PHYSICAL REVIEW D VOLUME 33, NUMBER 9 1 MAY 1986

QCD at collider energies

A. Nicolaidis
Theoretical Physics Department, University of Thessaloniki, Thessaloniki, Greece

G. Bordes
Laboratoire de Physique Corpusculaire, Collège de France, Paris, France
(Received 8 July 1985)

We examine available experimental distributions of transverse energy and transverse momentum, obtained at the CERN $p\bar{p}$ collider, in the context of quantum chromodynamics. We consider the following. (i) The hadronic transverse energy released during $W^{\pm}$ production. This hadronic transverse energy is made out of two components: a soft component which we parametrize using minimum-bias events and a semihard component which we calculate from QCD. (ii) The transverse momentum of the produced $W^{\pm}$. If the transverse momentum (or the transverse energy) results from a single gluon jet we use the formalism of Dokshitzer, Dyakonov, and Troyan, while if it results from multiple-gluon emission we use the formalism of Parisi and Petronzio. (iii) The relative transverse momentum of jets. While for $W^{\pm}$ production quarks play an essential role, jet production at moderate p_T and present energies is dominated by gluon-gluon scattering and therefore we can study the Sudakov form factor of the gluon. We suggest also how through a Hankel transform of experimental data we can have direct access to the Sudakov form factors of quarks and gluons.

I. INTRODUCTION

The CERN $p\bar{p}$ collider has provided a new energy domain where we can try to detect new physical phenomena and at the same time we can test the already existing theories. Quantum chromodynamics (QCD), the theory of strong interactions, has passed successfully all the proposed experimental tests and it is highly desirable to find out if QCD still describes the hadronic world at the momentum scale $Q \sim 100$ GeV of the $p\bar{p}$ collider. At this high momentum scale nonperturbative phenomena and subleading terms are controlled and perturbative QCD emerges as a powerful theory. Furthermore, our task is facilitated, since, because of calorimetric techniques employed at the $p\bar{p}$ collider, we can study and analyze jets, which to most physicists are synonymous with quarks and gluons.

The different experiments testing QCD at the collider can be grouped into three classes. (i) Those experiments which are more or less repetitions of experiments carried out already at lower energies, such as jet production, Drell-Yan, transverse-momentum distributions of gauge bosons, etc.[1] (ii) Experiments which make use of the axial-vector couplings of the weak gauge bosons to extract more information about the dynamics of strong interactions. In Ref. 2 it was indicated how the angular distribution of dileptons arising from Z^0 decay can help us to fix the spin of the gluon. (iii) Experiments which rely on calorimetric techniques and full coverage of the angular domain. We have here in mind the measurements of hadronic transverse energy (E_T) released during a hard process.

In this paper we examine mostly experiments which belong in the last category. Namely, we study the hadronic transverse energy accompanying W production. The hadronic transverse energy is made up of two components: a soft component associated with the spectator hadronic system and a semihard component associated with the gluons radiated from the $q\bar{q}$ pair producing the W. We parametrize the soft component by taking into account that the soft E_T is of the same origin as the E_T observed in minimum-bias events. For the semihard component, the novel calorimetric technique allows us to know if the semihard E_T is composed mainly of a single gluon jet or if it is composed instead of many gluonic jets. In the first case we calculate the semihard E_T using the formalism of Dokshitzer, Dyakonov, and Troyan (DDT), while in the second case we apply the formalism of Parisi and Petronzio. The details of our calculation and the comparison to the UA1 experimental data are presented in Sec. II. In Sec. III we study the transverse-momentum distribution of the W gauge boson. Only the semihard component is present and again we consider two cases: the q_T of the W is balanced by a single jet, or many jets. W production proceeds through quarks, while jet production of moderate p_T proceeds through gluons. We study therefore the relative q_T distribution of dijets in order to obtain information about the Sudakov form factor of the gluon. We indicate also that a Hankel transform of experimental q_T distributions gives us the product of parton distribution functions and Sudakov form factors in impact parameter space. Knowledge of parton distribution functions allows then to extract directly the Sudakov form factor. In Sec. IV we present our conclusions; we point out the limits of our approach and we discuss related works that have appeared already in the literature.

