Path integrals and the solution of the Schwinger model in curved space-time

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We use the path-integral formalism to derive the solution of the Schwinger model in curved space-time. We show that the nature of flat—space-time solutions persists even in the presence of a background gravitational field.

I. INTRODUCTION

The Schwinger model[1] is an exactly soluble field-theoretical model which has been studied extensively in flat space. It is the theory of quantum electrodynamics in $1+1$ dimensions and provides a simple example of dynamical breakdown of symmetry. The gauge fields in this theory acquire mass as a result of symmetry breaking and the electric charge is confined.

In the past few years people have become interested in studying quantum field theories in curved space-time. An amusing question to ask in this connection is whether a nontrivial structure of space-time affects the qualitative behavior of quantum field theories. In particular, in the case of exactly soluble models one can ask whether the nature of the solution changes by the introduction of a curved background. More precisely one can ask whether curvature affects long-distance properties of solutions such as confinement.

Intuitively, of course, one would expect that if fermions are confined in a theory in flat space-time, they would continue to be confined in a curved background. This is because gravitation, being an attractive force, would only help in confinement. Quantitatively, however, an attempt was made[2] to study this question in the case of the Schwinger model in curved space-time by examining the equations of motion and the results were inconclusive. Namely, what was found was that although the gauge fields still acquired the same mass as in the case of flat space-time, the question of confinement was not an easy one to answer since the current-current correlations had a difficult form in a curved background.

Recently we studied the solubility of various two-dimensional models in flat space-time in the path-integral formalism.[3] It is of interest, therefore, to see whether these methods generalize easily to curved space-time and, if so, whether they lead to a more unique conclusion in a simple way than the study of the equations of motion had revealed. Our conclusions are that the path-integral method generalizes readily to curved space-time and the nature of flat—space-time solutions persists even in the presence of curvature. More specifically, we show that just as in the case of flat space-time, one can integrate out the fermion fields completely from the generating functional leaving an effective action only in terms of the photon field and the background gravitational field. The photon field acquires a mass whose value is precisely the same as the flat—space-time case.

In deriving these results we make use of the ideas of Fujikawa[4] that the axial anomaly is related to the noninvariance of the fermionic measure under a chiral transformation. Normally to derive the anomaly in flat space-time one uses a momentum representation which is not available in general in the presence of a gravitational field. In Sec. II we show how a local momentum representation can be used to derive the anomaly equation simply because we are interested in the short-distance behavior and hence can use the Riemann normal coordinates. In Sec. III we calculate the change in the fermionic measure using the ζ-function method and show that it leads to the same answer as the momentum-space calculation. In this section the model is solved and some concluding remarks are given in Sec. IV.

II. MOMENTUM-SPACE CALCULATION

The Schwinger model in curved space-time is described by the Lagrangian

$$L=-\tfrac{1}{4}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}+i\bar{\psi}e^{\mu a}\gamma_a D_\mu\psi\ , \tag{2.1}$$

where the zweibein fields $e^{\mu a}$ satisfy the relations[5]

$$\begin{aligned} &e^{\mu a}e_a^{\nu}=g^{\mu\nu},\quad e^{\mu a}e_\mu^{b}=\eta^{ab}\ ,\\ &e^{\mu a}e_{\nu a}=\delta^\mu_\nu,\quad e^{\mu a}e_{\mu b}=\delta^a_b\ . \end{aligned} \tag{2.2}$$

Note that the world indices are raised and lowered with the Minkowski metric whereas the Riemann indices are raised or lowered with the metric. The curved-space Dirac matrices are obtained from the flat-space ones as

$$\gamma^\mu(x)=e^{\mu a}\gamma_a\ . \tag{2.3}$$

The electromagnetic field strengths are given by

$$F_{\mu\nu}=D_\mu A_\nu-D_\nu A_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu$$

and the covariant derivative acting on the spinors is defined to be

$$\begin{aligned} D_\mu\psi&=(\partial_\mu+\tfrac{1}{2}\omega_\mu^{ab}\sigma_{ab}-ieA_\mu)\psi\\ &=(\nabla_\mu-ieA_\mu)\psi\quad (e=\text{electric charge})\ . \end{aligned} \tag{2.4}$$

