 2}$ and $\pi^+_{\mu 2}$ decays have placed limits on the admixture of right-handed currents (RHC's) by treating the neutrino as massless. Recently there has been a good deal of activity to look for the effects of a finite neutrino mass,[3] mixings,[4-6] and neutrino oscillations.[7] Further, any theory incorporating RHC's must involve a finite neutrino mass in a consistent manner.[8] We have, therefore, reexamined the muon longitudinal polarization[1,2] with the inclusion of a neutrino mass, neutrino mixings, and RHC contributions.[8-11] The discussion also includes contributions of these factors to the ratio

$$R_M = B(M^+ \to e^+\nu_e)/B(M^+ \to \mu^+\nu_\mu) ,$$

as well as to the electron polarization in M_{e2} decays.

The effective charged-current weak-interaction Lagrangian for the M^+_{l2} decay with the inclusion of neutrino mixings and the RHC contribution may be written as[9,11,12]

$$\mathscr{L}(G/2^{1/2})V_L f_M \sum_{i=1}^{3} [\, U^{L\dagger}_{li}\bar{u}_{\nu i}(k_i)q(1-\gamma_5)v_l(p) + C_R U^{R\dagger}_{li}\bar{u}_{\nu i}(k_i)q(1+\gamma_5)v_l(p)] , \tag{1}$$

where

$$G = 2^{1/2}g_L{}^2/8M_L{}^2, \quad C_R = g_R{}^2M_L{}^2V_R/g_L{}^2M_R{}^2V_L .$$

g_L and g_R are, respectively, the field strengths for left-handed (LH) and right-handed (RH) gauge fields in the generally known left-right-symmetric $SU(2)_L \times SU(2)_R \times U(1)$ model; M_L and M_R are the masses of W_L and W_R gauge bosons; V_L and V_R are the Kobayashi-Maskawa mixings for LH and RH quarks which constitute the meson; U^L_{li} and U^R_{li} are the LH and RH neutrino mixing parameters, respectively, and the other symbols have their usual meanings. Following the usual analysis,[13] we assume $U^L_{li} = U^R_{li} \equiv U_{li}$. The expressions for the longitudinal polarization (P_{l^+}) of the l^+, and the ratio of the branching ratios, $R_M = B(M^+ \to e^+\nu_e)/B(M^+ \to \mu^+\nu_\mu)$, are given by

$$P_{l^+}(M_{l2}) = \frac{-\sum_i |U_{li}|^2(1-C_R{}^2)(\delta_l-\delta_i)\lambda^{1/2}(1,\delta_l,\delta_i)}{\sum_i |U_{li}|^2\{(1+C_R{}^2)[\delta_l+\delta_i-(\delta_l-\delta_i)^2]-4C_R(\delta_l\delta_i)^{1/2}\}} , \tag{2}$$

and

$$R_M = \sum_i F_{ei} \Big/ \sum_i F_{\mu i} , \tag{3}$$

where

$$F_{li} = |U_{li}|^2\{(1+C_R{}^2)[\delta_l+\delta_i-(\delta_l-\delta_i)^2] - 4C_R(\delta_l\delta_i)^{1/2}\}\lambda^{1/2}(1,\delta_l,\delta_i) ,$$

$$\lambda(x,y,z) = x^2+y^2+z^2-2(xy+yz+zx) ,$$

$$\delta_l = m_l{}^2/m_M{}^2, \quad \delta_i = m_i{}^2/m_M{}^2 .$$

In the limit of massless neutrinos, $\delta_i = 0$ for all i, and $\lambda^{1/2}(1,\delta_l,0) = (1-\delta_l)$. Then, Eq. (2) reduces to

$$P_{l^+}(M_{12}) = -\frac{1-C_R{}^2}{1+C_R{}^2} , \tag{4}$$

which is the same expression as that used in Ref. 1 to infer the RHC contribution in $K^+_{\mu 2}$ decay. Further, for $C_R = 0$, this expression gives the usual limit $P_{l^+}(M_{12}) = -1$. However, it may be noted that in the limit $\delta_i = 0$, $C_R \neq 0$, the ratio R_M [Eq. (3)] reduces to the value[14]

$$R_M = \delta_e(1-\delta_e)^2/\delta_\mu(1-\delta_\mu)^2 , \tag{5}$$

and is not affected by the presence of the RHC contribution.

