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Coupling the spinning string to an antisymmetric tensor field and quarks with spin

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We present a locally supersymmetric action for a spinning string interacting with an antisymmetric tensor field. This action incorporates in particular the boundary conditions. Then, we present a first physical application of this action in the context of a string model of mesons; explicitly, we show how it leads at the classical level to effective quarks with spin.

In a recent work,[1] which was a new attempt at describing mesons as strings (see Ref. 2 for a previous work along this line) we studied a Nambu string[3] coupled to a U(1) gauge field, the system being described by the action S (see the Appendix for a detailed explanation of the notations):

$$S=\eta \int \frac{e}{2} g^{\alpha\beta}\partial_\alpha y^\mu \partial_\beta y_\mu d^2\tau - \frac{M_0}{2}\int \epsilon^{\alpha\beta}\partial_\alpha y^\mu \partial_\beta y^\nu \widetilde{F}_{\mu\nu} d^2\tau + \frac{1}{16\pi}\int F_{\mu\nu}F^{\mu\nu}d^4x \;, \tag{1}$$

where, as usual,

$$F_{\mu\nu}=\partial_\mu A_\nu - \partial_\nu A_\mu \tag{2a}$$

and

$$\widetilde{F}_{\mu\nu}=\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma} \;. \tag{2b}$$

In particular, we showed in Ref. 1 that S leads to two unitarily equivalent descriptions of mesons, first, as a charged Nambu string as immediately read off from the Lagrangian and second, as an effective quark-antiquark pair driven, in a suitable approximation, by a semirelativistic Schrödinger equation.

However, the quarks we obtained from the preliminary model are spinless; therefore the model itself, in spite of the fact that it generalizes the successful model of Eichten *et al.*[4] and allows one to calculate a satisfactory spectrum of the $S=1$ at $J=J_{\max}$ mesons,[5] is not realistic enough to ensure a detailed description of nature. In particular, no spin interactions appear in the semirelativistic Schrödinger Hamiltonian we get in Ref. 1 and it is well known that these cannot be neglected (think, for instance, of the ρ-π splitting).

To overcome this shortcoming we therefore addressed ourselves to the problem of generalizing the action S to a new action S_N from which we get effective quarks with spin. It is the purpose of this paper to present a solution of this problem.

In fact the whole solution may be summed up in a word: supersymmetrization.

Indeed, as is easily forseeable from the association of the method we used in Ref. 1 to obtain the effective quark-antiquark pair on one hand, and of the work[6] of Berezin and Marinov and of Casalbuoni on the other, we will show that the new action S_N is just the supersymmetric extension of S; i.e., it is the action which describes the Neveu-Schwarz-Ramon spinning string[7] interacting in a supersymmetric way with a U(1) gauge field A_μ [in Ref. 6, Berezin and Marinov, and Casalbuoni (BMC) showed how to describe a free particle of spin $\frac{1}{2}$ by using a generalization with both bosonic and fermionic (Grassmann) coordinates of the usual quantum mechanics]. Now, we see on (1) that the U(1) field A_μ enters only through its field strength $F_{\mu\nu}$ which is an antisymmetric tensor field so that, ignoring the kinetic term $(1/16\pi)\int F_{\mu\nu}F^{\mu\nu}d^4x$ which will play no role, we realize that the procedure of supersymmetrization of S will work equally well if we replace $F_{\mu\nu}$ by any antisymmetric tensor field $B_{\mu\nu}$ which does not transform under supersymmetry. Such an action $S_N(B_{\mu\nu})$ is deserving of a potentially larger field of application, especially in those times where string models in general get a new impetus under the impulse of the work of Green and Schwarz;[8] we will consider this more general case in the first part of this paper. Then, in the second part, we will show that if we specify to $\widetilde{F}_{\mu\nu}$ as defined in (2b) we can introduce effective quarks, in the sense of Ref. 1, with a classical spin in the manner of Berezin and Marinov, and Casalbuoni. This second part may be thought of as a first application of the action $S_N(B_{\mu\nu})$, the study of a spinning string in background fields being another possibility.[9]

I. SUPERSYMMETRIC ACTION FOR A STRING INTERACTING WITH ANTISYMMETRIC TENSOR FIELD

First, we will restrict ourselves to the orthogonal gauge where $e^a_\alpha=\eta^a_\alpha$ and compute the action S_N up to boundary terms. This means that $S_N(B_{\mu\nu})$ will be invariant under a limited set of local supersymmetry transformations, which will be specified below, up to a pure divergence term.

Second, using the superspace formalism of Salam and Strathdee[10] and Zumino[11] we will introduce the correct boundary contributions so that $S_N(B_{\mu\nu})$ will be exactly invariant under these supersymmetry transformations.

Third, we will generalize the action we obtained above in the orthogonal gauge, to an action invariant under any local supersymmetry transformation. This will be the complete action describing a string interacting with an antisymmetric tensor field $B_{\mu\nu}$ in a manner invariant under any local transformation of supersymmetry we are looking for.

