Spectral-function sum rules for *W* boson in the weak- and strong-coupling versions of the standard model

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(Received 15 October 1985)

The QCD-like sum-rule method is applied to the *W* boson in the strong-coupling version of the standard model. For comparison the same methods are applied to the perturbative [Glashow-Salam-Weinberg (GSW)] version of the model. It is shown that the method works in the GSW model with good accuracy independent of the continuum threshold if the definition of the so-called sum-rule window is changed. In the strong-coupling model, one cannot "predict" the *W*-boson mass because the vacuum expectation values (VEV's) are unknown. Instead, a restriction on VEV's coming from the experimental data can be found when some additional assumptions about the *W'* contribution are made. These restrictions can be useful in the application of the sum-rule method to other channels (Higgs, isoscalar vector, excited fermions, leptoquarks, etc.). The vacuum structure in the strong-coupling model may differ considerably from the GSW model.

I. INTRODUCTION

There is significant interest in models containing "new physics," especially now that the intermediate bosons have been discovered.[1,2] Among these are composite models of electroweak interactions.[3–10] In these models, not only fermions but also the *W* and *Z* bosons can be made up of "preons" with spin 0 and $\frac{1}{2}$. Since fundamental methods of calculation are not available for these types of models, it seems to be interesting to use semiphenomenological methods for analysis of the spectrum.

In the present paper we consider the application of the Shifman-Vainshtein-Zakharov (SVZ) -like sum-rule method[11] (SRM) to the strong-coupling version of the standard model (SCSM) of electroweak interactions, and its more familiar perturbative realization [Glashow-Salam-Weinberg (GSW)]. In particular, we study the *W*-boson channel of these models. For some discussion of the status of this particular composite model, see Ref. 12.

The SRM within general composite models of electroweak interactions has been considered in Refs. 13–20. It is a powerful tool in QCD (see, for example, the Refs. 21–24). Our work differs from previous studies of electroweak interactions using the SRM in two important points. First, we propose a different definition of the "sum-rule window" based on a different criterion for convergence. Since the spectrum and couplings of the SCSM differ dramatically from QCD, it is not surprising that the definition of the sum-rule window should be changed. In particular, the continuum threshold for bosonic correlation functions occurs at a mass (typically twice the lightest fermion mass) much lower than the lowest pole term (e.g., the *W* boson). Second, we check our assumptions and methods by verifying that we reproduce the standard model in the GSW limit. Of course, in the GSW model one can calculate everything without the SRM. We study the SRM for the GSW model to be certain that we calculate correctly in the SCSM and to check if there is a complementarity limit from the SCSM to the GSW model.

The SCSM Lagrangian is formally identical with the GSW model. The difference is in the values of the parameters. Λ_H is large and the parameters of the scalar potential $V(\phi)$ are such that no spontaneous symmetry breaking occurs, $\langle\phi\rangle=0$; the $SU(2)_L$ gauge symmetry is exact and confining. All physical particles are $SU(2)_L$ singlets. The left-handed fermions "consist of" a left-handed fundamental fermion and a scalar; the *W* and *Z* bosons are bound states of scalars; right-handed fermions are pointlike. The SCSM can reproduce all the low-energy weak phenomena.[12]

The SCSM is similar to QCD in its main features: both are confining and asymptotically free. They differ in that the chiral symmetries of the SCSM must not break spontaneously in order that (composite) quark and lepton masses be much less than $G_F^{-1/2}$. Standard models for quark dynamics (e.g., the bag model or the nonrelativistic quark model) cannot be adopted to this realization of chiral symmetry. On the other hand, it appears to present no fundamental problem for the sum-rule method. The SRM in QCD shows us[21–24] that VEV's are responsible for the mass-spectra creation, the perturbative growth of $\alpha_S(M)$ at small M gives only small corrections to the zeroth-order approximation and has nothing to do with m_ρ, m_p, etc. Numerically, it means that $\Lambda_S < m_\rho$ (in fact, $\Lambda_S = 100$–150 MeV $\ll m_\rho$ had been used in most of the SRM analyses in QCD); i.e., $\alpha_S(M)$ is small at M of the order of typical hadronic masses. The main assumption of this paper is that this is the case for the SCSM. What does it mean for Λ_H? If we ignore all other interaction contributions (Yukawa coupling, self-coupling of the scalar fields, strong interaction, and electromagnetic interaction) to the renormalization-group equation for α_H and take into account only the gauge-boson interaction, then in the leading-log approximation, we get for α_H

$$\alpha_H(M)=\frac{\pi}{b\ln\frac{M^2}{\Lambda_H{}^2}}\ ,$$

$$b=\tfrac{1}{4}(\tfrac{22}{3}-\tfrac{12}{3}-\tfrac{1}{6})\approx 0.792\ .$$

If one demands $\alpha_H(80\text{ GeV})\lesssim 1$, then $\Lambda_H\lesssim 10$ GeV. Such a small value for Λ_H may seem to be a difficulty for the SCSM. This feeling comes from the numerical analogy with QCD. In QCD the characteristic scale which determines the orbital excitation energy, or the inverse radius of the composite particle (ρ meson,proton,...) is numerically related to Λ_S as $m_{R'}-m_R\sim K\Lambda_S$, where $K\sim 3$–5, $\Lambda_S\sim 100$–150 MeV. If we pursue a direct analogy between QCD and SCSM, then we would expect $m_{M^*}-m_W\sim K\Lambda_H\sim 30$–50 GeV, where m_{M^*} is the mass of an exotic (with respect to the standard model) particle. Such light exotics seem to be excluded by experiment.

At first sight this seems to be a serious problem for the sum-rule method and the SCSM in general. We believe there is no problem. Instead, the analogy is faulty and the fault lies in the numerical coincidence between Λ_S and $1/R$ (the nuclear radius) in QCD. R measures the distance scale at which confining effects become important; $1/\Lambda_S$ measures the distance scale at which a prescription-dependent definition of the coupling becomes infinite. The former is a physical (measurable) parameter; the latter may or may not be an accurate measure of the mass scale of the theory. They are related by some (unknown) renormalization-group equations in the strong-coupling domain, which may differ markedly between $SU(2)_L$ and SU(3). In fact, in lowest order, the coupling runs much more slowly in $SU(2)_L$ than in SU(3), so although $\Lambda_H\sim 10$ GeV the "confinement radius," i.e., the scale at which the $SU(2)_L$ becomes strong is ~ 80 GeV. Thus, we believe it to be natural to expect $m_{M^*}-m_W\sim(10$–$15)\Lambda_H$ in the SCSM. Since Λ_H may not be a good measure of the excitation spectrum it may be better to pursue a different analogy with QCD. In QCD typically $m_{R'}-m_R\sim K(|\langle\bar{q}q\rangle|)^{1/3}$, where $K\sim 1$–3, and $\langle\bar{q}q\rangle\approx-(250\text{ MeV})^3$ is the light-quark condensate (one could use the constituent-quark mass instead). Suppose that the SCSM is analogous: $m_{M^*}-m_W\sim K(\langle\phi^2\rangle)^{1/2}$ where $K\sim 1$–3, $\langle\phi^2\rangle\gtrsim(170\text{ GeV})^2$ (see the analysis of the VEV's in the SCSM in Sec. III). This case, where not only the mass of the ground state but also the excitations are determined by the VEV's (scalar and gluonic) is very natural from the SRM point of view.

These arguments notwithstanding it may be that Λ_H is larger, $\Lambda_H\sim 50$ GeV and $\alpha_H(m_W)$ is large. Then, the SR analysis cannot be carried too close to m_W. The lower bound of the sum-rule window (SRW) may correspond to those M where $\alpha_H(M_1)\sim 1$ (the one-loop formula gives $M_1\sim 4.5m_W$ for $\Lambda_H\sim 50$ GeV), but not to those M where the power corrections become too large. This does not decrease the SRW to zero (see Sec. III) but it creates a problem of accuracy in the W mass determining from the SRM (if we knew the VEV's) or in the VEV's estimations (if we knew the W-boson parameters). For at $M\gtrsim M_1\sim 4.5m_W$, $\exp(-m_W{}^2/M^2)$ is close to 1 and is not sensitive to any value $m_W\lesssim 150$ GeV. This case resembles the situation with the π meson in QCD: if $\Lambda_H\sim m_W$ then one cannot "calculate" m_W from the SRM [but it does not exclude the use of the SRM for the analysis of more heavier "exotic" states, see case (IV) in Sec. III].

The purpose of this paper is to see how and why the SRM works for the W boson in the GSW model and to get some estimates on VEV's in the SCSM (unfortunately, one cannot use the SRM in the opposite direction, i.e., to predict the m_W, because the VEV's are unknown).

This work is organized as follows. In Sec. II the SRM in the GSW model is considered. It is shown that to extract not only the value of the W mass but also its coupling constant g_{Wff}, one must use the current $J_\mu(x)=-(i/2)\phi^\dagger(x)\overleftrightarrow{D}_\mu\phi(x)$ (see the notation below). Then, a very short review of the QCD-like SRM is given and the method is applied to the W boson. The attempt to use the SRM by SVZ (Ref. 11) without any changes is shown to fail. In the "sum-rule window" (SRW) defined as SVZ prescribe, the left-hand side (LHS) of the SR has nothing to do with the right-hand side (RHS). Therefore, a new definition of the SRW is suggested. In this SRW, the LHS is equal to the RHS with the accuracy $\approx 4\%$ if the parameters on both sides are taken from the GSW model.

In Sec. III we proceed with the SR analysis in the SCSM. Unlike QCD where there is a lot of experimental data to fix the VEV's, or the GSW model where the vacuum structure is simple, in the SCSM the VEV's are unknown beforehand. So, one can only try to evaluate the VEV's from the experimental data on W boson (not vice versa). This information about VEV's can be useful in the SR analysis for the other channels in the SCSM. The question of whether or not the SCSM can be distinguished from the GSW model from the SRM point of view is discussed and the assumptions used in the paper are quoted.

In Appendix A the expression for a massless scalar propagator in x space in an external gauge field is calculated up to second order. The use of this expression in Appendix B makes the vacuum polarization calculations for some currents in the SCSM much easier.

II. SUM RULE FOR W BOSON IN THE GSW MODEL

First of all, let us derive the current which creates the W boson from the vacuum in GSW and SCSM. The kinetic term in the scalar piece of the Lagrangian is the same for both models:

$$\mathscr{L}^\phi=(D_\mu\phi)^\dagger(D^\mu\phi)=\tfrac{1}{2}\,\mathrm{Tr}[(D_\mu\Omega)^\dagger D^\mu\Omega]\ . \quad (1)$$

Here

$$\phi=\begin{bmatrix}\phi_1\\ \phi_2\end{bmatrix},\quad \phi^\dagger=(\phi_1^*\ \ \phi_2^*)$$

is the scalar doublet [under $SU(2)_L$ gauge group], $D_\mu=\partial_\mu-ig\omega_\mu-ig'YB_\mu$ is the covariant derivative, $\omega_\mu=\omega_\mu^aT^a$ and B_μ, T^a, and Y, g, and g' are gauge fields, generators, and coupling constants for the $SU(2)_L$ and U(1) groups, respectively ($Y=\frac{1}{2}$ for the scalar doublet). The use of the matrix

$$\Omega=\begin{bmatrix}\phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^*\end{bmatrix}$$

reveals explicitly the global $SU(2)_R$ invariance of $\mathcal{L}^\phi$:

$$\mathcal{L}^\phi \to \mathcal{L}^\phi \text{ when } \Omega\to\Omega h \ , \quad h=e^{-i\beta^a T^a}, \ \beta^a=\text{const} \ . \tag{2}$$

The isovector Noether current corresponding to Eq. (2) is

$$J_\mu^a=-\frac{\delta\mathcal{L}^\phi}{\delta(\partial_\mu\Omega)}\delta_a\Omega-\frac{\delta\mathcal{L}^\phi}{\delta(\partial_\mu\Omega^\dagger)}\delta_a\Omega^\dagger = -\frac{i}{2}\text{Tr}[T^a\Omega^\dagger(D_\mu\Omega)-(D_\mu\Omega)^\dagger\Omega T^a] \ . \tag{3}$$

After some algebra, one can get

$$J_\mu^{(+)}\equiv J_\mu^1+iJ_\mu^2=-i\phi^T\epsilon D_\mu\phi \ , \tag{4}$$

$$J_\mu^{(-)}\equiv J_\mu^1-iJ_\mu^2=i\phi^\dagger D_\mu\epsilon\phi^*=(J_\mu^{(+)})^* \ , \tag{5}$$

$$J_\mu^3=-\frac{i}{2}\phi^\dagger\overleftrightarrow{D}_\mu\phi \ . \tag{6}$$

Here

$$\phi^T=(\phi_1\ \phi_2), \quad \phi^*=\begin{bmatrix}\phi_1^*\\ \phi_2^*\end{bmatrix} ,$$

$$\epsilon=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}, \quad \overleftrightarrow{D}_\mu=\overleftrightarrow{\partial}_\mu-2ig\omega_\mu-2ig'YB_\mu \ .$$

In the SCSM $SU(2)_L$ and $SU(2)_R$ symmetries are exact (we are ignoring electromagnetism and Yukawa couplings), in the GSW model they both are broken. In both versions of the model the vacuum (and the Lagrangian) is symmetric under transformations $\Omega\to e^{i\alpha\cdot\tau}\Omega\, e^{-i\alpha\cdot\tau}$, $\Psi_L^a\to e^{i\alpha\cdot\tau}\Psi_L^a$, $\omega_\mu\to e^{i\alpha\cdot\tau}\omega_\mu e^{-i\alpha\cdot\tau}$, $\alpha=$const, Ψ_L^a is a fundamental left-handed fermion. We will refer to this symmetry as (weak) isospin.

