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Signatures of a second heavy neutral vector boson in e^+e^- annihilation

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Some grand unification models predict the existence of a second massive neutral vector boson which couples to fermions in much the same way as the well-known Z boson in the standard model of electroweak interactions. Among the possibilities, e^+e^- annihilations at high energies may provide a means of observing the effects of this particle. We analyze the process $e^+e^- \to f\bar{f}$ in the context of a $SU(2)\times U(1)\times U(1)$ theory with a view to study the anomalies in the angular distribution of the differential cross section. Two models are worked out as illustrations and low-energy predictions compared with current data from SLAC and DESY.

INTRODUCTION

From a theoretical point of view it is reasonable to ask if the standard (Salam-Weinberg) model of electroweak interactions is only a close low-energy approximation to a more general unified theory of interactions. Search for a candidate grand unification theory[1–4] (GUT) of interactions is consequently of considerable interest and currently popular models predict a simple extension to the standard $SU(2)\times U(1)$ theory at low energies: the gauge group is $SU(2)\times U(1)\times U(1)$ and therefore requires the existence of a second massive neutral vector boson (referred to herein as Z_2) due to the extra U(1) factor.

An example is the group SO(10) which, being one rank higher than SU(5), has a second (heavy) neutral vector boson which couples to a specific charge that is the conserved charge of $U(1)_A$ when

$$SO(10)\to SU(5)\times U(1)_A \ .$$

Another example is the group

$$E_6 \supset SO(10)\times U(1)_B$$

[here the Z_2 associated with the $U(1)_B$ couples with equal strength to all known light fermions]. In the following we examine the possibility of measuring the effects of the second massive vector boson, either now, or in the near future, through a more detailed study of the experimental data on e^+e^- annihilations. In principle either deep-inelastic scattering or e^+e^- annihilations may be used to detect anomalies in the standard model. However, we have analyzed the latter for a simple reason: the process is easier to work with than an inclusive process of the type $ep\to q\bar{q}+X$, where q is some heavy quark.

Because the number of parameters arising from the Higgs sector is large we take the point of view that it is reasonable to search only for crude signatures of the particle and then to test individual models with experiment in a more refined manner.

A general treatment of the dynamics of the theory is given in Ref. 5. In Sec. I we present a short review of the basics relevant to this discussion. Section II describes the kinematics of e^+e^- annihilations for an $SU(2)\times U(1)\times U(1)$ theory. In Sec. III we discuss a simple example [when the charge of the third U(1) factor does not contribute to the total charge] in the context of the SO(10) GUT, with intention to point out a procedure which may be easily extended to more complex situations. Other possibilities are discussed briefly in Sec. IV, and in a final section we present our results.

I. SU(2)×U(1)×U(1): A REVIEW

We follow somewhat closely the theoretical construction of Belanger and Feldman[5] and consequently it seems appropriate to briefly review what is relevant to our discussion.

The charge operator is taken to be

$$\hat{Q}=\hat{Q}_1+\hat{Q}_2+x\hat{Q}_3 \ , \tag{1.1}$$

where $\hat{Q}_1$ is the third component of isospin, $\hat{Q}_2$ and $\hat{Q}_3$ are the generators of the U(1) factors, and $x=1$ (0) corresponds to the charge $\hat{Q}_3$ contributing (not contributing) to the total charge. Masses are obtained for the neutral vector bosons in the standard way by introduction of an A Higgs field and spontaneous symmetry breaking. The mass matrix is[6]

$$(M)_{ij}=\sum_a g_i g_j \Phi_i^{(a)}\Phi_j^{(a)} \ , \tag{1.2}$$

where $\Phi_i^{(a)}=q_i^{(a)}\langle 0|\Phi^{(a)}|0\rangle$, q_i is the eigenvalue of $\hat{Q}_i$ and g_i are the coupling constants associated with each factor. Since only neutral components of $\Phi^{(a)}(x)$ couple to the neutral vector fields they obey the constraint (see Fig. 1)

$$\Phi_1+\Phi_2+x\Phi_3=0 \ . \tag{1.3}$$

Mass eigenstates result from diagonalizing the matrix $(M)_{ij}$. If $M_D{}^2$ is the matrix of the square masses of the square masses of the vector bosons then

$$M_D{}^2=V^TMV \ , \tag{1.4}$$

where V is the (orthogonal) diagonalizing matrix of general form

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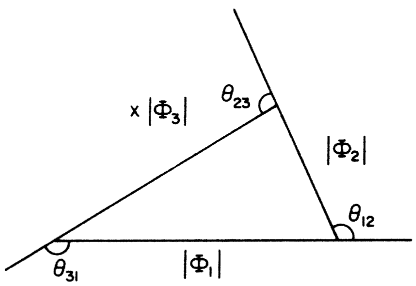