A. Action in the orthogonal gauge up to boundary terms

The notations are fully explained in the Appendix.

ξ^μ is a Majorana spinor field on the world sheet spanned in space-time by the string. The supersymmetric transformations we consider in this section are[7,11]

$$\begin{aligned} \delta y^\mu &= i\bar{\alpha}\xi^\mu , \\ \delta\xi^\mu &= \partial_\beta y^\mu \gamma^\beta \alpha + F^\mu \alpha , \\ \delta F^\mu &= i\bar{\alpha}\gamma^\beta \partial_\beta \xi^\mu , \end{aligned} \tag{3}$$

where α is the parameter of the transformation. It is a Majorana spinor defined on the sheet. It depends on τ^0 and τ^1, the two coordinates on the sheet and satisfies the equation

$$\gamma^\beta\gamma^\delta\partial_\beta\alpha=0, \quad \forall\delta=0,1 ;$$

that is, $\alpha^1(\tau^0,\tau^1)=\alpha^1(\tau^0-\tau^1)$ and $\alpha^2(\tau^0,\tau^1)=\alpha^2(\tau^0+\tau^1)$,

$$\alpha \equiv \begin{bmatrix} \alpha^1 \\ \alpha^2 \end{bmatrix} .$$

As M_0 is an arbitrary coupling constant which does not appear in (3) both the term generalizing the kinetic term

$$-\eta \left[-\tfrac{1}{2} \int \partial^\alpha y^\mu \partial_\alpha y_\mu d^2\tau \right] \tag{4}$$

and the one generalizing

$$-\eta \left[\frac{M_0}{2\eta} \int \epsilon^{\alpha\beta}\partial_\alpha y^\mu \partial_\beta y^\nu B_{\mu\nu}(y) d^2\tau \right] \tag{5}$$

should be separately invariant.

The kinetic term of the antisymmetric tensor field $B_{\mu\nu}(x)$ plays no role, as $B_{\mu\nu}(x)$ is supposed to be a scalar under the transformations of supersymmetry. Therefore we will not display it. In the following we pose $m=M_0/2\eta$. The term generalizing (4) is well known; it was introduced as early as 1971 by Neveu, Schwarz, and Ramond (NSR) in Ref. 7, and was first written by Zumino[11] in the form

$$-\eta \left[\int \left(-\tfrac{1}{2}\partial^\alpha y^\mu \partial_\alpha y_\mu - \tfrac{1}{2} i \bar{\xi}^\mu \gamma^\alpha \partial_\alpha \xi_\mu + \tfrac{1}{2} F^\mu F_\mu \right) d^2\tau \right] . \tag{6}$$

We obtained the term generalizing (5) after a lengthy but straightforward calculation:

$$\begin{aligned} -\eta m \int \Big[& \epsilon^{\alpha\beta}\partial_\alpha y^\mu \partial_\beta y^\nu B_{\mu\nu}(y) \\ & + \tfrac{1}{2}(\bar{\xi}^\rho \gamma_5 \gamma^\alpha \xi^\mu \partial_\alpha y^\nu + \bar{\xi}^\rho \gamma_5 \xi^\mu F^\nu) A_{\rho\mu\nu} \\ & - \frac{i}{8} \bar{\xi}^\eta \xi^\rho \bar{\xi}^\mu \gamma_5 \xi^\nu \partial_\eta A_{\rho\mu\nu} \Big] d^2\tau , \end{aligned} \tag{7}$$

where $A_{\rho\mu\nu}=\partial_\rho B_{\mu\nu}+\partial_\mu B_{\nu\rho}+\partial_\nu B_{\rho\mu}$ is the component of the exterior derivative $\mathbf{d}$ of the two-form of component $B_{\mu\nu}$. We give in the Appendix some useful formulas to compute the variation of (7). As indicated above, the variation of $S_N \equiv (6) + (7)$ leads to a pure divergence term. Explicitly it becomes

$$\begin{aligned} \delta S_N^1 = -\eta \Bigg\{ & \partial_\alpha \left[-\frac{i}{2}\partial_\beta y_\mu \bar{\alpha}\gamma^\alpha\gamma^\beta\xi^\mu + \frac{i}{2}F^\mu\bar{\alpha}\gamma^\alpha\xi_\mu \right] \\ & + m\,\partial_\alpha \left[2\epsilon^{\alpha\beta} i\bar{\alpha}\xi^\mu\partial_\beta y^\nu B_{\mu\nu} - \tfrac{1}{2}A_{\rho\mu\nu}\left[\epsilon^{\alpha\beta}\bar{\xi}^\rho\gamma_\beta\xi^\mu\bar{\alpha}\xi^\nu + \frac{i}{3}\bar{\xi}^\rho\gamma_5\gamma_\beta\xi^\mu\bar{\alpha}\gamma^\beta\gamma^\alpha\xi^\nu \right] \right] \Bigg\} . \end{aligned} \tag{8}$$

In usual field theories such a divergence is harmless as it is always assumed that all the fields vanish at infinity. However, in a string theory the boundary of the parameter strip ($\tau^0 \in]-\infty,+\infty[$, $\tau^1 \in [0,\pi]$) parametrizes the world lines of the ends of the string on which the fields of the theory ($y^\mu, \ldots$) are nontrivial so that the volume integral of a divergence is not zero in general and should be canceled. The next problem we have to face is therefore to cancel (8). That we will do by adding some contribution from the boundaries of the string to the action S_N as we will see now.