The normalization of the currents (4)–(6) is fixed by the current algebra:

$$[J_0^+(x),J_0^-(y)]_{x^0=y^0}=2J_0^3(x)\delta^3(\mathbf{x}-\mathbf{y}) \ . \tag{7}$$

In either version of the model the chiral symmetry remains unbroken so the matrix element of the current J_μ^a between light fermions is

$$\langle F_L(p')\,|\,\mathbf{J}_\mu(0)\,|\,F_L(p)\rangle=\overline{U}_L(p')\gamma_\mu\frac{\tau}{2}U_L(p)\mathcal{F}(q^2) \tag{8}$$

$[q^2=(p'-p)^2]$, independent of whether the fermion is composite (SCSM) or fundamental (GSW). Here $F_L(p)$ is the physical left-handed fermion state with momentum $p\,(e_L,\mu_L,\nu_e,\nu_\mu,\dots)$; U_L are Dirac spinors. In both versions of the model

$$\mathcal{F}(0)=1 \ . \tag{9}$$

In the SCSM, Eq. (9) follows from the (unbroken) global $SU(2)_R$ invariance and the fact that the composite fermions are $SU(2)_R$ doublets. [$F_L\sim\Omega^\dagger\Psi_L$, where Ψ_L is a fundamental fermion transforming as a (2,1) under $SU(2)_L\times SU(2)_R$, transforms as a doublet under (2).] In the GSW model, Eq. (9) follows from the fact that in the unitary gauge the current $\mathbf{J}_\mu$ is proportional to ω_μ (see below).

When q^2 approaches $m_W{}^2$, $\mathcal{F}(q^2)$ becomes

$$\mathcal{F}(q^2)\sim\frac{F_W m_W{}^2 g_{Wf\bar f}}{q^2-m_W{}^2} \ , \tag{10}$$

where F_W is the (dimensionless) coupling of the W boson to $\mathbf{J}_\mu$ and $g_{Wf\bar f}$ is the on-shell $Wf\bar f$ coupling. In the GSW model

$$F_W g_{Wf\bar f}=1 \tag{11}$$

(at the tree level) and $g_{Wf\bar f}$ is g, the $SU(2)_L$ gauge coupling. F_W is defined by

$$\langle\, W^\pm(p,\lambda)\,|\,J_\mu^\pm(x)\,|\,0\,\rangle=e^{ip\cdot x}\epsilon_\mu^\lambda(p)F_W m_W{}^2\sqrt{2} \ . \tag{12}$$

To prove (11) write $J_\mu^\pm$ [Eqs. (4) and (5)] in unitary gauge where

$$\phi(x)=U(x)\phi'(x)$$

and

$$\phi'(x)=\begin{bmatrix}0\\ \frac{1}{\sqrt{2}}[v+\rho(x)]\end{bmatrix} ,$$

$$U(x)=\exp[i\xi^a(x)T^a/v] \ .$$

Then

$$J_\mu^\pm=\frac{g\,(v+\rho)^2}{4}\omega_\mu^\pm\sqrt{2} \ , \tag{13}$$

where $\omega_\mu^\pm$ is the field of the $SU(2)_L$ gauge boson, identified directly with the physical $W^\pm$ in the GSW model. Using $gv^2/4=m_W{}^2/g$, Eq. (11) follows upon combining Eqs. (12) and (13). In the SCSM, Eq. (11) is true in the limit that exotic composite particles are heavy and weakly coupled to F_L (Ref. 12). It is the deviation of $F_W g_{Wf\bar f}$ from unity which the SRM relates to the condensate of the SCSM.

In fact, one can use any currents creating a W boson from the vacuum for the SR analysis. Whatever is used, its coupling to fermions as well as to W bosons must be related to experimental observables. Now let us turn to the SR analysis with the current (6). Everywhere below we will consider the SR for intermediate bosons ignoring electromagnetism. Then, because of the isotopic invariance of the vacuum, the SR's for the charged and neutral bosons are the same (up to isotopic factor 2), which implies the same masses for $W^\pm$ and W^3. Then we take into account electromagnetism perturbatively, and the W^3 and B get mixed and give the right mass for Z (Refs. 7, 9, 10, and 12). The advantage of this point of view is that it allows us to examine the Z boson in the same way both in the GSW model and in the SCSM (for details see Ref. 12). We also convinced ourselves that in the standard model one can get the right Z mass from the SR for the current J_μ^3 with electromagnetism taken into account from the very beginning [then, new VEV's like $(g')^2\langle(\phi^\dagger Y\phi)^2\rangle$, $g^2(g')^2\langle\phi^\dagger T^a\phi\phi^\dagger YT^a\phi\phi^\dagger Y\phi\rangle$, and so on appear on the RHS].

In short, the idea of SRM by SVZ is the following.[11,21–24] The polarization operator of the appropriate current is considered:

$$\Pi^W_{\mu\nu}(x,y)=\langle 0\,|\,TJ^a_\mu(x)J^a_\nu(y)\,|\,0\rangle\ ,$$

$$\Pi^W_{\mu\nu}(q)=i\int d^4x\, e^{iqx}\Pi^W_{\mu\nu}(x,0)=(q_\mu q_\nu-g_{\mu\nu}q^2)\Pi^W(q^2).$$

At large $Q^2=-q^2>0$, one can look at $\Pi^W(q^2)$ from two different points of view. The first one is to parametrize $\mathrm{Im}\Pi^W(S)$ phenomenologically in terms of some simple model and then use dispersion relations (with subtractions if necessary). This point of view we will call LHS (left-hand side) hereafter. Thus,

$$\Pi^W(q^2)=\int_0^\infty dS\frac{(-Q^2)}{S(S+Q^2)}\frac{1}{\pi}\mathrm{Im}\Pi^W(S)+\text{polynomial}\ ,$$
$$\frac{1}{\pi}\mathrm{Im}\Pi^W(S)=F_W{}^2m_W{}^2\delta(S-m_W{}^2)+a\theta(S-t_c)\ . \qquad (14)$$

Here, t_c is the continuum threshold and a is a factor which is defined below. As one can see, the zeroth-width approximation is used for the lowest resonance contribution and all other contributions [for example, the one from the W-physical-Higgs-boson, W-two-physical-Higgs-bosons intermediate states, the presence of which in the GSW model can be seen from Eq. (13)] are parametrized by the θ function.

The other, "theoretical," side of the SR (this side will be called RHS, right-hand side) is the Wilson operator-product expansion[25,26] (OPE) generalized by SVZ (Ref. 11) for the nonperturbative dynamics. The RHS is the usual perturbative series in $\alpha_H=g^2/4\pi$ together with a series of nonperturbative power corrections due to the VEV's allowed in the theory. For the current (6), the RHS looks like

$$\Pi^W(q^2)=C_1\langle 1\rangle+C_{\phi^2}\frac{\langle\phi^2\rangle}{q^2}+C_{\phi^4}\frac{\langle(\phi^\dagger T^a\phi)^2\rangle}{q^4}+C_{\omega^2}\frac{\langle\omega^2\rangle}{q^4}+\cdots\ . \qquad (15)$$

The graphical representations of C_1, C_{ϕ^2}, C_{ϕ^4}, and C_{ω^2} are shown in Figs. 1(a)–1(d), respectively. The coefficient in front of the unit operator contains all the perturbative contributions (PC). In a simplified version of the OPE, the coefficients in front of the power-correction contributions (PCC) are calculated in perturbation theory, while it is assumed that the perturbation-theory contribution to VEV's is absent. In some theories this simplified version of the OPE can lead to paradoxes. But it works for our case. For the theoretical status of OPE, see Refs. 27 and 28 and references therein.

In Eq. (15) we included only the lowest-dimension VEV's in the GSW and SCSM model:

$$\langle\phi^2\rangle=\langle 0\,|\,\phi_1^*\phi_1+\phi_2^*\phi_2\,|\,0\rangle\ ,$$

$$\langle(\phi^\dagger T^a\phi)^2\rangle=\left\langle 0\left|\sum_{a=1}^{3}\phi^\dagger T^a\phi\phi^\dagger T^a\phi\right|0\right\rangle\ ,$$

$$\langle\omega^2\rangle=\langle 0\,|\,\omega^a_{\mu\nu}\omega^a_{\mu\nu}\,|\,0\rangle\ ,$$

$$\omega^a_{\mu\nu}=\partial_\mu\omega^a_\nu-\partial_\nu\omega^a_\mu+g\epsilon^{abc}\omega^b_\mu\omega^c_\nu\ .$$

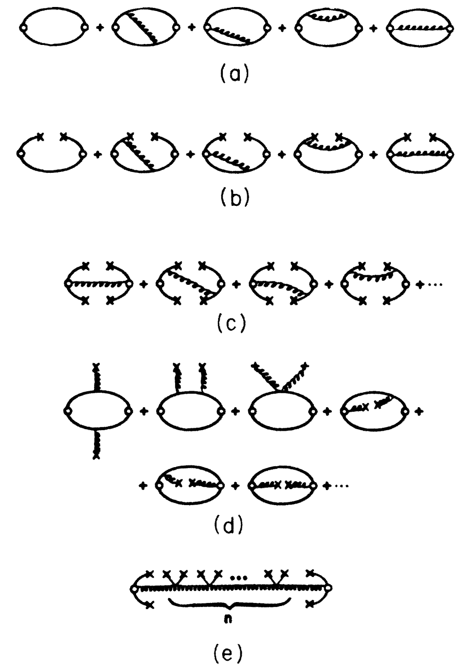

FIG. 1. Diagrams of lower-dimension VEV's and lower power in α_H, contributing to the OPE of the two-point function of the current creating a W boson from the vacuum. (a) and (b) Contributions proportional to the unit operator and $\langle\phi^2\rangle$ together with their first correction. (c) and (d) Contributions proportional to $\langle(\phi^\dagger T^a\phi)^2\rangle$ and $\langle\omega^2\rangle$. (e) The only diagram (to leading order in α_H) proportional to the scalar VEV of dimension $d=2n+4$ and to the α_H^{n+1} in the Landau gauge. Solid line is scalar field, waved line is hypergluonic field, and circle is the current creating a W boson. Broken lines mean nonperturbative interaction with the vacuum.

The VEV's can depend slightly on Q^2 due to the anomalous dimensions of composite operators. There is no fermion VEV in our case because we assume the chiral symmetry of the model remains unbroken. The zeroth-order contribution to C_1 is $C_1=-a_0\ln(Q^2/\mu^2)$, $a_0=(96\pi^2)^{-1}\approx 0.001$ (see Appendix B). The next correction to C_1 is[20] $(-a_0)(9\alpha_H/4\pi)\ln(Q^2/\mu^2)$. As we shall see, one can neglect this correction. Then, according to the usual model for continuum, the factor a on the LHS [see Eq. (14)] has to be $a=a_0$.

The method for extracting the mass and the coupling constant by SVZ in QCD (Ref. 11) is the following. The numerical values for VEV's are taken either from "something else" (the quark VEV $\langle\bar qq\rangle$ is determined by current algebra) or from the SR analysis in some channels (gluonic VEV is fixed by the charmonium sum rule). The higher dimension ($d=6$) quark VEV's are reduced to the square of the VEV for $\langle\bar qq\rangle$ with the help of the "factorization hypothesis" (also called the "vacuum-dominance hypothesis"). For example,

$$\langle 0\,|\,\bar q\gamma_\mu t^a q\bar q\gamma_\mu t^a q\,|\,0\rangle=-\tfrac{16}{9}\langle 0\,|\,\bar qq\,|\,0\rangle^2\ .$$

Usually, the coefficients in front of PCC [like C_{ϕ^4} and C_{ω^2} in Eq. (15)] are proportional to α_S (α_H in our case).

So, one needs the value of α_S (or Λ_S) at the resonance region. This information also comes from external sources (the J/ψ decays and so on). Thus, the RHS ("theoretical") is known after some experimental data are used and additional assumptions are made. To provide a sense to the series [Eq. (15)] $(-q^2)$ has to be not too small.

In our case the phenomenological LHS contains few parameters [F_W, m_W, t_c in Eq. (14)] which describe the lowest resonance contribution (RC) (the W boson in our case) and all the other (continuum) contributions (CC). The Borel (Laplace) transformation is applied to both sides of the SR to nullify subtraction terms, to suppress the CC compared to the lowest state contribution and to suppress the contributions of high-order terms in the OPE:

$$L_{M^2}\Pi^W(q^2) = \lim_{\substack{n,Q^2\to\infty \\ Q^2/n=M^2=\text{const}}} \frac{(Q^2)^n}{(n-1)!}\left[-\frac{d}{dQ^2}\right]^n \Pi^W(Q^2) \,. \tag{16}$$

Now the LHS and the RHS look like

$$\text{LHS} = \frac{F_W{}^2 m_W{}^2}{M^2} e^{-m_W{}^2/M^2} + a\, e^{-t_c/M^2} \,, \tag{17}$$

$$\text{RHS} = a - C_{\phi^2}\frac{\langle\phi^2\rangle}{M^2} + C_{\phi^a}\frac{\langle(\phi^\dagger T^a\phi)^2\rangle}{M^4} + C_{\omega^2}\frac{\langle\omega^2\rangle}{M^4} + \cdots \,. \tag{18}$$

Where (at which M^2) do these expressions have to be equal? Theoretically, if we were able to take into account all intermediate states of LHS and all the perturbative and nonperturbative contributions to RHS, both sides would be the same at any M^2. But we cannot go to small M^2 because we ignored the high-power contributions and higher-order perturbative terms in C_1 (α_H grows at small M^2). We cannot go to very large M^2 either, because there the CC becomes larger than RC [see Eq. (17)]. So, we can trust our SR only in the sum-rule window (SRW),

$$M_{\min}{}^2 \lesssim M^2 \lesssim M_{\max}{}^2 \,,$$

in which, on one hand, the series [Eq. (18)] makes sense (this determines $M_{\min}{}^2$) and, on the other hand, the CC is small (this determines $M_{\max}{}^2$). This very qualitative definition of the SRW was specified in QCD by saying that the power-correction contributions have to be $\lesssim 30\%$ of the perturbative contribution, and the continuum contribution has to be $\lesssim 30\%$ of the perturbative contribution (in QCD it happened that the resonance contribution on LHS at $M^2 \sim M_R{}^2$ is of the order of the perturbative contributions on RHS).