FIG. 1. Constraints on the Higgs vectors defined in Eq. (1.3).

$$V=\begin{bmatrix} s_\theta & c_\theta c_\alpha & c_\theta s_\alpha \\ c_\theta c_\phi & -s_\theta c_\alpha c_\phi - s_\alpha s_\phi & -s_\theta s_\alpha c_\phi + s_\phi c_\alpha \\ c_\theta s_\phi & -s_\theta c_\alpha s_\phi + s_\alpha c_\phi & -s_\theta s_\alpha s_\phi - c_\alpha c_\phi \end{bmatrix} . \qquad (1.5)$$

Clearly θ is the Weinberg angle; ϕ is related to the coupling strengths via

$$V_{11}=\frac{e}{g_1}=s_\theta\ ,\quad V_{21}=\frac{e}{g_2}=c_\theta c_\phi\ ,$$
$$V_{31}=x\frac{e}{g_3}=c_\theta s_\phi\ . \qquad (1.6)$$

Now using Eq. (1.4) and the assumption that[6]

$$(M_D{}^2)_{11}=0\ ,\quad (M_D{}^2)_{22}=M_{Z_1}{}^2\ ,\quad (M_D{}^2)_{33}=M_{Z_2}{}^2\ , \qquad (1.7)$$

we find $[M^2=(M)_{11}]$

$$\tan^2(\alpha)=\frac{\left[\dfrac{M^2}{c_\theta{}^2}-M_{Z_1}{}^2\right]}{\left[M_{Z_2}{}^2-\dfrac{M^2}{c_\theta{}^2}\right]}\ . \qquad (1.8)$$

There are five independent parameters which appear in the Lagrangian, viz.,

$$M,\ \theta,\ \phi,\ \theta_{13},\ \frac{s_{23}}{s_{12}}\ , \qquad (1.9)$$

where $s_{ij}=\sin(\theta_{ij})$ and θ_{ij} are defined in Fig. 1. The heavy-neutral-vector-boson currents are

$$jz_1=\left[-\frac{i}{2\cos(\theta)}\right]g_1\bar{\psi}\gamma^\mu(c_V-c_A\gamma_5)\psi\ , \qquad (1.10a)$$

$$jz_2=\left[-\frac{i}{2\cos(\theta)}\right]g_1\bar{\psi}\gamma^\mu(c_V'-c_A'\gamma_5)\psi\ . \qquad (1.10b)$$

Here we have chosen to express both in terms of the coupling constant g_1 which, taking the charged-current and neutral-current interactions to be of the same relative strengths, is related to the Fermi coupling G_F by

$$\frac{G_F}{\sqrt{2}}=\frac{g_1{}^2}{8M_W{}^2}\ . \qquad (1.11)$$

The vector and axial-vector coupling coefficients c_V, c_A, c_V', c_A' can be obtained from Eqs. (1.5) and (1.6):

$$c_V=\left[c_\theta V_{12}-\frac{s_\theta}{c_\phi}V_{22}\right]T^3+2\frac{s_\theta}{c_\phi}Q_fV_{22}+x\left[\frac{s_\theta}{s_\phi}V_{32}-\frac{s_\theta}{c_\phi}V_{22}\right]Y'\ , \qquad (1.12a)$$

$$c_A=\left[c_\theta V_{12}-\frac{s_\theta}{c_\phi}V_{22}\right]T^3\ , \qquad (1.12b)$$

$$c_V'=\left[c_\theta V_{13}-\frac{s_\theta}{c_\phi}V_{23}\right]T^3+2\frac{s_\theta}{c_\phi}Q_fV_{23}+x\left[\frac{s_\theta}{s_\phi}V_{33}-\frac{s_\theta}{c_\phi}V_{23}\right]Y'\ , \qquad (1.12c)$$

$$c_A'=\left[c_\phi V_{13}-\frac{s_\theta}{c_\phi}V_{23}\right]T^3\ , \qquad (1.12d)$$

where T^3 is the third component of isospin, Q_f is the fermionic charge due to $\hat{Q}_3$, i.e.,

$$\hat{Q}_3=\tfrac{1}{2}\hat{Y}'\ . \qquad (1.13)$$

Inasmuch as the dynamics are concerned these quantities would be complete if choices of the parameters in (1.9) could be made without ambiguity, but at the current level of understanding this is not possible. Indeed we must make arbitrary, if educated, "guesses," or use specific GUT'S for plausible candidates.