B. Boundary terms

In order to discover the correct boundary term to be added to S_N we insist on writing it in the superspace formalism. Therefore we introduce the scalar superfield

$$x^\mu(\tau^\alpha,\theta^\alpha) = y^\mu + i\bar{\theta}\xi^\mu + \frac{i}{2}\bar{\theta}\theta F^\mu$$

with

$$x'^\mu(\tau'^\alpha = \tau^\alpha + i\bar{\alpha}\gamma^\alpha\theta, \theta'^\alpha = \theta^\alpha - \alpha^\alpha) = x^\mu(\tau^\alpha,\theta^\alpha)$$

and the covariant derivative in superspace $D_A=\bar{E}\,^{\alpha}_{A}D_\alpha$, where E^α_A is the flat zweibein[11] generalizing e^α_a in the orthogonal gauge (see the Appendix for further notations) and we look at the simplest action one can think of to be S_N. That is, we consider

$$S_x^1=-\eta\left[\frac{1}{4}\int d^2\tau\, d^2\theta\, D_A x^\mu \eta^{AB} D_B x_\mu + \frac{m}{2i}\int d^2\tau\, d^2\theta\, D_A x^\mu \gamma_5^{AB} D_B x^\nu B_{\mu\nu}(x)\right]. \tag{9}$$

It is easy to show that indeed S_x^1 is just

$$S_x^1=S_N^1+\frac{m\eta}{2}\int d^2\tau\, \partial_\alpha(\bar{\xi}^\mu\gamma_5\gamma^\alpha\xi^\nu)B_{\mu\nu} \tag{10}$$

which means that S_x^1 is the same as S_N^1 up to boundary terms. But in the superfield formalism we know how to deal with the contributions at the boundaries to obtain an exactly invariant action. Indeed, it is easy to show, following the method given (for the NSR string) by Ademollo *et al.* in Ref. 12, that if we add to S_x^1 the boundary term

$$S_B^1=\int d^2\tau\, d^2\theta\, \bar{\theta}\theta[\delta(t^1)-\epsilon\delta(\tau^1-\pi)]\frac{i}{2}\mathscr{L}(x)\,,$$

where $\mathscr{L}(x)$ is the Lagrangian in S_x^1 and where ϵ may be either ± 1, then the action $S_N^1=S_x^1+S_B^1$ is exactly invariant under (3), provided we impose on the parameter of the supersymmetry transformations α the restrictions

$$\gamma^{a=1}\alpha+\bar{\epsilon}\alpha=0 \tag{11}$$

with $\bar{\epsilon}=1$ for $\tau^1=0$ and $\bar{\epsilon}=\epsilon$ for $\tau^1=\pi$. These restrictions may also be written

$$\alpha^1+\alpha^2=0 \quad\text{at}\quad \tau^1=0$$

and

$$\alpha^1+\epsilon\alpha^2=0 \quad\text{at}\quad \tau^1=\pi\,.$$

It is easy to convince oneself that there is no way of avoiding the restrictions (11). In the case of the NSR string, the choice $\epsilon=+1$ corresponds to the Ramond boundary conditions for ξ_μ and the choice $\epsilon=-1$ corresponds to the Neveu-Schwarz ones.

So, the correct supersymmetric generalization of the action S in the orthogonal gauge becomes, if we make explicit the component fields of the superfield,

$$S_N^2=S_N^1-\eta\left[-\frac{m}{2}\int\partial_\beta(\bar{\xi}^\mu\gamma_5\gamma^\beta\xi^\nu)B_{\mu\nu}d^2\tau+\int d^2\tau[\delta(\tau^1)-\epsilon\delta(\tau^1-\pi)]\left[\frac{i}{4}\bar{\xi}^\mu\xi_\mu+\frac{m}{2}\bar{\xi}^\mu\gamma_5\xi^\nu B_{\mu\nu}\right]\right] \tag{12a}$$

or

$$S_N^2=S_N^1-\eta\left[-\left[\epsilon\int_f d\tau_f\frac{i}{4}\bar{\xi}^\mu\xi_\mu-\int_i d\tau_i\frac{i}{4}\bar{\xi}^\mu\xi_\mu\right] -\frac{m}{2}\left[\int_f d\tau_f(\bar{\xi}^\mu\gamma_5\gamma^{\beta=1}\xi^\nu+\epsilon\bar{\xi}^\mu\gamma_5\xi^\nu)B_{\mu\nu}-\int_i d\tau_i(\bar{\xi}^\mu\gamma_5\gamma^{\beta=1}\xi^\nu+\bar{\xi}^\nu\gamma_5\xi^\nu)B_{\mu\nu}\right]\right]. \tag{12b}$$

For the purpose of obtaining effective quarks with a classical spin this action would have been sufficient; in fact, we will use it in the second part of the paper; however, for the more general purpose of studying the supersymmetric interaction of a spinning string and an antisymmetric tensor field, and even for the meson model, as a forthcoming paper[13] will prove, we had better know its complete form; that is, we had better avoid the orthogonal gauge restriction. That is why we now turn to the construction of the complete action.