Provided a sum-rule window can be found one can then proceed to fit the LHS parameters (F_W, m_W, t_c) to reproduce the RHS. By means of the SRM much hadron physics can be explained and, sometimes a new phenomenon has been predicted.[11,21–24]

Now let us turn to the SR analysis for the W boson in the GSW model. In Appendix B, the two-point function $\Pi^W_{\mu\nu}$ is calculated [see Eq. (B8)]. So the coefficients C_i in Eq. (15) are

$$C_1 = -\frac{1}{96\pi^2}\ln\frac{Q^2}{\mu^2}\,, \quad C_{\phi^2} = -\tfrac{1}{2}\,,$$

$$C_{\phi^4} = -g^2\,, \quad C_{\omega^2} = \frac{g^2}{192\pi^2}\,.$$

Then the SR [Eqs. (17) and (18)] takes the form

$$\text{LHS} = \frac{F_W{}^2 m_W{}^2}{M^2} e^{-m_W{}^2/M^2} + a\, e^{-t_c/M^2}$$
$$= \text{RHS} = a + \frac{\langle\phi^2\rangle}{2M^2} - \frac{g^2\langle(\phi^\dagger T^a\phi)^2\rangle}{M^4} + \frac{g^2\langle\omega^2\rangle}{192\pi^2 M^4}\,,$$
$$a = (96\pi^2)^{-1} \approx 0.001\,05\,. \tag{19}$$

How does this SR work in the GSW model? The field ϕ gets a VEV

$$\langle\phi\rangle = \begin{bmatrix} 0 \\ v/\sqrt{2} \end{bmatrix}.$$

The physical Higgs ρ and the field ω_μ (which is considered as elementary) have no VEV's, $\langle\rho\rangle = 0$, $\langle\omega^2\rangle = 0$. Thus, ignoring perturbative contributions to $\langle\rho^2\rangle$, $\langle\rho^4\rangle$, $\langle\omega^2\rangle$ compared to the v^2, v^4 (for the importance of the perturbative contributions in VEV's and for examples of a few simple models, see Ref. 28) one obtains for the VEV's in the GSW model

$$\langle\omega^2\rangle = 0\,, \quad \langle\phi^2\rangle = \frac{v^2}{2}\,,$$
$$\langle(\phi^\dagger T^a\phi)^2\rangle = \frac{1}{4}\left[\frac{v^2}{2}\right]^2 = \tfrac{1}{4}\langle\phi^2\rangle^2\,. \tag{20}$$

Besides, in the GSW, where there are only VEV's from the scalar field, one can use[20] (see also Appendix B) the next power-correction term on RHS [Eq. (15)]:

$$C_{\phi^6}\frac{\langle\phi^\dagger T^a\phi\phi^\dagger T^b\phi\phi^\dagger T^a T^b\phi\rangle}{q^6}\,, \quad C_{\phi^6} = -2g^4\,.$$

By the same assumptions as in Eq. (20), one can get for this new VEV

$$\langle\phi^\dagger T^a\phi\phi^\dagger T^b\phi\phi^\dagger T^a T^b\phi\rangle = \frac{1}{16}\left[\frac{v^2}{2}\right]^3\,. \tag{21}$$

The approximations of Eqs. (20) and (21) may be called naturally "tree approximations." The q^2 dependence of VEV's in the GSW model is very slow because $\alpha_H^0 = g^2/4\pi$ is a very flat function of q^2.

As it had been shown previously in the GSW model, $F_W = 1/g$ ($m_W = gv/2$); therefore, the SR is finally

$$\frac{F_W{}^2 M_W{}^2}{M^2} e^{-m_W{}^2/M^2} + a\, e^{-t_c/M^2} = a + \frac{v^2}{4M^2} - \frac{g^2 v^4}{16M^4} + \frac{g^4 v^6}{16\times 8M^6}\,, \tag{22}$$

$$\frac{v^2}{4M^2} e^{-g^2v^2/4M^2} + a\, e^{-t_c/M^2} = a + \frac{v^2}{4M^2} - \frac{g^2 v^4}{16M^4} + \frac{g^4 v^6}{16\times 8M^6}\,. \tag{23}$$

Since a is very small (it is 12 times smaller than the corresponding term for the ρ meson SR in QCD) the attempt to define the SRW in analogy with QCD fails. If we adopt the definition of a SRW described above for QCD to the present case, we obtain

$$8000m_W{}^2 \lesssim M^2 \lesssim 0.83t_c$$

and at these M^2

$$\text{RHS} \gtrsim a, \quad \text{LHS} \lesssim 0.6a \ . \tag{24}$$

The mass scale of the GSW model is set by the scalar VEV, there is a $1/M^2$ term with a large coefficient in front of it (unlike in QCD). So, not only is the SRW far away from the mass we wish to estimate, but the LHS has nothing to do with the RHS in the SRW.

Therefore, let us give a new definition of the SRW for the W-boson SR. Let us determine $M_{\min}{}^2$ by the condition that at $M^2=M_{\min}{}^2$ the contribution of the last term in the $1/M^2$ series is of the order of 50% of the whole series:

$$\frac{g^4v^6}{16\times 8M_{\min}{}^6} \approx 0.5\left[\frac{v^2}{4M_{\min}{}^2}-\frac{g^2v^4}{16M_{\min}{}^4}\right],$$
$$M_{\min}{}^2 \approx 1.6m_W{}^2 \ . \tag{25}$$

The idea of this definition is to make sense of the $1/M^2$ series by comparing the terms of the series with each other, but not with the (small) perturbative contribution. One can give a lot of other definitions with such an idea in mind. All such definitions give $M_{\min}{}^2 \sim m_W{}^2$. For example, one can demand for the last term in the series to be of the order of 50% of the first, main one:

$$\frac{g^4v^6}{16\times 8M_{\min}{}^6} \approx 0.5\frac{v^2}{4M_{\min}{}^2}, \quad M_{\min}{}^2 \approx m_W{}^2 \ .$$

Let us determine $M_{\max}{}^2$ by the condition that at $M^2=M_{\max}{}^2$ the continuum contribution is of the order of 30% of the resonant contribution in LHS:

$$a\,e^{-t_c/M_{\max}{}^2} \approx \frac{0.3v^2}{4M_{\max}{}^2}e^{-g^2v^2/4M_{\max}{}^2} \ . \tag{26}$$

The solution of Eq. (26) grows if t_c grows. So, the lower bound for $M_{\max}{}^2$ is obtained if $t_c=0$:

$$M_{\max}{}^2 \approx 0.3\frac{v^2}{4a} \approx 700m_W{}^2 \ .$$

The SRW definition given above makes the SRM by SVZ applicable to the GSW model without any further changes. There is no contradiction like Eq. (24) anymore; the LHS is equal to the RHS in the window:

$$m_W{}^2 \lesssim M^2 \lesssim 700m_W{}^2 \tag{27}$$

with good accuracy. Really, in this interval for any $t_c \gtrsim 0$ the LHS and RHS differ from each other by less than 26% (the RHS is about 26% larger at $M^2=m_W{}^2$, at $M^2=2m_W{}^2$ this difference is $\approx 3\%$, at $M^2 \sim 700m_W{}^2$ the difference is smaller than 3%, if $0 \lesssim t_c \lesssim 100m_W{}^2$). If one decreases the SRW,

$$2m_W{}^2 \lesssim M^2 \lesssim 100m_W{}^2 \ , \tag{28}$$

then the difference between the LHS and RHS is less than 4% at any $t_c \geq 0$ and the CC is less than 4% of the resonant contribution even at $M^2 \approx M_{\max}{}^2 = 100m_W{}^2$ (see Fig. 2).

Besides, from the new SRW definition, it follows that within the SRW we can neglect not only the higher-order perturbative corrections to the RHS [$\sim\alpha_H(M^2)$, $\alpha_H{}^2(M^2)$, etc.] but the main one [a on the RHS in Eqs. (22) and (23)]. We conclude, therefore, that the SRM does work perfectly in the GSW model of weak interactions, provided the definition of the SRW is suitably modified.

A few comments are in order. The first remark is about the connection of the point of view on the SRM for the GSW model presented here and the finite-energy sum-rule constraints in Refs. 18 and 20. The consideration of the first three terms on the RHS of Eqs. (22) and (23) suggests the idea that the whole series on RHS is the expansion of

$$\frac{v^2}{4M^2}\exp\left[-\frac{g^2v^2}{4M^2}\right].$$

The first three terms in the expansion of this exponential are exactly the first three terms on the RHS of Eqs. (22) and (23). Equating the coefficients in front of equal powers of $1/M^2$ in Eq. (22) in the interval [Eq. (28)], where one can neglect the CC on the LHS and the PCC on the RHS, we get

$$F_W{}^2m_W{}^2=\frac{v^2}{4},$$
$$-F_W{}^2m_W{}^4=-\frac{g^2v^4}{16}, \tag{29}$$
$$\tfrac{1}{2}F_W{}^2m_W{}^6=\frac{g^4v^6}{16\times 8}.$$

These equations are identities in the GSW model where

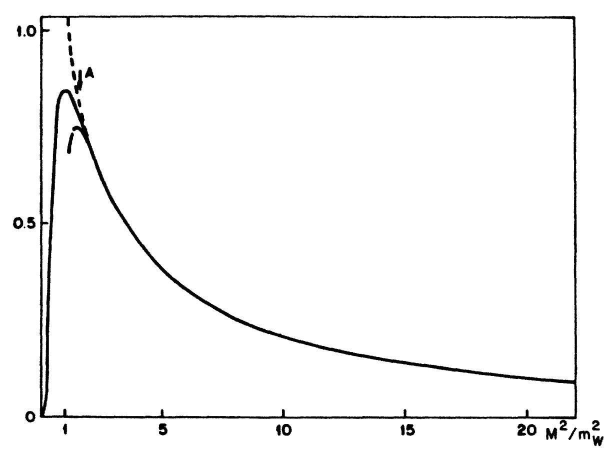

FIG. 2. LHS of the sum rule (22) and (30) (solid line) with the parameters $m_W=81.5$ GeV, $F_W{}^2=2.28$ (values corresponding to the GSW model). RHS of (22) (short-dashed line), RHS of (30) (long-dashed line) with the parameters $C_2=m_W{}^2F_W{}^2$, $C_4=1.73m_W{}^4\approx 0.76F_W{}^2m_W{}^4$. The W' and continuum contributions can be neglected, see the text. Arrow A indicates the lower bound of the sum-rule window defined in text.

$m_W = gv/2$ and, as it has been shown, $F_W = 1/g$. Note that Eqs. (29) are very different from the finite-energy sum-rule (FESR) constraints in Refs. 18 and 20. The FESR constraints in Refs. 18 and 20 can be obtained by equating the coefficients in front of equal powers of $1/M^2$ at $M^2 \to \infty$, i.e., there where one cannot ignore the continuum contribution. As a result, the left-hand sides of Eqs. (29) depend on t_c and that entails the dependence of the solution (F_W and m_W) on t_c. Therefore, the standard value of m_W is obtained in Ref. 20 only for $t_c = 0$. Moreover, in Refs. 18 and 20 a current which is two times larger than (6) was used for the *W*-boson SR. So, there is no statement in Ref. 20 of how to get the value for $g_{Wf\bar{f}}$ from the SR analysis. Further, the desire of the author of Ref. 18 to consider the SR with $t_c > 0$ (i.e., $t_c \neq 0$) immediately leads him to large values of t_c ($t_c \sim 1$ $\text{TeV}^2 \approx 156 m_W{}^2$). This is so because the FESR constraints in Ref. 18 have no solutions with $\langle \omega^2 \rangle > 0$, if

$$t_c \lesssim m_W{}^2 \left[3a + \left[24a^2 + \frac{5a}{g^2} \right]^{1/2} \right] \Big/ (4a + 12a^2 g^2)$$

$$\approx 28 m_W{}^2 \, .$$

The large t_c leads the author of Ref. 18 to a large value for the $SU(2)_L$ gauge field VEV, $\alpha_H \langle \omega^2 \rangle \sim (1/4\pi^2) t_c{}^2$ in the SCSM (otherwise, the mass $m_W{}^2$ cannot be much smaller than t_c).

The point of view on the SRM suggested in the present paper allows us to get the correct values for m_W and $g_{Wf\bar{f}}$ in the GSW model independent of the choice of t_c.

