

## Proposed experimental test of Bell's inequality in nuclear beta decay

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A  $\beta$  decay experiment is proposed for testing Bell's inequality, related to hidden-variables alternatives to quantum mechanics. The experiment uses Mott scattering for spin polarization analysis of internal conversion electrons. Beta-decay electrons, in cascade with the conversion electrons, are longitudinally polarized due to parity violation in the weak interaction. So simply detecting the  $\beta$  electron direction effectively measures the spin. A two-particle spin-spin correlation can thus be investigated and related, within certain assumptions, to Bell's inequality. The example of  $^{203}\text{Hg}$  decay is used for a calculation of expected results. Specific problems related to nuclear structure and experimental inconsistencies are also discussed.

### I. INTRODUCTION

The history of the theory of quantum mechanics has been filled with controversy over the probabilistic, non-deterministic results obtained from the theory.<sup>1-8</sup> The philosophical implications of this theory are intellectually challenging and sometimes seemingly contradictory, but a consistent view of nature is contained in quantum mechanics.<sup>9,10</sup> Fifty years ago, Einstein, Podolsky, and Rosen<sup>1</sup> initiated the attack on the nondeterministic nature of the quantum theory that continues today.<sup>8</sup> In spite of the obvious success of quantum mechanics in describing physical phenomena, the statistical nature of quantum predictions may not be inherently fundamental. There may be an underlying hidden-variables theory which is not probabilistic in its action.

Until 1965, speculations on this topic were entirely theoretical, with no relevant experiment that could be performed to resolve the questions. The means of investigation were *Gedankenexperimente*, such as a two-particle system with spatially separated spin polarimeters proposed by Bohm.<sup>4</sup> The 1965 paper by Bell<sup>6</sup> radically changed this situation and directly led to tests of the quantum theory versus a broad class of deterministic-type theories called local, hidden-variables theories. The locality principle can be summarized as follows: *Consider a pair of particles that interacted in the past but now are spatially separated. The result of measurements on one particle cannot depend on what is done to the other particle.* Direct violation of locality would imply, for example, any information transmission at  $v > c$  or action at a distance. Bell showed that no local, deterministic, hidden-variables theory can give complete agreement with all the statistical predictions of quantum mechanics. This is expressed in what is called Bell's inequality; the specific form appropriate for this proposed experiment will be discussed in Sec. III.

Extension of Bell's original work quickly led to a proposed realizable experimental situation.<sup>11</sup> The stringent requirements for a most general experimental test have also been delineated.<sup>7</sup> Presently, all tests have required certain assumptions, albeit some quite weak, to satisfy

these requirements. A number of experiments, reaching extremely fine precision, have been performed with results that verify (within those certain assumptions) the quantum theory as the correct, complete description.<sup>12</sup> These have all been very difficult experiments to execute. Effectively, each one measures a two-particle, spin-spin correlation function in two separated polarimeters, the general scheme used in Bohm's *Gedankenexperiment*. The vast majority of the experiments have used a pair of correlated visible-light photons obtained from atomic electron cascade transitions in excited atoms. The latest results are quite conclusively in favor of the quantum theory.<sup>13,14</sup> The decay of positronium into two correlated annihilation  $\gamma$  rays has also been used to investigate local, hidden-variables theories.<sup>15</sup> Again, experiment points to the quantum interpretation. Finally, low-energy proton-proton scattering was used, in a single instance, to test for hidden variables.<sup>16</sup> The result here is again in agreement with the quantum theory.

Is the case now closed? Is the probabilistic quantum theory the complete, final description, as suggested in Ref. 12? Maybe not. First, there has been recent controversy<sup>8</sup> over the validity of a new proof that all experimental tests of Bell's inequality have been incomplete. The proof purports to show that the class of hidden-variables theories tested against quantum mechanics is not the complete set of local, deterministic theories but actually an uninteresting subset. Second, some of our most fundamental notions of what constitutes probability, and how that affects Bell's inequality, have also recently been questioned.<sup>17</sup> Finally, experimentally spin correlations can be a tricky, subtle business. This is aptly pointed out in Ref. 16 where it is concluded that "this shows that it is necessary to test the inequality of Bell in very different experimental conditions." In this paper, I propose an experimental technique for determining two-particle spin-spin correlations in an entirely new system: nuclear  $\beta$  decay with forces mediated by the weak interaction. This basic idea is not new, but the actual realization of a  $\beta$  decay experiment with two polarimeters measuring correlated pairs has been achieved only once<sup>18</sup> and worked then very crudely. The essential point that is new here is to use the known, parity-violating

known, parity-violating properties of the weak interaction to help conduct a spin-spin correlation experiment. As a specific example, a promising, although not necessarily optimum, candidate experiment will be presented.