C. Complete action

To obtain this action we proceed along the same lines as before and thus first begin by ignoring the boundary terms and construct the action S_N^3, which simplifies to S_N^1 in the orthogonal gauge, using the method Deser and Zumino used in Ref. 14.

The supersymmetry transformations now become[11,14]

$$\begin{aligned}
&\delta y^\mu=i\bar{\alpha}\xi^\mu\,,\\
&\delta\xi^\mu=F^\mu\alpha+\partial_\alpha y^\mu\gamma^\alpha+i\bar{\xi}^\mu\chi_\alpha\gamma^\alpha\alpha\,,\\
&\delta F^\mu=i\bar{\alpha}\gamma^\alpha\partial_\alpha\xi^\mu+\delta F_+\,,\\
&\delta e_\alpha a=-2i\bar{\alpha}\gamma^a\chi_\alpha\,,\\
&\delta\chi_\alpha=-\left[\partial_\alpha-\frac{i}{2}[\omega_\alpha(e)-2\bar{\chi}_\alpha\gamma_5\gamma^\beta\chi_\beta]\gamma_5\right]\alpha=-D_\alpha\alpha\,,
\end{aligned} \tag{13}$$

where α can be any function of τ^0 and τ^1, χ_α is the analog of the gravitino field in supergravity.[15] It is a Majorana vector spinor on the sheet spanned by the string; $\omega_\mu(e)$ is the spin connection on the sheet. δF_+ is a complicated expression depending on the χ_α we give in the Appendix.

We obtain, after some calculations,

$$S_N^3=-\eta\left[\int\left[-\tfrac{1}{2}eg^{\alpha\beta}\partial_{\alpha}y^{\mu}\partial_{\beta}y_{\mu}-\frac{i}{2}\bar{\xi}^{\mu}\gamma^{\alpha}\partial_{\alpha}\xi_{\mu}-ie\bar{\chi}_{\alpha}\gamma^{\beta}\gamma^{\alpha}\xi^{\mu}\partial_{\beta}y_{\mu}-\frac{e}{4}\bar{\xi}^{\mu}\xi_{\mu}\bar{\chi}_{\alpha}\gamma^{\beta}\gamma^{\alpha}\chi_{\beta}+\frac{e}{2}F_{\mu}F^{\mu}\right]\right.$$
$$+m\int\left[e\epsilon^{\alpha\beta}\partial_{\alpha}y^{\mu}\partial_{\beta}y^{\nu}B_{\mu\nu}+\frac{e}{2}(\bar{\xi}^{\rho}\gamma_5\gamma^{\alpha}\xi^{\mu}\partial_{\alpha}y^{\nu}+\bar{\xi}^{\rho}\gamma_5\xi^{\mu}F^{\nu})A_{\rho\mu\nu}-\frac{i}{8}e\bar{\xi}^{\eta}\xi^{\rho}\bar{\xi}^{\mu}\gamma_5\xi^{\nu}\partial_{\eta}A_{\rho\mu\nu}\right.$$
$$\left.\left.+\frac{i}{6}e\bar{\xi}^{\rho}\gamma_5\gamma_{\beta}\xi^{\mu}\bar{\xi}^{\nu}\gamma^{\alpha}\gamma^{\beta}\chi_{\alpha}A_{\rho\mu\nu}\right]\right]\,. \tag{14}$$

The quantity in the first set of brackets of this expression is already known.[14] The variation of S_N^3 under the transformation (13) gives a pure divergence which generalizes (8). In shorthand we write

$$\delta S_N^3=-\eta[\partial_{\alpha}(e\cdot)+\partial_{\alpha}(\tfrac{1}{2}e\bar{\xi}^{\mu}\gamma^{\alpha}\gamma^{\beta}\alpha\bar{\chi}_{\beta}\xi_{\mu})]\ . \tag{15}$$

The point stands for the whole term under the derivative sign in (8). Next using the superfield formalism we get

$$S_x^2=S_N^3+\frac{\eta m}{2}\int\partial_{\beta}(e\bar{\xi}^{\mu}\gamma_5\gamma^{\beta}\xi^{\nu}B_{\mu\nu})d\tau\ . \tag{16}$$

Concerning this step we should say that we did not start from the most general form of the super-zweibein imposing afterward certain "kinematic" constraints on the supertorsion as done by Howe for the spinning string in Ref. 16; instead we adopted a strategy similar to the one he adopted in Ref. 17. That is, guided by the result (10) in the orthogonal gauge, we introduced an ansatz for the super-zweibein which leads to S_N up to a divergence term and takes the additional divergence term as the generalization of the one in (10). The expression of our ansatz is given in the Appendix.