The second remark concerns taking into account the VEV's of arbitrary dimension in the standard model to leading order in g^2 and, therefore, to extend the lower bound of the SRW. In Appendix B (see also Ref. 16) we present a calculation of the coefficient in front of the scalar VEV of dimension $d = 2n + 4$, $n \ge 1$ for the OPE of Eq. (15). It is shown there that in leading order in g^2 the OPE in the GSW model is [instead of (15)]

$$\Pi^W(q^2) = C_1 \langle 1 \rangle + C_{\phi^2} \frac{\langle \phi^2 \rangle}{q^2} + C_{\phi^4} \frac{\langle (\phi^\dagger T^a \phi)^2 \rangle}{q^4} + \sum_{n=1}^{\infty} C_d \frac{\text{VEV}^{(d)}}{(q^2)^{n+2}} \, ,$$

where

$$C_d = -(g^2)^{n+1}/2^n, \quad d = 2n + 4 \, ,$$

$$\text{VEV}^{(d)} = \langle \phi^\dagger T^a \phi (\phi^\dagger \phi)^n \phi^\dagger T^a \phi \rangle \, .$$

Applying the Borel transformation (16) one can get, for the RHS in the standard model,

$$\text{RHS} = a + \frac{\langle \phi^2 \rangle}{2M^2} - \frac{g^2 \langle (\phi^\dagger T^a \phi)^2 \rangle}{M^4} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} (g^2)^{n+1}}{(n+1)! (M^2)^{n+2} 2^n} \text{VEV}^{(d)} \, .$$

Using the tree approximation for $\text{VEV}^{(d)}$ one gets, analogous to (20) and (21),

$$\text{VEV}^{(d)} = \left[\frac{v^2}{2} \right]^n \left[\frac{1}{2} \frac{v^2}{2} \right]^2 = \left[\frac{v^2}{2} \right]^{n+2} \frac{1}{4} = (\langle \phi^2 \rangle)^{n+2} \tfrac{1}{4} \, .$$

Then on the RHS of Eq. (23) we have

$$\text{RHS} = a + \frac{v^2}{4M^2} - \frac{g^2 v^4}{16 M^4} + \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(n+1)!} \left[\frac{g^2 v^2}{4M^2} \right]^{n+1} \frac{v^2}{4M^2} \, .$$

This is exactly the expansion of the first exponential in the LHS of Eq. (23). So, for example, in the SRW $0.16 m_W{}^2 \lesssim M^2 \lesssim 100 m_W{}^2$, the SR is obeyed with the accuracy of the order 4% for any $t_c \ge 0$. Note that both the lower and upper bounds of the SRW are determined now by the condition of the CC to be less than 4% of the resonant contribution. There is no doubt, of course, that the SR for the *W* boson in the standard model would be an identity at any M^2 if we would calculate all radiative corrections to the RHS, perturbative contributions to the scalar VEV's, the finite width of the *W* boson and all other intermediate states on the LHS (instead of a simple continuum model $a\, e^{-t_c/M^2}$).

III. SUM RULE FOR THE $J^P(I) = 1^-(1)$ CHANNEL IN THE SCSM

As was mentioned before, the Lagrangians of the SCSM and GSW models are the same except for the scalar potential. Therefore, the RHS of the SR for the SCSM model is identical to Eq. (19) (if we neglect the scalar field self-couplings), where $g^2 = 4\pi\alpha_H$, with α_H unknown but not too large, for example, $\alpha_H \lesssim 0.7$. There is no advantage to knowing C_{ϕ^6} since other power corrections (like $C_{\omega^3}, C_{\phi^2\omega^2}$) of the same dimension are not calculated.

In composite models of weak interactions, a rich spectrum of new particles may exist. To be definite, pick out the first-excited-state contribution (*W'* boson) from the whole "soup" of the continuum contributions:

$$\frac{F_W{}^2 m_W{}^2}{M^2} e^{-m_W{}^2/M^2} + \frac{F_{W'}{}^2 m_{W'}{}^2}{M^2} e^{-m_{W'}{}^2/M^2} + a\, e^{-t_c/M^2} = a + \frac{C_2}{M^2} - \frac{C_4}{M^4} \, , \quad (30)$$

where

$$C_2 = \tfrac{1}{2} \langle \phi^2 \rangle \, ,$$

$$C_4 = 4\pi\alpha_H \langle (\phi^\dagger T^a \phi)^2 \rangle - \frac{\alpha_H}{48\pi} \langle \omega^2 \rangle \, ,$$

$F_{W'}$ is the *W'* coupling constant to the current (6), α_H is $SU(2)_L$ strong running coupling constant, and $m_{W'}$ is the *W'*-boson mass.

"Complementarity" in gauge theories with scalars in the fundamental representation, as described by 't Hooft[29] and Dimopoulos, Raby, and Susskind,[30] leads us to expect that the SR in the SCSM reduce to the GSW results in an appropriate limit. (For further discussion of complementarity in the SCSM see Ref. 12.) This limit is obtained by letting the parameters on both sides of the SR go to their GSW values [$\alpha_H \to \alpha_H^0 \approx 0.032$, $\langle \omega^2 \rangle \to 0$, $\langle (\phi^\dagger T^a \phi)^2 \rangle \to \frac{1}{4}(v^2/2)^2$, $\langle \phi^2 \rangle \to v^2/2$, $F_{W'} \to 0$ and/or $m_{W'} \to \infty$]. If the SCSM is to describe nature, the left-hand side of the SR must be rather close to the GSW limit (since the GSW model works well) without the fundamental parameters being close to their GSW values. This is clearly possible since C_2 and C_4 [see Eq. (30)] can clearly have (approximately) their GSW values even though $\langle \omega^2 \rangle \neq 0$, $\langle (\phi^\dagger T^a \phi)^2 \rangle \neq \frac{1}{4}\langle \phi^2 \rangle^2$, and $\alpha_H \gg 0.032$. The situation when $\langle (\phi^\dagger T^a \phi)^2 \rangle > \frac{1}{4}\langle \phi^2 \rangle^2$ is preferred from the point of view of composite models of weak interaction. This preference is based on the analogy with QCD.

As mentioned above, in QCD the vacuum-dominance (or factorization) hypothesis (VDH) works rather well. It allows one to reduce the high-dimension quark VEV's to the lowest one. For our case it means that in doing the contraction of the scalar fields ϕ_i and ϕ_k^* inside the VEV $\langle (\phi^\dagger T^a \phi)^2 \rangle$ one need take into account only the vacuum intermediate state and ignore all the others:

$$\langle 0 | \cdots \phi_i \cdots \phi_k^* \cdots | 0 \rangle = \tfrac{1}{2}\delta_{ik} \langle \phi^2 \rangle \langle 0 | \cdots | 0 \rangle \ .$$

We get

$$\langle \phi^\dagger T^a \phi \phi^\dagger T^a \phi \rangle = \tfrac{1}{4}\langle \phi^2 \rangle^2 T^a_{ik} T^a_{lm} \delta_{mi} \delta_{kl} = \tfrac{1}{4}\langle \phi^2 \rangle^2 \tfrac{3}{2} \ ,$$

that is as much as $\frac{3}{2}$ times larger than the tree approximation [see Eq. (20)]. The VDH seems more natural in the composite model. If there is no Higgs mechanism, the lowest-dimension scalar VEV is $\langle \phi^2 \rangle$ not $\langle \phi \rangle$, so then there is no preferred direction:

$$\langle \phi_1^* \phi_1 \rangle = \langle \phi_2^* \phi_2 \rangle = \tfrac{1}{2}\langle \phi^2 \rangle \ ,$$

$$\langle (\phi^\dagger T^1 \phi)^2 \rangle = \langle (\phi^\dagger T^2 \phi)^2 \rangle = \langle (\phi^\dagger T^3 \phi)^2 \rangle \ ,$$

and so on, whereas in the GSW model

$$\langle \phi_1^* \phi_1 \rangle = 0, \ \ \langle \phi_2^* \phi_2 \rangle = \frac{v^2}{2} \neq 0 \ ,$$

$$\langle (\phi^\dagger T^1 \phi)^2 \rangle = \langle (\phi^\dagger T^a \phi)^2 \rangle = 0 \ ,$$

$$\langle (\phi^\dagger T^3 \phi)^2 \rangle = \frac{1}{4}\left[\frac{v^2}{2}\right]^2 \neq 0 \ ,$$

and so on. If the VDH is used, then the GSW limit $F_W \to F_W^0 = 1/g_{Wf\bar f} \approx 1/0.63$, $m_W^2 \to g^2 v^2/4$, $F_{W'} \to 0$ (or $m_{W'} \to \infty$) is obtained if

$$\langle \phi^2 \rangle \to \frac{v^2}{2}, \ \ \langle \omega^2 \rangle \to 12\pi^2 v^4 \left[\frac{3}{2} - \frac{\alpha_H^0}{\alpha_H}\right] .$$

Now let us turn to the SR analysis in the SCSM itself. The parameters on the RHS of the SR (α_H and VEV's) are *a priori* unknown in the SCSM. So there is no way to predict the parameters of the LHS ($m_W, F_W, \ldots$) as in QCD. Therefore, let us assume that $\alpha_H(M^2 \sim m_W^2)$ is not large [for example, $\alpha_H(m_W^2) \lesssim 0.7$] and is a smooth function of M^2, so that one can neglect the M^2 dependence of VEV's through the anomalous dimensions and try to find "experimental values" for VEV's from the SR [Eq. (30)].

What is known about the LHS from the experiment? F_W is no longer $1/g_{Wf\bar f}$, because the normalization condition for the weak form factor of composite fermions [Eq. (8)] with the W' contribution taken into account becomes

$$F_W g_{Wf\bar f} + F_{W'} g_{W'f\bar f} = 1 \ . \tag{31}$$

The experimental value of m_W from Ref. 31 is $81.5 \pm 1 \pm 1.5$ GeV. The W-boson coupling constant $g_{Wf\bar f}$ and the W' coupling constant are related to the Fermi constant

$$\frac{g_{Wf\bar f}^{\,2}}{m_W^{\,2}} + \frac{g_{W'f\bar f}^{\,2}}{m_{W'}^{\,2}} = \frac{8}{\sqrt{2}} G_F \ . \tag{32}$$

The analysis of the experimental restrictions on the parameters of the phenomenological model with two composite $W(Z)$ and $W'(Z')$ bosons and isoscalar vector boson Y was carried out in Ref. 32 (see also Ref. 33). The allowed two-dimensional region for the parameters $K^2 = (F_{W'}/F_W)^2$, $\mu^2 = m_W^{\,2}/m_{W'}^{\,2}$, was found to be dependent on $r^2 = (g_{W'f\bar f}/g_{Wf\bar f})^2$. We will use the restrictions for K^2, μ^2 at given r^2 to estimate the allowed values for F_W and $F_{W'}$ through the equations

$$F_{W'}^{\,2} = K^2 F_W^{\,2} \ , \tag{33}$$

$$F_W^{\,2} = \frac{1 + r^2\mu^2}{4\sqrt{2} G_F m_W^{\,2} (1 + rK)^2} \ . \tag{34}$$

The available experimental data (on m_W, m_Z, $\sin\theta_W, \ldots$) do not impose very strong constraints on the parameters of the first excited intermediate boson.[32] Of course, when the total width of W and Z (it determines $g_{Wf\bar f}$) and $\Gamma_{Z \to e^+ e^-}$ (it determines F_W) are measured, then Eqs. (31) and (32) will give good bounds on the W' parameters.

Meanwhile, let us consider four different sets of W' parameters not excluded by the data and which cover all the interesting possibilities of the SCSM vacuum structure. The first two are those with a large two-particle-channel coupling constant: $g_{W'f\bar f} \approx g_{Wf\bar f}$ (i.e., $r^2 = 1$); the last ones are those with small (or medium?) $g_{W'f\bar f} \approx 0.22 g_{Wf\bar f}$ (i.e., $r^2 = 0.05$). In both cases, we will consider "small" and "large" W'-boson masses. In all cases we will be interested in the maximum possible W' contributions. (Small changes in the maximum possible W' contribution do not affect qualitative conclusions on the SCSM vacuum.)

(1) The W' boson is strongly coupled to the fermion-antifermion channel: $g_{W'f\bar f} = +g_{Wf\bar f}$ (the plus sign is for definiteness); $m_{W'}^{\,2}$ is small; for example, $m_{W'}^{\,2} = 3 m_W^{\,2}$.

The maximum allowed K^2 in this case is ≈ 0.023 (Ref. 32), and according to Eq. (33) the maximum W' contribution to the LHS of Eq. (30) is proportional to $(F_{W'}^{\,2})_{\max} = 0.023 F_W^{\,2}$, i.e., is less than 7% of W contribution even at large $M^2 \sim 100$–$200 m_W^{\,2}$. But at such M^2 the continuum contribution (which is of the order of $a = 0.001$) is $\lesssim 4\%$ of the W contribution, so there is no meaning in separating the W' contribution from the whole "soup" of the continuum. On the other hand, at small $M^2 \sim$ few $m_W{}^2$ the W' contribution (which is $\lesssim 3.5\%$ of the W one) also can be neglected because the RHS is poorly known here. Therefore, in the case under consideration, one can neglect the W' contribution on the LHS of the SR, Eq. (30), when determining C_2 and C_4. From Eq. (34) at $r^2=1$, $K^2=0.023$, $\mu^2=\frac{1}{3}$, $m_W=81.5\pm1\pm1.5$ GeV (Ref. 31) one gets $F_W{}^2 \approx 2.28$. In Fig. 2, the graphs for the LHS with the parameters listed and for the RHS with $C_2=2.28m_W{}^2 \approx F_W{}^2 m_W{}^2$, $C_4=1.73m_W{}^4 \approx 0.76F_W{}^2 m_W{}^4$ are shown and the lower bound of the SRW is indicated.