In the neutrino sector, hierarchical[15] (H) and Kobayashi-Masakawa (KM) mixings with the Barger[5] and Silverman-Soni[6] parametrization have been widely used for neutrinos in low-energy processes.[16,17] We use, for hierarchical mixing,[15,17]

$$|U_{e3}|^2=0.0003, \quad |U_{\mu 3}|^2=0.059 ,$$

and for KM mixing[6,17]

$$|U_{e3}|^2=0.044, \quad |U_{\mu 3}|^2=0.0016 .$$

The full mixing matrices are given in Ref. 17. The current experimental limits on neutrino masses are 20 eV $< m(\nu_e) < 46$ eV,[3] $m(\nu_\mu) < 0.5$ MeV,[18] and $m(\nu_\tau) < 125$ MeV.[19] In order to have an order-of-magnitude estimate, we take ν_e to be mostly ν_1, ν_μ to be ν_2, and ν_τ as ν_3. This gives $\delta_1 < 1.09\times10^{-7}$, $\delta_2 < 1.28\times10^{-5}$, and $\delta_3 < 0.802$ for π^+ decays, $\delta_1 < 8.68\times10^{-9}$, $\delta_2 < 1.03\times10^{-6}$, and $\delta_3 < 0.064$ for K^+ decays. As such if we assume $\delta_1=\delta_2=0$, and use $|U_{l1}|^2+|U_{l2}|^2=1-|U_{l3}|^2$ (the unitarity condition[15]), the expression for polarization [Eq. (2)] reduces to

$$P_{l^+}(M_{12}) = -\frac{(1-C_R^2)[(1-|U_{l3}|^2)\delta_l(1-\delta_l)+|U_{l3}|^2(\delta_l-\delta_3)\lambda^{1/2}(1,\delta_l,\delta_3)]}{(1+C_R^2)\{(1-|U_{l3}|^2)\delta_l(1-\delta_l)+|U_{13}|^2[\delta_l+\delta_3-(\delta_l-\delta_3)^2]\}-4C_R|U_{l3}|^2(\delta_l\delta_3)^{1/2}} . \tag{6}$$

Again for the case of vanishing neutrino mass and RHC contribution (i.e., $U_{l3}=0$, $C_R=0$), the above expression gives the expected value of polarization: namely, $P_{l^+}(M_{l2})=-1$.

In Fig. 1, we display the theoretical curves for $P_{\mu^+}(K_{\mu 2})$, with different values of RHC factor C_R, and $m(\nu_3)$ [keeping $m(\nu_1)=0$, and $m(\nu_2)=0.5$ MeV] for the cases of H and KM mixings. The recent measurement of Hayano *et al.*[1] ($P_{\mu^+}=-0.971\pm0.047$) is shown as a vertical shaded strip. A comparison shows that this measurement is not only consistent with the $V-A$ prediction ($P_{\mu^+}=-1$) but it also admits various admixtures of RHC and $m(\nu_3)$ values. For H mixing, it permits $m(\nu_3)\leq 80$ MeV, with an RHC factor $C_R=0$, and $m(\nu_3)\leq 40$ MeV with $C_R=0.2$. However, for KM mixing, all kinematically allowed $m(\nu_3)$ values are permissible for the RHC factor $C_R \leq 0.2$. A similar comparison for the theoretical values of $P_{\mu^+}(\pi_{\mu 2})$, with the measurement of Carr *et al.*,[2] shows that $m(\nu_3)\leq 8$ MeV with $C_R \leq 0.04$ are allowed for both the H and KM mixings.

