Stochastic dynamical reduction theories and superluminal communication

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In theories which describe the reduction of the state vector as a physical process, the possibility exists, for certain experiments, of predictions which differ from those of quantum theory. These are "interrupted reduction interference" experiments, characterized by an interaction which triggers the reduction, followed rapidly (before the reduction is completed) by a measurement of interference between the superposed states that make up the state vector (possible examples: double Stern-Gerlach experiment, two-slit neutron interference). We consider the general class of stochastic reduction theories, and ask whether they allow superluminal communication by means of such experiments. We show that, if the state vector that precedes reduction is precisely reproducible, then superluminal communication can occur in certain circumstances, unless the off-diagonal elements of the density matrix decay exponentially, with a universal time constant. We also show, in that case, that no state vector ever reduces in a finite time, so such a theory is not satisfactory. However, superluminal communication can be avoided if reduction is triggered only in irreproducible state vectors, of such complexity that prior to reduction the "effective" density matrix, constructed from the ensemble of such state vectors and traced over the variables outside the experimenter's control, is diagonal. Then predictions are identical to those of quantum theory for "interrupted reduction interference" experiments and thus apparently for all experiments. The lesson of this paper is that the "effective" density matrix must always be used to make physical predictions in dynamical reduction theories. This supplies a resolution of the problem of reconciling state-vector reduction with relativity: even if the reduction dynamics is not relativistically invariant, its experimental predictions are. It also implies that the "effective" entropy increases during a measurement, but remains constant during reduction, which is the reverse of a common dictum.

I. INTRODUCTION: EXAMPLE OF A STOCHASTIC DYNAMICAL REDUCTION THEORY

When a measurement is described by quantum theory, the final result is a state vector

$$|\psi,t\rangle=\sum_n a_n(t)\,|\phi_n,t\rangle\ , \tag{1.1}$$

where the squared amplitudes $|a_n(t)|^2$ (corresponding to the different experimental outcomes described by $|\phi_n,t\rangle$) do not thereafter change with time.

The reduction of the state vector is the replacement of (1.1) by

$$|\psi,t\rangle=1\times e^{i\theta_m(t)}|\phi_m,t\rangle \tag{1.2}$$

if the measurement actually results in the mth outcome. Equation (1.2) then serves as the starting state vector for the description of a subsequent experiment.

Is the reduction of the state vector a necessary part of the formalism of quantum theory? So far, no resolution of this question has been given by experiment. Partly because of this, it is possible to apply a number of different interpretations to quantum theory, which may or may not require state-vector reduction. For example, if the state vector is interpreted as describing an ensemble of systems, state-vector reduction is not required (although it may be employed as a calculational convenience). Indeed, the great appeal of this interpretation is the avoidance of any time evolution other than the linear evolution of Schrödinger. But this interpretation renounces the possibility of physics describing, through quantum theory, what we know exists in nature, an individual physical system.

In this paper we wish to pursue consequences of another interpretation, that the state vector is in one-to-one correspondence with the physical state of an individual system in nature. Then, in order to maintain consistency of the interpretation with the formalism, the reduction of the state vector is necessary.

A dynamical reduction theory describes how the state vector evolves from (1.1) to (1.2), i.e., how all the amplitudes $a_n(t)$ vanish except one whose magnitude reaches 1. An instantaneous reduction [e.g., Eq. (1.1) holds for $t<0$, Eq. (1.2) holds for $t>0$] is a possible dynamics, but it is not a natural one, as physical systems generally do not undergo such discontinuous evolution (even a phase transition takes time). One is led to look for equations which describe the continuous evolution of (1.1) into (1.2).

Because dynamical reduction theories are still in a rudimentary stage, there is unfortunately still some abruptness involved, namely, the discontinuous replacement of the Schrödinger evolution by the reduction evolution. Presumably a single equation of motion ought to be all that is necessary. It should be the Schrödinger equation to a high degree of accuracy until the states $|\phi_n,t\rangle$ become "macroscopically distinct" (whatever that means—this important point will not be discussed here) and there-

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after be the dynamical reduction equation until the reduction is completed.

There is a hint that this may be possible. Remarkably, the Schrödinger equation itself, *properly written*, serves as an excellent example of a dynamical reduction theory.[1] Starting with

$$i da_n(t)/dt = \sum_m H_{nm}(t) a_m(t) \tag{1.3}$$

we introduce the squared amplitudes x_n and phases θ_n

$$a_n(t) \equiv \sqrt{x_n(t)}\, e^{i\theta_n(t)} \tag{1.4}$$

as well as the Hermitian matrix $B_{nm}(t)$ by $H_{nm}(t) \equiv dB_{nm}(t)/dt$, and rewrite Eq. (1.3) as two real equations:

$$dx_n = i^{-1} \sum_m (x_n x_m)^{1/2} (dB_{nm} e^{i(\theta_m - \theta_n)} - dB_{mn} e^{-i(\theta_m - \theta_n)}) , \tag{1.5a}$$

$$d\theta_n = -2^{-1} \sum_m (x_n{}^{-1} x_m)^{1/2} (dB_{nm} e^{i(\theta_m - \theta_n)} + dB_{mn} e^{-i(\theta_m - \theta_n)}) . \tag{1.5b}$$

Of course, if dB_{nm} is a continuous function of time, in the limit as $dt \to 0$ we recover the Schrödinger equation (1.3) that we started with. But if dB_{nm} changes abruptly and randomly over the time interval dt, then Eqs. (1.5) describe reduction dynamics in the limit $dt \to 0$, as we shall see. More precisely, we model this behavior by interpreting Eqs. (1.5) as Itô stochastic differential equations,[2] where $B_{nm}(t)$ is a Hermitian matrix of independent Brownian motion functions:

$$\langle dB_{nm}(t) \rangle = 0 , \quad \langle dB_{nm}(t) dB_{rs}(t) \rangle = \delta_{ns} \delta_{mr} \sigma_{nm}{}^2 dt . \tag{1.6}$$

Then it can be shown that the ensemble of solutions of Eqs. (1.5) obeys the diffusion equation[3]

$$\frac{\partial G}{\partial t} = \frac{1}{2} \sum_{nm} \sigma_{nm}{}^2 \left[\frac{\partial}{\partial x_n} - \frac{\partial}{\partial x_m} \right]^2 x_n x_m G + \frac{1}{2} \sum_{nm} \sigma_{nm}{}^2 \left[\frac{\partial^2}{\partial \theta_n{}^2} \frac{x_m}{x_n} G + \frac{\partial^2}{\partial \theta_n \partial \theta_m} G \right] , \tag{1.7}$$

where $G(\mathbf{x},\boldsymbol{\theta};\mathbf{x}_0,\boldsymbol{\theta}_0;t)$ is the probability density of solutions with initial values $\mathbf{x}(0) \equiv \mathbf{x}_0, \boldsymbol{\theta}(0) \equiv \boldsymbol{\theta}_0$.

Equations (1.5) or (1.7) describe a stochastic dynamical reduction theory. The occurrence of a particular experimental outcome, about which ordinary quantum theory has absolutely nothing to say, is here determined by the particular fluctuations of certain coefficients. A more complete theory (so far not known) ought to explain those fluctuations.

Parenthetically, it should be made clear that it is only as Itô equations in x_n, θ_n that the Schrödinger equation and the stochastic reduction equation assume the same form (1.5). As Itô equations for the amplitudes a_n, Eq. (1.5) is equivalent to

$$i da_n = \sum_m dB_{nm} a_m - \frac{1}{2a_n^*} \sum_m \sigma_{nm}{}^2 |a_m|^2 dt \tag{1.5c}$$

while as Stratonovich equations for the amplitudes, Eq. (1.5) is equivalent to

$$i \frac{da_n}{dt} = \sum_m \dot{B}_{nm} a_m + \frac{1}{2a_n^*} \sum_m \sigma_{nm}{}^2 (|a_n|^2 - |a_m|^2) . \tag{1.5d}$$

These are all nonlinear equations as they must be to produce the nonunitary reduction evolution.

The analysis of this paper will be based upon the reduction evolution of the ensemble of state vectors described by diffusion equations which appropriately generalize (1.7). Most of this analysis is in the Appendixes, the main body of the paper being devoted to quoting, illustrating, and discussing the results.

In the remainder of this section we will illustrate properties of stochastic dynamical reduction theories with the specific example (1.7). To see that its solution satisfactorily describes the reduction process, we first note [by multiplying (1.7) by $x_n x_m$ and integrating—by parts when necessary—over all $\mathbf{x},\theta$] that

$$d\langle x_n x_m \rangle / dt = -\sigma_{nm}{}^2 \langle x_n x_m \rangle, \quad n \neq m , \tag{1.8}$$

where

$$\langle f(\mathbf{x},\theta) \rangle \equiv \int d\mathbf{x}\, d(\boldsymbol{\theta}/2\pi) G(\mathbf{x},\boldsymbol{\theta};t) f(\mathbf{x},\boldsymbol{\theta}) . \tag{1.9}$$

According to Eq. (1.8), $\langle x_n x_m \rangle$ vanishes at $t = \infty$, and that is only possible if, for each state vector, all x_n's vanish except perhaps one. That nonzero x_n must achieve the value 1 because it follows from Eq. (1.7) that $G \sim \delta(1 - \sum_n x_n)$ at all times if it is true initially. Thus each state vector reduces.