The fitting of C_2 and C_4 is carried out as follows. In our case (unlike in QCD) the continuum which comes from the discontinuity of the perturbative diagrams is numerically very small. Therefore, one can work at almost "infinite" M^2, where all power corrections except the main one C_2/M^2 are practically zero. On the other hand, at such M^2 the LHS is almost a hyperbole:

$$\frac{F_W{}^2}{M^2}\exp\left[-\frac{m_W{}^2}{M^2}\right] \approx \frac{F_W{}^2}{M^2} .$$

So, the equation LHS($M^2 \sim 100m_W{}^2$) $=$RHS($M^2 \sim 100m_W{}^2$), which is true up to 4% (the CC are of the order of 4% at such M^2) gives

$$C_2=F_W{}^2 m_W{}^2(1\pm0.04) . \tag{35}$$

This equality reminds us of the finite-energy sum-rule constraint[18,20] but it does not contain t_c and is correct for any t_c (it is obtained not at $M^2 \to \infty$, where the continuum contribution is much larger than the resonant one, but at those M^2 where the continuum contribution is much less than the resonant one).

The next power correction C_4/M^4 comes into the game when we go to smaller M^2. One cannot go to too small M^2 because unknown higher power corrections become significant (C_6/M^6, etc.). The smaller M^2, the worse the accuracy of the RHS. Let us use the GSW model as a rough guide to the accuracy. There, the RHS is known at M^2 with the error of the order of $m_W{}^2/2M^2$, if only two terms are used. So let us require for LHS and RHS to coincide with each other at $M^2=3m_W{}^2$ (for example) up to 17%:

$$(1\pm0.17)\frac{F_W{}^2}{3}e^{-m_W{}^2/3m_W{}^2}=\frac{C_2}{3m_W{}^2}-\frac{C_4}{9m_W{}^4} . \tag{36}$$

The main point of Eq. (36) is to define clearly what we mean by accuracy of C_4. Equations (35) and (36) give, for C_4,

$$C_4=(0.85\pm0.42)F_W{}^2 m_W{}^4 . \tag{37}$$

Instead of Eq. (36) one could use other criteria, for example, the equality of the area under the LHS and RHS in the SRW, the equality of the LHS and RHS at $M^2=4m_W{}^2$ and so on. All these methods give similar values for C_4.

The lower bound of the SRW is indicated in Fig. 2, calculated according to the recipe given in Sec. II:

$$0.5\frac{C_2}{M_1{}^2}=\frac{C_4}{M_1{}^4},\quad M_1{}^2 \approx 1.6m_W{}^2 .$$

If we require for the continuum contribution to be $\lesssim 12\%$ of the resonant one, then the following SRW is obtained:

$$1.6m_W{}^2 \lesssim M^2 \lesssim 270m_W{}^2 .$$

As one can see from Fig. 2, agreement between LHS and RHS in the SRW is good. As was to be expected, the vacuum parameters (C_2 and C_4) obtained are close to the GSW values because the W' contribution has been neglected.

Now let us imagine what might have happened if someone told us that C_2 and C_4 are 1.5×10^4 GeV^2 and 0.76×10^8 GeV^4, respectively, and asked us to evaluate the spectrum in the $J^P(I)=1^-(1)$ channel. Solving Eqs. (35) and (36) one gets

$$\begin{aligned} &F_W{}^2=C_2(1\pm0.04)/m_W{}^2=2.5\times(1\pm0.5) ,\\ &m_W{}^2=\frac{C_4}{C_2(0.85\pm0.42)}=5960\ \text{GeV}^2(1\pm0.5) ,\\ &m_W \approx 77\ \text{GeV}(1\pm0.25) . \end{aligned} \tag{38}$$

From Eq. (31), one gets for $g_{Wf\bar f}$ (neglecting the W' contribution)

$$g_{Wf\bar f}=\frac{1}{F_W}=0.63(1\pm0.25) . \tag{39}$$

Using instead of Eqs. (35) and (36) a different fitting procedure and the same accuracy requirement, one would get numbers close to Eqs. (38) and (39) with the same accuracy.

The estimates [Eqs. (38) and (39)] were obtained by neglecting the W' contribution on the LHS. But we do not know *a priori* whether that is reasonable. Therefore, we would have to check how the W' contribution may affect Eqs. (38) and (39) and what can be said about the W' parameters. Trying to fit the SR, Eq. (30), with nonzero $F_{W'}{}^2$ one can get an upper bound on it. The $m_{W'}{}^2$ value seems to be unrestricted because it is impossible to investigate the small contribution on the LHS with only two terms on the RHS. Since the VEV's are unknown, we will not pursue these speculations further.

The estimates, Eqs. (35) and (37), do not change very much for any $m_{W'}{}^2 \lesssim 10m_W{}^2$, so we conclude that for these $m_{W'}{}^2$ and $g_{W'f\bar f}=g_{Wf\bar f}$ the experimental data constrain the SCSM vacuum parameters to be close to the ones in the GSW model:

$$\begin{aligned} &2C_2=\langle\phi^2\rangle=3\times10^4\ \text{GeV}^2(1\pm0.04) ,\\ &C_4=4\pi\alpha_H\langle(\phi^\dagger T^a\phi)^2\rangle-\frac{\alpha_H}{48\pi}\langle\omega^2\rangle\\ &\quad=0.76\times10^8\ \text{GeV}^4(1\pm0.5) . \end{aligned} \tag{40}$$

(2) Let us investigate the large $m_{W'}{}^2$ value case at $g_{W'f\bar f}=g_{wf\bar f}$, for example, $m_{W'}{}^2=100m_W{}^2$. Then, the maximum allowed K^2 is ≈ 0.0051 (Ref. 32) and according to Eq. (33), the maximum W' contribution is proportional to $(F_{W'}{}^2)_{\max}=0.0051F_W{}^2$. One cannot neglect such a contribution at large M^2 and it should be separated from the whole soup of the continuum at $M^2 \sim 100m_W{}^2$. From Eq. (34) at $r^2=1$, $K^2=0.0051$, $\mu^2=0.01$,

$m_W = 81.5 \pm 1 \pm 1.5$ GeV (Ref. 31) one gets $F_W{}^2 \approx 2.0$. In Fig. 3, the graphs for the LHS with the parameters listed and the RHS with $C_2 = 2.94 m_W{}^2 \approx (F_W{}^2 m_W{}^2 + F_{W'}{}^2 m_{W'}{}^2)$, $C_4 = 60.3 m_W{}^4 \approx 0.59 F_{W'}{}^2 m_{W'}{}^4$ are shown and the lower bound of the SRW is indicated. The fitting procedure is similar to the one described before, but now C_2 is determined by the behavior at $M^2 \sim 500$–$1000 m_W{}^2$ (not $\sim 100 m_W{}^2$, as before). C_4 is determined by the behavior at medium M^2, i.e., $M^2 \sim 100 m_W{}^2$ (not $\sim 3 m_W{}^2$ as before). The accuracy of C_2 is of the order of 12% (at $M^2 \sim 500 m_W{}^2$ the CC is about 12% of the resonant contributions), the accuracy of C_4 is about 70%. The SRW is

$$0.41 m_{W'}{}^2 = 41 m_W{}^2 \lesssim M^2 \lesssim 500 m_W{}^2 = 5 m_{W'}{}^2 ,$$

if one requires the CC to be less than 12% on the upper bound of the SRW.

As one can see, the vacuum parameters may considerably differ from the GSW values:

$$\langle \phi^2 \rangle = 4 \times 10^4 \text{ GeV}^2 (1 \pm 0.12) ,$$
$$4\pi\alpha_H \langle (\phi^\dagger T^a \phi)^2 \rangle - \frac{\alpha_H}{48\pi} \langle \omega^2 \rangle = 2.75 \times 10^9 \text{ GeV}^4 (1 \pm 0.7) . \quad (41)$$

Note, that as far as C_2 feels both resonances ($C_2 \approx F_W{}^2 m_W{}^2 + F_{W'}{}^2 m_{W'}{}^2$), C_4 is determined mostly by the heavier resonance contribution ($C_4 \approx 0.59 F_{W'}{}^2 m_{W'}{}^4$).

Now let us again imagine inverting the analysis. Suppose we know C_2 and C_4 and are going to say something about W and W' bosons. We will fail in trying to fit the LHS with a single resonance with $m_R{}^2 \approx m_W{}^2$, but it can be done with the parameters $F_R{}^2 \approx 3.75$, $m_R{}^2 \approx 20 m_W{}^2$. But then the normalization conditions (31) and (32) will be violated even if the errors in F_R and m_R are taken into account. So we have to include the second resonance in our fit. In any case these values for C_2 and C_4 would indicate the presence of a structure at large $M^2 \gtrsim 20 m_W{}^2$. To investigate both resonances more precisely we would need to know the next power corrections ($C_6/M^6, \ldots$) and, perhaps, the dependence of the VEV's on M^2 to extend the SRW. On the other hand, if we knew the W-boson parameters and C_2, C_4 we could predict something more definite about the W' boson.

(3) The W' boson is weakly coupled to the two-particle channel $r^2 = 0.05$, $g_{W'f\bar{f}} \approx 0.22 g_{Wf\bar{f}}$; the W' mass is medium, say, $m_{W'}{}^2 = 10 m_W{}^2$. This case of the medium coupling constant and mass of the W' boson seems most reasonable for the SCSM. Then the maximum allowed K^2 is ≈ 0.1 (Ref. 32) and the maximum W' contribution is proportional to $(F_{W'}{}^2)_{\max} = 0.1 F_W{}^2$. From Eq. (34) at $r^2 = 0.05$, $K^2 = 0.1$, $\mu^2 = 0.1$, $m_W = 81.5 \pm 1 \pm 1.5$ (Ref. 31) one gets $F_W{}^2 \approx 2.29$. In Fig. 4 the graphs for the LHS with the parameters listed and the RHS with

$$C_2 = 4.45 m_W{}^2 \approx F_{W'}{}^2 m_{W'}{}^2 + F_W{}^2 m_W{}^2 ,$$
$$C_4 = 14.75 m_W{}^4 \approx 0.633 F_{W'}{}^2 m_{W'}{}^4$$

are shown and the lower bound of the SRW is indicated. The accuracy of C_2 is $\sim 12\%$; the accuracy of C_4 is $\sim 60\%$. The SRW is

$$0.67 m_{W'}{}^2 = 6.7 m_W{}^2 \lesssim M^2 \lesssim 550 m_W{}^2 = 55 m_{W'}{}^2 ,$$

if one requires for the CC to be less than 12% on the upper bound of the SRW. Again, the vacuum parameters may differ considerably from the GSW values:

$$\langle \phi^2 \rangle = 5.9 \times 10^4 \text{ GeV}^2 (1 \pm 0.12) ,$$
$$4\pi\alpha_H \langle (\phi^\dagger T^a \phi)^2 \rangle - \frac{\alpha_H}{48\pi} \langle \omega^2 \rangle = 6.5 \times 10^8 \text{ GeV} (1 \pm 0.6) . \quad (42)$$

Again, C_2 feels both resonances; C_4 is determined almost

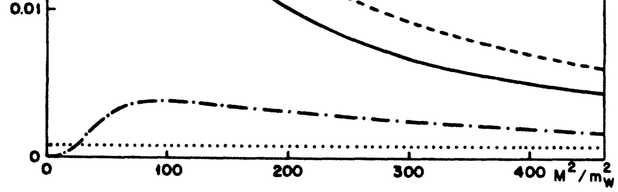

FIG. 3. LHS of the SR (30) (short-dashed line) with the parameters $m_W = 81.5$ GeV, $F_W{}^2 = 2.0$, $m_{W'}{}^2 = 100 m_W{}^2$, $F_{W'}{}^2 = 0.0051 F_W{}^2$. Solid line—$W$-boson contribution to the LHS; dashed-dotted line—W'-boson contribution to the LHS; dotted line shows the order of magnitude of the continuum contribution to the LHS and perturbative contribution to the RHS which were neglected. RHS of the SR (30) (long-dashed line) with the parameters $C_2 = 2.94 m_W{}^2 \approx (F_W{}^2 m_W{}^2 + F_{W'}{}^2 m_{W'}{}^2)$, $C_4 = 60.3 m_W{}^4 \approx 0.59 F_{W'}{}^2 m_{W'}{}^4$. Arrow A indicates the lower bound of the SRW defined in text.

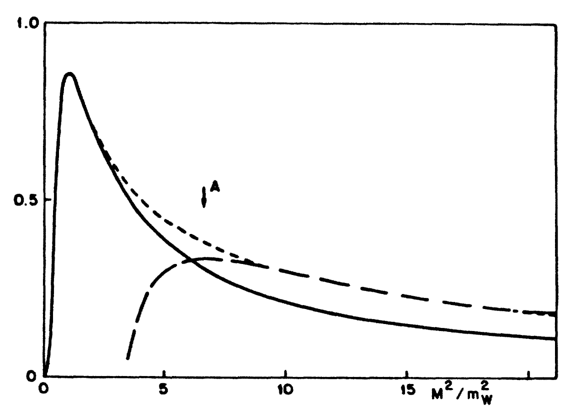

FIG. 4. LHS of the SR (30 (short-dashed line) with the parameters $m_W = 81.5$ GeV, $F_W{}^2 = 2.29$, $m_{W'}{}^2 = 10 m_W{}^2$, $F_{W'}{}^2 = F_W{}^2$. RHS of the SR (30) (long-dashed line) with the parameters $C_2 = 4.45 m_W{}^2 \approx F_W{}^2 m_W{}^2 + F_{W'}{}^2 m_{W'}{}^2$, $C_4 = 14.75 m_W{}^4 \approx 0.63 F_{W'}{}^2 m_{W'}{}^4$. Solid line—$W$-boson contribution to the LHS. The continuum contribution to the LHS and perturbative contribution to the RHS can be neglected, see text. Arrow A indicates the lower bound of the SRW defined in text.

by the heavier resonance. If we try to investigate the LHS having VEV's on the RHS according to (42) we would not succeed by doing this with a single resonance without violating the normalization conditions (31) and (32). If we know the W parameters and C_2, C_4 then we would be able to investigate the W' boson more precisely.