In Fig. 2, the theoretical variation of P_{e^+} with $m(\nu_3)$, for K_{e2}^+ and π_{e2}^+ decays, is shown for $C_R=0$ and 0.2. The curves for $P_{e^+}(\pi_{e2})$ almost coincide with the curves for $P_{e^+}(K_{e2})$. An interesting feature is that for $m(\nu_3)>1$ MeV the electron polarization values are drastically different for H and KM mixings irrespective of the presence or absence of RHC contributions. As such, this measure-

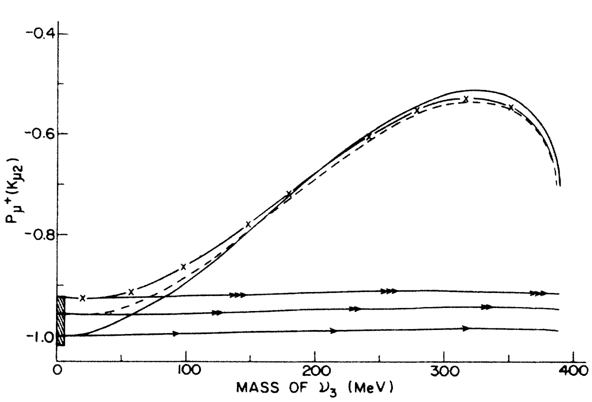

FIG. 1. The variation of muon longitudinal polarization with mass of the ν_3 neutrino, in the decay $K^+\to\mu^+\nu_\mu$. Solid curve, dashed curve, and cross curve are for $C_R=0$, 0.15, and 0.2, respectively, with H mixing. Single-arrow, double-arrow, and triple-arrow curves are for $C_R=0$, 0.15, and 0.2, respectively, with KM mixing.

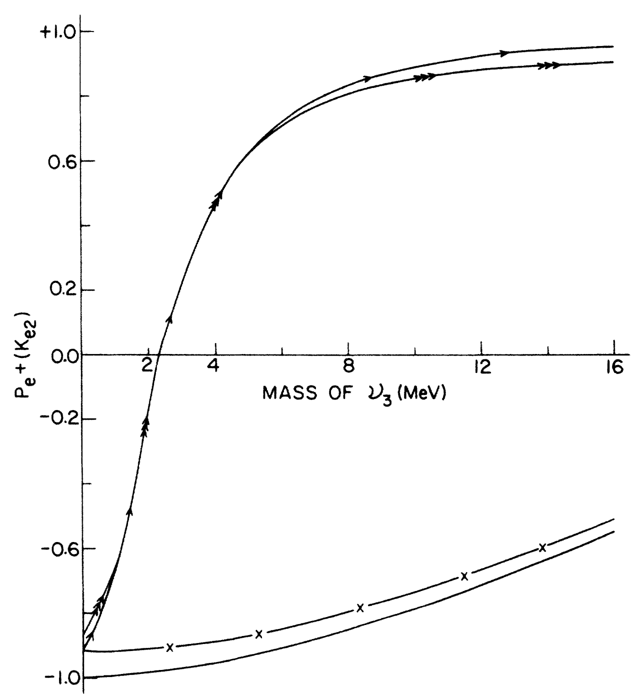

FIG. 2. The variation of electron longitudinal polarization with mass of ν_3 neutrino in the decay $K^+\to e^+\nu_e$. The description of the curves is identical to that given in Fig. 1. The curves for $\pi^+\to e^+\nu_e$ with H mixing and KM mixing, are not shown in the figure because these almost coincide with the corresponding curves for K^+ decay. In the limit of $m(\nu_3)=0$, $C_R=0$, the value of $|P_e|$ is slightly less than 1 for KM mixing because of our choice $m(\nu_2)=0.5$ MeV and the large mixing parameter.