II. GLUONIC TRANSVERSE ENERGY

Consider a quark which participates in a hard process characterized by a scale Q. We can define a probability $q(x,E_T,Q)$ where because of gluonic emission the quark ends up with a fraction x of its initial momentum and where the emitted gluons give a total transverse energy equal to E_T. The probability for n-gluon emission factorizes into the product of probabilities of one-gluon emission:

$$q(x,E_T,Q)=\sum_{n=0}^{\infty}\frac{1}{n!}\int\delta\left[\sum k_{Ti}-E_T\right]\prod_{i=1}^{n}V(\mathbf{k}_{Ti})d^2k_{Ti}\int\delta\left[x-\prod_{i=1}^{n}x_i\right]\prod_{i=1}^{n}P_{qq}(x_i)dx_i\ , \tag{1}$$

where $P_{qq}(x)$ are the usual Altarelli-Parisi kernels[3] and $V(k_T)$ is given by[4]

$$V(k_T)=\frac{1}{2\pi^2}\frac{\alpha_S(k_T{}^2)}{k_T{}^2}\ . \tag{2}$$

The x_i integrations decouple if we Mellin transform $q(x,E_T,Q)$ with respect to x. It was also first shown in Ref. 5 that the k_{Ti} integrations are much simplified if we replace the δ function by its integral representation:

$$\delta\left[\sum k_{Ti}-E_T\right]=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[it\left[\sum k_{Ti}-E_T\right]\right]dt\ . \tag{3}$$

Defining the probability $q(m,E_T,Q)$ by[6]

$$q(m,E_T,Q)=\int x^{m-1}q(x,E_T,Q)dx \tag{4}$$

and taking into account that[4]

$$\int_0^{1-k_T{}^2/Q^2}dx\,x^{m-1}P_{qq}(x)\simeq A_m-2C_F\ln\frac{k_T{}^2}{Q^2}\ , \tag{5}$$

where A_m is the usual anomalous dimension encountered in scaling violations and C_F is the quark-color factor, we obtain

$$q(m,E_T,Q)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{-itE_T}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}V(k_{Ti})\left[A_m-2C_F\ln\frac{k_{Ti}{}^2}{Q^2}\right]e^{itk_{Ti}}d^2k_{Ti}\ . \tag{6}$$

The k_{Ti} integration is bounded by E_T. Following the procedure outlined in Ref. 6 we write

$$L(m,E_T)=A_m\int^{E_T}V(k_T)e^{itk_T}d^2k_T\ , \tag{7}$$

$$S(t)=-2C_F\int^{E_T}V(k_T)\ln\frac{k_T{}^2}{Q^2}(e^{itk_T}-1)d^2k_T\ . \tag{8}$$

The factor -1 in Eq. (8) originates from the "+ prescription"[7] which takes into account virtual graphs. We find therefore

$$q(m,E_T,Q)=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\,e^{-itE_T}\exp[L(m,t)+S(t)]\ . \tag{9}$$

It is easy to find an approximate expression for L, Eq. (7),

$$L(m,t)\simeq\frac{A_m}{\pi}\int_{\mu}^{p}\frac{\alpha_s(k_T{}^2)}{k_T}dk_T=\frac{A_m}{2\pi b}\ln\left[\frac{\ln p^2/\Lambda^2}{\ln\mu^2/\Lambda^2}\right]\ , \tag{10}$$

where $p\equiv\min(E_T,1/t)$. We recognize that $\exp(L)$ represents scaling violations at a scale p. On the other hand, $\exp(S)$ is the Sudakov form factor in impact-parameter space. If $q(m,t,Q)$ is the Fourier transform of $q(m,E_T,Q)$ with respect to E_T, Eq. (9) gives us

$$q(m,t,Q)=\exp[L(m,t)+S(t)]\ . \tag{11}$$

If we include the initial quark distribution within the hadron, we can write

$$q(x,t,Q)=q(x,p)\exp[S(t)]\ . \tag{12}$$

The physical meaning of the above equations is quite clear: the Mellin-Fourier–transformed probability that a quark after gluonic emission provides an amount of E_T is a product of two factors, one factor containing scaling violations and the other factor representing the Sudakov form factor.