(4) The W' boson is weakly coupled to two-body channel, $r^2=0.05$, $g_{W'f\bar{f}} \approx 22 g_{Wf\bar{f}}$; the W' mass is large, say, $m_{W'}{}^2=100 m_W{}^2$. The maximum allowed K^2 is ≈ 0.09 (Ref. 32) and the maximum W' contribution is proportional to $(F_{W'}{}^2)_{\max} \approx 0.09 F_W{}^2$. From Eq. (34) at $r^2=0.05$, $K^2=0.09$, $\mu^2=0.01$, $m_W=81.5\pm1.0\pm1.5$ GeV (Ref. 31) one gets $F_W{}^2 \approx 2.14$. In Fig. 5, the graphs for the LHS and the RHS with $C_2=20.2 m_W{}^2 \approx 1.04 F_{W'}{}^2 m_{W'}{}^2$, $C_4=1162 m_W{}^4 \approx 0.6 F_{W'}{}^2 m_{W'}{}^4$ are shown. The accuracy of C_2 is $\sim 12\%$; the accuracy of C_4 is $\sim 50\%$. The SRW is

$$1.14 m_{W'}{}^2=114 m_W{}^2 \lesssim M^2 \lesssim 2550 m_W{}^2=25.5 m_{W'}{}^2 .$$

This case is similar to case (1) in the sense that one can take into account on the LHS only one resonance contribution. In this case it is W': the W contribution is $\lesssim 20\%$ of the W' contribution in the SRW. But in the normalization conditions (31) and (32) the W boson is dominating, so we cannot fit the LHS with a single resonance. Both C_2 and C_4 feel now almost only the W' boson; C_2 is determined by its coupling $F_{W'}{}^2$,C_4 by its mass, $m_{W'}{}^2$. Roughly speaking, the order of the heavier resonance mass is $\sim (C_4/C_2)^{1/2}$. The W boson is far away from the SRW, so to investigate it carefully one should know the next power corrections $(C_6/M^6, C_8/M^8, \ldots)$ to be able to extend the lower bound of the SRW up to few $M_W{}^2$. In the case under consideration the W boson resembles the π meson in QCD in the sense that its (mass)^2 is much less than the order of the lower dimension VEV's. The possible analogy between the W boson and the π meson suggests the idea that the W mass can be of the order of Λ_H—the mass scale where α_H becomes large. Then, as in QCD, one cannot "calculate" the W boson mass from the SRM.

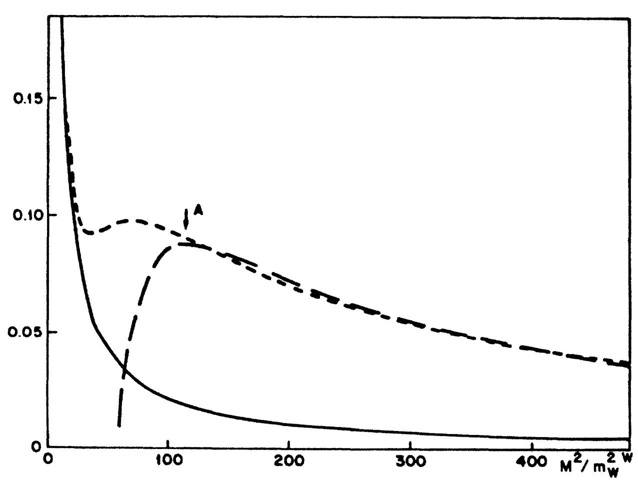

FIG. 5. The same as in Fig. 4, but for parameters m_W $=81.5$ GeV, $F_W{}^2=2.14$, $m_{W'}{}^2=100 m_W{}^2$, $F_{W'}{}^2=0.09 F_W{}^2$, $C_2=20.2 m_W{}^2 \approx 1.04 F_{W'}{}^2 m_{W'}{}^2$, $C_4=1162 m_W{}^4 \approx 0.6 F_{W'}{}^2 m_{W'}{}^4$.

As one can see from the cases considered, the experiment does not yet exclude the possibility that the VEV's in the SCSM are very different from the ones in the GSW model. This is encouraging because it is known from other approaches that the SCSM can account for weak phenomena without apparent fine-tuning of parameters.[12] In the SRM approach similarly it appears that the current data do not force the VEV's to be tuned to their GSW values. For the strong coupling of W' with $f\bar{f}$ and small masses $M_{W'}{}^2 \lesssim 10 m_W{}^2$ one can neglect the W' contribution to the LHS (more precisely not to separate it from the rest of continuum contributions) and C_2, C_4 are close to their GSW values. But still there is a possibility of having $\langle \omega^2 \rangle \neq 0$. For large $m_{W'}{}^2$ or weak $W' f\bar{f}$ coupling, values for C_4 (and even for C_2) which differ very much from the GSW VEV's are possible.

How can one distinguish between the SCSM and GSW models from the point of view of the SRM when the accuracy of the experiment improves? Of course, if the W' boson were found experimentally, the SCSM would be preferable and having W' parameters one would be able to say something definite about the VEV's. For example, if the LHS with the W' contribution measured would have two peaks (from the W and W' bosons) then one could estimate the next power-correction term, C_6/M^6.

But what if the experiment agrees with the GSW model better and better? Does it mean that the SCSM can be rejected from the SRM point of view? The answer is no, if we consider the SR only for $J^P(I)=1^-(1)$ channel. Indeed, the SCSM vacuum contains more parameters. Besides the scalar condensates $\langle \phi^2 \rangle$, $\langle (\phi^\dagger T^a \phi)^2 \rangle$, etc., it has VEV's $\langle \omega^2 \rangle$, $\langle \omega^3 \rangle$, etc., and mixing VEV's, for example, $\langle \phi^\dagger \omega^a_{\mu\nu} T^a \omega^b_{\mu\nu} T^b \phi \rangle$, etc. Even if combinations of VEV's of given dimension must obey relations, like Eq. (40) for C_4 (with better accuracy) these relations can be satisfied with $\langle \omega^2 \rangle \neq 0$, $\langle \omega^3 \rangle \neq 0$, etc. The SR analysis for other channels may be helpful in distinguishing the SCSM from the GSW model. If experiment will tell us that there are no excited fermions, leptoquarks, and so on up to very high energies, it would mean that the VEV's in the SCSM must obey a nontrivial system of relations, which obviously must have the solution with $\langle \omega^2 \rangle = 0$, $\langle \omega^3 \rangle = 0$, etc. (if the GSW model agrees with experiment). The existence of another solution with $\langle \omega^2 \rangle \neq 0$, $\langle \omega^3 \rangle \neq 0$, etc., would not be forbidden but would seem unnatural.

IV. CONCLUSIONS

We have considered the application of the QCD-like sum method[11] to the W boson in the GSW and SCSM of weak interactions. It is shown that the method works in the GSW model if one changes the definition of the sum-rule window: instead of the requirement for the nonperturbative power corrections to be small compared with the perturbative contributions (which are small in our case), one can require the last calculated power correction to be small compared with the first (main) one. In the sum-rule window so defined, the LHS and RHS are equal to each other with the accuracy $\sim 4\%$ independent of the continuum threshold for the GSW values of the parameters

m_W, $g_{Wf\bar{f}}$, $\langle\phi^2\rangle$, $\langle(\phi^\dagger T^a\phi)^2\rangle$, etc.

In the SCSM, the VEV's are unknown. Even the attempt to "calculate" the W-boson mass by means of the SRM may be wrong if $\alpha_H(m_W)$ is very large. But, if reasonable assumptions are made, then one can get some information on VEV's from experimental data. It is shown that the vacuum structure in the SCSM (i.e., VEV's) may differ considerably from the one in the GSW model and still reproduce the structure in the $J^P=1^-$ channel. Arguments are given that one cannot rule out the SCSM from the SRM point of view even if experiment excludes the existence of excited intermediate bosons up to rather high mass.

It is interesting to carry out the sum-rule analysis in the spirit suggested in the present paper in other channels: Higgs boson, isoscalar vector, fermion, leptoquark. At present, little is known about the vacuum and α_H in the composite models of weak interaction. But, in the future the SRM in composite models with accumulating experimental data and theoretical progress may become powerful and predictive as it is in QCD.

ACKNOWLEDGMENTS

We are grateful to M. Claudson, E. Farhi, C. Korpa, and Z. Ryzak for useful discussions. One of the authors (S.D.) is grateful to the International Research and Exchanges Board and the Center for Theoretical Physics at Massachusetts Institute of Technology for support and hospitality. This work was supported in part by funds provided by the U.S. Department of Energy under Contract No. DE-AC02-76ER03069.

APPENDIX A

In this appendix, we derive the expression for the massless scalar propagator in x space in the external gauge field in the Fock-Schwinger gauge:[34]

$$x^\mu \omega_\mu^a(x)=0\ . \tag{A1}$$

This expression simplifies the calculations of the coefficients in front of VEV's in the operator-product expansion in the SCSM (see examples in Appendix B) like the similar expression for the light-quark propagator[35,36] does in QCD (Ref. 35).

The calculations are similar to those in QCD (for a technical review, see Ref. 37). The gauge [Eq. (A1)] is useful because the gauge fields $\omega_\mu^a(x)$ can be expressed in terms of the field strength tensor $\omega_{\mu\nu}^a(0)$ and its derivatives at $x=0$ (Ref. 38):

$$\omega_\mu(x)=\tfrac{1}{2}x^\rho\omega_{\rho\mu}(0)+\tfrac{1}{3}x^\alpha x^\rho[D_\alpha\omega_{\rho\mu}(0)]\cdots\ ,$$

where $\omega_\mu=T^a\omega_\mu^a$, $\omega_{\mu\nu}=T^a\omega_{\mu\nu}^a$, T^a are generators of the gauge group [here SU(2)], D_α is the covariant derivative. We will use the operators with dimension ≤ 4, so the external field has a form

$$\omega_\mu(x)=\tfrac{1}{2}x^\rho\omega_{\rho\mu}(0)\ . \tag{A2}$$

To find the propagator up to second order, one has to solve the equation

$$\left[\frac{\partial}{\partial z_\mu}-ig\omega_\mu(x)\right]\left[\frac{\partial}{\partial z^\mu}-ig\omega^\mu(z)\right]D(z,y)=\delta^4(z-y) \tag{A3}$$

up to the terms $\sim g^2$. Here $D(z,y)$ is a 2×2 matrix, and g is the gauge coupling [the calculations can be easily extended to SU(N)].

The solution of Eq. (A3) in a perturbative series looks like

$$D(z,y)=D_0(z,y)+D_1(z,y)+D_2(z,y)+\cdots\ , \tag{A4}$$

where D_0 is the free propagator, $D_0(z,y)=-id_0/(z-y)^2$, $d_0=1/4\pi^2$, D_1 and D_2 are terms proportional to g and g^2. The requirement (A3) in zeroth, first, and second order gives

$$\begin{aligned}&\Box_z D_0(z,y)=\delta^4(z-y),\quad \Box_z=\frac{\partial^2}{\partial z^\mu\partial z_\mu}\ ,\\ &-2ig\omega_\mu(z)\partial_z^\mu D_0(z,y)+\Box_z D_1(z,y)=0,\quad \partial_z^\mu=\frac{\partial}{\partial z^\mu}\ ,\\ &-g^2\omega_\mu(z)\omega^\mu(z)D_0(z,y)\\ &\qquad -2ig\omega_\mu(z)\partial_z^\mu D_1(z,y)+\Box_z D_2(z,y)=0\ .\end{aligned} \tag{A5}$$

The calculations significantly simplify if we are interested only in the singlet part of $\omega_{\rho\mu}^a\omega_{\rho_1\mu_1}^b$ with respect to Lorentz and color indices (only such an operator may have a nonzero vacuum expectation):

$$\langle 0\,|\,\omega_{\rho\mu}^a\omega_{\rho_1\mu_1}^b\,|\,0\rangle=\frac{1}{3\times 12}\delta^{ab}(g_{\rho\rho_1}g_{\mu\mu_1}-g_{\rho\mu_1}g_{\mu\rho_1})\langle\omega^2\rangle\ , \tag{A6}$$

where $\omega_{\mu\nu}^a=\partial_\mu\omega_\nu^a-\partial_\nu\omega_\mu^a+g\epsilon^{abc}\omega_\mu^b\omega_\nu^c$ [for SU(2)], $\langle\omega^2\rangle=\langle 0\,|\,\omega_{\mu\nu}^a(0)\omega_{\mu\nu}^a(0)\,|\,0\rangle$.