It is worth mentioning here that much work has been done in the past on extended electroweak gauge theories of this type. For example, Refs. 7–11 have considered the case $x=1$ with all fermions assigned zero $\hat{Q}_3$ quantum numbers. References 12–14 treat the case $x=1$ with fermions of nonzero $\hat{Q}_3$ quantum numbers and in Refs. 15–17 we can find studies of $x=0$ theories.

II. e^+e^- ANNIHILATION: KINEMATICS

At sufficiently high center-of-mass-system (c.m.s.) energies it is hoped that the effects of Z_2 become large enough to manifest themselves quite strongly, though perhaps not strongly enough to appear in absolute measurements of the differential cross section. We therefore concentrate on the angular distribution.

The computation of the differential cross section for the process $e^+e^-\to f\bar{f}$ taking into account only lowest-order diagrams (Fig. 2) is straightforward with the obvious result (the masses of the fermions are taken to be negligible)[18]

$$\frac{4s}{\alpha}\frac{d\sigma}{d\Omega}=A_0(1+a_1)[1+\cos^2(\theta)]+B_0(1+b_1)\cos(\theta)\ , \qquad (2.1)$$

where we have extracted the coefficients A_0 and B_0, which have the same form as their counterparts in the standard model but differ only in the fact that the vector and axial-vector coefficients appearing in them is slightly

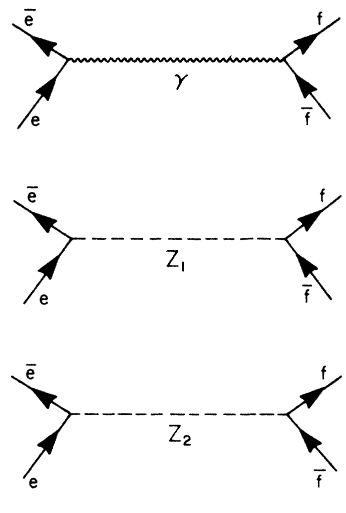

FIG. 2. Lowest-order diagrams for the process $e^+e^- \to f\bar{f}$.

different from those predicted therein. The measured asymmetry is then[19]

$$A=\frac{3}{8}\frac{B_0}{A_0}\left[\frac{1+b_1}{1+a_1}\right], \tag{2.2}$$

where the standard model predicts

$$A_{\rm st}=\frac{3}{8}\frac{B_0'}{A_0'} . \tag{2.3}$$

Here the primes mean that we replace the vector and axial-vector couplings in A_0 and B_0 by the corresponding values predicted by the Salam-Weinberg theory. The computation is tedious, but the result can be obtained in closed form; however, a few reasonable simplifications can be made.

(i) The propagators of the Z_1 and Z_2 bosons have been approximated by a free propagator with a complex pole at $s=M_Z{}^2+i\Gamma_Z M_Z$ where s is the square of the c.m.s. energy.

(ii) The mass of the Z_2 boson is well above c.m.s. energies so that the contact approximation is justified; therefore we have written

$$[(s-M_{Z_2}{}^2)+i\Gamma_{Z_2}M_{Z_2}]^{-1}=-\frac{1}{M_{Z_2}{}^2}\left[1+\frac{s}{M_{Z_2}{}^2}\right] . \tag{2.4}$$

The coefficients appearing in relation (2.1) are listed in the Appendix.

III. A SPECIFIC EXAMPLE

To perform numerical computations from the general theory given in the previous two sections, reasonable values must be assigned to the five independent parameters in (1.9). How one approaches this problem is a matter of taste but there are a few immediate possibilities.

(i) When mixing is weak, the mass of the Z_1 boson [which arises from the first U(1) factor] is close enough to the mass of the standard Z boson (Z_0) to permit the replacement

$$M_{Z_1}\sim 93.2 \text{ GeV} . \tag{3.1}$$

With this we can write

$$\frac{G_F}{\sqrt{2}}=\frac{g_1{}^2}{8M_{Z_1}{}^2\cos^2(\theta)} . \tag{3.2}$$

(ii) If we assume canonical symmetry breaking (i.e., if isospin is broken by Higgs fields that are isospin singlets or doublets) then $(M)_{11}=M_W{}^2$, the mass of the charged vector boson,[5] which in the limit of weak coupling, is approximately the experimental value of 81 GeV.