II.  $\beta$ -CONVERSION ELECTRON CORRELATION

Consider a typical case of nuclear  $\beta$  decay shown in Fig. 1. An unstable nucleus ( $^{203}\text{Hg}$ ) decays by emitting a  $\beta$ -decay electron ( $\beta^-$ ) leaving the daughter nucleus ( $^{203}\text{Tl}$ ) in a short-lived excited state. The usual mode of decay of this excited state is to emit a  $\gamma$  ray, leaving the Tl nucleus in the stable ground state. A competing process for  $\gamma$  emission is the internal conversion of atomic electrons. Here, instead of  $\gamma$  emission, an atomic electron is given the extra excited-state energy and appears as a fast electron with an energy equal to the  $\gamma$  energy minus the original atomic binding energy of the ejected electron. In  $^{203}\text{Hg}$  decay, this decay branch occurs  $\approx 15\%$ , with the other  $\approx 85\%$  as  $\gamma$  emission. The conversion electrons ( $e$ ) are in coincidence with the  $\beta$ -decay electrons that populate the excited state and hence they form a pair of correlated particles:  $\beta^-e$ .

Next, consider the spin properties of this correlated pair starting first with the  $\beta^-$  particle individually.  $\beta$  decay is an example of the weak interaction in nuclei. One important feature exhibited by the weak interaction is the maximal violation of parity invariance.<sup>19</sup> A direct result of the parity violation is the presence of a longitudinal spin polarization ( $P_L$ ) of the  $\beta^-$  particles:<sup>20</sup>

$$P_L \equiv \langle \sigma \cdot \hat{v} \rangle = \frac{N_+ - N_-}{N_+ + N_-} \cong - \left| \frac{v_\beta}{c} \right| = -\beta, \quad (1)$$

where  $\sigma$  are the Pauli spin matrices and  $N_+$  ( $N_-$ ) are the number of  $\beta^-$ 's with spin parallel (antiparallel) to their velocity  $v_\beta$ . This relationship has been found to be valid over a wide variety of nuclei<sup>21</sup> and over a large range of energies.<sup>22</sup> Although the longitudinal  $\beta^-$  polarization has never been measured in  $^{203}\text{Hg}$  decay (quite surprisingly), it

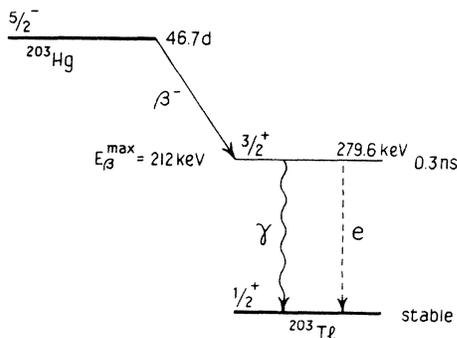


FIG. 1.  $^{203}\text{Hg}$   $\beta^-$  decay. The first forbidden  $\beta$  decay of  $^{203}\text{Hg}$  is depicted. The decay leaves the daughter nucleus  $^{203}\text{Tl}$  in a short-lived excited state. Internal conversion of atomic electrons ( $e$ ) competes with  $\gamma$ -ray emission to depopulate the excited state.

is improbable that Eq. (1) is incorrect by more than a few percent. The largest known deviation from  $\beta$  is roughly 25% for the somewhat pathological decay of RaE ( $^{210}\text{Bi}$ ).<sup>23</sup> There is no other nucleus with a well-documented deviation in the polarization from  $\beta$  of more than a few percent. Note that  $P_L = \beta$  is a fairly large polarization, e.g., at 100 keV,  $P_L = 0.55$ .