Consider now the W production at the collider, which proceeds through a $q_i\bar{q}_j$ annihilation. W is accompanied by large transverse energy and the semihard component, following our formalism, will be given by

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=\int q(x_1,E_{T1},Q)\bar{q}(x_2,E_{T2},Q)\times\delta(E_{T1}+E_{T2}-E_T)dE_{T1}dE_{T2}\ . \tag{13}$$

Replacing the δ function by an expression similar to Eq. (3), we obtain

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\, e^{-itE_T}\int q(x_1,E_{T1},Q)e^{itE_{T1}}dE_{T1}\int \bar{q}(x_2,E_{T2},Q)e^{itE_{T2}}dE_{T2}\,. \tag{14}$$

Using Eq. (12), we find

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\, e^{-itE_T}q(x_1,p)\bar{q}(x_2,p)\exp[S(t)]\,. \tag{15}$$

Notice that the factor $S(t)$, due to gauge invariance,[8] appears only once. Equation (15) can be written also in the following form:

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=\frac{1}{\pi}\int_0^{\infty}dt\, q(x_1,p)\bar{q}(x_2,p) \times\exp\left[\int_\mu^{E_T}\phi(k_T)(\cos tk_T-1)dk_T\right] \times\cos\left[\int_\mu^{E_T}\phi(k_T)\sin tk_T dk_T-tE_T\right], \tag{16}$$

where

$$\phi(k_T)=-\frac{2C_F}{\pi}\frac{\alpha_s(k_T{}^2)}{k_T}\ln\frac{k_T{}^2}{Q^2}\,. \tag{17}$$

An expression for semihard E_T distribution containing only double logarithms has been given in Ref. 9. Notice that until now we have subscribed to the point of view of Parisi and Petronzio;[7] i.e., the total E_T results from *multiple*-gluon emissions. Unfortunately, along with the gluonic transverse energy associated with the hard process, a nonsignificant amount of E_T will be deposited by the spectator hadronic system. Already large amounts of E_T are observed in minimum-bias events.[10] A possible parametrization of the soft E_T distribution is provided by the relation[9]

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=a^2E_T\exp(-aE_T)\,, \tag{18}$$

where

$$a=\frac{2}{\langle E_T\rangle}\,. \tag{19}$$

In minimum-bias events we have $\langle E_T\rangle\simeq 20$ GeV leading to $a=0.1$ GeV^{-1}. When a W boson is produced the available energy for the spectator hadrons is reduced to $\sqrt{s}\simeq 400$ GeV. Assuming that the $\langle E_T\rangle$ scales with energy we find that $a\simeq 0.13$ GeV^{-1}. The final expression for E_T distribution is given by

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=\int\frac{1}{\sigma_0}\frac{\sigma_g}{dE_{T1}}\frac{1}{\sigma_0}\frac{d\sigma_s}{dE_{T2}} \times\delta(E_{T1}+E_{T2}-E_T)dE_{T1}dE_{T2}\,, \tag{20}$$

where $d\sigma_g/dE_T$ is given by Eq. (16) and $d\sigma_s/dE_T$ is given by Eq. (18). A Fourier transformation of Eq. (20) provides

$$f(t)=f_g(t)f_s(t)\,. \tag{21}$$

Taking into account that the Fourier transformation of Eq. (18) is

$$f_s(t)=\frac{a^2}{a^2+t^2}e^{i\theta}\,,\qquad \theta=\arctan\frac{2ta}{a^2-t^2}\,, \tag{22}$$

we finally obtain, using Eqs. (15), (16), (21), and (22),

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=\frac{1}{\pi}\int_0^{\infty}dt\, q(x_1,p)\bar{q}(x_2,p)\frac{a^2}{a^2+t^2} \times\exp\left[\int_\mu^{E_T}\phi(k_T)(\cos tk_T-1)dk_T\right] \times\cos\left[\int_\mu^{E_T}\phi(k_T)\sin tk_T dk_T-tE_T+\theta\right]. \tag{23}$$

The above equation is our main result. In Fig. 1 we compare our formulas with the available experimental data.[11] Both the magnitude and the shape of the E_T distribution is well reproduced. (We used $\mu=1$ GeV. Our numerical calculations are not sensitive to this precise value.)