Finally, we have to add the term S_B:

$$S_B^2=-\eta\left[\int d^2\tau\left[\frac{i}{4}x\bar{\xi}^{\mu}\xi_{\mu}+\frac{m}{2}x\bar{\xi}^{\mu}\gamma_5\xi^{\nu}B_{\mu\nu}\right]\right.$$
$$\left.\times[\delta(\tau^1)-\epsilon\delta(\tau^1-\pi)]\right] \tag{17}$$

with

$$x^2=-g_{a=0,a=0}$$

to get an action $S_N^4=S_x^2+S_B^2$ which is exactly invariant under any local supersymmetry transformation provided its parameter satisfies

$$e\gamma^{a=1}\alpha+\bar{\epsilon}x\alpha=0 \tag{18a}$$

for both $\tau^1=0$ and $\tau^1=\pi$. This spinorial equation reduces in fact to the single equation

$$\frac{\alpha^1}{f}+\bar{\epsilon}f\alpha^2=0\ ,$$

where (18b)

$$xf^2=e_{\alpha=0}^{a=0}-e_{\alpha=0}^{a=1}\ .$$

The meaning of x which appears in (17) is clear. Indeed $g_{a=0,a=0}$ is just the metric induced by $g_{\alpha\beta}$ along the world lines of the edges of the string and thus x^2 is just the analog of $e^2=-\det(g_{\alpha\beta})$. The meaning of f which appears in (18b) is the following: it is just the right quantity to build from a spinor, and a zweibein a Lorentz invariant (in fact, we can build a pair of Lorentz invariants). The proof of this property is easy and will appear in Ref. 13.

For memory, let us recall that, exactly as the usual complete action of Deser and Zumino[14] [obtained from (14) by choosing $m=0$], the action S_N has a much larger gauge invariance than simply the local supersymmetric invariance. It has also a local Lorentz invariance under Lorentz transformations in the tangent space of the sheet, a general reparametrization invariance, a local Weyl invariance, and at last a local invariance under $\chi_{\alpha}\to\chi_{\alpha}+\gamma_{\alpha}\phi$ (super-Weyl invariance).

In a forthcoming paper,[13] we will study in detail the consequences of these symmetries, especially in what concerns the other possible contributions at the boundaries. In particular, we will prove that they enable us to give a first-order formulation of a spinning string with massive ends, which will generalize some known results for the bosonic string in the orthogonal gauge.[18] Moreover, we will solve exactly, after a suitable choice of gauge, the classical equations of motion of this system. These symmetries are also of crucial importance for the canonical quantization of the action S_N^4.

Before leaving this first section let us sum up its main results: (1) the building of the complete action (S_N^4) of a spinning string, interacting in a locally supersymmetric way with an antisymmetric tensor field; (2) the incorporation of all the boundary contributions into the action so that all the boundary conditions for the motion of the string and, in particular, those for the variable ξ_{μ}, belong now to the set of its equations of motion.

We now turn to the second part of this paper. As stated in the Introduction it may be thought of as a first application of the calculations we have just explained but, in fact, it was rather their initial motivation.

II. EFFECTIVE QUARKS WITH SPIN

We will explain in this section how the action (12) leads to effective quarks with spin if we insert for $B_{\mu\nu}$ the tensor $\tilde{F}_{\mu\nu}$ (2b). We will use the action in the orthogonal gauge (12) as our starting point. It is simpler than S_N and enough to get the result we are looking for.

The section is organized as follows. To begin with, we recall from Ref. 1 the key equations for the existence of the effective quarks; then, we compute the corresponding equations for the action (12), discuss their interpretation in the light of the work of BMC (Ref. 6) and conclude.

A. Key equations from Ref. 1

It follows from Ref. 1 that the key equations for the existence of effective quarks are the equations of motion of the U(1) gauge field $A_\mu(x)$. These equations are

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu \,, \qquad \partial_\mu F^{\mu\nu}=-4\pi\partial_\mu \widetilde{G}^{\,\mu\nu} \,, \tag{19}$$

or

$$\widetilde{F}_{\mu\nu}=\widetilde{H}_{\mu\nu}+4\pi M_0 G_{\mu\nu}$$

with

$$\partial_\mu H^{\mu\nu}=0 \,, \qquad \partial_\mu \widetilde{H}^{\,\mu\nu}=-4\pi M_0(d_f^\nu-d_i^\nu) \,, \tag{20}$$

and

$$d_{i,f}^\nu=\int_{i,f}\frac{dy^\mu}{ds}\delta^4(x-y)ds$$

and

$$G^{\mu\nu}=\int_{\text{sheet}}\frac{\partial(y^\mu,y^\nu)}{\partial(\tau^0,\tau^1)}\delta^4(x-y)d^2\tau \,.$$

i and f are the two ends of the string.