Moreover, each state vector reduces in a finite time. This is not clear from the exponential decay of $\langle x_n x_m \rangle$, which could be due to x_n's decaying exponentially. This would mean that each state vector is in a superposition (all x_n's nonzero) at all finite times. Such behavior is unacceptable in a dynamical reduction theory, whose goal is to produce a unique macroscopic state vector in a finite time. However, the exponential decay of $\langle x_n x_m \rangle$ could also be due to the complete reduction of all but an exponentially decreasing number of state vectors. That is what occurs in this case, for it can be shown that the mean time for complete reduction to occur is finite[4] (also see Appendix C).

The diffusion equation for the x_n's in Eq. (1.7) arises in some other circumstances. Equation (1.7) (with $\sigma_{nm}{}^2 = \sigma^2$ and no θ dependence) has been known for some time to population geneticists as the diffusion approximation to the "Wright-Fisher model" of genetic drift.[5] Geneticists consider the evolution of a large finite population of individuals possessing a few alleles (variant genes such as eye colors), where x_n is the frequency of the nth allele in the population. The evolutionary model is that the next generation of alleles is determined by randomly sampling the previous generation with replacement (the model is called "neutral" because it employs no mutation or natural selection). Of interest are the statistics of when a particular allele becomes "fixed," that is, when the whole population possesses that allele—when it is what we call "reduced."

There is another model that produces the same diffusion equation, to which we prefer to compare the reduction process.[6] This is a "gambler's ruin game," in which x_n represents the nth gambler's fraction of the total money possessed by all gamblers in the game. The $(n-m)$th pair of gamblers toss a fair coin every Δt sec, exchanging $\Delta x_n = -\Delta x_m \sim \pm(x_n x_m \Delta t)^{1/2}$ of the money, depending on the outcome of the toss. Eventually one of the gamblers wins all the money. Thus we may think of the various possible outcomes of an experiment as competing in "the reduction game" until one of them wins.

Of course not only must the state vector reduce, but the tested predictions of quantum theory must be achieved in a satisfactory theory. If a measurement is completed at $t=0$, quantum theory predicts that $x_n(0)$ is the probability of the nth experimental outcome. If the reduction starts at $t=0$, the nth outcome must "win" a fraction $x_n(0)$ of the "games." This is ensured because [from (1.5a) or (1.7)] the evolution is a "martingale" or "fair game,"

$$d\langle x_n \rangle /dt = 0 , \tag{1.10}$$

and it follows from (1.10) that $x_n(0) \equiv \langle x_n(0) \rangle = \langle x_n(\infty) \rangle = 1 \times$(probability that the nth outcome "wins").

This result, in the context of the two models cited above, is well known (the fixation probability of an allele equals its initial frequency, the probability a gambler wins equals his initial fraction of the game's wealth).

Although the "martingale" property (1.10) ensures that $x_n(0) = \langle x_n(\infty) \rangle$, which is all that is needed for the above argument, it has a further important consequence, ensuring agreement with the predictions of quantum theory for the overwhelming majority of experiments[7] (see Sec. III for a more precise statement). Because of this valuable behavior, we have suggested[8] that any dynamical reduction theory should satisfy Eq. (1.10), calling it the "constant mean" hypothesis. It is not a property of the first dynamical reduction theory constructed by Bohm and Bub,[9] which achieved $x_n(0) = \langle x_n(\infty) \rangle$ by causal (nonstochastic) evolution.

Another important property possessed by the diffusion equation (1.7) is that the solutions are independent of the choice of arbitrary phase factors incorporated in the basis states $|\phi_n, 0\rangle$, as in usual quantum theory. If $|\phi_n, 0\rangle$ is replaced by $e^{-i\Delta_n}|\phi_n, 0\rangle$ in the initial state vector (1.1), the amplitude $a_n(0)$'s initial phase angle $\theta_n(0)$ is replaced by $\theta_n(0)+\Delta_n$. Invariance of physical results under such phase translations will certainly be achieved if G is a function of $\theta-\theta_0$. This is the case if the diffusion and drift coefficients are independent of θ, as in Eq. (1.7). Actually, all that is required is a weaker condition on the density matrix, which is discussed in Appendix A. We note that the evolution of an *individual* state vector may depend upon the choice of initial phase angles $\theta_n(0)$. A dynamical reduction theory may give significance to phase angles belonging to an individual state vector that is absent in ordinary quantum theory, as we have commented elsewhere.[8] However, the behavior of the *ensemble* of state vectors, which is what the experimenter sees, should be phase translation invariant.

Other useful properties of the dynamical equations (1.5) are that the physical results are invariant under state vector renormalization (uniform rescaling of the x_n's) and that separated uncorrelated systems can reduce independently.[8] These are not properties of all such theories. Indeed, there is a family of stochastic dynamical reduction theories[3] which obey the (Stratonovich) stochastic differential equations

$$i\, da_n/dt = \sum_m \dot{B}_{nm} a_m^{*r} a_n{}^r / a_n^* . \tag{1.11}$$

It is only for $r=1$ that these properties hold [in this case the Itô dynamical equations are identical to Eqs. (1.5) except for a minus sign on the right-hand side of (1.5a); Eq. (1.7) is the diffusion equation also].

II. GISIN'S STOCHASTIC DYNAMICAL REDUCTION THEORY

Recently, Gisin has published another stochastic dynamical reduction model.[10] The Itô dynamical equations are

$$dx_n = x_n \sum_m \left[\frac{x_m}{s} - \delta_{nm} \right] dB_m , \tag{2.1a}$$

$$d\theta_n = 0 , \tag{2.1b}$$

where $s \equiv \sum_n x_n$ and where the $B_m(t)$ are real independent Brownian motion functions ($\langle dB_n dB_m \rangle = \delta_{nm}\sigma^2 dt$: actually Gisin chose $\sigma^2 = 1$, but we introduce a time scale for later purposes). The associated diffusion equation is readily found to be

$$\frac{\partial G}{\partial t} = \frac{\sigma^2}{4} \sum_{nm} \left[\frac{\partial}{\partial x_n} - \frac{\partial}{\partial x_m} \right]^2 x_n x_m \times \left[\frac{x_n}{s} + \frac{x_m}{s} - \sum_k \left[\frac{x_k}{s} \right]^2 \right] G . \tag{2.2}$$

To see that the ensemble of solutions reduces, we calculate from Eq. (2.2) that

$$d\langle \sqrt{x_n x_m} \rangle /dt = -\frac{\sigma^2}{4} \langle \sqrt{x_n x_m} \rangle, \quad n \neq m . \tag{2.3}$$

As in the case of Eq. (1.8), it follows, for each state vector, that all x_n's vanish except perhaps one at $t=\infty$, and the appearance of the derivatives in (2.2) in the combination $(\partial/\partial x_n - \partial/\partial x_m)$ ensures that $G \sim \delta(1-s)$ if that is true initially. Finally, the model obeys the constant mean hypothesis (1.10), so the probabilities predicted by quantum theory should be obtained for completely reduced state vectors.

However, in this model no state vector reduces in a finite time. We pointed out this serious defect in a Comment[11] (see also Appendix C for another discussion of this important point). In his response[12] Gisin acknowledged the defect, but raised an interesting new issue. He first notes that in his model the "density matrix undergoes a closed linear evolution." He then goes on to assert, without proof, "... in models which deny superluminal communication the density matrix evolves in a closed form (i.e., $\dot{\rho}$ depends only upon ρ and not upon a particular mixture of pure states). Models in which, on the contrary, the evolution of ρ depends upon a particular mixture (such as the models referred to by Pearle in his Comment) predict superluminal communication, and thus contradict relativity." He concludes, "The challenge now is thus to combine the existing models, keeping the attractive characteristics of each (finite reduction time, closed evolution of the density matrix, and thus no superluminal communication, etc.) without introducing undesirable features."

In this paper we are going to examine the conditions under which superluminal communication can or cannot occur in stochastic dynamical reduction theories. We first distinguish between two hypotheses about the nature of the state vector at the onset of reduction, the "reproducible" and the "irreproducible." The reproducible hypothesis assumes that the state vector (1.1) immediately preceding reduction can be precisely reproduced by an experimenter. The irreproducible hypothesis is that, immediately prior to the onset of reduction, neither the phases $\theta_n(0)$ nor the states $|\phi_n,0\rangle$ are precisely under the experiment's control in otherwise identically performed experiments, so that the effective density matrix is diagonal. We shall make three major points.