Here σ_{ab} are flat—space-time matrices defined as

$$\sigma_{ab}=\tfrac{1}{4}[\gamma_a,\gamma_b] \tag{2.5}$$

and are the generators of the Lorentz group. The spin connections are defined in terms of the zweibein fields as

$$\omega_{\mu ab}=\tfrac{1}{2}[e_a^{\nu}(\partial_\mu e_{\nu b}-\partial_\nu e_{\mu b})+e_a^{\rho}e_b^{\sigma}(\partial_\sigma e_{\rho c})e_\mu^c-(a\leftrightarrow b)] \,. \tag{2.6}$$

The action for the Schwinger model in curved space-time has the form

$$S=\int d^2x\,\sqrt{-g}\,L \,, \tag{2.7}$$

where $g=\det g_{\mu\nu}$. Consequently the generating functional can be written as

$$Z=\int DA_\mu D\bar\psi D\psi e^{iS} \,. \tag{2.8}$$

Our metric convention in flat space-time is that of Bjorken and Drell. Namely, $\eta^{00}=1$ and $\eta^{11}=-1$. γ^0 is Hermitian with square $+1$ whereas γ^1 is anti-Hermitian with square -1. $\gamma_5=\gamma^0\gamma^1$ is Hermitian with square $+1$.

Let us now examine the effect of an infinitesimal chiral redefinition of the fermion fields, namely,

$$\begin{aligned}\psi\to\psi'&=e^{-i\gamma_5\epsilon(x)}\psi=[1-i\epsilon(x)\gamma_5]\psi \,,\\ \bar\psi\to\bar\psi'&=\bar\psi e^{-i\gamma_5\epsilon(x)}=\bar\psi[1-i\epsilon(x)\gamma_5] \,.\end{aligned} \tag{2.9}$$

Under this transformation

$$\begin{aligned}\delta S&=-\int d^2x\,\sqrt{-g}\,[\partial_\mu\epsilon(x)\bar\psi(x)\gamma^\mu\gamma_5\psi]\\ &=+\int d^2x\,\sqrt{-g}\,\partial_\mu\epsilon(x)J_5^\mu(x)\\ &=-\int d^2x\,\epsilon(x)\partial_\mu[\sqrt{-g}\,J_5^\mu(x)]\\ &=-\int d^2x\,\sqrt{-g}\,\epsilon(x)D_\mu J_5^\mu(x) \,,\end{aligned} \tag{2.10}$$

where $J_5^\mu=\bar\psi\gamma_5\gamma^\mu\psi$. However, the generating functional should be invariant under any field redefinition and hence

$$\begin{aligned}\delta Z&=\int DA_\mu D\bar\psi D\psi\, i\delta S e^{iS}\\ &=-i\int DA_\mu D\bar\psi D\psi\left[\int d^2x\,\sqrt{-g}\,\epsilon(x)D_\mu J_5^\mu(x)\right]e^{iS}\\ &=0 \,.\end{aligned} \tag{2.11}$$

This leads to the naive conservation of the axial-vector current given by

$$D_\mu J_5^\mu(x)=0 \,. \tag{2.12}$$

However, as is well understood by now, under this field redefinition, the fermionic measure changes nontrivially and this leads to the anomalous behavior of the axial-vector current conservation.