The effective quarks appear in these equations as the magnetic monopoles which contribute to $\partial_\mu\widetilde{H}^{\,\mu\nu}$. They are the sources of the charged currents $-4\pi(d_f^\nu-d_i^\nu)$. The quarks are effective in the sense that at the level of the action (1) (put it in the orthogonal gauge) what we have is truly a charged string [it is not possible for the coupling (5) to be replaced by a coupling on the ends of the string only, contrary to what happens if we use $F_{\mu\nu}$ instead of $\widetilde{F}_{\mu\nu}$]; nevertheless, at the level of the equations of motion, it is physically clearer to think of the system as a quark-antiquark pair (effective quark and antiquark) interacting with each other than to think of it as a string interacting with a U(1) gauge field. The comparison of (19) and (20) is very suggestive in this respect. (See Ref. 1 for more details.) In (20) the tensor field $G_{\mu\nu}$ characterizes the singular field along the string. It is the covariant generalization of the magnetic field along an infinitely thin solenoid (no edge effects).

Let us now look at the analog of (19) and (20) for the action (12).

B. Effective quarks with spin

We vary the gauge field $A_\mu(x)$ in the action (12) which we complete with the usual kinetic term $(1/16\pi)\int F_{\mu\nu}F^{\mu\nu}d^4x$ for A_μ. We get

$$\int \delta F_{\mu\nu}\left[(\partial^\mu C^\nu-\partial^\nu C^\mu)+\frac{M_0}{4}(\widetilde{D}_f^{\mu\nu}-\widetilde{D}_i^{\mu\nu}) +\frac{M_0}{2}\widetilde{G}^{\,\mu\nu}+\frac{1}{8\pi}F^{\mu\nu}\right]=0 \,,$$

where $G^{\mu\nu}$ is the same as before:

$$D_{i,f}^{\mu\nu}=\int_{i,f}d\tau\,\delta^4(x-y)(\bar{\xi}^{\,\mu}\gamma_5\gamma^1\xi^\nu+\epsilon\bar{\xi}^{\,\mu}\gamma_5\xi^\nu)$$

and

$$C_\nu=\tfrac{1}{2}\int \epsilon_{\nu\alpha\beta\gamma}\Gamma^{\alpha\beta\gamma}\delta^4(x-y)d^2\tau -\tfrac{1}{2}\partial_\eta\int \epsilon_{\nu\alpha\beta\gamma}\Gamma^{\eta\alpha\beta\gamma}\delta^4(x-y)d^2\tau$$

with

$$\Gamma^{\alpha\beta\gamma}=-\frac{M_0}{4}(\bar{\xi}^\alpha\gamma_5\gamma^\delta\gamma^\beta\partial_\delta y^\gamma+\bar{\xi}^{\,\alpha}\gamma_5\xi^\beta F^\gamma)$$

and

$$\Gamma^{\eta\alpha\beta\gamma}=\frac{M_0}{16}i\bar{\xi}^{\,\eta}\xi^\alpha\bar{\xi}^{\,\beta}\gamma_5\xi^\gamma \,,$$

or

$$F^{\mu\nu}=-8\pi(\partial^\mu C^\nu-\partial^\nu C^\mu)-2\pi M_0(\widetilde{D}_f^{\mu\nu}-\widetilde{D}_i^{\mu\nu}) -4\pi M_0\widetilde{G}^{\,\mu\nu}+H^{\mu\nu}$$

with

$$\partial_\mu H^{\mu\nu}=0 \,, \qquad \partial_\mu\widetilde{H}^{\,\mu\nu}=4\pi M_0(d_f^\nu-d_i^\nu) -2\pi M_0\partial_\mu(\widetilde{D}_f^{\mu\nu}-\widetilde{D}_i^{\mu\nu}) \,. \tag{21}$$

We see on these equations that the effective quarks are now characterized by the source terms $-4\pi M_0(d_f^\nu-d_i^\nu)-2\pi M_0\partial_\mu(\widetilde{D}_f^{\mu\nu}-\widetilde{D}_i^{\mu\nu})$ which is exactly what we expect for effective quarks with a classical spin in the manner of Berezin and Marinov, and Casalbuoni.