First, in Appendix A we show that, under the reproducible hypothesis, superluminal communication can occur unless the off-diagonal elements of the density matrix exponentially decay with a universal time constant [the diagonal elements of course are constant, by (1.10)]. So, it is not so much that "the density matrix evolves in a closed form" which characterizes nonsuperluminal communication, as a very specific time evolution. Gisin's model has precisely this time evolution. Since, in the basis $|\phi_n\rangle$,

$$\rho_{nm} \equiv \langle a_n a_m^* \rangle = \langle \sqrt{x_n x_m}\, e^{i(\theta_n-\theta_m)} \rangle \quad (2.4a)$$

$$= \langle \sqrt{x_n x_m} \rangle e^{i(\theta_n-\theta_m)} \quad (2.4b)$$

[where (2.4b) obtains because the phase angles remain constant according to (2.1b)], it follows from (2.3) that

$$d\rho_{nm}/dt = -\frac{\sigma^2}{4}\rho_{nm}, \quad n\neq m \ . \quad (2.5)$$

Second, in any stochastic dynamical reduction theory which obeys (2.5), all state vectors take infinite time to reduce (see Appendixes B and C). It turns out that Gisin's model is the essentially unique model obeying Eq. (2.5) in which the angles θ_n do not evolve: we show that no angular evolution can make the infinite reduction time finite. Thus it appears that a satisfactory theory cannot be found following Gisin's concluding suggestion.

Third, to obtain an acceptable stochastic dynamical reduction theory, in which no superluminal communication occurs and in which the reduction time is finite, we turn to the irreproducible hypothesis. We show, in Sec. V, under this hypothesis, that superluminal communication does not occur for the theory in Sec. I (which is one of a broader class of theories), which therefore does not "contradict relativity." On the contrary, in Sec. VI, it is argued that this theory together with the irreproducibility hypothesis provides a relativistically invariant set of predictions even though the reduction dynamics of any particular state vector is not relativistically invariant. This provides an unexpected answer to the question of how to construct a relativistically invariant reduction: it is not necessary.

The next two sections discuss the measurement process in a dynamical reduction theory, and introduce and illustrate the results of the Appendixes.

III. INTERRUPTED REDUCTION EXPERIMENTS

A dynamical reduction theory is designed to produce the same predictions as quantum theory *provided the reduction goes to completion.* Therefore it is to a sequence of measurements separated by less than the characteristic reduction time ("interrupted reduction" experiments) that we must look to seek predictions different from quantum theory's.

In most sequences of measurements, the basis vectors describing the various possible sequences of outcomes are orthogonal. This is because each measurement in the sequence usually involves a macroscopic apparatus that is correlated to each outcome, and the macroscopic apparatus states are orthogonal. It turns out that in these most common circumstances, the constant mean hypothesis (1.10) ensures agreement with quantum theory's predictions.[7] We review the argument here.

Consider the state vector (1.1) describing a just completed measurement at time $t=0$, which commences reducing. At some time T the density matrix corresponding to the ensemble of such state vectors is

$$\rho(T) = \sum_{nm} \langle a_n a_m^* \rangle_T \, |\phi_n,T\rangle\langle\phi_m,T| \quad (3.1a)$$

$$= \sum_n |a_n(0)|^2 \, |\phi_n,T\rangle\langle\phi_n,T| + \sum_{n\neq m} \langle a_n a_m^* \rangle_T \, |\phi_n,T\rangle\langle\phi_m,T| \quad (3.1b)$$

using the constant mean hypothesis (1.10). If T is so large that reduction is completed, then $\langle a_n a_m^* \rangle_T \to 0$ $(n\neq m)$, and the diagonal density matrix describes a mixture identical to that obtained by procedures generally used in applying quantum theory. However, suppose that T is less than the mean reduction time, and another measurement is performed at T. The state vector $|\phi_n,T\rangle$ evolves into

$$|\phi_n,T^+\rangle = \sum_m b_{nm} \, |\phi_{nm},T\rangle \ , \quad (3.2)$$

where $|\phi_{nm},T\rangle$ refers to the nth and mth outcomes of the first and second measurements, respectively. We are supposing here that the measurement takes place over such a short duration compared to the reduction time that it can be regarded as an instantaneous measurement. [The state vectors $|\phi_n,t\rangle$, $|\phi_{nm},t\rangle$, etc., evolve according to the usual Schrödinger dynamics, whereas the amplitudes $a_n(t)$, etc., evolve according to the reduction dynamics, analogous to what takes place in the interaction picture.]

Thereafter each state vector in the ensemble continues reducing, with

$$|\psi,t\rangle = \sum_{nm} a_{nm}(t)\,|\phi_{nm},t\rangle, \quad t \geq T \,, \tag{3.3a}$$

$$|a_{nm}(T)|^2 = |a_n(T)|^2\,|b_{nm}|^2 \,, \tag{3.3b}$$

and density matrix for the ensemble given by

$$\rho(t) = \sum_{nmrs} \langle a_{nm}a_{rs}^* \rangle_t\,|\phi_{nm},t\rangle\langle\phi_{rs},t| \;. \tag{3.4}$$

We could consider subsequent additional measurements, but this example is typical. When the reduction goes to completion, the off-diagonal terms in (3.4) vanish and the density matrix is

$$\rho(t) = \sum_{nm} \langle |a_{nm}|^2\rangle_t\,|\phi_{nm},t\rangle\langle\phi_{nm},t| \tag{3.5a}$$

$$= \sum_{nm} \langle |a_{nm}|^2\rangle_T\,|\phi_{nm},t\rangle\langle\phi_{nm},t| \tag{3.5b}$$

$$= \sum_{nm} \langle |a_n|^2\rangle_T\,|b_{nm}|^2\,|\phi_{nm},t\rangle\langle\phi_{nm},t| \tag{3.5c}$$

$$= \sum_{nm} |a_n(0)|^2\,|b_{nm}|^2\,|\phi_{nm},t\rangle\langle\phi_{nm},t| \;, \tag{3.5d}$$

where (3.5b) follows from (3.5a) by the constant mean hypothesis (1.10), (3.5c) follows because (3.3b) is true for every amplitude in the ensemble, and (3.5d) follows again because of (1.10). The result is identical to quantum theory's result.

To obtain possible disagreement with quantum theory predictions, we must therefore consider interrupted reduction experiments where a later experiment in the sequence measures interference between states produced by an earlier experiment (an "interrupted reduction interference" experiment). If such experimental tests are to be possible, the state vector prior to reduction must be reproducible.

Moreover, we must make certain hypotheses concerning the cause of the onset of the reduction process. It is not unreasonable to hypothesize that the rate of the reduction "competition" between two states increases as the states become more macroscopically distinct. Perhaps it is an increasing function of the spatial separation between the particles in the states, and/or an increasing function of the mass of the particles. Thus we might assume that a Stern-Gerlach magnet's separation of the packets describing a spinning particle triggers reduction if the packets are separated far enough and if the particle is massive enough. If a second Stern-Gerlach magnet later recombines the packets (a "double-Stern-Gerlach" experiment) whose amplitudes have been altered during the reduction "competition," a subsequent spin-projection measurement should obtain results different from those of quantum theory.

Another possible experimental test of interrupted reduction interference, a two-slit neutron interference experiment, has actually been performed.[13] If we assume that the neutron is massive enough and the slit separation (≈ 0.01 cm) large enough, the two packets separated by the slits may be supposed to compete in the reduction "game." When they reinterfere at the detector, the altered amplitudes of the packets should result in a "washed-out" interference pattern.[7] The nonobservation of such a phenomenon to 1% accuracy in this experiment translated to a lower limit of 8 sec on the mean reduction time.[13]

To conclude, since superluminal communication is in disagreement with the predictions of quantum theory, and since only interrupted reduction interference experiments may produce disagreement with the predictions of quantum theory, it is to such experiments we must turn for the possibility of superluminal communication.

IV. CORRELATED INTERRUPTED REDUCTION INTERFERENCE EXPERIMENTS AND SUPERLUMINAL COMMUNICATION

We consider two experimenters together with their respective apparatuses, so far separated that no communication with the speed of light can pass from one to the other during the duration of their experiments and the subsequent reduction. We suppose that the state vector describing their situation at $t=0$ is a correlated one

$$|\psi,0\rangle = \sum_{rl} a_{rl}(0)\,|r,0\rangle\,|l,0\rangle \;, \tag{4.1}$$

where $|r,t\rangle$ and $|l,t\rangle$ denote the basis states describing, respectively, the equipment of the "right experimenter" and the "left experimenter."

We consider any interrupted reduction interference sequence of measurements by both experimenters. Suppose the density matrix after the reduction has been completed is calculated, and the trace is taken over, say, the left-experimenter basis. The resulting density matrix which describes the right-experimenter experience should be independent of anything the left experimenter has done, if there is to be no superluminal communication. In particular, it should be independent of the times the left experiments have been performed, and which left superpositions have been measured.

To illustrate these considerations, consider the following sequence of measurements. We suppose that both left and right experimenters have by prearranged agreement each completed an experiment at $t=0$ where the macroscopically distinguishable states are $\{|r,t\rangle\},\{|l,t\rangle\}$, respectively, and so the state vector (4.1) begins to reduce. At time τ the left experimenter performs an instantaneous experiment such that $|l,\tau\rangle$ evolves into $|l,\tau^+\rangle = \sum_{\bar{l}} u_{\bar{l}l}^L\,|\bar{l},\tau^+\rangle$ (u^L is some unitary transformation), where $\{|\bar{l},t\rangle\,|r,t\rangle\}$ are the macroscopically distinguishable states that compete in the reduction game thereafter. At time $T>\tau$, the right experimenter does likewise, producing competing states $|\bar{r},t\rangle$. Following this, the reduction is permitted to proceed to completion. (We could consider more general sequences, but the results obtained will not differ.)