To calculate the change in the fermionic measure let us Wick rotate to a space with Euclidean signature. We do this by letting $x^0\to-ix^4$, $\gamma^0\to i\gamma^4$, and $D_0\to iD_4$. Furthermore, let us choose the eigenstates of the Dirac operator in this space as

$$\not{D}\phi_n=\gamma_\mu D_\mu\phi_n(x)=\gamma_\mu(\nabla_\mu-ieA_\mu)\phi_n(x)=\lambda_n\phi_n(x) \,. \tag{2.13}$$

We assume that the eigenstates satisfy the orthonormality and completeness relations given by

$$\begin{aligned}&\int d^2x\,g^{1/2}\phi_n^\dagger(x)\phi_m(x)=\delta_{nm} \,,\\ &\sum_n\phi_n(x)\phi_n^\dagger(y)=g^{-1/2}\delta^2(x-y) \,.\end{aligned} \tag{2.14}$$

We can, therefore, expand the fermionic variables in these basis states so that

$$\psi(x)=\sum_n a_n\phi_n(x),\quad \bar\psi(x)=\sum_n\phi_n^\dagger(x)b_n \tag{2.15}$$

and

$$D\psi=\prod_n da_n,\quad D\bar\psi=\prod_n db_n \,, \tag{2.16}$$

where a_n and b_n are elements of the Grassmann algebra. We can also expand the chirally rotated variables in this basis so that

$$\psi'(x)=\sum_n a_n'\phi_n(x)=\sum_{n,m}C_{nm}a_m\phi_n(x) \,, \tag{2.17}$$

where

$$C_{nm}=\delta_{nm}-i\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_m(x) \tag{2.18}$$

and

$$D\psi'=\prod_n da_n'=(\det C_{nm})^{-1}\prod_n da_n=(\det C_{nm})^{-1}D\psi \,. \tag{2.19}$$

To calculate the Jacobian of the transformation, note that

$$\begin{aligned}\det C_{nm}&=\exp(\mathrm{Tr}\ln C_{nm})\\ &=\exp\left[-i\sum_n\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x)\right] \,.\end{aligned} \tag{2.20}$$

The exponent in the above expression is divergent and can be regulated as

$$\begin{aligned}i\sum_n\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x)&=\lim_{M^2\to\infty} i\sum_n\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x)e^{-\lambda_n{}^2/M^2}\\ &=\lim_{M^2\to\infty} i\sum_n\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5 e^{-\not{D}^2/M^2}\phi_n(x)\\ &=\lim_{\substack{M^2\to\infty\\ x'\to x}} i\int d^2x\,g^{1/2}\epsilon(x)\mathrm{Tr}\gamma_5 e^{-\not{D}^2/M^2}g^{-1/2}\delta^2(x-x')\\ &=\lim_{\substack{M^2\to\infty\\ x'\to x}} i\int d^2x\,\epsilon(x)\mathrm{Tr}\gamma_5 e^{-\not{D}^2/M^2}\delta^2(x-x') \,.\end{aligned} \tag{2.21}$$

Normally in flat space-time one would go to the Fourier representation or the plane-wave representation and evaluate the expression. In curved space, however, a global definition of the momentum representation is not available. On the other hand, we are only interested in the short-distance behavior of the expression and hence can introduce the Riemann normal coordinates[6]

$$y^\mu=(x-x')^\mu .$$

In terms of these coordinates one can introduce a local momentum representation and the exponent becomes

$$i\sum_n \int d^2x\, g^{1/2}\epsilon(x)\phi_n(x)\gamma_5\phi_n(x) = \lim_{\substack{M^2\to\infty\\ y\to 0}} i\int d^2x\, g^{1/2}\frac{d^2k}{(2\pi)^2}\epsilon(x)\mathrm{Tr}\gamma_5 e^{-\not{D}^2/M^2}e^{-ik\cdot y} . \tag{2.22}$$

The trace can be evaluated using the identity

$$\not{D}^2=-D_\mu D_\mu-\tfrac{1}{12}R-\frac{e}{2}g^{-1/2}\epsilon_{\mu\nu}\gamma_5 F_{\mu\nu} \tag{2.23}$$

and the value of the exponent then becomes

$$\begin{aligned} i\sum_n \int d^2x\, g^{1/2}\epsilon(x)\phi_n(x)\gamma_5\phi_n(x) &= i\frac{e}{2}\frac{1}{2\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu} \\ &= \frac{ie}{4\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu} . \end{aligned} \tag{2.24}$$