To prove this statement we just have to correctly identify the spin variables at the boundaries. This we do by noting that by virtue of the restrictions (11) on α we can impose $\theta^1+\epsilon\theta^2=0$ at $\tau^1=0$ and $\tau^1=\pi$ on the superspace coordinates θ. Then the scalar superfield $x^\mu(\tau^a,\theta^a)$ for the spinning string reduces at the boundaries to a scalar superfield for a spinning particle.[19] That is, defining

$$\widetilde{\theta}=\theta^1-\epsilon\theta^2/\sqrt{2} \text{ and } \widetilde{\xi}^{\,\mu}=\xi^{\mu 2}+\epsilon\xi^{\mu 1}/\sqrt{2}$$

we have at each end

$$x^\mu(\tau^0,\widetilde{\theta})=y^\mu(\tau^0)+i\widetilde{\theta}\widetilde{\xi}^{\,\mu} \,,$$

$$\bar{\xi}^{\,\mu}\gamma_5\gamma^1\xi^\nu+\epsilon\bar{\xi}^{\,\mu}\gamma_5\xi^\nu=i2\widetilde{\xi}^{\,\mu}\widetilde{\xi}^{\,\nu}$$

(note that this last equality is true without any conditions on ξ_μ^1 and ξ_μ^2, in particular, we do not need $\xi^{\mu 1}-\epsilon\xi^{\mu 2}=0$), and

$$D_{i,f}^{\mu\nu}=\int_{i,f}\delta^4(x-y)2i\widetilde{\xi}^{\,\mu}\widetilde{\xi}^{\,\nu}d\tau \,.$$

Moreover, the transformation laws (3) imply the following transformation laws for y^μ and $\widetilde{\xi}_\mu$ at the boundaries:

$$\delta y^\mu=i\widetilde{\alpha}\widetilde{\xi}^{\,\mu} \,,$$

$$\delta\widetilde{\xi}^{\,\mu}=-\widetilde{\alpha}\frac{2}{2\tau^0}y^\mu \,,$$

where

$$\tilde{\alpha}=\frac{\alpha^1-\bar{\epsilon}\alpha^2}{\sqrt{2}} .$$

It is now straightforward to verify, by redoing the calculation of Refs. 6 and 19 for these transformation laws, that the source term in (21) is indeed the correct one for particles with a classical spin described in the BMC method by $\tilde{\xi}_\mu$ [in particular, the effective quarks have no anomalous electric moments at the classical level (remember they are effective magnetic monopoles)].

We may further check that it is physically relevant to make the superfield of the spinning string reduce to the superfield of a spinning particle at the boundaries by looking at the boundary equations of motion for the spinor $\tilde{\xi}_\mu$. They write

$$\tfrac{1}{2}(\xi^{\mu\underline{2}}-\bar{\epsilon}\xi^{\mu\underline{1}})(\delta\xi_\mu{}^{\underline{2}}+\bar{\epsilon}\delta\xi_\mu{}^{\underline{1}}) - m(\xi^{\mu\underline{2}}+\bar{\epsilon}\xi^{\mu\underline{1}})(\delta\xi^{\nu\underline{2}}+\bar{\epsilon}\delta\xi^{\nu\underline{1}})\tilde{F}_{\mu\nu}=0 \qquad (22)$$

for each end, and as it should for a spinning particle, they depend only on the variation of $\tilde{\xi}_\mu$.

Thus we have shown the action (12) leads to effective quarks with a classical spin $\tilde{\xi}_\mu$ but still we do not know precisely what spin.

In fact, a complete answer to this question is not possible before the quantization of the model, which is still under study. Nevertheless, we can already indicate from our first results on this problem that a spin-$\frac{1}{2}$ realization of the quantum Clifford algebra associated with the Grassmann coordinates ξ_μ is indeed compatible with the set of Poisson brackets or rather Dirac brackets one gets in the process of the canonical quantization of S_N^2 or S_N^4. This makes us think we are on the right track to deduce, from a well-defined model, a Hamiltonian for mesons which incorporates the spin interactions, and we may suspect from Ref. 1 that these interactions will be essentially of the Breit-Fermi type in a suitable approximation, the Lorentz transformation properties of the confining potential being fixed by the model. However, we still have to prove that things indeed work as we suspect they work.

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APPENDIX

A point on the surface spanned by the string as it moves in space-time is characterized by two parameters τ^α, $\alpha\in\{0,1\}$ with $\tau^1\in[0,\pi]$. Its position in Minkowski space is defined by $y^\mu(\tau^0,\tau^1)$. The dimension of Minkowski space need not be specified, $0\le\mu\le d-1$.

We pose

$$\eta_{ab}=\begin{bmatrix}-1 & 0\\ 0 & 1\end{bmatrix},\quad \epsilon_{ab}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},$$

where a denotes a flat index. We have $\{\gamma^a,\gamma^b\}=2\eta^{ab}$ and $\gamma_5=i\gamma_0\gamma_1$. We have $\gamma^a\gamma^b=\eta^{ab}-i\epsilon^{ab}\gamma_5$, $\gamma^a\gamma^b\gamma_a=0$, and $c^{-1}\gamma_5^T C=-\gamma_5$, where C is the charge-conjugation matrix. In the Majorana representation

$$\gamma^0=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},\quad \gamma^1=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},$$

and $C=\gamma^0$.