In Appendix A we consider this case, and show that the only evolution of the off-diagonal density matrix consistent with nonsuperluminal communication is exponential decay with a universal time constant.

Here we will be content to illustrate how superluminal communication can occur when the off-diagonal density-matrix elements decay exponentially, but with different time constants λ_1 and λ_2 for the time intervals $0 \le t \le \tau$, and $\tau \le t \le T$, respectively. This could be the case if the left experiment at time τ produces states $\{ |\bar{l}\rangle \}$, which are more macroscopically distinctive than the states $\{ |l\rangle \}$, as hypothesized in the previous section.

The density matrix for $0 \le t \le \tau$ in this case is

$$\rho = \sum_{rlr'l'} a_{rl}(0) a^*_{r'l'}(0) |r,t\rangle |l,t\rangle \langle r',t| \langle l',t| [e^{-\lambda_1 t} + \delta_{rr'}\delta_{ll'}(1-e^{-\lambda_1 t})] \tag{4.2}$$

and for $\tau^+ \le t \le T$

$$\rho = \sum_{rlr'l'\bar{l}\bar{l}'} a_{rl}(0) a^*_{r'l'}(0) u^L_{\bar{l}l} u^{L\dagger}_{l'\bar{l}'} |r,t\rangle |\bar{l},t\rangle \langle r',t| \langle \bar{l}',t| [e^{-\lambda_1 \tau} + \delta_{rr'}\delta_{ll'}(1-e^{-\lambda_1 \tau})][e^{-\lambda_2(t-\tau)} + \delta_{rr'}\delta_{\bar{l}\bar{l}'}(1-e^{\lambda_2(t-\tau)})] . \tag{4.3}$$

The density matrix following the right experiment is similarly found by replacing $|r,t\rangle$ in (4.3) by $\sum u^R_{\bar{r}r} |\bar{r},t\rangle$ and treating $\langle r',t|$ similarly, letting t become T in the second set of brackets in (4.3), and multiplying inside the summation by $[e^{-\lambda_3(t-T)} + \delta_{\bar{r}\bar{r}'}\delta_{\bar{l}\bar{l}'}(1-e^{-\lambda_3(t-T)})]$. When $t=\infty$, this last term becomes $\delta_{\bar{r}\bar{r}'}\delta_{\bar{l}\bar{l}'}$, and the sum over $\bar{r}'$ and $\bar{l}'$ can be performed with the resulting diagonal density matrix

$$\rho = \sum_{rlr'l'\bar{r}\bar{l}} a_{rl}(0) a^*_{r'l'}(0) u^L_{\bar{l}l} u^{L\dagger}_{l'\bar{l}} u^R_{\bar{r}r} u^{R\dagger}_{r'\bar{r}} |\bar{r}\rangle |\bar{l}\rangle \langle \bar{r}| \langle \bar{l}| \times [e^{-\lambda_1\tau - \lambda_2(T-\tau)} + \delta_{rr'}(e^{-\lambda_1\tau} - e^{-\lambda_1\tau-\lambda_2(T-\tau)}) + \delta_{rr'}\delta_{ll'}(1-e^{-\lambda_1\tau})] . \tag{4.4}$$

When the left trace is taken over this density matrix we finally obtain

$$\mathrm{Tr}_L \rho = \sum_{rr'l} a_{rl}(0) a^*_{r'l}(0) u^R_{\bar{r}r} u^{R\dagger}_{r'\bar{r}} |\bar{r}\rangle \langle \bar{r}| [e^{-(\lambda_1-\lambda_2)\tau - \lambda_2 T} + \delta_{rr'}(1-e^{-(\lambda_1-\lambda_2)\tau-\lambda_2 T})] . \tag{4.5}$$

We see that Eq. (4.5) depends upon τ, so the right experimenter superluminally receives the message of when the left experimenter performed his experiment, unless $\lambda_1 = \lambda_2$, i.e., unless there is a universal reduction rate.

However, a theory with exponentially decaying off-diagonal density-matrix elements with a universal time constant λ^{-1} is not satisfactory on another score. It is shown in Appendixes B and C that the most general stochastic reduction theory giving rise to such behavior never completely reduces in a finite time. Essentially what happens is that the exponentially decaying behavior requires the diffusion to proceed so slowly in the neighborhood of the boundary $x_n = 0$ that it can never be reached.

V. HOW TO AVOID SUPERLUMINAL COMMUNICATION

Thus we turn to the hypothesis of irreproducible state vectors for a satisfactory means of avoiding superluminal communication yet retaining a finite reduction time. That is, we hypothesize that when an experimenter repeatedly performs an experiment, the state vector corresponding to the physical system at $t=0$, immediately prior to reduction, cannot be precisely reproduced. In particular, while the magnitude corresponding to the nth outcome, $|a_n(0)| = [x_n(0)]^{1/2}$ in Eq. (1.1), is reproducible, the phases $\theta_n(0)$ are not reproducible, and moreover each is uniformly and independently distributed over the interval 0 to 2π. Neither is the state $|\phi_n, 0\rangle$ precisely reproducible, although its macroscopic behavior (e.g., its "pointer position") is.

It is a common hypothesis to suppose that such irreproducibility is the result of the measurement process, whether because of uncontrollable interaction with the environment, as argued by Zeh,[14] Zurek,[15] and others, or because of the complexity of the measurement apparatus.[16] What we are adding here is the assumption that such irreproducibility is either a prerequisite or a corequisite for the onset of the dynamical reduction process.

In this case, an experimenter is confronted with an ensemble of possible state vectors:

$$|\psi,t\rangle_\lambda = \sum_n \sqrt{x_n(t)}\, e^{i\theta_{n,\lambda}(t)} |\phi_n,t\rangle_\lambda \tag{5.1}$$

each with probability p_λ. Therefore the density matrix that must be used by the experimenter to describe his experience is the "effective" density matrix

$$\rho(t) = \sum_{nm} [x_n(t) x_m(t)]^{1/2} \times \sum_\lambda p_\lambda e^{i[\theta_{n,\lambda}(t) - \theta_{m,\lambda}(t)]} \mathop{\mathrm{Tr}}_{\mathrm{ext}} |\phi_n,t\rangle_\lambda {}_\lambda\langle \phi_m,t| . \tag{5.2}$$

In Eq. (5.2) the trace is over extraneous and/or external states which are outside the experimenter's control.

Our hypothesis is that at $t=0$ the sum over λ in (5.2) vanishes unless $n=m$. This could be because the states $|\phi_n,0\rangle_\lambda$ and $|\phi_m,0\rangle_\lambda$, corresponding to different outcomes as they do, engender different states in extraneous or external variables (differences in one environmental particle's state could make the trace vanish). It could be because of the randomness of the phases, even though the states might differ very little in the external or extraneous states. It could be because of a combination of the two effects.

Thus it is only the diagonal elements of the effective density matrix that do not vanish at $t=0$. Moreover, this should be true for $t>0$ as the reduction takes place. The diffusion equation describes the evolution of each state vector regardless of λ and therefore it describes the whole ensemble's diffusion, which proceeds from an initial distribution with each phase uniformly and randomly distributed. Such a random-phase distribution is maintained by the diffusion equation, provided the diffusion coefficients are independent of θ_n as in all models presented to date. This, together with the unitary evolution of the states $|\phi_n,t\rangle_\lambda$, argues that, for $t>0$,

$$\rho(t)=\operatorname*{Tr}_{\text{ext}}\sum_\lambda p_\lambda \sum_n \langle x_n(t)\rangle\,|\phi_n,t\rangle_{\lambda\,\lambda}\langle\phi_n,t| \tag{5.3a}$$

$$=\operatorname*{Tr}_{\text{ext}}\sum_\lambda p_\lambda \sum_n x_n(0)\,|\phi_n,t\rangle_{\lambda\,\lambda}\langle\phi_n,t|\,,\quad t\geq 0\,, \tag{5.3b}$$

where we have used the constant mean hypothesis (1.10) in going from (5.3a) to (5.3b).

We see that no superluminal communication can be achieved in these circumstances. The effective density matrix (5.2) before, during, and after reduction is identical to that obtained by applying quantum theory without any reduction to the same ensemble with the same hypothesis, and this never leads to superluminal communication. Another way to view this is to regard the inner sum in (5.3b) as a density matrix associated with a single state vector (the λth) in which the off-diagonal elements decay with universal time constant but zero initial values. Then there is no superluminal communication, as we have argued in this paper.

We regard the theory described in Sec. I, together with the irreproducible hypothesis, as a satisfactory dynamical reduction theory in so far as there is no superluminal communication, and the reduction time is finite.

VI. CONCLUDING REMARKS

In Sec. III we argued that the only experimental tests that can distinguish between a dynamical reduction theory and quantum theory are interrupted reduction interference experiments. Such tests depend upon state vector reproducibility. In order to avoid superluminal communication we have had to hypothesize state vector irreproducibility, implying that the only appropriate density matrix to use to describe an experiment is the "effective" one, (5.2), which is diagonal during the reduction process. This means that no interrupted reduction interference experiments—and therefore no experiments—can distinguish between the two theories.

