PHYSICAL REVIEW D VOLUME 33, NUMBER 8 15 APRIL 1986

Consistent method of truncating the electron self-energy in nonperturbative QED

P. Rembiesa
The Citadel, Military College of South Carolina, Charleston, South Carolina 29409
(Received 24 June 1985; revised manuscript received 2 December 1985)

A nonperturbative method of solving the Dyson-Schwinger equations for the fermion propagator is considered. The solution satisfies the Ward-Takahashi identity, allows multiplicative regularization, and exhibits a physical-mass pole.

I. INTRODUCTION

Methods based on finding approximate solutions to Dyson-Schwinger (DS) equations play a prominent role among efforts to find nonperturbative solutions in realistic field models. Their origin dates as far back as to the 1960s when Johnson, Baker, and Willey applied the truncated expansion to the equations of quantum electrodynamics.[1] In the following we shall use the term "truncation" to describe any procedure which breaks the infinite hierarchy of DS equations by replacing one of the kernels with either a subclass of diagrams of its Feynman expansion or, by using asymptotic estimates or other argument, making a postulate about its form.

In the original approach[1-3] the truncation is performed in both the photon propagator and vertex function. Kernels of integral equations for the electron propagator are built with use of the free photon propagator and, less justifiably, a vertex made of a selected subclass of Feynman diagrams.

In the first order of such an expansion, the self-energy function $\Sigma(S;p)$ in the equation for the electron propagator

$$S^{-1}(p)=A(p)\gamma^{\mu}p_{\mu}-B(p)$$

$$=\gamma^{\mu}p_{\mu}-m_0+\Sigma(S;p) \qquad (1.1)$$

collects only contributions represented in Fig. 1. An insertion is made only on the fermion line while the photon propagator and the vertex function are represented by their respective free-field forms. The integrand of the Feynman amplitude does not depend on angular variables and the integration over angles can be easily performed. In Euclidean space, after taking traces and using formulas listed in the Appendix one obtains

$$A(x)-1=(1-G)g\left[x^{-2}\int_0^x y^2A(y)D^{-1}(y)dy+\int_x^{\infty}A(y)D^{-1}(y)dy\right] \qquad (1.2)$$

and

$$B(x)-m_0=(4-G)(g/2)\left[x^{-2}\int_0^x yB(y)D^{-1}(y)dy+\int_x^{\infty}B(y)D^{-1}dy\right], \qquad (1.3)$$

where $g=e^2/32\pi^2$ and $D=A^2x+B^2$.

One can use the above equations and their derivatives with respect to x to eliminate integrals from the equations obtained by taking second derivatives of (1.2) and (1.3). It produces a system of nonlinear differential equations:

$$[x^2A(x)]''+A'(x)=(G-1)gA(x)D^{-1}(x), \qquad (1.4)$$

$$[xB(x)]''=(G-4)(g/2)B(x)D^{-1}(x). \qquad (1.5)$$

If we consider the Minkowski counterpart of the above equations and introduce

$$x=-\exp(t),$$

$$F(t)=A(t)+\exp(-t/2)B(t),$$

$$H(t)=A(t)-\exp(-t/2)B(t),$$

then (1.4) and (1.5) assume a very symmetric form:

$$F''+2F'=\tfrac{3}{8}(H-F)+(G-2)(3g/4)H^{-1}+(4-G)(g/4)F^{-1}, \qquad (1.6)$$

$$H''+2H'=\tfrac{3}{8}(F-H)+(G-2)(3g/4)F^{-1}+(4-G)(g/4)H^{-1}. \qquad (1.7)$$

A simple argument based on mechanical analogy shows that this expansion technique requires modification. First, we notice that (1.6) and (1.7) may be regarded as a system of equations of motion for a two-dimensional, strongly damped motion under an external, rather peculiar, time-independent force.