ξ^μ is a two-dimensional Majorana spinor and also a Minkowski vector. Its components $(\xi^\mu)^{\underline{a}}$ belong to a Grassmann space. For any Majorana spinor χ we pose $\bar{\chi}_{\underline{a}}=\chi^{\underline{b}}C_{\underline{b}\underline{a}}$. The Fierz rearrangement formula for any four Majorana two-component anticommuting spinors is

$$\bar{\lambda}M\phi\bar{\psi}N\chi=-\tfrac{1}{2}(\bar{\lambda}M0_iN\chi)(\bar{\psi}0^i\phi)$$

with $0_i=(1,\gamma_5,\gamma_a)$.

We have

$$A_{\rho\mu\nu}=\partial_\rho\beta_{\mu\nu}+\partial_\mu B_{\nu p}+\partial_\nu B_{\rho\mu} .$$

In the particular case of $B_{\mu\nu}=\tilde{F}_{\mu\nu}$ we have $A_{\rho\mu\nu}=\epsilon_{\rho\mu\nu\eta}\partial_\tau F^{\tau\eta}$,

$$\bar{\xi}^\eta\delta\xi^\rho\bar{\xi}^\mu\gamma_5\xi^\nu\partial_\eta A_{\rho\mu\nu}=\tfrac{1}{4}\delta(\bar{\xi}^\eta\xi^\rho\bar{\xi}^\mu\gamma_5\xi^\nu)\partial_\eta A_{\rho\mu\nu} ,$$

$$(\bar{\xi}^\rho\gamma^a\xi^\mu\bar{\xi}^\nu\gamma_5\gamma^b\xi^\eta+\bar{\xi}^\rho\gamma_5\gamma^a\xi^\mu\bar{\xi}^\nu\gamma^b\xi^\eta)\partial_\eta A_{\rho\mu\nu}=0 ,$$

and for any Majorana spinor λ

$$\bar{A}\xi^\omega\bar{\xi}^\eta\xi^\rho\bar{\xi}^\mu\gamma_5\xi^\rho\partial_\omega\partial_\eta A_{\rho\mu\nu}=0 .$$

e_a^α in the inverse zweibein α denotes a curved index, that is, a two-dimensional Lorentz index:

$$e=\det e_\alpha^a,\quad g^{\alpha\beta}=e_a^\alpha e_b^\beta\eta^{ab} ,$$

$$\epsilon^{\alpha\beta}=e_a^\alpha e_b^\beta\epsilon^{ab},\quad \gamma^\alpha=e_a^\alpha\gamma^a .$$

We have

$$A\equiv(a,\underline{a}),\quad \eta^{AB}=\begin{bmatrix}\epsilon^{ab} & 0\\ 0 & (c^{-1})^{\underline{a}\,\underline{b}}\end{bmatrix},$$

$$\gamma_5^{AB}=\begin{bmatrix}\eta^{ab} & \\ & (\gamma_5 c^{-1})^{\underline{a}\,\underline{b}}\end{bmatrix} .$$

We have

$$\delta F_+^\nu=i\bar{\alpha}\gamma^\alpha\left[-\frac{i}{2}\omega_\alpha(e)\gamma_5\xi^\nu+\gamma^\beta\chi_\alpha\partial_\beta y^\nu+\chi_\alpha F^\nu\right]+\frac{\bar{\alpha}\xi^\nu}{2}\bar{\chi}_\alpha\gamma^\beta\gamma^\alpha\chi_\beta .$$

We choose the ansatz

$$E_A^{\alpha}=\begin{bmatrix} e_a^{\beta}(\delta_\beta^\alpha+2i\bar{\theta}\gamma^\alpha\chi_\beta) & 0 \\ ie_b^{\alpha}(\bar{\theta}\gamma^b)_{\underline{a}}-\frac{1}{2}\bar{\theta}\theta(\bar{\chi}_\beta\gamma^\alpha\gamma^\beta)_{\underline{a}} & \delta_{\underline{a}}^{\underline{\alpha}}(1+\frac{1}{4}\bar{\theta}\theta\bar{\chi}_\beta\gamma^\delta\gamma^\beta\chi_\delta)+i(\bar{\theta}\gamma^\beta)_{\underline{a}}\chi_\beta^{\underline{\alpha}} \end{bmatrix}$$

and

$$S_x^2=-\eta\tfrac{1}{4}\int d^2\theta\, d^2\tau\, E(E_A^\alpha\mathscr{D}_\alpha x^\mu\eta^{AB}E_B^\beta\mathscr{D}_\beta x_\mu)+\frac{m}{2i}\int d^2\theta\, d^2\tau\, E(E_A^\alpha\mathscr{D}_\alpha x^\mu\gamma_5^{AB}E_B^\beta\mathscr{D}_\beta x^\nu)\widetilde{F}_{\mu\nu}(x)$$

with

$$\alpha=(\alpha,\underline{\alpha}),\quad E=\mathrm{SDet}E_\alpha^A,\quad \mathscr{D}_\alpha\equiv\left[\partial_\alpha-\frac{i\gamma_5}{2}\omega_\alpha(e),\partial_{\underline{a}}\right].$$

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