This has bad and good aspects. The possibility of experimental tests is a stimulation of physics activity (as we have seen in the past decade of excitement over testing Bell's inequality), and this is lacking. Moreover, all that is left to enable one to choose between identically predictive theories are philosophical or aesthetic grounds. While physicists do make choices on these grounds, unless dynamical reduction theories are developed to the point where they are more strikingly compelling than at present, most physicists will opt for the simpler structure of quantum theory.

On the other hand, the results of this paper unexpectedly give support to a possible resolution of the problem of reconciling relativity with reduction, whose difficulty has been pointed out by a number of authors.[17] (It is interesting that considerations of the requirements for nonsuperluminal communication may shed light on the broader question of relativistic invariance.) An unreduced state vector or density matrix undergoes only unitary evolution, and one knows how to describe that evolution in another Lorentz frame. However, during reduction the evolution is not unitary, and no prescription is known for a Lorentz-invariant transformation. Furthermore, one can argue that if there are state vectors describing spatially extended systems which in one frame undergo unitary evolution followed by reduction, in another frame part of the state vector may evolve unitarily and part reduce, and no consistent dynamics of this kind is known.

The dynamics of the stochastic reduction theory described in Sec. I takes place with respect to a parameter t which is presumably the time in some preferred frame (perhaps the frame comoving with the Universe's expansion, perhaps the center-of-mass frame—it has so far not been found necessary to prescribe it). The interaction between amplitudes playing the reduction "game" is instantaneous and can be nonlocal. Moreover, the dynamics is different when translated to another Lorentz frame. This is obviously a nonrelativistic description.

Nonetheless, we have shown that it gives rise to predictions identical to those of quantum theory in this preferred frame. In a relativistic quantum theory, predicted results in any frame may be Lorentz transformed to another frame. We can adopt this same prescription for the dynamical reduction theory. Thus we have a prescription for predicting experimental results in any Lorentz frame, and those results are identical to the results of the relativistic quantum theory. So the dynamical reduction theory, although not relativistic in its formulation, is relativistic in its predictions.

The results of this paper also suggest a different answer to the problem of determining when it is that entropy increases during a measurement than that given by von Neumann.

According to von Neumann, a measurement is described by an evolving state vector, and the entropy $S=-k\,\mathrm{Tr}\rho\ln\rho$ is zero during the measurement. It is only reduction, which according to von Neumann requires the replacement of the pure density matrix corresponding to the state vector by a mixed density matrix corresponding to the experimental outcomes, that brings about the increase in entropy.

From the point of view adopted here, one is forced to use the effective density matrix (5.2) to describe experiments. During the measurement, when each individual state vector evolves unitarily, the effective density matrix does not evolve unitarily because it is constructed by taking the partial trace of a unitarily evolving density matrix. Using von Neumann's expression, the entropy increases during the measurement. However, during the reduction, when each individual state vector does not evolve unitarily, the effective density matrix (5.3) does, and the entropy remains constant.

One final comment: based upon considerations of Bell's inequality, some people believe that there should be a hidden-variables dynamics behind quantum theory, in which correlated particles communicate superluminally, but an associated statistical behavior masks the superluminal communication so that it cannot be used by experimenters. In this paper we have seen an example of how, in a dynamical reduction theory, the possibility of superluminal communication is masked by the statistical behavior required by the irreproducible hypothesis. Perhaps there is a connection.

APPENDIX A

In this appendix we shall show that the condition of no superluminal communication implies that the off-diagonal elements of the density matrix must necessarily exponentially decay with a universal time constant, in stochastic reduction theories.

The argument is somewhat involved, so we shall number the separate main points.

(1) Such theories give rise to diffusion equations whose most general form [incorporating the constant mean hypothesis (1.10)] is

$$\frac{\partial G}{\partial t}=\frac{1}{2}\sum_{nm}\left[\frac{\partial^2}{\partial x_n\partial x_m}b_{nm}+\frac{\partial^2}{\partial x_n\partial\theta_m}c_{nm}+\frac{\partial^2}{\partial\theta_n\partial\theta_m}h_{nm}\right]G-\sum_n\frac{\partial}{\partial\theta_n}v_nG\ , \qquad (A1)$$

where G is the density function. If the initial state vector $|\psi,0\rangle=\sum_s a_{0s}|s\rangle$ is reproducible, then G is the Green's function with the initial condition

$$G(\mathbf{a},\mathbf{a}^*;\mathbf{a}_0,\mathbf{a}_0^*;t)=\prod_s\delta(x_s-x_{0s})\delta(\theta_s-\theta_{0s})\ , \qquad (A2)$$

where $d\mathbf{a}\,d\mathbf{a}^*=da_1\cdots da_1^*\cdots=dx_1\cdots d\theta_1\cdots$ is the volume element in $\mathbf{x},\boldsymbol{\theta}$ space.

G also satisfies the backward (adjoint) diffusion equation

$$\frac{\partial G}{\partial t}=\frac{1}{2}\sum_{nm}\left[b_{0nm}\frac{\partial^2}{\partial x_{0n}\partial x_{0m}}+c_{0nm}\frac{\partial^2}{\partial x_{0n}\partial\theta_{0m}}+b_{0nm}\frac{\partial^2}{\partial\theta_{0n}\partial\theta_{0m}}\right]G+\sum_n v_{0n}\frac{\partial}{\partial\theta_{0n}}G \qquad (A3)$$

(b_{0nm} means the arguments $\mathbf{x},\boldsymbol{\theta}$ in b_{nm} are replaced by $\mathbf{x}_0,\boldsymbol{\theta}_0$, etc.).

(2) Each element of the density matrix

$$D_{nm}(\mathbf{a}_0,\mathbf{a}_0^*;t)\equiv\langle a_na_m^*\rangle_t=\int d\mathbf{a}\,d\mathbf{a}^*a_na_m^*G(\mathbf{a},\mathbf{a}^*;\mathbf{a}_0,\mathbf{a}_0^*;t) \qquad (A4)$$

also satisfies the backward diffusion equation (A3), as can be seen by taking the derivative of (A4) with respect to t, and using (A3). The initial condition is

$$D_{nm}(\mathbf{a}_0,\mathbf{a}_0^*;0)=\sqrt{x_{0n}x_{0m}}e^{i(\theta_{0n}-\theta_{0m})} \qquad (A5)$$

which follows from (A4) and (A2).

Now, we require invariance of experimental results (in particular, for interrupted reduction interference experiments—for other experiments the reduction of the density matrix to diagonal form ensures that the phase invariance plays no role) under arbitrary fixed phase transformations of the basis vectors $|\phi_n,t\rangle\to e^{-\Delta_n}|\phi_n,t\rangle$, as discussed in Sec. I. This means that the density-matrix elements (A4) must be related to the density-matrix elements describing the evolution with translated phases by

$$D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0+\boldsymbol{\Delta};t)=D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;t)e^{i\Delta_n-i\Delta_m}\ . \qquad (A6)$$

(We have replaced $\mathbf{a}_0,\mathbf{a}_0^*$ by $\mathbf{x}_0,\boldsymbol{\theta}_0$ as D_{nm}'s arguments for clarity.)

$D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0+\boldsymbol{\Delta};t)$ also satisfies the backward diffusion equation (A3), where the argument $\boldsymbol{\theta}_0$ in the diffusion and drift coefficients is replaced by $\boldsymbol{\theta}_0+\boldsymbol{\Delta}$. Substituting (A6) into this equation, we find that $D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;t)$ satisfies it also. Taking the derivative with respect to Δ_k, say, and letting Δ_k go to zero, we find

$$0=\frac{1}{2}\sum_{rs}\left[b'_{0rs}\frac{\partial^2}{\partial x_{0r}\partial x_{0s}}+c'_{0rs}\frac{\partial^2}{\partial x_{0r}\partial\theta_{0s}}+h'_{0rs}\frac{\partial^2}{\partial\theta_{0r}\partial\theta_{0s}}\right]D_{nm}+\sum_s v'_{0s}\frac{\partial D_{nm}}{\partial\theta_{0s}}\ , \qquad (A7)$$

where $b'_{0rs}\equiv\partial b_{rs}/\partial\theta_{0k}$, etc.

If (A7) is evaluated at as many different times as there are coefficients, we get a sufficient number of homogeneous equations to solve for the coefficients. *Provided the matrix composed of derivatives of D is nonsingular,* the solution is $b'=c'=h'=v'=0$, and so the coefficients are independent of the angles. Then an expansion of D_{nm} in a Fourier exponential series in the angles, and substitution into the backward diffusion equation, reveals that each Fourier coefficient—in particular, the coefficient of $e^{i\theta_{0n}-i\theta_{0m}}$—satisfies a closed differential equation. This, combined with the initial condition (A5), yields the explicit angular dependence of D_{nm}:

$$D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;t)=\mu_{nm}(\mathbf{x},t)e^{i(\theta_{0n}-\theta_{0m})}\ . \qquad (A8)$$

However, the matrix composed of derivatives of D may not be nonsingular. Nonetheless, we can achieve the result (A8) in the following way. By taking the derivative of the backward diffusion equation for D_{nm} with respect to θ_{0k}, and using (A7), we find that $\partial D_{nm}/\partial\theta_{0k}$ also satisfies the backward diffusion equation. The initial condition here is

$$\partial D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;0)/\partial\theta_{0k} = i\sqrt{x_{0n}x_{0m}}(\delta_{kn}-\delta_{km})e^{i(\theta_{0n}-\theta_{0m})} \quad \text{(A9a)}$$

$$= iD_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;0)(\delta_{kn}-\delta_{km}) \; . \quad \text{(A9b)}$$

The solution of the diffusion equation with a given initial condition is unique: because of this, the linearity of the equation, and the equality (up to a multiplicative constant) of the initial conditions for D_{nm} and D'_{nm}, Eq. (A9b), we can extend (A9b) to $t>0$:

$$\partial D_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;t)/\partial\theta_{0k} = iD_{nm}(\mathbf{x}_0,\boldsymbol{\theta}_0;t)(\delta_{kn}-\delta_{km}) \; . \quad \text{(A10)}$$

The solution of Eq. (A10) with initial condition (A5) is (A8).