(iii) From a specific GUT and the mass scale of symmetry breaking, the angle θ and the ratio g_3/g_1 are exactly determined as are the eigenvalues of the charge $\hat{Q}_3$.

Here, as an example of the above possible simplifications, we have analyzed the case $x=0$. From Eq. (1.5) and (1.6) it is clear that we must take the limit $\phi\to 0$ in such a way that

$$\frac{x}{s_\phi}\to\frac{g_3}{e}c_\theta . \tag{3.3}$$

g_3 is now a free parameter so that reexpressing relations (1.12) in terms of g_3 we have

$$c_V=c_V^{(0)}c_\alpha+\frac{g_3}{e}(s_\theta s_\alpha)Y' , \tag{3.4a}$$

$$c_A={}_{A}^{(0)}c_\alpha , \tag{3.4b}$$

$$c_V'=c_V^{(0)}s_\alpha+\frac{g_3}{e}(s_\theta c_\alpha)Y' , \tag{3.4c}$$

$$c_A'=c_A^{(0)}s_\alpha , \tag{3.4d}$$

where

$$e_V^{(0)}=T^3-2Q_f\sin^2(\theta) \tag{3.5}$$

and

$$c_A^{(0)}=T^3 \tag{3.6}$$

are the couplings predicted by the standard model if $s_\theta{}^2$ is taken to be the observed value ($s_\theta{}^2\sim\frac{1}{4}-\frac{1}{5}$). Obviously if the eigenvalue of $\hat{Q}_3$ is zero, Z_2 couples very weakly because, from Eq. (1.8), $c_\alpha\sim 1$ unless the mass of Z_2 is very close to M_W. The case $Y'\neq 0$ is slightly more interesting. To illustrate the problem we turn to the SO(10) GUT for values of g_3 and Y' after requiring that we retain the successful numerical prediction ($s_\theta{}^2=0.23$) of the SU(5) GUT. SO(10) has several advantages over the group SU(5) as a GUT,[20,21] which is our reason for choosing it in this illustrative example. However, what is important to this work is that the prediction $s_\theta{}^2=0.23$ can be accommodated by SO(10) provided that the mass scale at which SO(10)$\to$SU(5)$\times$U(1) is the same as that at which SU(5) further breaks into SU(3)$\times$SU(2)$\times$U(1). Unfortunately, the ability of SO(10) to resolve any problems regarding proton decaying is then lost; nevertheless we shall assume here that $s_\theta{}^2=0.23$. Now after suitable normalization of the generators of the group and the second hypercharge, we have[22,23]

$$g_3 = \left[\frac{5}{3}\right]^{1/2} g_1 \tan(\theta) \tag{3.7}$$

(if the breaking is assumed to take place at the Planck mass, then g_3 would be about $\frac{2}{3}$ the above value) and

$$Y' = \frac{2}{\sqrt{10}}[5I_{3R} + 3(I_{3L} - Q_f)] \ . \tag{3.8}$$

We have taken values for the Z_2 mass ranging from 500 GeV to 2 TeV and computed the vector and axial-vector couplings for quarks and leptons, some of which are listed in Table I. Figures 3(a)—3(c) are the plots of the difference $A_{\text{pred}} - A_{\text{st}}$ as a function of the c.m.s. energy, for leptons and quarks, where A_{pred} and A_{st} are the asymmetries in the angular distribution of the differential cross section predicted by this model and the standard model, respectively. M_{Z_2} is taken to be 1 TeV. Low-energy data[19] from SLAC and DESY are found to be entirely compatible with these, and it is only at higher energies that one begins to observe a significant difference between the predictions of this model and those of the Salam-Weinberg theory. For example, at a c.m.s. energy of 80 GeV the asymmetry predicted by this model is about 19% greater than the corresponding prediction of the standard model for $e^+e^- \to$leptons [see Fig. 3(a)]. Furthermore, unless M_{Z_2} is very close to M_W the difference will increase only slowly with decreasing M_{Z_2}.