The novel calorimetric technique permits us to isolate the "jetty" events, i.e., those events where semihard E_T is dominated by a single gluon jet. In order to analyze that sample of events we adopt the approach of Dokshitzer, Dyakonov, and Troyan;[12] i.e., we assume that the semihard E_T is made up of a single gluon and all other gluons are soft and have to be integrated over. Taking into account the ordering in the k_T variable,[12] we find that the E_T distribution will be proportional to

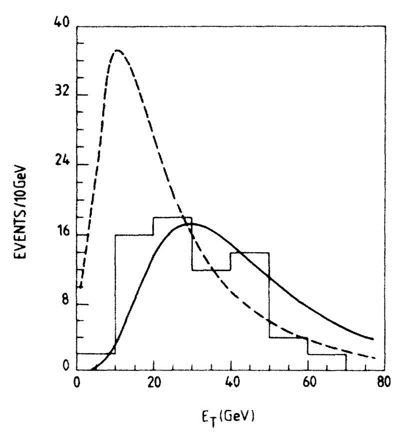

FIG. 1. The hadronic-transverse-energy distribution associated with W production. Dashed curve represents the semihard component, Eq. (16), solid curve the full E_T distribution, Eq. (23). The data are from Ref. 11.

$$\omega(E_T)\int_{\mu}^{E_T}\omega(k_{Tn})dk_{Tn}\int_{\mu}^{k_{Tn}}\omega(k_{Tn-1})dk_{Tn-1}\cdots\int_{\mu}^{k_{T2}}\omega(k_{T1})dk_{T1}\,,$$
$$\omega(k_T)=\frac{\alpha_S(k_T)}{k_T}\left[-2C_F\ln\frac{k_T{}^2}{Q^2}\right]. \tag{24}$$

The final expression is given by

$$\frac{1}{\sigma_0}\frac{d\sigma}{dE_T}=q(x_1,E_T)\bar{q}(x_2,E_T)\frac{1}{\pi}\frac{\alpha_s(E_T)}{E_T}\left[-2C_F\ln\frac{E_T{}^2}{Q^2}\right]\exp\left[\frac{-C_F}{\pi b}\ln\frac{E_T{}^2}{\mu^2}+\frac{C_F}{\pi b}\ln\frac{Q^2}{\Lambda^2}\ln\left[\frac{\ln E_T{}^2/\Lambda^2}{\ln\mu^2/\Lambda^2}\right]\right], \tag{25}$$

where Q is the mass of W. The above distribution, normalized to the observed jetty events, is compared to the experimental data[13] in Fig. 2. At large E_T we observe the $1/E_T$ tail, while at small E_T the Sudakov form factor takes over and suppresses the spectrum. In the same figure we have plotted the leading-logarithmic version of the first order in α_s perturbative calculation, i.e., Eq. (25) without the exponential factor. It is remarkable that the leading-logarithmic formula (dashed curve) is very close to the exact perturbative calculation presented in Ref. 14.