(3) Now, as in Sec. IV, we consider a correlated state vector which at $t=0$ is

$$|\psi,0\rangle = \sum_{rl} a_{0rl}|r,0\rangle|l,0\rangle \; , \quad \text{(A11)}$$

where $|r\rangle$ and $|l\rangle$ describe widely separated experiments "to the right" and "to the left." We suppose that both right and left experimenters, by prearrangement, have just completed experiments at $t=0$ thereby triggering the reduction process for $t>0$. The density-matrix elements are, by (A4),

$$D_{rlr'l'}(\mathbf{a}_0,\mathbf{a}_0^*;t) = \int d\mathbf{a}\,d\mathbf{a}^* a_{rl}a_{r'l'}^* G(\mathbf{a},\mathbf{a}^*;\mathbf{a}_0,\mathbf{a}_0^*;t) \; . \quad \text{(A12)}$$

Because these theories treat all amplitudes equivalently, the diffusion equation and the initial conditions and therefore G are invariant under the exchange of any pair of amplitudes

$$a_{rl}\leftrightarrow a_{sm}, \quad a_{rl}^*\leftrightarrow a_{sm}^*, \quad a_{0rl}\leftrightarrow a_{0sm}, \quad a_{0rl}^*\leftrightarrow a_{0sm}^* \; . \quad \text{(A13)}$$

(If there are numerical coefficients in the diffusion equation depending upon the indices, it is understood that they must be exchanged, too.)

The result (A8) of point (2) of this appendix reads, in this context,

$$D_{rlr'l'}(\mathbf{a}_0,\mathbf{a}_0^*;t) = \mu_{rlr'l'}(\mathbf{x}_0,t)e^{i(\theta_{0rl}-\theta_{0r'l'})} \; . \quad \text{(A14)}$$

The symmetry in G implies symmetry in the μ's: each $\mu_{rlr'l'}$ is invariant under exchange of all x_0's except those with rl or $r'l'$ indices, and $\mu_{rlr'l'}$ $[(r,l)\neq(r',l')]$ can be turned into $\mu_{sms'm'}$ $[(s,m)\neq(s',m')]$ by the exchange (A13) and a similar exchange with primed indices (μ_{rlrl} can similarly be turned into μ_{smsm}).

In point (5) of this appendix we show that nonsuperluminal communication implies that

$$\sum_l D_{rlr'l}(\mathbf{a}_0,\mathbf{a}_0^*;t) = \sum_l D_{rlr'l}(\mathbf{a}_0\mathbf{u},\mathbf{a}_0\mathbf{u}^*;t) \; , \quad \text{(A15a)}$$

where $\mathbf{a}_0\mathbf{u}$ is an abbreviation for $\sum_l a_{0rl}u_{\bar{l}l}^L$ for *arbitrary* unitary transformation u^L. This comes from the condition that an arbitrary left experiment at an arbitrary time be undetected by the right experimenter. Similarly

$$\sum_r D_{rlrl'}(\mathbf{a}_0,\mathbf{a}_0^*;t) = \sum_r D_{rlrl'}(\mathbf{u}\mathbf{a}_0,\mathbf{u}\mathbf{a}_0^*;t) \quad \text{(A15b)}$$

(where $\mathbf{u}\mathbf{a}_0$ represents $\sum_r u_{\bar{r}r}^R a_{0rl}$).

(4) We now have enough information to develop the main argument. Equation (A15a) requires that $\sum_l D_{rlr'l}$ be a function only of the invariants under left-unitary transformations, i.e., only of $\sum_l a_{0sl}a_{0s'l}^*$. But according to Eq. (A14)

$$\sum_l D_{rlr'l}(\mathbf{a}_0,\mathbf{a}_0^*;t) = \sum_l \mu_{rlr'l}(\mathbf{x}_0,t)(x_{0rl}x_{0r'l})^{-1/2}a_{0rl}a_{0r'l}^* \; . \quad \text{(A16)}$$

Taking derivatives of (A16) with respect to its possible arguments, the invariants, we find the μ's must be restricted in form to

$$\mu_{rlr'l}(\mathbf{x}_0;t) = \mu_{rr'}\left[\cdots \sum_m x_{0sm}\cdots;t\right](x_{0rl}x_{0r'l})^{1/2} \; , \quad r\neq r' \; . \quad \text{(A17)}$$

For $r=r'$ there is a much weaker constraint; but, regardless, we know $D_{rlrl}=x_0$ by the constant mean hypothesis (1.10).

The symmetry enjoyed by the μ's means that we can promote the simplification of (A17) to any pairs of unidentical indices, and (A14) becomes

$$D_{rlr'l'} = \mu_{rr'}\left[\cdots \sum_m x_{0sm}\cdots;t\right]a_{0rl}a_{0r'l'}^* \; , \quad (r,l)\neq(r',l') \; . \quad \text{(A18a)}$$

The same argument based upon (A15b) leads to

$$D_{rlr'l'} = \bar{\mu}_{ll'}\left[\cdots \sum_s x_{0sm}\cdots;t\right]a_{0rl}a_{0r'l'}^* \; , \quad (r,l)\neq(r',l') \; . \quad \text{(A18b)}$$

The only way $D_{rlr'l'}$ can depend upon the left sums in (A18a) and the right sums in (A18b) is if the μ's depend only upon the single double sum $\sum_{sm}x_{0sm}=1$ and t, i.e., on t alone:

$$D_{rlr'l'} = D(t)a_{0rl}a_{0r'l'}^*, \quad (r,l)\neq(r',l') \quad \text{(A19)}$$

[with $D(0)=1$ implied by the initial condition on $D_{rlr'l'}$].

Using the Chapman-Kolmogorov equation expressing the Markov nature of the diffusion process

$$G(\mathbf{a},\mathbf{a}^*;\mathbf{a}_0,\mathbf{a}_0^*;t) = \int d\mathbf{b}\,d\mathbf{b}^* G(\mathbf{a},\mathbf{a}^*;\mathbf{b},\mathbf{b}^*;t-\tau) \times G(\mathbf{b},\mathbf{b}^*;\mathbf{a}_0,\mathbf{a}_0^*;\tau) \; , \quad \text{(A20)}$$

where τ is arbitrary, we obtain, from (A12),

$$D_{rlr'l'} = \int d\mathbf{b}\,d\mathbf{b}^* D_{rlr'l'}(\mathbf{b},\mathbf{b}^*;t-\tau)G(\mathbf{b},\mathbf{b}^*;\mathbf{a}_0,\mathbf{a}_0^*;\tau) \; . \quad \text{(A21)}$$

By substituting (A19) into (A21) we find

$$D_{rlr'l'}(\mathbf{a}_0,\mathbf{a}_0^*;t) = D(t-\tau)D_{rlr'l'}(\tau) \; . \quad \text{(A22)}$$

Using (A19) once more allows us to conclude

$$D(t)=D(t-\tau)D(\tau) \text{ or } D(t)=e^{-\lambda t} \quad \text{(A23)}$$

which completes the demonstration that superluminal communication implies exponential decay of the off-diagonal density-matrix elements.