The propagator will possess a physical pole if, for some t, $A^2-C^2=0$, i.e., if $F=0$ or $H=0$. It would require that for $F\to 0$ the "force" is attractive towards the H, or

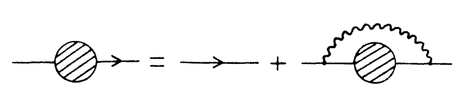

FIG. 1. Graphical representation of the first order of the truncated Dyson-Schwinger equation.

for $G \to 0$ towards the F axis. We immediately see that it does not happen unless $G > 4$ which disallows the existence of physical poles in all other gauges, a result confirmed by numerical work and attributed to the violation of gauge invariance.[3]

Two successful methods to alleviate this problem both leave the photon propagator unmodified and incorporate the Ward-Takahashi identity as an additional equation for the vertex function. It allows us to break the infinite hierarchy of the Dyson-Schwinger equations without expanding the vertex in a perturbative series of Feynman diagrams.

The first method, developed by Delburgo and West,[4] is based on Salam's gauge technique.[5] The method explicitly refers to the spectral representation of the propagator and vertex functions and it uses an ansatz that respective spectral density functions are identical. Equations for the spectral density function turn out to be linear and, for some gauges, exactly solvable. The transverse part of the vertex may be modified so that the resulting theory is multiplicatively renormalizable.[6]

Another method, proposed by Broyles and co-workers[7] uses an assumption that the dominating contribution to the DS equations comes from the zero-momentum-transfer part of the vertex function. Replacing the vertex by its zero-momentum-transfer part $\Gamma^\mu(p,p) = \partial S^{-1}(p)/\partial p_\mu$ takes care of the integrations, which may then be performed by parts, and leads to a system of differential equations which, in the Landau gauge, are exactly solvable.

In this paper we present another expansion scheme. It satisfies the basic requirements of consistency: it preserves the Ward identity; equations for the propagator generated by this expansion permit the use of the multiplicative regularization and their solutions exhibit physical poles on the Minkowski part of the real axis.

We do not attempt to build any systematic expansion as a series in powers of some new expansion parameter. As a matter of fact, in a simple field theory like QED, there may be no such parameter other than the coupling constant α. The failure of the conventional perturbation technique to describe effects such as the dynamical symmetry breaking is often attributed to the possibility that in the limit $\alpha \to 0$ the exact solutions of the theory do not reproduce the $\alpha = 0$ (free-field) solutions. The examples of Klauder phenomena[8] offer numerous illustrations that, even on the classical level, in similar cases perturbative methods are still adequate if the expansion is carried over around the $\alpha \to 0$ (pseudofree) instead of the free ($\alpha = 0$) solution. Since the exact solutions are not known, many attempts to find nonperturbative solutions in quantum field theory are motivated by a hope that quasifree limits can be found with the help of insight gained from the study of desired physical properties and mathematical structures of underlying theories. In the expansion schemes we mentioned above, some results of the standard perturbative expansion are used to provide that insight. In the method of Ref. 4 the first-order approximation is defined by the requirement that on-mass-shell Green's functions reproduce Born terms of the perturbative expansion. In the approach of Ref. 7 the expansion starts with the infrared-limit values of the vertex function defining the first order.

Our procedure, although similar in spirit, relies on a different ansatz. For any value of the charge parameter, the introduction of interaction terms into the Lagrangian changes the character of solutions by establishing, via the DS equations, relations between various Green's functions. In particular, the first equation of the DS hierarchy will relate the form factors of the vertex Γ^μ with form factors A and B in the electron propagator $A(p)\not{p} + B(p)$. We assume that form factors of Γ can be expanded in a functional series in powers of A and B. We further assume that the "quasifree" theory, or the first order of our approximation will contain only linear terms of that expansion. We determine this (nonunique) linear relation using the Ward-Takahashi identity C and P invariance as well as some additional symmetry requirements. The postulated form of Γ^μ is then substituted into the DS equation. Taking appropriate traces of the resulting equations and performing the integrations over angles which, like in the original truncated expansion, can be performed explicitly we obtain a pair of coupled integral equations for A and B. Searching for poles, we solve them numerically for some values of the gauge parameter and coupling constant and find that such poles exist in the desired locations on the Minkowski part of the real axis.