III. TRANSVERSE MOMENTUM OF W AND DIJETS

The transverse-momentum ($\mathbf{q}_T$) distribution of W is directly related to the semihard gluonic radiation (no soft component is present). There are already numerous articles[15] about the $\mathbf{q}_T$ distribution of W. We simply outline here the essential steps of our derivation. We define a probability $q(x,\mathbf{k}_T,Q)$ where a quark participating in a hard process characterized by a scale Q, after gluonic emission, ends up with a fraction x of its momentum and a transverse momentum $\mathbf{k}_T$ with respect to its original direction of motion. The Fourier-transformed probability $q(x,\mathbf{b},Q)$ is given by[6]

$$q(x,\mathbf{b},Q)=q\left[x,\frac{1}{b^2}\right]\exp[S(\mathbf{b})]\,, \tag{26}$$

where

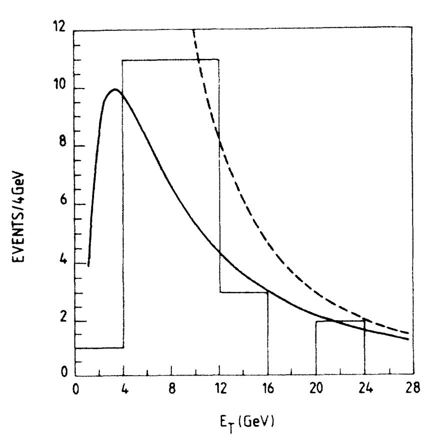

FIG. 2. Transverse-energy distribution of jets in W events. Dashed curve represents the leading-logarithmic result of the first order in α_s calculation, while solid curve represents the leading-logarithmic result to all orders in α_s. Data are from Ref. 13.

$$S(\mathbf{b})=\int^{Q}V(k_T)\left[-2C_F\ln\frac{k_T{}^2}{Q^2}\right](e^{i\mathbf{k}_T\cdot\mathbf{b}}-1)d^2k_T\,. \tag{27}$$

Both the quark and the antiquark contribute to the observed $\mathbf{q}_T$ spectrum of the W:

$$\frac{1}{\sigma_0}\frac{d\sigma}{d^2q_T}=\int q(x_1,\mathbf{k}_{T1},Q)\bar{q}(x_2,\mathbf{k}_{T2},Q)\times\delta^2(\mathbf{k}_{T1}+\mathbf{k}_{T2}-\mathbf{q}_T)d^2k_{T1}d^2k_{T2}\,. \tag{28}$$

Following the steps outlined in the preceding section and in Ref. 6, we obtain

$$\frac{1}{\sigma_0}\frac{d\sigma}{d^2q_T}=\frac{1}{(2\pi)^2}\int d^2b\,e^{-i\mathbf{q}_T\cdot\mathbf{b}}q\left[x_1,\frac{1}{b^2}\right]\times\bar{q}\left[x_2,\frac{1}{b^2}\right]\exp[S(\mathbf{b})]\,. \tag{29}$$

Integrating over the angular domain we have

$$\frac{1}{\sigma_0}\frac{d\sigma}{q_Tdq_T}=\int_0^{\infty}b\,db\,J_0(bq_T)q\left[x_1,\frac{1}{b^2}\right]\times\bar{q}\left[x_2,\frac{1}{b^2}\right]\exp[S(b^2)]\,. \tag{30}$$

The calorimetric technique allows us again to isolate the "jetty events," i.e., those events where the W is balanced by a single gluon jet. For these events a formula similar to Eq. (25) is valid, with E_T replaced by q_T. In Fig. 3 we can see the agreement between our formulas and the available experimental data.[13]

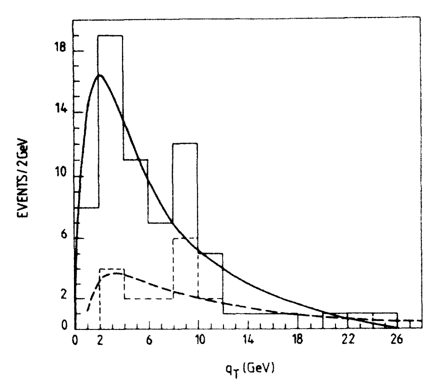

FIG. 3. The transverse-momentum distribution of W. Solid curve is obtained through Eq. (30). Dashed histogram and dashed curve represent the jetty events. Data are from Ref. 13.