(5) The remaining chore is to prove Eq. (A15a). We suppose that the left experimenter chooses a time τ at which to perform another experiment before the reduction has been completed. The experiment is "instantaneous" (i.e., its duration is much less than the reduction time). It is an interference experiment, so that a new basis of macroscopically distinguishable state vectors $|\bar{l}\rangle$ compete in the reduction "game" for $t>\tau$. The density matrix at τ is

$$D=\sum_{rlr'l'} |r\rangle|l\rangle\langle r'|\langle l'| \int d\mathbf{a}\, d\mathbf{a}^* a_{rl} a^*_{r'l'} G(\mathbf{a},\mathbf{a}^*;\mathbf{a}_0,\mathbf{a}_0^*;\tau) \quad \text{(A24)}$$

and at τ^+, when $|l\rangle$ has evolved into $\sum_{\bar{l}} |\bar{l}\rangle\langle\bar{l}|u|l\rangle$, is

$$\begin{aligned} D&=\sum_{r\bar{l}r'\bar{l}'} |r\rangle|\bar{l}\rangle\langle r'|\langle\bar{l}'| \int d\mathbf{a}\, d\mathbf{a}^* b_{r\bar{l}} b^*_{r'\bar{l}'} G(\mathbf{a},\mathbf{a}^*;\mathbf{a}_0,\mathbf{a}_0^*;\tau) \\ &=\sum_{r\bar{l}r'\bar{l}'} |r\rangle|\bar{l}\rangle\langle r'|\langle\bar{l}'| \int d\mathbf{b}\, d\mathbf{b}^* b_{r\bar{l}} b^*_{r'\bar{l}'} G(\mathbf{b}\mathbf{u}^\dagger,\mathbf{b}\mathbf{u}^{\dagger *};\mathbf{a}_0,\mathbf{a}_0^*;\tau)\, , \end{aligned} \quad \text{(A25)}$$

where $b=au$ $(b_{r\bar{l}}=\sum_l a_{rl}u^L_{l\bar{l}})$ and $d\mathbf{b}\,d\mathbf{b}^*=d\mathbf{a}\,d\mathbf{a}^*$ because the transformation is unitary. The evolution continues until time T (still less than the reduction time) when the density matrix is

$$D=\sum_{r\bar{l}r'\bar{l}'} |r\rangle|\bar{l}\rangle\langle r'|\langle\bar{l}'| \int d\mathbf{c}\, d\mathbf{c}^*\, d\mathbf{b}\, d\mathbf{b}^* c_{r\bar{l}} c^*_{r'\bar{l}'} G(\mathbf{c},\mathbf{c}^*;\mathbf{b},\mathbf{b}^*;T-\tau) G(\mathbf{b}\mathbf{u}^\dagger,\mathbf{b}\mathbf{u}^{\dagger *};\mathbf{a}_0,\mathbf{a}_0^*;\tau)\, . \quad \text{(A26)}$$

At time T^+, the r experimenter makes an interference experiment resulting in the macroscopically distinguishable basis $|\bar{r}\rangle$. Thereafter the reduction proceeds to completion in the $|\bar{r}\rangle|\bar{l}\rangle$ basis, so that only the diagonal elements remain. We take the trace over the left basis to see the density matrix used by the right experimenter to describe his experience:

$$\operatorname*{Tr}_l D=\sum_{\bar{r}} |\bar{r}\rangle\langle\bar{r}| \sum_{rr'\bar{l}} u^R_{\bar{r}r} u^{R\dagger}_{r'\bar{r}} \int d\mathbf{c}\, d\mathbf{c}^* d\mathbf{b}\, d\mathbf{b}^* c_{r\bar{l}} c^*_{r'\bar{l}} G(\mathbf{c},\mathbf{c}^*;\mathbf{b},\mathbf{b}^*;T-\tau) G(\mathbf{b}\mathbf{u}^\dagger,\mathbf{b}\mathbf{u}^{\dagger *};\mathbf{a}_0,\mathbf{a}_0^*;\tau)\, . \quad \text{(A27)}$$

The condition of nonsuperluminal communication means that the right experimenter must be unaware of the time of the left experiment and of the left experimenter's choice of basis $\{|\bar{l}\rangle\}$. That is, Eq. (A27) must be independent of τ and of u^L. Setting τ first equal to 0 and then equal to T, and using the initial condition (A2), we find

$$\begin{aligned} 0&=\sum_{rr'} u^R_{\bar{r}r} u^{R\dagger}_{r'\bar{r}} \int d\mathbf{c}\, d\mathbf{c}^* \sum_{\bar{l}} c_{r\bar{l}} c^*_{r'\bar{l}} [G(\mathbf{c}\mathbf{u}^\dagger,\mathbf{c}\mathbf{u}^{\dagger *};\mathbf{a}_0,\mathbf{a}_0^*;T)-G(\mathbf{c},\mathbf{c}^*;\mathbf{a}_0\mathbf{u},\mathbf{a}_0\mathbf{u}^*;T)] \\ &=\sum_{rr'} u^R_{\bar{r}r} u^{R\dagger}_{r'\bar{r}} \left[\sum_{\bar{l}} D_{r\bar{l}r'\bar{l}}(\mathbf{a}_0,\mathbf{a}_0^*;T)-\sum_{\bar{l}} D_{r\bar{l}r'\bar{l}}(\mathbf{a}_0\mathbf{u},\mathbf{a}_0\mathbf{u}^*;T) \right]\, , \end{aligned} \quad \text{(A28)}$$

where we have used $d\mathbf{c}\,d\mathbf{c}^*=d\mathbf{c}\,\mathbf{u}^\dagger d(\mathbf{c}\mathbf{u}^\dagger)^*$ and $\sum c_{r\bar{l}} c^*_{r'\bar{l}}=\sum_{\bar{l}} (cu^\dagger)_{r\bar{l}} (cu^\dagger)^*_{r'\bar{l}}$ in evaluating the first integral.

Equation (A28) has the form $\langle\bar{r}|A|\bar{r}\rangle=0$, where $|\bar{r}\rangle$ is an arbitrarily chosen basis because the right experimenter is free to choose his experiment. Thus the term in large parentheses in (A28) vanishes, and this is Eq. (A15a).

APPENDIX B

In this appendix we prove that exponential decay of the off-diagonal density-matrix elements in a stochastic dynamical reduction theory implies that each state vector takes infinite time to reduce.

We start with the observation that the density matrix (A4) satisfies the backward diffusion equation (A3). Putting the presumed form

$$D_{mn}(\mathbf{a},\mathbf{a}^*;t)=a_m a_n^* e^{-\lambda t}=\sqrt{x_m x_n}\, e^{i(\theta_m-\theta_n)} e^{-\lambda t}\, , \quad n\neq m \quad \text{(B1)}$$

into this most general diffusion equation (for simplicity we drop the zero subscripts) with zero drift in $\mathbf{x}$,

$$\frac{\partial D_{mn}}{\partial t}=\frac{1}{2}\sum_{rs}\left[b_{rs}\frac{\partial^2}{\partial x_r \partial x_s}+c_{rs}\frac{\partial^2}{\partial x_r \partial\theta_s} +h_{rs}\frac{\partial^2}{\partial\theta_r\partial\theta_s}\right] D_{mn}+\sum_r v_r \frac{\partial D_{mn}}{\partial\theta_r} \quad \text{(B2)}$$

and taking the real part of the result leaves us with the following constraints on the diffusion coefficients:

$$-\lambda=\frac{1}{4}\left[\frac{b_{nm}}{x_m x_n}-\frac{1}{2}\frac{b_{nn}}{x_n{}^2}-\frac{1}{2}\frac{b_{mm}}{x_m{}^2}\right] +(h_{nm}-\tfrac{1}{2}h_{nn}-\tfrac{1}{2}h_{mm}),\quad n\neq m\, . \quad \text{(B3)}$$

In addition, since the diffusion coefficients b_{nm} are obtained from stochastic differential equations, with

$$b_{nm} = \langle dx_n dx_m \rangle / dt \tag{B4}$$

and since $\sum_m x_m = s$ where s is constant is required, we have the additional equation

$$\sum_m b_{nm} = 0 \; . \tag{B5}$$

In Eqs. (B3) and (B5) we have sufficient equations to solve for all b_{nm}. We define

$$\lambda_{nm} \equiv \lambda(1-\delta_{nm}) - H_{nm} \; , \tag{B6a}$$

$$H_{nm} \equiv \tfrac{1}{2}(h_{nn} + h_{mm} - 2h_{nm}) \; . \tag{B6b}$$

The solution is readily found to be

$$\begin{aligned} b_{nm} &= 4x_n x_m \left[-\lambda_{nm} + \sum_k (\lambda_{nk} + \lambda_{mk}) \frac{x_k}{s} - \sum_{kl} \lambda_{kl} \frac{x_k x_l}{s^2} \right] \\ &= 4\lambda x_n x_m \left[\sum_k \frac{{x_k}^2}{s^2} - \frac{x_n}{s} - \frac{x_m}{s} + \delta_{nm} \right] + 4x_n x_m \left[H_{nm} - \sum_k (H_{nk} + H_{mk}) \frac{x_k}{s} + \sum_{kl} H_{kl} \frac{x_k x_l}{s^2} \right] . \end{aligned} \tag{B7}$$

First consider the case where $h_{nm} = 0$ (no angular diffusion). We see that the b_{nm} in Eq. (B7) are identical to the diffusion coefficients in Eq. (2.2). Thus we have shown that the diffusion equation arising from Gisin's model is the unique one (without angular diffusion) that gives rise to exponential decay of the off-diagonal density-matrix elements.

We show in Appendix C that this diffusion equation describes a state vector that never completely reduces; i.e., the vector $\mathbf{x}$ can approach but never reach any one of the hyperplanes $x_n = 0$ that bound the region of diffusion $\sum_m x_m = 1$. This is because $b_{nn} \sim {x_n}^2$ for small x_n, and such a diffusion rate is too slow in the neighborhood of the boundary to reach it in a finite time.

Thus the question remaining to be answered is whether a choice of $h_{nm} \neq 0$ can be made so that the additional terms in Eq. (B7) produce a less rapid decrease of b_{nm} in the neighborhood of $x_n = 0$ (e.g., $b_{nn} \sim {x_n}^{2-\epsilon}$, $\epsilon > 0$).

The answer to this question is no. We show below that the extra terms in (B7) make a negative contribution to b_{nn}. If such terms are to dominate the small x_n behavior, then b_{nn} will be negative for sufficiently small x_n. But, by (B4), b_{nn} must be positive. Thus it is impossible that nonzero h_{nm} can render the reduction time finite.