TABLE I. Vector and axial-vector couplings for $e^+e^- \to f\bar{f}$ for the model $x = 0$, $Y' = +0.98$ for leptons and -0.316 for quarks. We have taken Weinberg angle-$\sin^2(\theta_W) = 0.023$, Fermi coupling$= 1.16 \times 10^{-5}$ GeV^{-2}, and $M_{Z_2} = 1$ TeV.

Fermion type	c_V	c_A	c'_V	c'_A
Leptons	-0.04	-0.50	-0.67	0.00
Up quarks	$+0.20$	$+0.50$	$+0.22$	0.00
Down quarks	-0.35	-0.50	$+0.22$	0.00

IV. OTHER POSSIBILITIES

Numerical analysis of $x = 1$ theories is more complex because of the ambiguities in the Higgs structure; howev-

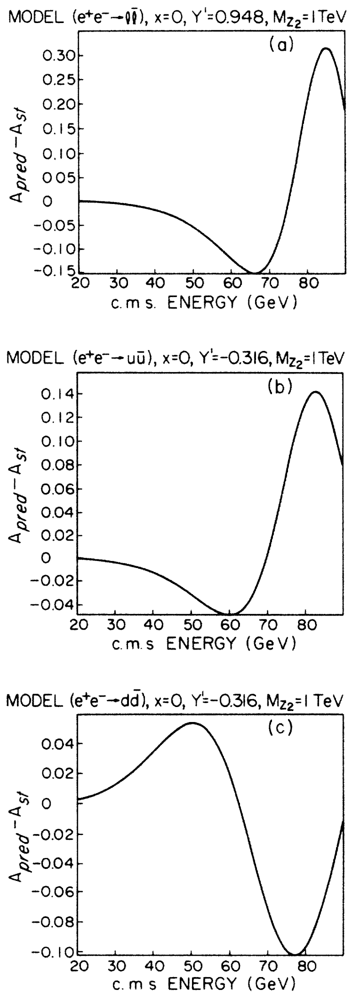

FIG. 3. $A_{\text{pred}} - A_{\text{st}}$ vs c.m.s. energy for the process $e^+e^- \to f\bar{f}$ taking into account a second massive vector boson (Z_2) in lowest-order diagrams for the case $x = 0$, $Y' = 0.948$ (-0.316) for leptons (quarks) and $\sin^2(\theta) = 0.23$.

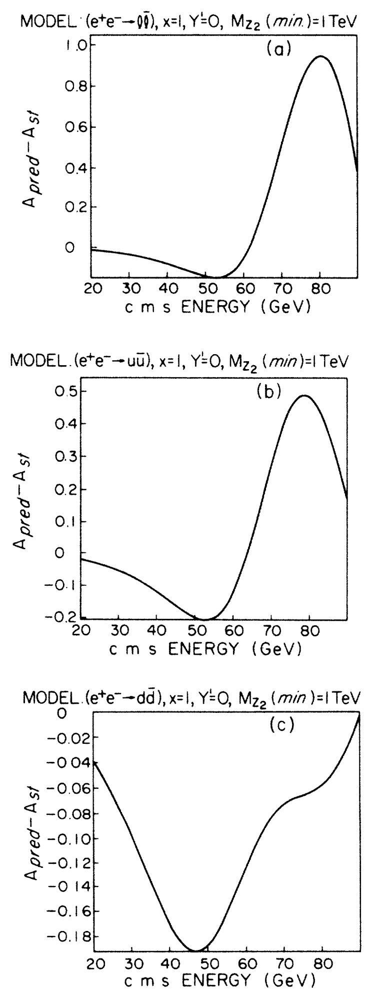

FIG. 4. $A_{\text{pred}} - A_{\text{st}}$ vs c.m.s. energy as in Figs. 3(a)—3(c), but for the case $x = 1$ and $Y' = 0$.

TABLE II. Same as Table I, for the model $x=1$, $Y'=0$, M_{Z_2} (minimum) $=1$ TeV.