Now, consider the polarization of the conversion electron ( $e$ ) emitted after the  $\beta^-$  particle. Observing the  $\beta^-$  from the decay will leave the excited nuclear state polarized along the direction of the  $\beta^-$  velocity; one would expect the conversion electrons emitted from the excited state to be polarized along the same direction. This was recognized<sup>24</sup> soon after parity violation was discovered and experimental investigations searching for this effect ensued.

The technique used by all the experimental investigations was to analyze  $e$ 's emitted perpendicular to the  $\beta^-$  direction, resulting in a transverse spin polarization of the  $e$ 's. The magnitude of the transverse polarization (the direction implied is the  $\beta^-$  direction, this is crucial later) is given by

$$P_T^e = K \left[ \frac{v_\beta}{c} \right] \sin \theta_{\beta e}, \quad (2)$$

where  $\theta_{\beta e}$  is the angle between the velocities of  $\beta^-$  and  $e$ , chosen to be  $\theta_{\beta e} \approx \pi/2$ . The factor  $v_\beta/c$  is simply the

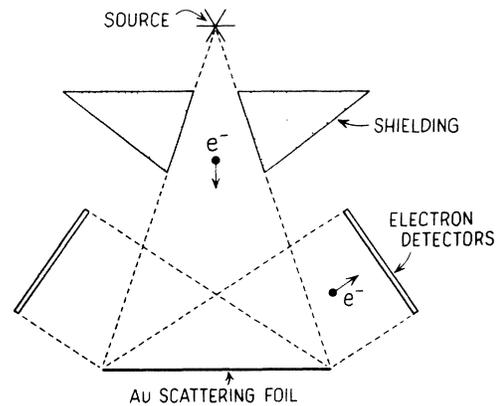


FIG. 2. Mott scattering. Electrons emitted downward from the source are allowed to scatter in a thin Au foil. Electrons that are scattered in a back-angle direction are detected in two thin detectors placed symmetrically left and right of the foil. If the electrons emitted by the source are transversely polarized in or out of the page, then equal numbers will not scatter into each detector. Defining the observed asymmetry of the number scattered right ( $N_R$ ) vs number left ( $N_L$ ):

$$A = \frac{N_R - N_L}{N_R + N_L},$$

the transverse polarization  $P_T$  is found from  $A = S_{an} P_T$ , where  $S_{an}$  is the polarimeter analyzing power. The analyzing power is, in general, a function of the electron energy and scattering angle and the Au foil thickness.

longitudinal polarization of the preceding  $\beta^-$  and  $K$  is a constant determined by nuclear structure. Measurements of  $K$  have been performed on a number of different nuclei. The results are displayed<sup>25</sup> in Table I. Note, in particular, that the measurements of  $K$  in  $^{203}\text{Hg}$  decay are quite discrepant with one another, roughly consistent in

magnitude but differing in sign. No measurement clearly indicates  $K(^{203}\text{Hg})=0$ , however. In what follows, I will somewhat arbitrarily assume the results of Ref. 26 are correct and use those values.<sup>27</sup>

The reason is not known for the large number of conflicting results in the various measurements of  $K$ . All ex-

TABLE I. Conversion electron polarization results. For  $e$  selection,  $M$  denotes magnetic spectrometer and  $S$  denotes scintillation counters. The asterisk in the last column indicates that the value is possibly correlated with the data on the line immediately above.