The weak-boson production proceeds through quarks and all phenomena described above are due to gluonic radiation from the incoming quarks. On the contrary, jet production at moderate transverse momenta at the collider proceeds through gluon-gluon collisions. At the parton level we expect the total transverse momentum of the dijet system ($\mathbf{p}_T$) to be equal to zero. Any deviation of $\mathbf{p}_T$ from zero is attributed to gluonic radiation from the incoming gluons. In Fig. 4 is shown the projected $\mathbf{p}_T$ and it is compared with the theoretical prediction (details of the theoretical formula can be found in Ref. 6). The agreement is satisfactory and provides further evidence that the gluons, being "more colored" than the quarks, give broader distributions in p_T (Ref. 5).

Throughout the paper we treated gluonic radiation assuming that (i) only the $1/k_T$ tails are important, (ii) each gluon emission is independent of the others, so that the whole process is a Markoff process,[17] and (iii) we imposed energy-momentum conservation through the δ functions in their integral representations. With respect to the last point we would like to remark that the δ functions, via Fourier transformations, lead us from the k_T space to impact-parameter space and then back again to momentum space. All our formulas about q_T or E_T distributions are amenable to Fourier transformations and in the impact-parameter space our equations have a simple form. With respect to q_T distribution, Eq. (30), we can use the Hankel theorem

$$f(x)=\int_0^\infty J_0(xu)\sqrt{xu}\,du\int_0^\infty J_0(uy)\sqrt{uy}\,f(y)dy \quad (31)$$

and then we obtain

$$\begin{aligned} F(a)&\equiv\int\frac{1}{\sigma_0}\frac{d\sigma}{dq_T}J_0(aq_T)dq_T \\ &=q\left[x_1,\frac{1}{a^2}\right]\bar{q}\left[x_2,\frac{1}{a^2}\right]\exp[S(a^2)]\;. \end{aligned} \quad (32)$$

Since the quark distributions are well known for a large range of x,Q values, we can deduce from a Hankel transformation of experimental q_T distributions, Eq. (32), the Sudakov form factor in impact-parameter space.

IV. CONCLUSIONS

We examined in this paper mostly the calorimetric experiments carried out at the $p\bar{p}$ collider which detect the hadronic transverse energy. The total E_T measured in W production is made up of the semihard gluonic radiation and of the soft component registered in minimum-bias events. The soft component, present in hadronic collisions, is not present in γ-γ collisions at e^+e^- machines. We provided a closed-form expression for the total E_T distribution which is in reasonable agreement to the available experimental data. Our results are also corroborated by recent Monte Carlo calculations.[18]

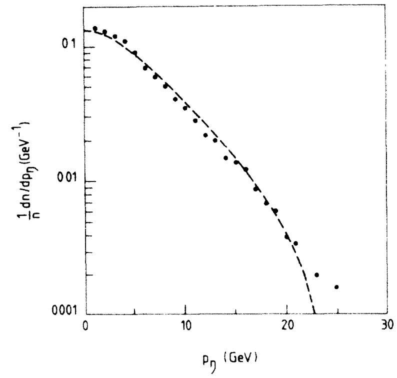

FIG. 4. The component (P_η) of the transverse momentum of the dijets. Data are from Ref. 16.

The novel calorimetric technique allows us to separate the jetty events, i.e., those events which are dominated by a single gluon jet. For these events we proposed a DDT-type formula which contains the $1/k_T$ tail and which at the same time is infrared safe (no divergence) at small k_T. The satisfactory comparison to the experimental data indicates that the cross sections are indeed dominated by the $1/k_T$ tails, which are contained in our leading-logarithmic formula. Clearly if we insist upon having a large k_T value, then our prescription fails and the whole perturbative result has to be taken into account. Such a procedure, incorporating the full first order in the α_s expression, has been implemented in Refs. 19 and 20.

We analyzed also the q_T distributions for W and for dijets which are free from any soft contribution, and we emphasized the interplay of color factors when we move from W production to jet production. We indicated also that once we have high statistics data it would be desirable to Hankel transform the q_T distributions in order to obtain the Sudakov form factor. Altogether we feel that in the new kinematical range, QCD still provides a viable description of hadronic phenomena.