Fermion type	c_V	c_A	c_V'	c_A'
Leptons	-0.04	-0.5	-0.98	$+0.32$
Up quarks	$+0.20$	$+0.5$	).54	-0.33
Down quarks	-0.35	-0.5	-0.11	$+0.33$

TABLE IV. Results for $e^+e^- \to u\bar{u}$.

c.m.s. energy (GeV)	A_{st} (%)	A_{meas} (%)	A_{pred} (%)	$\lvert A_{st}-A_{pred}\rvert$ (%)
Model: $x=0$, $Y'=-0.316$; and $M_{Z_2}=1$ TeV				
29	-9.0	-25 ± 18	-9.3	0.3
35	-13.7	-13 ± 10[a]	-14.4	0.7
80	-56.6		-43.4	13.2
Model: $x=1$, $Y'=0$; $c_{13}=0$ and M_{Z_2} (minimum) $=1$ TeV				
29	-9.0	-25.0 ± 18	-13.9	4.9
35	-13.7	-13 ± 10[a]	-21.7	8.0
80	-56.6		-9.9	46.7

[a]At 34.6-GeV c.m.s. energy.

er, it is useful to distinguish between two possible types of theories. (i) Models with $Y'=0$ for all known fermions and (ii) models with $Y'\neq 0$ for all (some) fermions.

The central problem is to assign values to the angles between the Higgs vectors and the three mixing angles, but there is an interesting fact that may be used to obtain crude candidates for the mixing angle ϕ. From Eq. (1.6),

$$\theta_{12}+\theta_{23}+\theta_{31}=2\pi \tag{4.1a}$$

and

$$\frac{|\Phi_1|}{s_{23}}\,\frac{|\Phi_2|}{s_{31}}\,\frac{|\Phi_3|}{s_{12}}\,, \tag{4.1b}$$

it is possible to find a value for ϕ (for fixed θ_{13} and s_{23}/s_{12}) which makes M_{Z_2} minimum[5]

$$1-2c_\theta{}^2 s_\phi{}^2=\frac{2s_\theta{}^2 c_{13}}{\overline{\eta}}\,, \tag{4.2}$$

where $\overline{\eta}=s_{23}/s_{12}$. This is model dependent and we have analyzed the first category above by taking $c_{13}=0$ which, from Eq. (4.2) gives the condition

$$2c_\theta{}^2 s_\phi{}^2=1\,. \tag{4.3}$$

Table II lists the values of the vector and axial-vector couplings computed for $s_\theta{}^2=0.23$ and ϕ given by Eq. (4.3), while Figs. 4(a)–4(c) plot the difference $A_{pred}-A_{st}$ as a function of the c.m.s. energy for both leptons and quarks. It is worth noting, however, that the condition for minimum mass is only a convenience. Other choices of ϕ have been made in the past. Nevertheless, this is an extreme example of a model that will predict asymmetries differing greatly from the standard model (about 88.5% at c.m.s. energy of 80 GeV for $e^+e^-\to$leptons with $M_{Z_2}=1$ TeV).

V. CONCLUSIONS

We have proposed the possibility of observing the effects of a new massive vector boson in the asymmetry of the angular distribution in e^+e^- annihilations and some models are found to predict large enough deviations from the standard model as to be tested directly. Despite the difficulties in assigning values to the parameters of the theory, a greater emphasis on the "fingerprints" of this particle could yield interesting results. Computations have been performed for two cases: (a) $x=0$ in the context of the SO(10) GUT, and (b) $x=1$ but the eigenvalue of the third charge operator is zero and the mass of Z_2 is minimum.

Tables III–V summarize our results; the low-energy data have been taken from Naroskas' review (Ref. 19).

A very heavy Z_2 naturally minimizes the effects of coupling, which is the reason for our conservative choice. Although greater differences are expected for low Z_2 masses a careful study of Eq. (A1)–(A3) and (1.7)–(1.12) indicates that the change is relatively small except if the mass of the Z_2 is close to the mass of the charged vector boson. If this were indeed the case, however, interference effects, of which we have no evidence from either UA1 or UA2, should occur at around a c.m.s. energy of 81 GeV. For this reason we feel justified in using the contact approximation.

Supersymmetric models have not been considered and,

TABLE III. Results for $e^+e^- \to l\bar{l}$.

c.m.s. energy (GeV)	A_{st} (%)	A_{meas} (%)	A_{pred} (%)	$\lvert A_{st}-A_{pred}\rvert$ (%)
Model: $x=0$, $Y'=0.948$, and $M_{Z_2}=1$ TeV				
29	-6.0	-6.3 ± 0.9	-6.4	0.4
35	-9.2	-12.4 ± 3.1*	-10.1	0.9
80	-73.7		-55.0	18.7
Model: $x=1$, $Y'=0$; $c_{13}=0$ M_{Z_2} (minimum) $=1$ TeV				
29	-6.0	-6.3 ± 0.9	-9.3	3.3
35	-9.2	-12.4 ± 3.1[a]	-14.6	5.4
80	-73.7		$+14.8$	88.5

[a]At 34.7-GeV c.m.s. energy.