II. THE VERTEX FUNCTION

We want to break the infinite hierarchy of DS equations without resorting to a partial Feynman expansion or *a priori* restricting the vertex function to any particular kinematic region. Since leaving the photon propagator unaltered is justifiable by arguments based on the requirement of proper asymptotic behavior[1] we shall alter the scheme by including corrections to the vertex function and use the DS equations represented in Fig. 2. The general Lorentz-invariant vertex function $\Gamma^\mu(k,p)$ is characterized by 12 form factors $f_i(p,k)$ being scalar functions of incoming and outgoing electron momenta k and p and momentum transfer k-p. Let us write it in the form

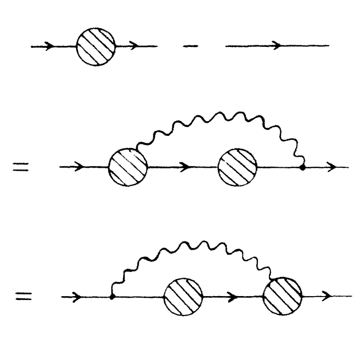

FIG. 2. The Dyson-Schwinger equation for the fermion propagator modified by adding corrections to the vertex function.

$$\Gamma^\mu(k,p)=f_1(k,p)\gamma^\mu+f_2(k,p)(k-p)^\mu+f_3(k,p)(k+p)^\mu+f_4(k,p)(\not p p^\mu-\not k k^\mu)$$
$$+f_5(k,p)(\not p p^\mu+\not k k^\mu)+f_6(k,p)(\not p k^\mu-\not k p^\mu)+f_7(k,p)(\not p k^\mu+\not k p^\mu)$$
$$+f_8(k,p)\sigma^{\mu\nu}(k-p)_\nu+f_9(k,p)\sigma^{\mu\nu}(k+p)_\nu+f_{10}(k,p)\epsilon^{\mu\nu\rho\eta}p_\nu k_\rho\gamma_\eta\gamma^5$$
$$+f_{11}(k,p)\sigma^{\alpha\beta}k_\alpha p_\beta(k-p)^\mu+f_{12}(k,p)\sigma^{\alpha\beta}k_\alpha p_\beta(k+p)^\mu\ . \tag{2.1}$$

The vertex function defined by (2.1) satisfies the requirements of invariance under the P transformation:

$$\gamma^0\Gamma^\mu(k,p)\gamma^0=\Gamma_\mu(Pk,Pp)\ .$$

The requirement of C invariance

$$C\Gamma_\mu(k,p)C^{-1}=-\Gamma_\mu^T(-p,-k)$$

implies that form factors f_1, f_3, f_5, f_7, and f_{12} are symmetric while f_2, f_4, f_6, f_8, f_9, f_{10}, and f_{11} are antisymmetric functions of momenta k and p.

The Ward-Takahashi (WT) identity

$$(p-k)_\mu\Gamma^\mu(k,p)=A(k)\not k-A(p)\not p+B(p)-B(k) \tag{2.2}$$

imposes additional constraints on form factors f_i. We postulate that

$$f_{10}(k,p)=0\ , \tag{2.3}$$

and, substituting (2.1) into (2.2), we find that

$$2f_9(k,p)+(k-p)^2f_{11}(k,p)+(k^2-p^2)f_{12}(k,p)=0\ , \tag{2.4}$$

$$(k-p)^2f_2(k,p)+(k^2-p^2)f_3(k,p)=B(p)-B(k)\ , \tag{2.5}$$

and

$$f_1(k,p)+(k^2-kp)(f_4(k,p)+f_5(k,p))$$
$$+(kp-p^2)(f_6(k,p)+f_7(k,p))=A(k)\ . \tag{2.6}$$