ACKNOWLEDGMENTS

This work was made possible by an exchange program supported by the Greek Ministry of Research and Technology and the French Ministry of External Affairs. We thank Professor M. Froissart for a careful reading of the manuscript and helpful remarks. One of us (A.N.) would like to thank the Laboratoire de Physique Corpusculaire (Collège de France) for providing kind hospitality.

[1]For a review, see G. Altarelli, in Proceedings of the Erice School of Subnuclear Physics, 1983 (unpublished); M. Jacob, in *Proceedings of the XXII International Conference on High Energy Physics, Leipzig, 1984*, edited by A. Meyer and E. Wieczorek (Academie der Wissenschaften der DDR, Zeuthen, DDR, 1984).

[2]N. Arteaga-Romero, A. Nicolaidis, and J. Silva, Phys. Rev. Lett. **52**, 172 (1984).

[3]G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).

[4]A. Nicolaidis, Lett. Nuovo Cimento **25**, 58 (1979); in *Quarks,*

Gluons, and Jets, proceedings of the XIV Rencontre de Moriond, Les Arcs, Savoie, France, 1979, edited by J. Tran Thanh Van (Editions Frontiéres, Dreux, France, 1979), p. 221.

[5]G. Bordes and A. Nicolaidis, Phys. Rev. D **22**, 2152 (1980).

[6]G. Bordes and A. Nicolaidis, Phys. Lett. **114B**, 175 (1982).

[7]G. Parisi and R. Petronzio, Nucl. Phys. **B154**, 427 (1979).

[8]A. Bassetto, in *Perturbative QCD and High Energy Physics,* proceedings of the 13th Summer School in Particle Physics, Gif-sur-Yvette, 1981 (Institut National de Physique Nucléaire et Physique Particules, Paris, 1982), p. 145.

[9]F. Halzen, A. Martin, D. Scott, and M. Tuite, Z. Phys. C **14**, 351 (1982).

[10]Y. Giraud-Héraud, in *Proceedings of the Hadronic Session of the XVIII Rencontre de Moriond, La Plagne, Savoie, France, 1983,* edited by J. Tran Thanh Van (Editions Frontiéres, Gif-sur-Yvette, 1983), p. 125.

[11]J. Rohlf, in *Proceedings of the XXII International Conference on High Energy Physics, Leipzig, 1984* (Ref. 1), Vol. II, p. 12.

[12]Yu. L. Dokshitzer, D. I. Dyakonov, and S. I. Troyan, Phys. Rep. **58**, 269 (1980).

[13]S. Geer, in *Proceedings of the Santa Fe Meeting of the Division of Particles and Fields of the American Physical Society, 1984,* edited by T. Goldman and M. N. Nieto (World Scientific, Singapore, 1985), p. 482.

[14]S. Geer and W. J. Stirling, Phys. Lett. **152B**, 373 (1985).

[15]F. Halzen, A. D. Martin, and D. M. Scott, Phys. Rev. D **25**, 754 (1982); A. Nakamura, G. Pancheri, and Y. Srivastava, Z. Phys. C **21**, 243 (1984); P. Aurenche and R. Kinnunen, Phys. Lett. **135B**, 493 (1984); G. Altarelli *et al.*, Nucl. Phys. **B246**, 12 (1984).

[16]P. Bagnaia *et al.*, Phys. Lett. **144B**, 283 (1984).

[17]S. Chandrasekhar, Rev. Mod. Phys. **15**, 1 (1943).

[18]R. Odorico, Phys. Rev. D **31**, 49 (1985).

[19]G. Altarelli *et al.*, Nucl. Phys. **B246**, 12 (1984); G. Altarelli, R. K. Ellis, and G. Martinelli, Z. Phys. C **27**, 617 (1985).

[20]J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. **B250**, 199 (1985); C. Davies, B. Webber, and W. Stirling, *ibid.* **B256**, 413 (1985).