Thus the change in the measure for ψ can be written as

$$\begin{aligned} D\psi&=\det C_{nm}D\psi' \\ &=\exp\left[-\frac{ie}{4\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu}(x)\right]D\psi' . \end{aligned} \tag{2.25}$$

One can show in a straightforward manner that the measure for $\bar\psi$ also changes by an equal amount, i.e.,

$$D\bar\psi=\exp\left[-\frac{ie}{4\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu}(x)\right]D\bar\psi' \tag{2.26}$$

so that the total change in the measure under an infinitesimal chiral transformation reads as

$$D\bar\psi D\psi=D\bar\psi' D\psi'\exp\left[-\frac{ie}{2\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu}(x)\right] \tag{2.27}$$

This implies that rotating back to the Minkowski signatures, the anomaly equation is given by

$$D_\mu J_5^\mu=-(-g)^{-1/2}\frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}$$

or (2.28)

$$\partial_\mu(\sqrt{-g}J_5^\mu)=-\frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu} .$$

This is precisely the covariantized form of the flat—space-time anomaly equation.

III. ζ-FUNCTION REGULARIZATION METHOD

As we have argued in the previous section, use of the momentum representation is not free from criticism in curved space. The standard form of calculation in such cases involves the ζ-function regularization method[7] which we will now describe. The calculation is done in three parts. First, we will rederive the anomaly equation under an infinitesimal chiral transformation justifying the momentum-space calculation. Then we will make a finite chiral transformation so that the fermions decouple from the electromagnetic field. And finally we evaluate the generating functional for the fermion field interacting with the background gravitational field using the ζ-function regularization. Of course, we could have evaluated the generating functional for the fermion field explicitly, even when it is interacting with the photon field. However, we choose to split up the calculation in this particular fashion so that analogy with the corresponding flat-space calculation[8] is maintained at every stage.

We have seen from Eq. (2.19) that the change in the fermionic measure under an infinitesimal chiral transformation involves the quantity

$$\det C_{nm}=\exp\left[-i\sum_n\int d^2x\, g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x)\right] . \tag{3.1}$$

From the form of the eigenvalue equation

$$\not{D}\phi_n=\lambda_n\phi_n$$

it follows that

$$\not{D}^2\phi_n=\lambda_n{}^2\phi_n \tag{3.2}$$

and we define a generalized ζ function associated with the operator $\not{D}^2$ as

$$\zeta(s,x)=\sum_n\frac{\phi_n(x)\phi_n^\dagger(x)}{\lambda_n{}^{2s}} . \tag{3.3}$$

It is clear that in terms of this function, the exponent can be written as

$$i\sum_n\int d^2x\, g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x) = i\int d^2x\, g^{1/2}\epsilon(x)\mathrm{Tr}\gamma_5\zeta(0,x) . \tag{3.4}$$

To evaluate this expression, let us study the heat equation associated with the operator $\not{D}^2$. That is, we want to study the equation

$$\frac{d}{d\tau}K(x,y,\tau)+\not{D}^2K(x,y,\tau)=0$$

with the condition

$$K(x,y,0)=g^{-1/2}(x)\delta^2(x-y) . \tag{3.5}$$

Note here that the operator $\not{D}^2$ acts with respect to the first coordinate and τ is the proper time parameter of Schwinger and DeWitt.[9] The function $K(x,y,\tau)$ is easy to find and has the form

$$K(x,y,\tau)=\sum_n e^{-\lambda_n{}^2\tau}\phi_n(x)\phi_n^\dagger(y) . \tag{3.6}$$

Furthermore, it is now straightforward to express the generalized ζ function in terms of the heat kernel, i.e.,

$$\zeta(s,x)=\frac{1}{\Gamma(s)}\int_0^\infty d\tau\,\tau^{s-1}K(x,x,\tau)\ . \tag{3.7}$$