Of course, the WT equation alone does not determine the transverse part of the vertex unambiguously. Indeed, Eq. (2.4) is the only constraint at hand on form factors responsible for terms proportional to $\sigma^{\mu\nu}$. In the following we shall adopt an ansatz that the anomalous magnetic-moment part of the vertex function is calculable as a higher-order correction to the vertex function and, in the first order, simply ignore the corresponding part $\Gamma_A^\mu(k,p)$ of $\Gamma^\mu(k,p)$:

$$\Gamma_A^\mu(k,p)=\psi_1(k,p)\sigma^{\mu\nu}(k-p)_\nu+\psi_2(k,p)\sigma^{\alpha\beta}k_\alpha p_\beta(k-p)^\mu+\psi_3(k,p)\sigma^{\alpha\beta}k_\alpha p_\beta(k+p)^\mu$$
$$-\tfrac{1}{2}[(k-p)^2\psi_2(k,p)+(k^2-p^2)\psi_3(k,p)]\sigma^{\mu\nu}(k+p)_\nu\ ,$$

with still undetermined functions $\psi_i(k,p)$. Form factors ψ_1 and ψ_3 are symmetric; ψ_2 is antisymmetric in k and p.

The longitudinal part contains other remaining form factors restricted by conditions (2.5) and (2.6). We presume that the vertex-function form factors f_i are expansible in a power series in the propagator form factors A and B. According to (2.5) and (2.6), in the first order of such expansion, functions f_2 and f_3 will be linear in B, while f_1 and f_4–f_7 will be linear in A.

The solution to (2.5) may be written in the form

$$f_2(k,p)=\xi[B(p)-B(k)]/(k-p)^2$$
$$+(k^2-p^2)\phi(k,p)/(k-p)^2\ ,$$
$$f_3(k,p)=(1-\xi)[B(p)-B(k)]/(k^2-p^2)+\phi(k,p)\ ,$$

where ξ is an arbitrary constant and while $\phi(k,p)$ is arbitrary and symmetric in k and p. As $k\to p$,

$$\Gamma^\mu(p,p)=\partial\Sigma(p)/\partial p_\mu\ . \tag{2.7}$$

Since, in the same limit,

$$[B(p^2)-B(k^2)]/(p^2-k^2)\to dB(p^2)/dp^2$$

and in order to avoid additional singularities as $(p-k)^2\to 0$, we take

$$f_2(k,p)=0\ ,$$
$$f_3(k,p)=[B(p)-B(k)]/(k^2-p^2)\ . \tag{2.8}$$

We find it convenient to define

$$f(k,p)=f_1(k,p)\ ,$$

$$g(k,p)=f_4(k,p)+f_5(k,p)\ ,$$

$$h(k,p)=f_6(k,p)+f_7(k,p)\ .$$

Then, after eliminating $f(k,p)$ with use of (2.6), the remaining terms in Γ^μ can be written in the form

$$\tilde\Gamma(k,p)=\tfrac{1}{2}[A(k)+A(p)]\gamma^\mu-\tfrac{1}{2}\{[(k^2-kp)g(k,p)+(p^2-kp)g(p,k)]-[(kp-p^2)h(k,p)+(kp-k^2)h(p,k)]\}\gamma^\mu$$
$$+g(k,p)\not k k^\mu+g(p,k)\not p p^\mu+h(k,p)\not k p^\mu+h(p,k)\not p k^\mu\ . \tag{2.9}$$

According to (2.7), in the limit of $k \to p$,

$$\tilde{\Gamma}(p,k) \to A(p)\gamma^\mu + 2A'(p)\not{p}p^\mu . \quad (2.10)$$

It is reasonable to assume that, at least in the first order, $\tilde{\Gamma}^\mu$ is symmetric in k and p. Therefore, and in order to satisfy Eq. (2.10) we take

$$g(k,p) = h(k,p) = \tfrac{1}{2}[A(k) - A(p)]/(k^2 - p^2) , \quad (2.11)$$

and the full zero-order vertex function is

$$\begin{aligned}\Gamma_0^\mu(k,p) = {} & \tfrac{1}{2}[A(k)+A(p)]\gamma^\mu \\ & + \tfrac{1}{2}[A(k)-A(p)](\not{k}+\not{p})(k+p)^\mu/(k^2-p^2) \\ & + [B(p)-B(k)](k+p)^\mu/(k^2-p^2) . \quad (2.12)\end{aligned}$$