Then, as one can verify, the solutions of (A5) are the functions

$$\begin{aligned}&D_0(z,y)=-\frac{id_0}{r^2},\quad r^2=(z-y)^2\ ,\\ &D_1(z,y)=-d_1\frac{\omega_{\beta\mu}z^\beta y^\mu}{r^2},\quad d_1=\frac{2g}{(4\pi)^2}\ ,\\ &D_2(z,y)=id_2[-\tfrac{3}{2}r^2+2f(z,y)]\ ,\\ &d_2=\frac{g^2\langle\omega^2\rangle}{12(16\pi)^2}\ ,\\ &f(z,y)=\frac{z^2y^2-(zy)^2}{r^2}\ .\end{aligned}$$

This answer can be obtained as follows. Let us multiply Eq. (A3) by $D_0(x,z)$ and integrate over d^4z:

$$\int d^4z\,D_0(x,z)[\Box_z-g^2\omega_\mu(z)\omega^\mu(z)-2ig\omega_\mu(z)\partial_z^\mu]D(z,y)=D_0(x,y)\ .$$

Here $\Box_z$ and ∂_z^μ are acting on the right, the relation $\partial_\mu^z\omega^{a\mu}(z)=0$, which is true in the gauge (A1), was used. After integrating by parts and having in mind the equation for the free propagator, $\Box_z D_0(x,z)=\delta^4(x-z)$, one gets

$$D(x,y)=D_0(x,y)+\int d^4z\, D_0(x,z)[g^2\omega_\mu(z)\omega^\mu(z)+2ig\omega_\mu(z)\partial_z^\mu]D(x,y)\ . \tag{A7}$$

From the integral equation (A7) one gets the standard perturbative series

$$\begin{aligned} &D(x,y)=D_0(x,y)+D_1(x,y)+D_2(x,y)+\cdots\ ,\\ &D_1(x,y)=2ig\int d^4z\, D_0(x,z)\omega_\mu(z)\partial_z^\mu D_0(z,y)\ ,\\ &D_2(x,y)=D_2^{(1)}(x,y)+D_2^{(2)}(x,y)=g^2\int d^4z\, D_0(x,z)\omega_\mu(z)\omega^\mu(z)D_0(z,y)+2ig\int d^4z\, D_0(x,z)\omega_\mu(z)\partial_z^\mu D_1(z,y)\ . \end{aligned} \tag{A8}$$

The divergent integrals in Eq. (A8) have to be regularized in some way. We perform the detailed calculations for $D_1(x,y)$ in the dimensional regularization scheme. The calculations for $D_2(x,y)$ are similar but more cumbersome. The explicit calculation yields

$$D_1(x,y)=-ig\, d_0^2\omega_{\rho\mu}\int d^4z\frac{z^\rho}{(x-y)^2}\frac{2(y^\mu-z^\mu)}{(z-y)^4}\equiv -ig\, d_0^2\omega_{\rho\mu}I^{\rho\mu}(x,y)\ .$$

The integral $I^{\rho\mu}$ in the dimensional regularization is (after transition to Euclidean space and then back)

$$\begin{aligned} I^{\rho\mu}&=i\int d^dz_1\frac{z_1^\rho 2(y_1^\mu-z_1^\mu)}{(x_1-z_1)^2(z_1-y_1)^4}=-i\frac{\partial}{\partial y_{1\mu}}\int d^4z_1\frac{z_1^\rho}{(x_1-z_1)^2(z_1-y_1)^2}\\ &=-\frac{i\pi^2}{2}\frac{\partial}{\partial y_{1\mu}}\left[(x_1^\rho+y_1^\rho)\left[\frac{2}{\epsilon}-\ln\frac{(x_1-y_1)^2}{x_0{}^2}\right]\right]\\ &=-\frac{i\pi^2}{2}\left\{g^{\rho\mu}\left[\frac{2}{\epsilon}-\ln-\left[\frac{(x-y)^2}{x_0{}^2}\right]\right]+\frac{2(x^\rho+y^\rho)(y^\mu-x^\mu)}{(x-y)^2}\right\}. \end{aligned}$$

Here $\epsilon=4-d$, d is dimension of the Euclidean space, x_1,y_1,z_1 are Euclidean vectors, $x_1^0=ix^0$, $x_1^k=x^k$, $(x_1)^2=-(x)^2=-(x^0)^2+(\mathbf{x})^2$, and $x_0^2>0$ is the normalization point. Taking into account the antisymmetry of $\omega_{\mu\nu}$ we get

$$D_1(x,y)=-\frac{2gd_0^2\pi^2}{r^2}\omega_{\beta\mu}x^\beta y^\mu=-d_1\omega_{\beta\mu}\frac{x^\beta y^\mu}{r^2}\ ,$$

$$d_1=\frac{2g}{(4\pi)^2}\ .$$

Note that $D_1(x,y)\neq D_1(y,x)$. Adding the possible vacuum expectation

$$i\langle\phi^2\rangle/2\equiv\frac{i}{2}\langle 0|\phi_1^*\phi_1(0)+\phi_2^*\phi_2(0)|0\rangle$$

one gets

$$D(x,y)=\frac{i\langle\phi^2\rangle}{2}-\frac{id_0}{r^2}-\frac{d_1\omega_{\beta\mu}x^\beta y^\mu}{r^2}+id_2[-\tfrac{3}{2}r^2+2f(x,y)]\ , \tag{A9}$$

$$D(y,x)=\frac{i\langle\phi^2\rangle}{2}-\frac{id_0}{r^2}+\frac{d_1\omega_{\beta\mu}x^\beta y^\mu}{r^2}+id_2[-\tfrac{3}{2}r^2+2f(x,y)]\ . \tag{A10}$$

APPENDIX B

Here we calculate the vacuum expectation of some currents in the SCSM. Although all momentum-space correlation functions considered are known,[19,20] the method of working in x space seems to be easier. We demonstrate it in the examples of W-boson, Higgs, and fermion ($J=\frac{1}{2}$) channels.

We will use the propagators (A9) and (A10) from Appendix A, and the massless quark propagator[36] which for the SU(2) gauge group is

$$S(x,y)=\frac{2d_0\hat{r}}{r^4}-\frac{id_1(\hat{r}\gamma^\alpha\gamma^\beta-\gamma^\beta\gamma^\alpha\hat{r})\omega_{\alpha\beta}}{4r^2}+\frac{2id_1\hat{r}\omega_{\beta\mu}x^\mu y^\rho}{r^4}-\frac{d_2f(x,y)\hat{r}}{4r^2}\ , \tag{B1}$$

where $d_0,d_1,d_2,f(x,y),\omega_{\beta\mu}$ are defined in Appendix A, $r_\mu=x_\mu-y_\mu$, $\hat{r}=r_\mu\gamma^\mu$.

1. Calculation in the $J^P(I)=1^-(1)$ channel

At first we calculate the correlation function for the currents (4) and (5) using the current (6) for W^3 boson because it is easier. The W^3 boson mixes with the photon and gives the Z (Refs. 7, 9, 10, and 12), but without electromagnetism the correlators for $W^{1,2}$ and W^3 are the same. More precisely, the scalar field VEV of dimension 4 for the changed W bosons is $-\langle\phi^T\epsilon T^a\phi\phi^\dagger T^a\epsilon\phi^*\rangle$ instead of $\langle(\phi^\dagger T^a\phi)^2\rangle$. Up to isotopic factor of 2, these VEV's are the same if we apply the vacuum-dominance hypothesis or the tree approximation. In fact, the isotopic invariance of the vacuum in both standard model and SCSM provides the following relation for the scalar VEV's:

$$\langle(\phi^\dagger Y\phi)^2\rangle=\langle(\phi^\dagger T^a\phi)^2\rangle=-\tfrac{1}{2}\langle\phi^T\epsilon T^a\phi\phi^\dagger T^a\epsilon\phi^*\rangle\ .$$

Diagrams with the lower dimension VEV's and low order in α_H are pictured in Figs. 1(a)–1(d). The diagrams in Fig. 1(a) are the usual perturbative series. The first diagram in Fig. 1(a) is zeroth order in α_H; the last four are the corrections $\sim\alpha_H$ calculated in Ref. 20. As was mentioned before, in our SRW the main perturbative contribution is small, so we will not take into account corrections to it. The main diagram proportional to $\langle\phi^2\rangle$ is the first on Fig. 1(b). Again, we will not take into account corrections to it [last diagrams in Fig. 1(b)]. The diagrams in Fig. 1(c) are proportional to $\alpha_H\langle(\phi^\dagger T^a\phi)^2\rangle$. In the Landau gauge, where the hypergluonic propagator is transverse [i.e., in momentum space $D^{ab}_{\mu\nu}=(-i\delta^{ab}/K^2)(g_{\mu\nu}-K_\mu K_\nu/K^2)$] the last three diagrams in Fig. 1(c) are zero because each of them contains at least one vertex $\sim K_e D_{\mu e}=0$. The first diagram in Fig. 1(c) is calculated below. The diagrams in Fig. 1(d) are proportional to $\alpha_H\langle\omega^2\rangle$.

To calculate

$$\Pi^W_{\mu\nu}(q)=i\int d^4x\, e^{iqx}\langle 0\,|\,TJ^3_\mu(x)J^3_\nu(0)\,|\,0\rangle\ ,$$

let us calculate

$$\Pi^W_{\mu\nu}(x,y)=\langle 0\,|\,TJ^3_\mu(x)J^3(y)\,|\,0\rangle\ ,$$

$$J^3_\mu=-\frac{i}{2}\phi^\dagger(x)\overleftrightarrow{D}_\mu\phi(x)\ ,$$

$$\overleftrightarrow{D}_\mu=\vec{D}_\mu-\overleftarrow{D}^\mu,\quad \vec{D}_\mu=\partial^x_\mu-ig\omega_\mu(x)\ ;$$

$$\overleftarrow{D}^\mu=\overleftarrow{\partial}{}^x_\mu+ig\omega_\mu(x)$$

and then make a Fourier transformation. After some algebra

$$\Pi^{(\phi^4)}_{\mu\nu}(q)\equiv i\int d^4x\, e^{iqx}\Pi^{(\phi^4)}_{\mu\nu}(x,0)=ig^2\langle\phi^\dagger T^a\phi\phi^\dagger T^b\phi\rangle(-i)\delta^{ab}\frac{1}{q^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right]=-\frac{q^2\langle(\phi^\dagger T^a\phi)^2\rangle}{q^4}(q_\mu q_\nu-q^2g_{\mu\nu})\ . \tag{B2}$$

To get $\Pi^W_{\mu\nu}(x,0)$ in the Fock-Schwinger gauge, one has to take derivatives first and then put $y=0$. The last two terms in (B2) are zero because $\omega_\nu(y=0)=0$ [see (A2)], if ω_μ is a classical field. If ω_μ is a quantum field, then the last term in (B2) describes the first diagram in Fig. 1(c), which is the only diagram $\sim\alpha_H\langle\phi^4\rangle$ in the Landau gauge:

$$\Pi^{(\phi^4)}_{\mu\nu}(x,y)=g^2\langle\phi^\dagger T^a\phi\phi^\dagger T^b\phi\rangle\langle T\omega^a_\mu(x)\omega^b_\nu(y)\rangle\ .$$

So one gets the answer at once in momentum space:

$$\Pi^{(\phi^4)}_{\mu\nu}(q)\equiv i\int d^4x\, e^{iqx}\Pi^{(\phi^4)}_{\mu\nu}(x,0)=ig^2\langle\phi^\dagger T^a\phi\phi^\dagger T^b\phi\rangle(-i)\delta^{ab}\frac{1}{q^2}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right]$$

$$=-\frac{q^2\langle(\phi^\dagger T^a\phi)^2\rangle}{q^4}(q_\mu q_\nu-q^2g_{\mu\nu})\ .$$

One can also calculate (to leading order in g^2) the coefficient in front of the scalar VEV of arbitrary dimension $d=2n+4$, $n\geq 1$. The simplest way is to work in the Landau gauge. Then, any diagram containing a vertex proportional to $\Delta\mathscr{L}_1(x)=\phi^\dagger(x)ig\omega_\mu\partial^\mu\phi(x)+\text{H.c.}$, where at least one of the fields ϕ "goes to vacuum," is nullified. As we are going to evaluate the coefficient proportional to the lowest power of g^2 at given dimension of the scalar VEV, we have to break both scalar field lines so there is the only diagram left, which is shown in Fig. 1(e).

To calculate it, let us notice that any term proportional to the lowest power in g^2 in the Landau gauge comes from the expression:

$$\Pi^{\text{scalar}}_{\mu\nu}(x,y)=g^2\left\langle T\phi^\dagger(x)\omega_\mu(x)\phi(x)\exp\left[i\int dz\,\Delta\mathscr{L}(z)\right]\phi^\dagger(y)\omega_\nu(y)\phi(y)\right\rangle\ ,$$

where

$$\Delta\mathscr{L}(z)=g^2\phi^\dagger(z)\omega_\rho(z)\omega^\rho(z)\phi(z)\ .$$

Then the term of dimension $d=2n+4$ pictured in Fig. 1(e) is

$$\Pi^d_{\mu\nu}(x,y)=g^2\frac{i^n}{n!}\left\langle T\phi^\dagger(x)\omega_\mu(x)\phi(x)\int dz_1\Delta\mathscr{L}(z_1)\int dz_2\Delta\mathscr{L}(z_2)\cdots\int dz_n\Delta\mathscr{L}(z_n)\phi^\dagger(y)\omega_\nu(y)\phi(y)\right\rangle\ .$$

Counting the number of ω^a_ρ contractions one gets in momentum space

$$\Pi^d_{\mu\nu}(q)=i\int d^4x\, e^{iqx}\Pi^d_{\mu\nu}(x,0)=ig^2\frac{(-1)^{n+1}}{(q^2)^{n+1}}\left[g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right](g^2)^n\frac{i^n}{2^n n!}n!\,\text{VEV}^{(d)}=-\frac{(g^2)^{n+1}}{2^n(q^2)^{n+2}}(q_\mu q_\nu-q^2q_{\mu\nu})\text{VEV}^{(d)},$$

where

$$\mathrm{VEV}^{(d)}=\langle\phi^\dagger T^a(\phi^\dagger\phi)^n\phi^\dagger T^a\phi\rangle \ .$$

To calculate the diagrams in Fig. 1 proportional to unity, $\langle\phi^2\rangle$ and $\alpha_H\langle\omega^2\rangle$, it is easier to work in x space using the propagators (A9) and (A10) and then making a Fourier transformation.