Decay	$K$ -polarization coefficient	$e$ shell	$e$ selection	Reference
$^{131}\text{I}$	$-0.23 \pm 0.07$	$K$	$M$	a
	$-0.32 \pm 0.06$		$M$	b
$^{141}\text{Ce}$	$-0.39 \pm 0.10$	$K$	$M$	a
$^{144}\text{Ce}$	$+0.47 \pm 0.18$	$K$	$M$	a
$^{153}\text{Sm}$	$-0.20 \pm 0.06$	$K$	$M$	b
	$-0.07 \pm 0.07$	$L$ (103)	$M$	b
	$+0.21 \pm 0.07$	$L$ (70)	$M$	b
$^{166}\text{Ho}$	$-0.01 \pm 0.02$	$L$	$M$	b
$^{170}\text{Tm}$	$+0.49 \pm 0.06$	$L$	$M$	c
	$-0.52 \pm 0.12$	$L$	$M$	a
	$+0.19 \pm 0.05$	$L$	$M$	b
	$+0.34 \pm 0.14$	$L$	$S$	d
$^{186}\text{Re}$	$+0.37 \pm 0.13$	$K$	$M$	b
$^{188}\text{Re}$	$+0.18 \pm 0.07$	$K$	$M$	b
$^{198}\text{Au}$	$-0.18 \pm 0.07$	$K$	$M$	a
	$-0.28 \pm 0.04$	$K$	$M$	e
	$+0.43 \pm 0.17$	$K$	$M$	f
$^{199}\text{Au}$	$+0.26 \pm 0.09$	$K$	$M$	a
$^{203}\text{Hg}$	$+0.35 \pm 0.10$	$K$	$S$	g
	$-0.32 \pm 0.09$	$K$	$M$	c
	$+0.50 \pm 0.10$	$K$	$M$	h
	$+0.30 \pm 0.08$	$K$	$M$	a*
	$+0.59 \pm 0.09$	$K$	$M$	e
	$+0.62 \pm 0.08$	$K$	$M$	i*
	$-0.42 \pm 0.18$	$K$	$S$	j
	$-0.31 \pm 0.04$	$K$	$S$	k
	$+0.66 \pm 0.10$	$K$	$M$	l
	$-0.33 \pm 0.07$	$K$	?	m
$^{233}\text{Pa}$	$-0.16 \pm 0.06$	$L$	$S$	k
	$-0.48 \pm 0.14$	$K$	$M$	c
	$+0.41 \pm 0.09$	$K$	$M$	a

<sup>a</sup>B. Blake *et al.*, Nuovo Cimento **25**, 942 (1962).

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<sup>c</sup>M. E. Vishnevskii *et al.*, Zh. Eksp. Teor. Fiz. **38**, 1424 (1960) [Sov. Phys JETP **11**, 1029 (1960)]; Nucl. Phys. **18**, 122 (1960).

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<sup>k</sup>Reference 26.

<sup>l</sup>S. K. Bhattacharjee *et al.*, Phys. Lett. **2**, 347 (1962).

<sup>m</sup>S. Buhl, Proceedings of the Conference on Beta Spectroscopy and Nuclear Structure, Groningen, 1968 (unpublished), p. 112, quoted in J. J. Van Rooijen *et al.*, Nucl. Phys. **A167**, 421 (1971).

periments used the same method of spin-polarization analysis, namely, Mott scattering. Mott scattering is elastic electron-nucleus, Coulomb scattering.<sup>28</sup> Optimization of the technique requires high- $Z$  nuclei for targets (typically thin Au foils) and back-angle scattering ( $\approx 120^\circ$ ) of electrons.<sup>29</sup> Mott scattering is sensitive to the transverse spin polarization of the electrons and generates a right-left scattering asymmetry proportional to that polarization (see Fig. 2).

The various  $K$  experimenters did use different methods to energy select the conversion electrons for polarization analysis ( $M$  and  $S$  in Table I). Most experiments employed a magnetic  $\beta$ -ray spectrometer for this purpose, a few experiments used the pulse-height distribution in the Mott detectors as their energy selector. This latter technique is especially appropriate for  $^{203}\text{Hg}$  decay since the  $\beta^-$  spectrum is fairly well separated in energy from the conversion lines. Still, this difference in the energy-selection method does not explain all the discrepancies in the  $K$  measurements. Considering only those measurements done with magnetic spectrometers, there is still disagreement between the experimental values. Further work in this area is clearly required to resolve these problems.

### III. PROPOSED EXPERIMENT

Setting aside the consistency problem of the experimental results for  $K$ , I wish to consider an extension of these experiments in the vein of Bohm's *Gedankenexperiment*. The essential point is that two correlated electrons, prepared in the singlet state, can be detected in two spatially separated detectors and a spin-spin correlation function can be determined by simply varying an angle  $\phi$ . It is presently not known what fraction of  $\beta^-e$  pairs from  $^{203}\text{Hg}$  decay are in the singlet state. This, in principle, can be determined from experiment (see Ref. 30). The prescription for handling the triplet contribution is given in Ref. 16. If a substantial triplet contribution exists in  $^{203}\text{Hg}$  decay, another isotope should be chosen. Since the proposed experiment is presented as an example, the triplet contribution problem is not fundamental here and it is assumed the singlet dominates.