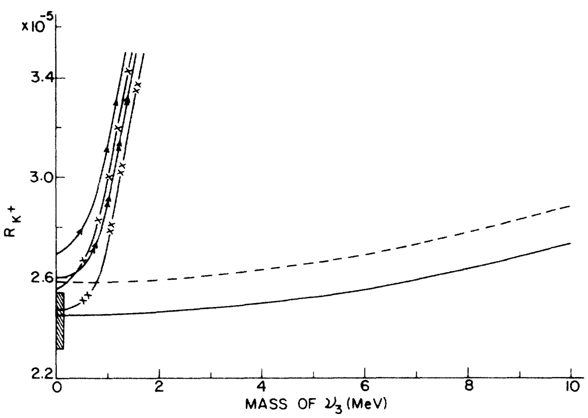

FIG. 3. The variation of the ratio R_{K^+} with mass of ν_3. Radiatively corrected R_{K^+} is taken as $(1-0.05)R_{K^+}^{\text{uncorr}}$ (see text). Solid and dashed curves are with and without radiative corrections, respectively, for H mixing and $C_R=0$. The curves for $C_R=0.2$, with and without radiative corrections for H mixing (not shown here) almost coincide with the corresponding curves for $C_R=0$. The single- and double-arrow curves are for $C_R=0$ and 0.2, with KM mixing and no radiative corrections. Single-cross and double-cross curves are for $C_R=0$ and 0.2, with KM mixing and with the inclusion of radiative corrections.

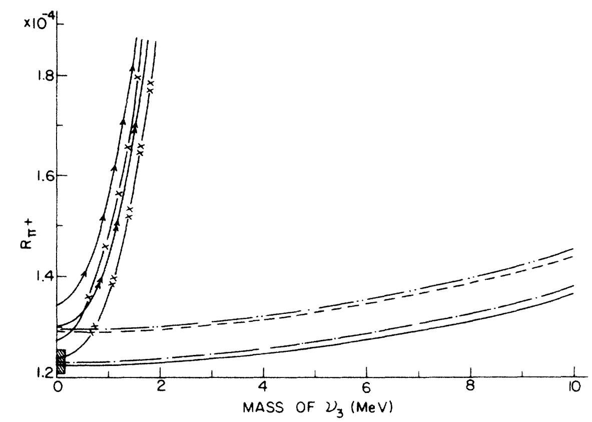

FIG. 4. The variation of R_{π^+}. Single-dot-dash and double-dot-dash curves are for $C_R=0.2$ with H mixing and with and without radiative corrections, respectively. The description of other curves is identical to that given in Fig. 3.

ment may be a useful place to distinguish between the nature of neutrino mixings. To our knowledge no such measurements are reported on P_{e^+}.

Another parameter which is very sensitive to the type of neutrino mixing is R_M. The expression given in Eq. (3) does not incorporate radiative corrections. As emphasized by Shrock,[11] these corrections with the inclusion of finite neutrino mass are uncertain because of meson structure functions. However, in order to have an idea of the dependence of R_M on C_R, $m(\nu_3)$, and neutrino mixings, we assume[11,20] the radiative corrections to be of the order of 5%. The resulting curves for R_{K^+} and R_{π^+} are plotted in Figs. 3 and 4, respectively. The experimental values,[21] $R_{K^+}=(2.43\pm0.11)\times10^{-5}$ and $R_{\pi^+}=(1.232\pm0.024)\times10^{-4}$ are shown as vertical shaded strips. The R_{K^+} and R_{π^+} values completely rule out KM mixings for any value of $m(\nu_3)>1$ MeV and RHC contributions $C_R\le0.2$. However, hierarchical mixing with $m(\nu_3)<5$ MeV and a large RHC is admitted by both the R_{K^+} and R_{π^+} values.

We, therefore, conclude that the recently measured value[1] of longitudinal polarization of a muon in $K_{\mu2}^+$ decay could admit a finite neutrino mass $m(\nu_3)\le40$ MeV with a RHC contribution $C_R\le0.2$, both for hierarchical as well as KM mixings. The ratio R_M could be a useful parameter for ascertaining the nature of neutrino mixings. The presently known values of R_M do not favor KM mixing of massive neutrinos with $m(\nu_3)>1$ MeV.

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