We take an arbitrary vector ξ and using (B7) form the scalar product $\xi \cdot b \cdot \xi$:

$$\sum_{nm} \xi_n b_{nm} \xi_m = 4\lambda \sum_n {u_n}^2 - 4 \sum_{nm} u_n h_{nm} u_m \; , \tag{B8}$$

where

$$u_n \equiv x_n \left[\frac{\mathbf{x}}{s} \cdot \xi - \xi_n \right] . \tag{B9}$$

Now, $h_{nm} = \langle d\theta_n d\theta_m \rangle / dt$ is a positive-definite matrix. Thus $-4 \sum_{nm} u_n h_{nm} u_m$ is negative definite. When we choose $\xi_n = \delta_{nk}$ we see that the contribution to b_{kk} from the second term in (B8) is negative, which concludes our argument.

APPENDIX C

In this appendix we show that the diffusion process described by Eq. (A1) never reduces if each $b_{nn} \sim x_n^r$, $r \geq 2$ near $x_n = 0$.

Consider the general diffusion process described by

$$\frac{\partial G}{\partial t}(\mathbf{y};\mathbf{y}_0;t) = \sum_{nm} \frac{\partial^2}{\partial y_n \partial y_m} b_{nm}(\mathbf{y}) G - \sum_n \frac{\partial}{\partial y_n} v_n(\mathbf{y}) G \tag{C1}$$

in a region surrounded by an absorbing boundary Σ. We first outline the derivation of Dynkin's equation[2] for the mean absorption time $m(\mathbf{y}_0)$ for a diffusion that starts from $\mathbf{y}_0$:

$$\begin{aligned} -1 = &\sum_{nm} b_{nm}(\mathbf{y}_0) \frac{\partial^2}{\partial y_{0n} \partial y_{0m}} m(\mathbf{y}_0) \\ &+ \sum_n v_n(\mathbf{y}_0) \frac{\partial}{\partial y_{0n}} m(\mathbf{y}_0) \; , \end{aligned} \tag{C2}$$

where $m(\mathbf{y}_0)$ vanishes on Σ.

Equation (C1) can be regarded as a conservation of probability equation $\dot{G} + \nabla \cdot \mathbf{J} = 0$ with the probability current density

$$J_n(\mathbf{y};t) \equiv \left[-\sum_m \frac{\partial b_{nm}}{\partial y_m}(\mathbf{y}) + v_n(\mathbf{y}) \right] G \; . \tag{C3}$$

Since G obeys the backward diffusion equation

$$\begin{aligned} \frac{\partial G}{\partial t}(\mathbf{y},\mathbf{y}_0;t) = &\sum_{nm} b_{nm}(\mathbf{y}_0) \frac{\partial^2 G}{\partial y_{0n} \partial y_{0m}} \\ &+ \sum_n v_n(\mathbf{y}_0) \frac{\partial G}{\partial y_{0n}} \end{aligned} \tag{C4}$$

we can operate on both sides of this equation with the operator in large parentheses of (C3), so J_n as well as the current to the boundary $I(\mathbf{y}_0;t) \equiv \int_\Sigma \mathbf{J} \cdot d\mathbf{A}$ satisfy this same backward diffusion equation. The mean reduction time is $m(\mathbf{y}_0) \equiv \int_0^\infty t I(\mathbf{y}_0;t) dt$. By multiplying the backward diffusion equation for I by t and integrating over all

t, Eq. (C2) is obtained ($\int_0^\infty t\dot{I}\,dt=-\int_0^\infty I\,dt=-1$ on the assumption that all the probability flows eventually to the boundary).

The one-dimensional diffusion equation on $0\le x\le 1$

$$\frac{\partial G}{\partial t}=\sigma^2\frac{\partial^2}{\partial x^2}x^r(1-x)^r G \tag{C5}$$

arises from stochastic dynamical reduction theories of two states with $x_1=x, x_2=1-x$: the theory described in Sec. I has $r=1$, Gisin's theory described in Sec. II has $r=2$, and the theories of Eq. (1.11) have arbitrary r. Equation (C2) for the mean reduction time

$$-1=\sigma^2 x_0^r(1-x_0)^r\frac{\partial^2}{\partial x_0^{\,2}}m(x_0) \tag{C6}$$

can be solved exactly, in this case, with the solution

$$m(x_0)=\frac{1}{\sigma^2}\left[(1-x_0)\int_0^{x_0}\frac{dy}{y^{r-1}(1-y)^r}+x_0\int_{x_0}^1\frac{dy}{y^r(1-y)^{r-1}}\right]. \tag{C7}$$

For example, for $r=0$ and 1 we have, from (C7),

$$m(x_0)=\tfrac{1}{2}\sigma^{-2}x_0(1-x_0)$$

and (C8)

$$-\sigma^2[x_0\ln x_0+(1-x_0)\ln(1-x_0)]\ ,$$

respectively, but the solution is infinite for $r\ge 2$. This should be interpreted as meaning that the mean reduction time is infinite, even though, strictly speaking, the problem we have posed is not defined [since (C7) does not satisfy the boundary conditions at $x_0=0,1$].

To illustrate this point, consider the diffusion (C5) for $r=2$ on the interval $(\epsilon,1-\epsilon)$ with absorbing boundaries at $x=\epsilon,1-\epsilon$. Then Eq. (C6) and the boundary conditions can be satisfied:

$$m(x_0)=\sigma^{-2}(1-2\epsilon)[\ln(1-\epsilon)-\ln\epsilon]-\sigma^{-2}(1-2x_0)[\ln(1-x_0)-\ln x_0]\ . \tag{C9}$$

We observe that as $\epsilon\to 0$, the mean reduction time from any fixed point x_0 in the interval grows without bound. What is happening is that for $r\ge 2$, the region in the neighborhood of the boundary acts like a quagmire: the diffusion proceeds toward the boundary, but there the diffusion coefficient $\sigma^2x^r(1-x)^r$ is so small that the diffusing points can never escape, nor can they reach the boundary in a finite time.

We wish to apply these ideas to the more general diffusion equation (A1) arising from stochastic reduction theories. We consider the diffusion within the (hyper)plane $x_1+x_2+\cdots+x_N=1$ bounded by the (hyper)planes $x_1=0, x_2=0, \ldots, x_N=0$. For $N=3$ this is a triangular area, for $N=4$ a tetrahedral volume, etc. We wish to focus on the diffusion behavior in the neighborhood of one of the boundary planes $x_n=0$. If there is no point near any boundary plane from which the diffusion can reach the boundary in a finite time, then no x_n can vanish, and the state vector never reduces.

Consider diffusion in a small (hyper)cylindrical volume bounded by the end "caps" $x_1=0$ and $x_1=\Delta$, and the surface

$$(x_2-\alpha_2)^2+\cdots+(x_N-\alpha_N)^2+(\theta_1-\beta_1)^2+\cdots+(\theta_N-\beta_N)^2=\epsilon^2\ ,$$

i.e., a cylinder with center axis line $0\le x_1\le\Delta, x_2=\alpha_2,\ldots,\theta_N=\beta_N$. If ϵ is small enough, we can consider that all diffusion and drift coefficients are only functions of x_1, since $x_2\approx\alpha_2,\ldots,\theta_N\approx\beta_N$ are approximately constant. Suppose we consider diffusion with absorbing boundary conditions at $x_1=0$ and $x_1=\Delta$ but reflecting boundary conditions everywhere else (i.e., $\mathbf{J}\cdot\hat{\mathbf{n}}=0$, where $\hat{\mathbf{n}}$ is the boundary normal) if there is no angular drift ($v_n=0$). If the angular drift is nonvanishing, we will impose periodic boundary conditions on the θ_n's, i.e., current exiting one side of the cylinder in a θ direction enters from the other side. In either case only end caps $x_1=0,\Delta$ and not the (hyper)cylindrical surface receives a net nonzero probability current.

Furthermore, suppose that the initial probability density distribution is uniform in the (hyper)disc $x_1=x_{01}$ $(0\le x_{01}\le\Delta)$ and zero elsewhere. Then the probability density thereafter will be uniform in each disc $x_1=$const, since a solution G of Eq. (A1) in the cylinder can be found that depends only upon x_1 and t, and satisfies the initial and boundary conditions, and such a solution is unique. The diffusion equation and initial condition satisfied by this solution are

$$\frac{\partial G}{\partial t}(x_1,t)=\frac{1}{2}\frac{\partial^2}{\partial x_1^{\,2}}b_{11}(x_1)G(x_1,t)\ , \tag{C10a}$$

$$G(x_1,0)=\delta(x_1-x_{01})\ , \tag{C10b}$$

where we have suppressed the dependence of b_{11} on the constants $\alpha_2,\ldots,\beta_N$.

The problem of whether the plane $x_1=0$ can be reached is thus reduced to the one-dimensional problem we discussed earlier. Thus if in the neighborhood of every point on each bounding plane $x_n=0$ the relevant diffusion coefficient $b_{nn}\sim x_n^r$ with $r\ge 2$, then there is no place on the boundary that can be reached in a finite time, and the reduction time is infinite.

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