The proposed apparatus is shown in Fig. 3. The quantity determined in previous measurements of  $K$  would be denoted here as  $K(\phi=0)$ ; that is, they measured the transverse polarization of the conversion electron along the direction of the  $\beta^-$  electron. Extending this notion to  $\phi \neq 0$ , for example, with  $\phi = \pi/2$ , one measures conversion electron transverse polarization perpendicular to the  $\beta^-$  direction. This quantity  $K(\phi = \pi/2)$  is expected to be very small compared to  $K(\phi=0)$ , if not zero.<sup>31</sup> The proposed experiment is a device that simultaneously measures  $K(0)$ ,  $K(\pi/4)$ , and  $K(\pi/2)$  from a single source. The multiple detector scheme not only provides the triple simultaneous measurements, enhancing the statistical rate, but it also effectively eliminates problems associated with instrumental asymmetries present in the apparatus.<sup>32</sup> Energy selection of the conversion electrons is performed after scattering in the Au foil using pulse-height information in the Mott detectors. The energy spectrum, derived

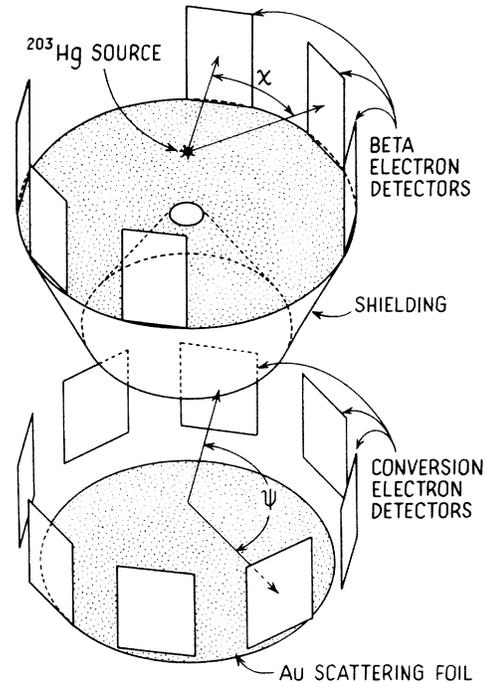


FIG. 3. Proposed apparatus. A source of  $^{203}\text{Hg}$  is positioned above a hole in the shielding leading downward. Conversion electrons ( $e$ ) emitted downward have their transverse polarization analyzed by Mott scattering. The back-scattered  $e$ 's are detected in one of eight detectors surrounding the Au foil.  $\beta$ -decay electrons, in cascade with the  $e$ 's, are detected in one of six detectors surrounding the source. The rate of all coincidence events between an upper detector and a lower detector is recorded. The angle  $\phi$ , referred to in the text, is given by  $\phi = \psi - \chi - \pi/2$ .

from pulse-height analysis, is displayed in Fig. 4. Energy selection in this manner will also enhance the polarization analyzing power over the values observed in Ref. 29, since a major contaminant to the analyzing power in those measurements was the inelastic-scattering contribution to the elastic scattering. Energy selection can greatly minimize the dilution effect of inelastically scattered conversion electrons.<sup>33</sup>

Other systematic effects that will arise include (1) depolarization and scattering of electrons in the source,<sup>34</sup> (2) scattering of electrons from other surfaces, (3) Auger lines in the electron spectrum, (4) accidental, random coincidences due to finite coincidence resolving time, (5) cosmic-ray background, and (6) the  $\beta^-e$  directional correlation. Accidental coincidences can be counted concurrently with the true coincidences using delayed coincidence techniques. The anticipated  $S/N$  for the true to accidental rate is 3:1. The effect of scattered electrons and cosmic-ray events can be determined by running the experiment with no Mott-scattering foil in place. The possibility of a large  $\beta^-e$  directional correlation can be investigated using an Al scattering foil replacing the Au Mott-scattering foil. The polarization sensitivity of Al is about an order of magnitude smaller than Au (low  $Z$  as