III. DYSON-SCHWINGER EQUATIONS FOR THE PROPAGATOR

Finally, we are in a position to write the DS equation for the propagator. The equation was already represented in Fig. 2 and it has the form

$$A(k)\not{k} - B(k) = \not{k} - m + \Sigma(S;k) , \quad (3.1)$$

where

$$\Sigma(S;k) = ie^2 \int \Gamma^\mu(k,p)S(p)D_{\mu\nu}(k-p)\gamma^\nu \times d^4p(2\pi)^{-4} . \quad (3.2)$$

As we explained earlier, we approximate $D_{\mu\nu}$ by the free-field expression

$$D_{\mu\nu}(q) \approx (g_{\mu\nu} - Gq^\mu q^\nu/q^2)/q^2 . \quad (3.3)$$

The electron propagator is

$$S^{-1}(q) = A(q)\not{q} - B(q) , \quad (3.4)$$

and the vertex function is approximated by (2.12).

Integral equations for the form-factor functions $A(q)$ and $B(q)$ are generated by taking proper traces of the DS equation:

$$A(k)k^\mu = k^\mu - \tfrac{1}{4}\,\mathrm{Tr}[\Sigma(S;k)\gamma^\mu]$$

and

$$B(k) = m + \tfrac{1}{4}\,\mathrm{Tr}[\Sigma(S;k)] . \quad (3.5)$$

Four-dimensional integrals in $\Sigma(S;k)$ have an attractive feature that their integrands do not depend on angles, except through the $(k-p)^2$ term in the denominator of the photon propagator. Again, in the Euclidean space, integrations over angles can be performed explicitly with use of the method and formulas given in the Appendix. After some uninteresting calculations we end with a pair of integral equations which include only squares of Euclidean momenta $x = k^2$ and $y = p^2$:

$$\begin{aligned}[A(x)-1]x^2 = {} & (G-1)g\left[\int_0^x y^2 A_+(x,y)A(y) + x^2\int_x^\infty A_+(x,y)A(y)\right] \\ & + \tfrac{1}{2}g\int_0^x y^2[(5x+y)/(x-y) - 2G]A_-(x,y)A(y) \\ & + \tfrac{1}{2}gx^2\int_x^\infty [(5x+y)/(x-y) - 2G]A_-(x,y)A(y) \\ & + g\int_0^x y[(2x+y)/(x-y) - 2G]B_-(x,y)B(y) + 3gx^2\int_x^\infty B_-(x,y)B(y)/(x-y) , \quad (3.6)\end{aligned}$$

and

$$\begin{aligned}[B(x)-m]x = {} & (G-4)g\left[\int_0^x yA_+(x,y)B(y) + x\int_x^\infty A_+(x,y)B(y)\right] \\ & - g\int_0^x y[(2x+y)/(x-y) - G]A_-(x,y)B(y) - gx\int_x^\infty [(2x+y)/(x-y)+G]A_-(x,y)B(y) \\ & + 3g\int_0^x y^2 B_-(x,y)A(y)/(x-y) + gx\int_x^\infty [(x+2y)/(x-y)+2G]B_-(x,y)A(y) , \quad (3.7)\end{aligned}$$

where

$$A_\pm(x,y) = [A(x) \pm A(y)]/[A^2(y)y + B^2(y)] ,$$

$$B_\pm(x,y) = [B(x) \pm B(y)]/[A^2(y)y + B^2(y)] ,$$

and integrations are over dy.

An important feature of Eqs. (3.6) and (3.7) is that their right sides are invariant under the transformation

$$A \to Z^{-1/2}A_R, \quad B \to Z^{-1/2}B_R .$$

Leaving aside the question of the cutoff dependence (or independence) of Z, it means that possible divergencies in both equations may be removed by a standard multiplicative regularization procedure.

Choosing $Z^{-1/2} = A(0)$ and $m_R = m - B_R(0)$, after some algebra, we obtain

TABLE I. Position of the mass pole for some selected values of the gauge parameter G and coupling constant $g=e^2/128\pi^2$.