As is clear from the form of the heat kernel, this integral is damped for large τ, and the only significant contribution comes from small values of the parameter τ. The asymptotic expansion for $K(x,x,\tau)$ when $\tau\to 0^+$ is known,[9] and in two dimensions has the form

$$K(x,x,\tau)\underset{\tau\to 0^+}{\sim}\frac{1}{4\pi\tau}[a_0(x)+a_1(x)+a_2(x)\tau^2+O(\tau^3)]\ . \tag{3.8}$$

Here the coefficients $a_n(x)$ are scalar polynomials in various fields and the first few of them have been calculated using the method of coincidence limits.[9]

Putting the form of the heat kernel [Eq. (3.8)] into the definition of the ζ function we obtain

$$\begin{aligned}\zeta(0,x)&=\lim_{s\to 0}\frac{1}{\Gamma(s)}\int_0^\infty d\tau\,\tau^{s-1}\frac{1}{4\pi\tau}[a_0(x)+a_1(x)\tau\\&\qquad+a_2(x)\tau^2+O(\tau^3)]\\&=\frac{a_1(x)}{4\pi}\ .\end{aligned} \tag{3.9}$$

The coefficient function $a_1(x)$ for the operator $\not{D}^2$ has already been calculated[9] to be

$$a_1(x)=-\tfrac{1}{12}R+\frac{e}{2g^{1/2}}\gamma_5\epsilon_{\mu\nu}F_{\mu\nu}\ . \tag{3.10}$$

Using this form, one can write the exponent now as

$$\begin{aligned}i\sum_n\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x)&=i\int d^2x\,g^{1/2}\epsilon(x)\mathrm{Tr}\gamma_5\zeta(0,x)\\&=i\int d^2x\,g^{1/2}\epsilon(x)\mathrm{Tr}\gamma_5\frac{a_1(x)}{4\pi}\\&=i\int d^2x\,g^{1/2}\epsilon(x)\frac{1}{4\pi}\left[\frac{e}{g^{1/2}}\right]2\epsilon_{\mu\nu}F_{\mu\nu}\\&=\frac{ie}{4\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu}(x)\ .\end{aligned} \tag{3.11}$$

Thus we immediately see that

$$\begin{aligned}D\psi&=\det C_{nm}D\psi'\\&=\exp\left[-i\sum_n\int d^2x\,g^{1/2}\epsilon(x)\phi_n^\dagger(x)\gamma_5\phi_n(x)\right]D\psi'\\&=\exp\left[-\frac{ie}{4\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu}(x)\right]D\psi'\ .\end{aligned} \tag{3.12}$$

Similarly, the change in the measure for $\bar\psi$ also contributes an equal amount so that we can write

$$D\bar\psi D\psi=\exp\left[-\frac{ie}{2\pi}\int d^2x\,\epsilon(x)\epsilon_{\mu\nu}F_{\mu\nu}(x)\right]D\bar\psi' D\psi'\ . \tag{3.13}$$

This is, of course, the result obtained by the method of momentum representation and as we have seen before leads to the anomaly equation

$$\partial_\mu(\sqrt{-g}\,J_5^\mu)=-\frac{e}{2\pi}\epsilon^{\mu\nu}F_{\mu\nu}\ . \tag{3.14}$$

Let us next make a finite chiral transformation so that the fermion fields decouple from the photon field. To see that we can do this, let us follow as closely as we can to the flat–space-time case.[8] Note that the action for the Schwinger model is invariant under a local gauge transformation given by

$$\begin{aligned}\psi(x)&\to e^{+i\beta(x)}\psi(x)\ ,\\\bar\psi(x)&\to e^{-i\beta(x)}\bar\psi(x)\ ,\\A_\mu(x)&\to A_\mu(x)+\frac{1}{e}\partial_\mu\beta(x)\ .\end{aligned} \tag{3.15}$$