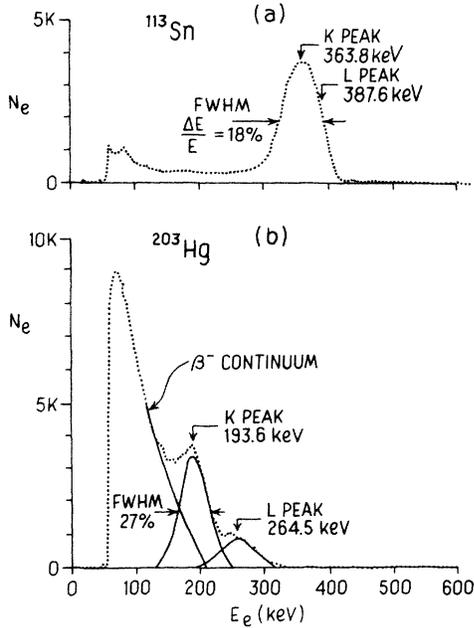


FIG. 4. Conversion-electron energy spectra. Energy spectra, obtained from pulse-height analysis of signals from a plastic scintillator detector, are displayed for decays of (a)  $^{113}\text{Sn}$  and (b)  $^{203}\text{Hg}$ . This detector is made with 1-mm-thick Pilot B plastic scintillator (barely thick enough for the Sn electrons, but quite adequate for the Hg electrons)  $\approx 5 \text{ cm}^2$  in area, coupled with epoxy glue to the face of an RCA 8850 photomultiplier tube. The  $^{113}\text{Sn}$  decay proceeds exclusively by electron capture; hence, there is no  $\beta$  continuum and only the conversion lines are seen. The  $^{203}\text{Hg}$  source is 6 mg/cm<sup>2</sup> thick and gave a counting rate of about 2 kHz in this detector. The conversion lines can be clearly seen above the  $\beta^-$  continuum.

opposed to high  $Z$ ).<sup>28</sup> If the directional correlation is causing a geometrical systematic effect, it will also appear with the same size in the Al measurement and can be corrected for.

Previous work with a Mott polarization analyzer, in a similar configuration to that proposed here,<sup>32</sup> allows a reliable estimate of the statistical efficiency of the apparatus shown in Fig. 3. This estimate also accounts for the various normalization (Al foil) and background (no foil) runs that are time consuming. For a continuous running time of three days, three polarization measurements (at  $\phi=0$ ,  $\pi/4$ ,  $\pi/2$ ) will be simultaneously obtained at the statistical level of  $\pm 0.012$  for each  $K(\phi)$ .

Previous measurements of  $K$  indicate<sup>27</sup> that  $K(\phi=0) = -0.31$ . It is assumed here that at  $\phi=\pi/2$ ,  $K \simeq 0$  (see Ref. 31). The interesting point is  $\phi=\pi/4$ . The quantum theory predicts the  $\phi$  dependence of  $K$  to be simply  $\cos\phi$ , giving

$$K^{\text{QM}}(\phi=\pi/4) = K(\phi=0)\cos\pi/4, \quad (3)$$

which is  $-0.221$  using the value for  $K(\phi=0)$  given above. Local, hidden-variables (HV) theories are required to satisfy Bell's inequality:<sup>7</sup>

$$|E(\hat{\mathbf{a}}, \hat{\mathbf{b}}) - E(\hat{\mathbf{a}}', \hat{\mathbf{b}})| + E(\hat{\mathbf{a}}, \hat{\mathbf{b}}') + E(\hat{\mathbf{a}}', \hat{\mathbf{b}}) \leq 2, \quad (4)$$

where  $\hat{\mathbf{a}}, \hat{\mathbf{a}}'$  are two directions of polarization analysis for the first particle and  $\hat{\mathbf{b}}, \hat{\mathbf{b}}'$  are the directions for the second particle.  $E(\hat{\mathbf{a}}, \hat{\mathbf{b}})$  is the expected value of  $\sigma_1 \cdot \hat{\mathbf{a}} \sigma_2 \cdot \hat{\mathbf{b}}$  (two spin measurements on a correlated pair of particles). The form of Bell's inequality for the present case with only coplanar polarization vectors is derived in Ref. 16 and is summarized in Table I of that paper. Bell's inequality requires