We restrict ourselves to terms in $\Pi_{\mu\nu}$ up to g^2, which are proportional to VEV's, so (B2) can be written in terms of the scalar propagator (A9) and (A10):

$$\begin{aligned}
&\Pi^W_{\mu\nu}(x,0)=\Pi^{(1)}_{\mu\nu}(x)+\Pi^{(2)}_{\mu\nu}(x)\ ,\\
&\Pi^{(1)}_{\mu\nu}(x)=\tfrac{1}{4}\mathrm{Tr}[\partial^x_\mu D(y,x)\partial^y_\nu D(x,y)+\partial^x_\mu D(x,y)\partial^y_\nu D(y,x)-\partial^x_\nu\partial^y_\nu D(x,y)D(y,x)-D(x,y)\partial^x_\mu\partial^y_\nu D(x,y)]\,|_{y=0}\ ,\\
&\Pi^{(2)}_{\mu\nu}(x)=-\frac{ig}{2}\mathrm{Tr}[\partial_\nu D(x,y)\omega_\mu(x)D(y,x)-D(x,y)\omega_\mu(x)\partial^y_\nu D(y,x)]\,|_{y=0}\ .
\end{aligned}\tag{B3}$$

The first and second derivatives of $D(x,y)$ and $D(y,x)$ from (A9) and (A10) are

$$\begin{aligned}
&D(x,0)=D(0,x)=\frac{i\langle\phi^2\rangle}{2}-\frac{id_0}{x^2}-\frac{3}{2}id_2x^2\ ,\\
&\partial^x_\mu D(y,x)\,|_{y=0}=\partial^x_\mu D(x,y)\,|_{y=0}=\frac{2id_0x_\mu}{x^4}-3id_2x_\mu\ ,\\
&\partial^y_\nu D(x,y)\,|_{y=0}+\frac{d_1\omega_{\beta\nu}x^\beta}{x^2}=\partial^y_\nu D(y,x)\,|_{y=0}-\frac{d_1\omega_{\beta\nu}x^\beta}{x^2}=-\frac{2id_0x_\nu}{x^4}+3id_2x_\nu\ ,\\
&\partial^x_\mu\partial^y_\nu D(x,y)\,|_{y=0}+\frac{d_1}{x^2}\left[\omega_{\mu\nu}-\frac{2\omega_{\beta\nu}x^\beta x_\mu}{x^2}\right]=\partial^x_\mu\partial^y_\nu D(y,x)\,|_{y=0}-\frac{d_1}{x^2}\left[\omega_{\mu\nu}-\frac{2\omega_{\beta\nu}x^\beta x_\mu}{x^2}\right]\\
&\qquad=\frac{2id_0}{x^4}\left[\frac{4x_\mu x_\nu}{x^2}-g_{\mu\nu}\right]+3id_2g_{\mu\nu}\ .
\end{aligned}$$

Putting these expressions in (B3), doing some algebra, taking the trace and grouping terms in $\Pi^{(1)}_{\mu\nu}$ and $\Pi^{(2)}_{\mu\nu}$ proportional to unity, $\langle\phi^2\rangle$ and $\langle\omega^2\rangle$, we get

$$\begin{aligned}
&\Pi^{(1)}_{\mu\nu}=\Pi^{(0)}_{\mu\nu}+\Pi^{(\phi^2)}_{\mu\nu}+\Pi^{(1)(\omega^2)}_{\mu\nu},\quad \Pi^{(2)}_{\mu\nu}=\Pi^{(2)(\omega^2)}_{\mu\nu}\ ,\\
&\Pi^{(0)}_{\mu\nu}(x)=2d_0^2\frac{(x^2g_{\mu\nu}-2x_\mu x_\nu)}{x^8}\ ,\\
&\Pi^{(\phi^2)}_{\mu\nu}(x)=-\frac{d_0\langle\phi^2\rangle}{x^6}(x^2g_{\mu\nu}-4x_\mu x_\nu)\ ,\\
&\Pi^{(1)(\omega^2)}_{\mu\nu}=-\frac{24d_0d_2}{x^4}x_\mu x_\nu\ ,\\
&\Pi^{(2)(\omega^2)}_{\mu\nu}=-\frac{gd_0d_1\langle\omega^2\rangle}{48x^2}(x^2g_{\mu\nu}-x_\mu x_\nu)\\
&\qquad\equiv-\frac{8d_0d_2}{x^4}(x^2g_{\mu\nu}-x_\mu x_\nu)\ .
\end{aligned}\tag{B4}$$

Note that as $\mathrm{Tr}(\omega_{\mu\nu})=0$, the terms in $D(x,y)$ and $D(y,x)$ proportional to d_1 do not contribute to $\Pi^{(1)}_{\mu\nu}$ and vice versa, only those terms contribute to $\Pi^{(2)}_{\mu\nu}$. Note also that, as was expected, each term in $\Pi_{\mu\nu}$ (proportional to unity, $\langle\phi^2\rangle,\langle\omega^2\rangle$) is transverse:

$$\frac{\partial}{\partial x_\mu}[\Pi^{(0)}_{\mu\nu}(x)]=\frac{\partial}{\partial x_\mu}[\Pi^{(\phi^2)}_{\mu\nu}(x)]=\frac{\partial}{\partial x_\mu}[\Pi^{(\omega^2)}_{\mu\nu}(x)]=0\ ,$$

where

$$\Pi^{(\omega^2)}_{\mu\nu}(x)=\Pi^{(1)(\omega^2)}_{\mu\nu}+\Pi^{(2)(\omega^2)}_{\mu\nu}=-\frac{8d_0d_2}{x^4}(x^2g_{\mu\nu}+2x_\mu x_\nu)\ .\tag{B5}$$

The graphic representation of $\Pi^{(0)}_{\mu\nu}$, $\Pi^{(\phi^2)}_{\mu\nu}$, and $\Pi^{(\omega^2)}_{\mu\nu}$ is shown in Figs. 1(a), 1(b), and 1(d), respectively. Note that in the gauge (A1) and at $y=0$ each of the last two diagrams in Fig. 1(d) is zero.

The transition to the momentum space is performed by means of the main formulas[36,39]

$$\begin{aligned}
&\int\frac{d^4x}{(x^2-i\epsilon)^n}e^{iqx}=\frac{i(-1)^n2^{4-2n}\pi^2}{\Gamma(n-1)\Gamma(n)}(q^2)^{n-2}\ln(-q^2),\ n\geq 2\ ,\\
&\int d^4x\frac{e^{iqx}}{(x^2-i\epsilon)}=-\frac{4\pi^2i}{q^2+i\epsilon}\ .
\end{aligned}\tag{B6}$$

One can get, for $\Pi^{(i)}_{\mu\nu}$,

$$\begin{aligned}
&\Pi^{(0)}_{\mu\nu}(q)=i\int d^4x\,e^{iqx}\Pi^{(0)}_{\mu\nu}(x)\\
&\qquad=(q_\mu q_\nu-q^2g_{\mu\nu})\left[-\frac{1}{96\pi^2}\ln(-q^2)\right]\ ,\\
&\Pi^{(\phi^2)}_{\mu\nu}(q)=i\int d^4x\,e^{iqx}\Pi^{(\phi^2)}_{\mu\nu}(x)\\
&\qquad=(q_\mu q_\nu-q^2g_{\mu\nu})\left[-\frac{\langle\phi^2\rangle}{2q^2}\right]\ ,
\end{aligned}\tag{B7}$$

$$\Pi^{(\omega^2)}_{\mu\nu}(q)=i\int d^4x\, e^{iqx}\Pi^{(\omega^2)}_{\mu\nu}(x)$$
$$=(q_\mu q_\nu-q^2 g_{\mu\nu})\frac{g^2\langle\omega^2\rangle}{192\pi^2q^4}\ .$$

Finally, the operator-product expansion for the two-point correlation function of the current creating W boson from the vacuum in the GSW and SCSM's up to the VEV's with dimension $d\le 4$ is

$$\Pi^W_{\mu\nu}(q)=(q_\mu q_\nu-q^2g_{\mu\nu})\left[-\frac{1}{96\pi^2}\ln(-q)^2-\frac{\langle\phi^2\rangle}{2q^2}-\frac{g^2\langle(\phi^\dagger T^a\phi)^2\rangle}{q^4}+\frac{g^2\langle\omega^2\rangle}{192\pi^2q^4}\right]. \quad \text{(B8)}$$

2. Current creating left fermions from the vacuum

Consider the two-point functions of the current:

$$J^a_f(x)=\phi^\dagger(x)\tfrac{1}{2}(1-\gamma_5)\psi^a(x)=\phi^\dagger(x)\psi^a_L(x)\ ,$$
$$a=1,2,\ldots,12\ \ (e_L,q_L)\ , \quad \text{(B9)}$$
$$\bar J^a_f(y)\equiv[J^a_f(y)]^\dagger\gamma_0=\bar\psi^a_L\phi(y)\ ,$$
$$\Pi^a_f(x,y)=\langle 0\,|\,TJ^a_f(x)\bar J^a_f(y)\,|\,0\rangle$$
$$=\langle T\phi^\dagger(x)\tfrac{1}{2}(1-\gamma_5)\psi(x)\bar\psi(y)\tfrac{1}{2}(1+\gamma_5)\phi(y)\rangle\ . \quad \text{(B10)}$$

Π^a_f is a 4×4 matrix. Terms proportional to unity $\langle\phi^2\rangle$ and $\langle\omega^2\rangle$ one can express by means of the propagators of the fermion (B1) and scalar (A9)

$$\Pi^a_f(x,y)=-\mathrm{Tr}[D(x,y)\tfrac{1}{2}(1-\gamma_5)S(y,x)\tfrac{1}{2}(1+\gamma_5)] \quad \text{(B11)}$$

(the trace acts on color indices only). As there are no derivatives in Π^a_f one can set one of the coordinates to be zero:

$$\Pi^a_f(x,0)=-\mathrm{Tr}[D(x,0)\tfrac{1}{2}(1-\gamma_5)S(0,x)]=-\tfrac{1}{2}(1-\gamma_5)\mathrm{Tr}\left[\left[\frac{i\langle\phi^2\rangle}{2}-\frac{id_0}{x^2}-\frac{3id_2x^2}{2}\right]\left[-\frac{2d_0\hat x}{x^4}\right]\right]$$
$$=\tfrac{1}{2}(1-\gamma_5)\left[\frac{2id_0\langle\phi^2\rangle\hat x}{x^4}-\frac{4id_0{}^2\hat x}{x^6}-\frac{6id_0d_2\hat x}{x^2}\right].$$

Making a Fourier transformation by means of formulas (B6) or their derivatives on q_μ one gets

$$\Pi^a_f(q)\equiv i\int d^4x\, e^{iqx}\Pi^x_f(x,0)$$
$$=\tfrac{1}{2}(1-\gamma_5)\hat q\left[-\frac{1}{16\pi^2}\ln(-q^2)-\frac{\langle\phi^2\rangle}{q^2}+\frac{g^2\langle\omega^2\rangle}{(16\pi)^2q^4}\right]+O\left[\frac{g^4\langle\phi^4\rangle}{q^4}\right]. \quad \text{(B12)}$$

3. Current creating Higgs boson from the vacuum

Consider the two-point functions of the current

$$J_H(x)=\phi^\dagger(x)\phi(x)\ ,$$
$$\Pi_H(x,y)=\langle 0\,|\,TJ_H(x)J_H(y)\,|\,0\rangle$$
$$=-\mathrm{Tr}[D(x,y)D(y,x)]\ .$$

There are no derivatives in Π_H, so using (A9) and (A10) one gets

$$\Pi_H(x,0)=-\mathrm{Tr}\left[\left[\frac{i\langle\phi^2\rangle}{2}-\frac{id_0}{x^2}-\frac{3id_2x^2}{2}\right]^2\right]$$
$$=\frac{2d_0{}^2}{x^4}-\frac{2d_0\langle\phi^2\rangle}{x^2}+6d_0d_2+\frac{\langle\phi^2\rangle^2}{2}-3d_2\langle\phi^2\rangle x^2\ .$$

The last three terms give a δ function or its derivative after Fourier transformations. We are interested only in the $q^2<0$ region, so we get

$$\Pi_H(q^2)=i\int d^4x\, e^{iqx}\Pi_H(x,0)$$
$$=-\frac{1}{8\pi^2}\ln(-q^2)-\frac{2\langle\phi^2\rangle}{q^2}+O\left[\frac{g^4\langle\omega^2\rangle}{q^4}\right]+O\left[\frac{g^4\langle\phi^4\rangle}{q^4}\right]. \quad \text{(B13)}$$

The last term in (B13) comes from the fact that the diagrams proportional to $g^2\langle\phi^4\rangle$ are zero for the same reason as the last three diagrams in Fig. 1(c).

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