$G \backslash g$	g_{em}	10^{-4}	10^{-3}	10^{-2}
-5	0.997	0.996	0.974	0.873
-2	0.997	0.996	0.969	0.828
-1	0.996	0.995	0.967	0.816
0	0.996	0.996	0.966	0.805
1	0.996	0.995	0.964	0.795
2	0.996	0.995	0.963	0.786
5	0.995	0.994	0.958	0.765
10	0.995	0.993	0.950	0.738

$$\begin{aligned}[A(x)-1]x^2 = {} & \tfrac{1}{2}gA(x)\int_0^x dy\,(x^2-6xy-3y^2)A(y)[A^2(y)y+B^2(y)]^{-1} \\ & -gB(x)\int_0^x dy\,(3x+y+2Gy)B(y)[A^2(y)y+B^2(y)]^{-1} \\ & +g\int_0^x dy\{\tfrac{1}{2}[3x^2+6xy-y^2+4G(y^2-x^2)]A^2(y)+[(3x+y+2Gy)B^2(y)]\}[A^2(y)y+B^2(y)]^{-1}\end{aligned} \tag{3.8}$$

and

$$\begin{aligned}[B(x)-m] = {} & A(x)g\int_0^x dy\,(6x-2y-2Gy)B(y)[A^2(y)y+B^2(y)]^{-1} \\ & -B(x)g\int_0^x dy\,(x+3y+2Gx)A(y)[A^2(y)y+B^2(y)]^{-1} \\ & +3g\int_0^x dy\,(x-y)A(y)B(y)[A^2(y)y+B^2(y)]^{-1}\,.\end{aligned} \tag{3.9}$$

IV. NUMERICAL RESULTS AND CONCLUSIONS

The system (3.8) and (3.9) was integrated numerically with use of a simple predict-correct method. Current values of A and B were used to calculate the values of integrals at $x+\Delta x$ by the open-end Newton method. Then $A(x+\Delta x)$ and $B(x+\Delta x)$ were recalculated with the use of new values of integrals. New values of A and B were used to recalculate the integrands by the Simpson method and then corrected values of A and B at $x+\Delta x$ were found. Both $A(x)$ and $B(x)$ turn out to change slowly and round-off errors related to the procedure can be easily kept under control by taking sufficiently small steps of integration.

Calculations were carried along the real axis for a few different values of parameters g and G. In all cases studied we found that physical poles exist on the Minkowski part of the real axis. Locations of these poles are shown in Table I. They turn out to be G dependent, a worrying fact but common in truncated expansions. It seems that this gauge dependence may be attributed to the fact that the transverse part of the propagator was omitted and the four form factors of the transverse part were arbitrarily set equal to zero. The problem of the transverse part of the propagator will be treated separately. Here, let us notice one encouraging feature of the subtracted equations (3.8) and (3.9): they contain only four types of integral terms which are proportional to the gauge parameter G. Having four form factors of the transverse part still undetermined, it is very likely that one can find their functional form such that they will be linear in A and B and further the dependence of (3.8) and (3.9) on G will be eliminated.

Other important problems which require separate discussion are the question of the IR and UV limits of the solutions and the applicability of this expansion to the question of the dynamical chiral-symmetry breaking. Postponing more detailed analysis let us only notice that since in the IR limit our vertex reproduces that of Ref. 7, we may expect a similar zero-momentum behavior in our model. Another nice property of our approximation is that in the chiral-invariant case, $B(x)=0$, Eq. (3.8) for $A(x)$ is solvable. This will help in the analysis of the Bethe-Salpeter eigenproblem and help in the discussion of the possibilities of chiral-symmetry breaking following the path of Ref. 9. Whether or not these features will be present in more complicated theories remains an open question.

ACKNOWLEDGMENT

This work was supported by Citadel Development Foundation Grant No. 21100-V731.

APPENDIX

Integrations over angles we performed in Secs. I and III were carried out with use of the standard method of angular averages. One first rotates momenta to the Euclidean space then expands the function $(p-k)^{-2}$ in terms of Gegenbauer polynomials

$$(p-k)^{-2}=\max(p,k)\sum_0 z^n C_n{}^1(\cos\omega)\,, \tag{A1}$$

where ω is the polar angle and $z=\min(p,k)/\max(p,k)$. Orthogonality conditions and recursion formulas for Gegenbauer polynomials allow us to reduce the problem of integration to simple algebra. We made use of the following integrals:

$$(2\pi)^{-4}\int d^4p\,F(p^2)(p-k)^{-2}=(64\pi^2)^{-1}\left[\int_0^x (y/x)F(y)dy+\int_x^\infty F(y)dy\right], \tag{A2}$$

$$(2\pi)^{-4}\int d^4p\,F(p^2)p^\mu(p-k)^{-2}=(k^\mu/128\pi^2)\left[\int_0^x (y/x)^2F(y)dy+\int_x^\infty F(y)dy\right], \tag{A3}$$

where $x=k^2$ and $y=p^2$. For integrals involving $(p-k)^{-4}$ we obtain

$$(2\pi)^{-4}\int d^4p\,(k^\mu-p^\mu)F(p^2)(p-k)^{-4}=(k^\mu/64\pi^2)\int_0^x (y/x^2)F(y)dy\;, \tag{A4}$$

$$(2\pi)^{-4}\int d^4p\,F(p^2)(k^2-kp)(p-k)^{-4}=(1/64\pi^2)\int_0^x (y/x)F(y)dy\;, \tag{A5}$$

$$(2\pi)^{-4}\int d^4p\,F(p^2)(p^2-kp)(p-k)^{-4}=(1/64\pi^2)\int_x^\infty F(y)dy\;, \tag{A6}$$

$$(2\pi)^{-4}\int d^4p\,p^\mu F(p^2)(k^2-kp)(k-p)^{-4}=(k^\mu/256\pi^2)\left[3\int_0^x (y/x)^2F(y)dy-\int_x^\infty F(y)dy\right], \tag{A7}$$

and, finally,

$$(2\pi)^{-4}\int d^4p\,p^\mu F(p^2)(p^2-kp)(k-p)^{-4}=(k^\mu/256\pi^2)\left[3\int_x^\infty F(y)dy-\int_0^x (y/x)^2F(y)dy\right]. \tag{A8}$$

[1]K. Johnson, M. Baker, and R. Willey, Phys. Rev. **163**, B111 (1964); K. Johnson, R. Willey, and M. Baker, Phys. Rev. Lett. **11**, 518 (1963); Phys. Rev. **163**, 1699 (1969); Phys. Rev. D **3**, 2516 (1971); **3**, 2541 (1971); **8**, 1110 (1973).

[2]R. Fukuda and T. Kugo, Nucl. Phys. **B117**, 250 (1976).

[3]D. Atkinson and D. W. E. Blatt, Nucl. Phys. **B151**, 342 (1979); D. Atkinson and M. P. Fry, *ibid.* **B15**, 301 (1979); D. Atkinson, M. P. Fry, and E. J. Luit, Lett. Nuovo Cimento **26**, 413 (1979).

[4]R. Delbourgo and P. West, J. Phys. A **10**, 1049 (1977); Phys. Lett. **72B**, 96 (1977); R. Delbourgo, Nuovo Cimento **49 A**, 484 (1979); D. Atkinson and H. A. Slim, *ibid.* **50A**, 555 (1979); H. A. Slim, Nucl. Phys. **B177**, 172 (1981); R. Delbourgo, B. W. Keck, and C. N. Parker, J. Phys. A **14**, 921 (1981).

[5]A. Salam, Phys. Rev. **130**, 1287 (1963).

[6]R. Delbourgo and R. Zhang, J. Phys. A **17**, 3593 (1984); J. E. King, Phys. Rev. D **27**, 1821 (1983).

[7]H. S. Green, J. F. Cartier, and A. A. Broyles, Phys. Rev. D **18**, 1102 (1978); J. F. Cartier, A. A. Broyles, R. M. Placido, and H. S. Green, *ibid.* **30**, 1742 (1984).

[8]J. R. Klauder, Acta Phys. Austriaca Suppl. **11**, 341 (1973); in *Proceedings of the International Symposium on Mathematical Problems in Theoretical Physics,* edited by H. Araki (Springer, Berlin, 1975).

[9]R. W. Haymaker and J. Perez-Mercader, Phys. Rev. D **21**, 1353 (1983); R. W. Haymaker and T. Matsuki, *ibid.* **33**, 1137 (1986).