TABLE V. Results for $e^+e^- \to d\bar{d}$.

c.m.s. energy (GeV)	A_{st} (%)	A_{meas} (%)	A_{pred} (%)	$\lvert A_{st}-A_{pred}\rvert$ (%)
Model: $x=0$, $Y'=-0.316$; and $M_{Z_2}=1$ TeV				
29	-17.6		-16.6	1.0
35	-26.3	-15.0 ± 22[a]	-24.2	2.1
80	-21.8		-31.0	9.2
Model: $x=1$, $Y'=0$; $c_{13}=0$; M_{Z_2} (minimum) $=1$ TeV				
29	-17.6		-26.6	9.0
35	-26.3	-15.0 ± 22[a]	-39.6	13.3
80	-21.8		-27.4	5.6

[a]At 34.6-GeV c.m.s. energy.

in fact, the cases worked out are only illustrations. Finally, it may be mentioned that it is possible to construct a Higgs sector that diminishes the anomalies caused by the Z_2; however, e^+e^- annihilations at high energies could provide a useful test of these theories or provide experimental bounds for the various parameters.

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APPENDIX

The coefficients appearing in Eq. (2.1) are obtained by straightforward computation using the vertex rules [Eq. (1.10)]

$$Z_1 \to f\bar{f} \quad \frac{i}{2\cos(\theta)} g_1 \gamma^\mu (c_V - c_A \gamma_5) , \tag{A1}$$

$$Z_2 \to f\bar{f} \quad \frac{i}{2\cos(\theta)} g_1 \gamma^\mu (c_V' - c_A' \gamma_5) , \tag{A2}$$

with the result that

$$A_0 = Q_f{}^2 + |r|^2 (c_A^{e^2} + c_V^{e^2})(c_A^{f^2} + c_V^{f^2}) - 2Q_f \mathrm{Re}(r) c_V^e c_V^f , \tag{A3a}$$

$$B_0 = 8|r|^2 c_V^e c_A^e c_V^f c_A^f - 4\,\mathrm{Re}(r) Q_f c_A^e c_A^f , \tag{A3b}$$

$$a_1 = \frac{1}{A_0} [\, |r'|^2 (c_A'^{e2} + c_V'^{e2})(c_A'^{f2} + c_V'^{f2}) - 2Q_f \mathrm{Re}(r') c_V'^e c_V'^f + 2\,\mathrm{Re}(rr')(c_V^e c_V'^e + c_A^c c_A'^e)(c_V^f c_V'^f + c_A^f c_A'^f)] , \tag{A3c}$$

$$b_1 = \frac{1}{B_0} [8|r'|^2 c_V'^e c_V'^f c_A'^e c_A'^f - 4Q_f \mathrm{Re}(r') c_A'^e c_A'^f + 4\,\mathrm{Re}(rr')(c_A'^e e_V^e - c_V'^e c_A^e)(c_A'^f c_V^f + c_V'^f c_A^f)] , \tag{A3d}$$

where we have defined

$$r = \frac{\sqrt{2} G_F M_Z{}^2}{(s - M_Z{}^2) + i\Gamma_Z M_Z} \left[\frac{4\pi s}{\alpha} \right] . \tag{A4}$$

The differential cross section is then

$$\frac{d\sigma}{d\Omega} = \frac{\alpha}{4s} \{ A_0 (1 + a_1)[1 + \cos^2(\theta)] + B_0 (1 + b_1) \cos(\theta) \} . \tag{A5}$$

In order to take color into account, the expression is multiplied by an overall factor of 3 for quarks, but naturally does not affect the angular asymmetry.

With Eqs. (1.6) and (4.1) we obtain

$$z + z' = \frac{s_\theta{}^2 + 2\overline{\eta} s_\phi{}^2 s_\theta{}^2 c_{13} + \overline{\eta}{}^2 s_\phi{}^2 (1 - c_\theta{}^2 s_\phi{}^2)}{\overline{\eta}{}^2 s_\phi{}^2 c_\phi{}^2} , \tag{A6}$$

$$zz' = s_\theta{}^2 s_{13}{}^2 / \overline{\eta}{}^2 s_\phi{}^2 c_\phi{}^2 , \tag{A7}$$

where

$$z = \frac{M_{Z_1}{}^2 c_\theta{}^2}{M^2} , \tag{A8}$$

$$z' = \frac{M_{Z_2}{}^2 c_\theta{}^2}{M^2} . \tag{A9}$$

Using Eq. (1.8) we find

$$1 = z c_\alpha{}^2 + z' s_\alpha{}^2 \tag{A10}$$

so that z and z' are simultaneously extremized with respect to ϕ. It is now convenient to extremize the quantity

$$\frac{z + z'}{zz'} \tag{A11}$$

from which Eq. (4.2) follows.