$$|K^{\text{HV}}(\phi=\pi/4)| \leq 0.5 \times |K(\phi=0)| \quad (5)$$

which is  $\geq -0.156$  using the same value for  $K(\phi=0)$  as in the quantum-mechanical (QM) case. Note the quantum prediction differs significantly from the Bell prediction (by  $\geq 0.065$ ) and if this experiment reaches the anticipated level of sensitivity (three measurements each with  $\pm 0.012$ ), it could easily discriminate between these predictions.

There are several assumptions required to derive the experimental result [Eq. (5)] from the general theoretical form [Eq. (4)].<sup>7,16</sup> Briefly, (1) the transmission ( $\approx 0.4\%$ ) and analyzing power ( $\approx 20\%$ ) of the Mott polarimeter in the proposed apparatus are both small compared to a perfect apparatus with 100% for each. I assume this does not effect the experimental result. (2) Another assumption is required which is analogous to the assumption concerning the quantum efficiency of the photomultiplier tubes in the atomic cascade experiments. Here, I assume the operation (efficiency and analyzing power) of the two polarimeters are independent of one another; that is, transmission and analyzing power are constant in one polarimeter regardless of what type of measurement the other polarimeter is performing. (3) The problem of triplet contributions (discussed earlier and in Refs. 16 and 30) requires another assumption. (4) A final assumption (referred to as the "polarization assumption" in the following) in this analysis is that using the parity-violating properties of the weak interaction as one polarimeter is equivalent to Bohm's scheme of two spatially separated polarimeters.

The polarization assumption seems justified if the required condition of the two measurements is a lack of determinacy of the two spin directions at the decay site. The spin direction of the  $\beta$  electron, while known to be along the velocity, is not determined until the firing of one of the upper  $\beta$  electron detectors. The upper  $\beta$  detectors then act, not only to find velocity directions, but also as spin analyzers.

Still, the polarization assumption is a strong assertion and an arguable point. The requirement of this assumption must somewhat diminish the strength of this experiment as a test of Bell's inequality. With the polarization assumption, it is, however, possible to obtain spatial separation between the choice of which axes for the two polarization determinations. This would be accomplished by making the overall size of the apparatus shown in Fig. 3 on the scale of  $\approx 1 \text{ m}$ , not an unreasonable requirement.

#### IV. CONCLUSIONS

The question of the inconsistencies of the conversion-electron polarization measurements, as shown in Table I, is still unanswered. This question and other nuclear structure questions<sup>30</sup> would need to be addressed before attempting the hidden-variables experiment proposed here. These problems deserve investigation in their own right, as well as their relation to Bell's inequality. Clearly, further work in this area would be useful.

Aside from these nuclear-structure reservations, there are further assumptions needed for this proposed experiment to be sensitive to Bell's inequality. The polarization assumption discussed at the end of Sec. III is particularly bothersome in this regard. It is perhaps necessary, in the strictest sense, to think of this experiment as testing both this assumption and Bell's inequality. The previous atomic experiments are certainly more powerful at verifying quantum predictions.

With all of these qualifications, I have shown that an experimental test of local, hidden-variables theories could be performed in the arena of nuclear  $\beta$  decay. The

parity-violating properties of the weak interaction have specifically been used to enable the measurement. The example of  $^{203}\text{Hg}$  decay and conversion-electron polarization has been discussed here. The same idea, of course, can be applied to other nuclear decays and to  $\beta^-$ - $\gamma$  cascades. Any experiment of this nature would be quite different from the previously performed hidden-variables tests. The most unique feature, the weak interaction, has historically led to many surprising and fruitful discoveries.

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correlation, which is not known for  $^{203}\text{Hg}$  (see Ref. 21, p. 1481). It is not even clear if  $^{203}\text{Hg}$  decay satisfies the so-called "quasiallowed" approximation ( $\xi$  approximation), see J. J. Van Rooijen *et al.*, Nucl. Phys. **A167**, 421 (1971). In spite of these reservations, I will assume the allowed result  $K(\phi=\pi/2)=0$ .

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