Therefore, to simplify our calculation we choose a covariantized Landau gauge, namely,

$$\nabla^\mu A_\mu(x)=0\ . \tag{3.16}$$

In two dimensions, the vector field can be decomposed into the longitudinal and transverse components as

$$A_\mu(x)=\nabla_\mu\rho(x)+\sqrt{-g}\,\epsilon_{\mu\nu}\nabla^\nu\sigma(x)\ . \tag{3.17}$$

With the gauge condition, Eq. (3.16), it is clear that the gauge field takes the form

$$A_\mu(x)=\sqrt{-g}\,\epsilon_{\mu\nu}\nabla^\nu\sigma(x)$$

so that

$$\sigma(x)=(\nabla^\mu\nabla_\mu)^{-1}\frac{1}{\sqrt{-g}}\epsilon^{\lambda\rho}F_{\lambda\rho}\ . \tag{3.18}$$

Furthermore, we can use the identity involving the two-dimensional γ matrices given by

$$\sqrt{-g}\,\epsilon_{\mu\nu}\gamma^\nu=\gamma_5\gamma_\mu$$

to write the Lagrangian density for the Schwinger model [see Eq. (2.1)] as

$$L=-\tfrac{1}{4}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}+i\bar{\psi}\gamma^{\mu}(\partial_{\mu}+\tfrac{1}{2}\omega_{\mu}^{ab}\sigma_{ab}-ie\gamma_5\nabla_{\mu}\sigma)\psi\ . \tag{3.19}$$

It is clear, therefore, that if we made a finite chiral redefinition of the fermions, namely,

$$\psi(x)=e^{ie\gamma_5\sigma(x)}\chi\ ,$$
$$\bar{\psi}(x)=\bar{\chi}(x)e^{ie\gamma_5\sigma(x)}\ ,$$

then the fermion fields and the photon field would decouple. Namely, the Lagrangian under this redefinition would become

$$L=-\tfrac{1}{4}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}+i\bar{\chi}\gamma^{\mu}(\partial_{\mu}+\tfrac{1}{2}\omega_{\mu}^{ab}\sigma_{ab})\chi\ . \tag{3.20}$$

Following Refs. 3 and 8, one can calculate the change in the measure under this finite chiral transformation from Eq. (3.13), and the result is (in Minkowski signatures)

$$D\bar{\psi}\,D\psi=\exp\left[\frac{ie^2}{2\pi}\int d^2x\,\sqrt{-g}\,A_{\mu}(x)A^{\mu}(x)\right]D\bar{\chi}\,D\chi\ . \tag{3.21}$$

This shows that the effective Lagrangian density for the Schwinger model when the fermions and the photon field decouple is given by

$$L=-\tfrac{1}{4}g^{\mu\lambda}g^{\nu\rho}F_{\mu\nu}F_{\lambda\rho}+\frac{e^2}{2\pi}g^{\mu\nu}A_{\mu}A_{\nu}+i\bar{\chi}\gamma^{\mu}\nabla_{\mu}\chi\ . \tag{3.22}$$

This shows that just as in the case of flat space-time, the fermions decouple from the photon field. The effective Lagrangian density shows that the photons have become massive with a mass $e/\sqrt{\pi}$ which is precisely the value obtained in flat-space calculations.

IV. CONCLUSION

We have tried to obtain the solution of the Schwinger model in curved space-time using the method of path integrals in the manner of Fujikawa. We have derived the anomaly equation for the axial-vector current using the momentum-space representation. We have rederived the anomaly equation using the ζ-function regularization. We have chirally rotated the fermion fields so that they decouple from the photon field. The photons in the theory do become massive with a mass given precisely by the flat—space-time value. The criterion for confinement is not clear in the presence of the background gravitational field. However, if being able to decouple the fermions is a signal for confinement, it happens in curved space-time also. We will conclude by saying that one would qualitatively expect all features of the flat—space-time solution to persist in a curved background since the background has no dynamics. We see it explicitly in our solution and we believe other soluble models also would display this.

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