[1]Howard Georgi and S. L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[2]Jogesh C. Pati and Abdus Salam, Phys. Rev. D **10**, 275 (1974).

[3]Savas Dimopoulos and Howard Georgi, Nucl. Phys. **B193**, 150 (1981).

[4]N. Sakai, Z. Phys. C **11**, 153 (1981).

[5]G. Belanger and G. Feldman, Johns Hopkins Report No. JHU-HET8406, 1984 (unpublished), and references therein.

[6]$(M)_{ij}$ is really the square of the mass matrix. Its eigenvalues are 0, $M_{Z_1}{}^2$, and $M_{Z_2}{}^2$, the masses of the neutral vector bosons of the theory.

[7]Howard Georgi and Steven Weinberg, Phys. Rev. D **17**, 275 (1978).

[8]E. H. Groot, G. J. Gounaris, and D. Schildknecht, Phys. Lett. **85B**, 399 (1979); Z. Phys. C **5**, 127 (1980).

[9]V. Barger, W. Y. Keung, and Ernest Ma, Phys. Rev. D **22**, 727 (1980); Phys. Rev. Lett. **44**, 1169 (1980).

[10]V. Barger, Ernest Ma, and K.Whisnant, Phys. Rev. Lett. **46**, 1501 (1981).

[11]V. S. Berezinsky and A. Yu. Smirnov, Phys. Lett. **94B**, 505 (1980).

[12]V. Barger, Ernest Ma, and K. Whisnant, Phys. Rev. D **26**,

2378 (1982). It is argued in Ref. 5 that the implication in the above that any $x=1$ theory may be transformed into an $x=0$ theory, though algebraically correct, does not make the point that in symmetry breaking the root vectors associated with the generators $\hat{Q}_i$ are expected to be orthogonal in root space. Having chosen $\hat{Q}_1$ as I_3 the weak hypercharge $\hat{Y}$ is $\hat{Y}/2=\hat{Q}_2+x\hat{Q}_3$. For $x\neq 0$, x may take any continuous value, thus changing all $\hat{Q}_3$ eigenvalues by a factor.

[13]Nilendra G. Deshpande and David Iskandar, Nucl. Phys. **B167**, 223 (1980).

[14]Aharon Davidson, Phys. Rev. D **20**, 776 (1979).

[15]K. Enqvist and J. Maalampi, Z. Phys. C **21**, 345 (1984).

[16]A. Zee and Jihn E Kim, Phys. Rev. D **21**, 1939 (1980).

[17]Aharon Davidson, Mehmet Koca, and Kameshwar C. Wali, Phys. Rev. D **20**, 1195 (1975).

[18]Francis Halzen and Alan D. Martin, *Quarks and Leptons: An Introductory Course in Modern Particle Physics* (Wiley, New York, 1984).

[19]Beate Naroska, in *Proceedings of the 1983 International Symposium on Lepton and Photon Interactions at High Energies, Ithaca, New York*, edited by D. G. Cassel and D. L. Kreinick (Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, 1983).

[20]Howard Georgi, in *Particles and Fields—1974*, proceedings of the Meeting of the APS Division of Particles and Fields, Williamsburg, Virginia, edited by C. E. Carlson (AIP Conf. Proc. No. 23) (AIP, New York, 1975), p. 575; H. Fritzch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).

[21]An interesting discussion of the history of SO(10) as a GUT is given by Howard Georgi, in *Proceedings of the 21st International Conference on High Energy Physics, Paris, 1982*, edited by P. Petiau and M. Porneuf [J. Phys. (Paris) Colloq. **43**, C3-705 (1982)].

[22]P. Langacker, Richard W. Robinett, and Jonathan L. Rosner, Phys. Rev. D **30**, 1470 (1984).

[23]P. Langacker, Phys. Rep. **72C**, 185 (1981).