Singularity-free cosmology: A simple model

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(Received 31 October 1985)

The nonminimal coupling of a self-interacting complex scalar field with gravity is studied. For a Robertson-Walker open universe the stable solutions of the scalar-field equations are time dependent. As a result of this, a novel spontaneous symmetry breaking occurs which leads to a varying effective gravitational coupling coefficient. It is found that the coupling coefficient changes sign below a critical "radius" of the Universe implying the appearance of repulsive gravity. The occurrence of the repulsive interaction at an early epoch facilitates singularity avoidance. The model also provides a solution to the horizon problem.

I. INTRODUCTION

In spite of its successes the standard big-bang cosmology suffers from many undesirable features. In recent years there has been considerable effort to overcome a few difficulties encountered in the standard model through the works of Guth,[1] Linde,[2] and others.[3,4] The most disturbing consequences of the standard model are the initial singularity and the concomitant horizon and flatness problems. As is well known, the Hawking-Penrose theorem[5] asserts that a singularity is a generic point in general relativity. Hence, in any general-relativistic model of the Universe a singularity cannot be avoided. It is believed that the singularity would disappear in a quantum theory of gravity.[6] In the absence of a complete theory of quantum gravity, it would be worthwhile to examine the singularity problem in the classical framework. As mentioned earlier, in the classical framework a pure general relativistic model would not give the desired results. However, a nonminimal coupling of other fields with gravity would lead to an adequate departure from Einstein's theory. In such theories one might hope to avoid the singularity.

In a previous paper[7] it was shown that when a scalar field is nonminimally coupled to the gravitational field, there is a phase transition at a critical temperature T_c. This is due to the spontaneous symmetry breaking of the scalar field. Associated with the phase transition is a change in the sign of the effective gravitational coupling coefficient (EGCC) which means that gravity becomes repulsive above the critical temperature T_c. The appearance of repulsive gravity indicates that the implosion, which would occur in the big-bang cosmology if one goes backward in time, cannot proceed unabatedly. This does not necessarily imply that the implosion would not be strong enough to overcome the repulsive field and reach the singularity. In this article we shall show that the Universe must have begun with a nonzero value of the cosmic scale factor. As a consequence of the removal of the singularity the horizon problem also disappears.

In the latter part of Ref. 7, a phase transition occurs due to the temperature dependence of the cosmological term. However, as we shall see, it is not necessary to start with a temperature-dependent, or constant, cosmological term to obtain repulsive gravity (or equivalently a change in the sign of EGCC). It is interesting to note that the quartic self-interaction of the scalar field and its nonminimal coupling with the metric field would suffice to bring about a change in the sign of EGCC.

In Sec. II the formalism is given and the field equations are derived. Some general features are pointed out but the analysis cannot be made complete without a choice of the background metric or a form for the matter energy-momentum tensor. By assuming that the background has a Friedmann-Robertson-Walker metric a novel spontaneous symmetry breakdown is demonstrated in Sec. III. This leads to a varying EGCC. Solution to the metric field equations is obtained in Sec. IV, which clearly shows that the singularity is avoided. A solution to the horizon problem is suggested in Sec. V.

II. FORMALISM

We consider a system comprising a massless, complex, self-interacting (quartic) scalar field ϕ nonminimally coupled to gravity, a metric field $g_{\mu\nu}$, and other matter fields. The Lagrangian for the system is[8]

$$\mathscr{L}=\sqrt{-g}\,[\,g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi-\sigma(\phi^*\phi)^2-\tfrac{1}{6}R\,(\phi^*\phi)+\kappa^{-1}R+\mathscr{L}_m\,]\;. \tag{2.1}$$

Here R is the curvature scalar and κ is the Einstein constant. The coupling constant σ is the only parameter in the theory. $\sqrt{-g}\,\mathscr{L}_m$ is the Lagrangian corresponding to all matter fields and has no interaction with the scalar field except through geometry.

The quartic self-interaction and the nonminimal coupling together play a crucial role in bringing about a spontaneous symmetry breakdown, and a change in sign of EGCC. The nonminimal coupling gives the required improvement over a general relativistic theory, as remarked by Callan, Coleman, and Jackiw.[9] As mentioned in Ref. 9 the theory retains all the predictions of general relativity.

The Lagrangian (2.1) is invariant under a global U(1) group and under the transformations $\phi \to \phi^*$ and $\phi \to -\phi$.

The scalar-field equations and the gravitational field equations are obtained from the Lagrangian (2.1). They are

$$\Box\phi + 2\sigma\phi^*\phi^2 + \tfrac{1}{6}R\phi = 0 \tag{2.2}$$

and its complex conjugate and

$$G_{\mu\nu} = -\tilde{\kappa}(\tilde{\theta}_{\mu\nu} + T_{\mu\nu}) . \tag{2.3}$$

Here $G_{\mu\nu}$ is the Einstein tensor and $\tilde{\kappa}$ is the effective gravitational coupling coefficient, and $\tilde{\theta}_{\mu\nu} + T_{\mu\nu}$ represents the energy-momentum tensor corresponding to the scalar field plus the rest of the matter fields, which together form the source of geometry. Explicitly,

$$\tilde{\kappa} = \kappa \left[1 - \frac{\kappa}{6}\phi^*\phi \right]^{-1} , \tag{2.4}$$

$$\tilde{\theta}_{\mu\nu} = \tfrac{1}{2}\left[\partial_\mu\phi^*\partial_\nu\phi + \partial_\nu\phi^*\partial_\mu\phi - g_{\mu\nu}[\partial_\rho\phi^*\partial^\rho\phi - \sigma(\phi^*\phi)^2] + \frac{g_{\mu\nu}}{3}\Box(\phi^*\phi) - \tfrac{1}{3}(\phi^*\phi)_{;\mu\nu} \right] , \tag{2.5}$$

and

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}}\left[\frac{\partial}{\partial g_{\mu\nu}}(\sqrt{-g}\,\mathcal{L}_m) - \frac{\partial}{\partial x^\alpha}\left[\frac{\partial(\sqrt{-g}\,\mathcal{L}_m)}{\partial g^{\mu\nu}_{,\alpha}} \right] \right] . \tag{2.6}$$

In the above expressions $\Box$ denotes the wave operator in curved space-time,

$$\Box = (1/\sqrt{-g}\,)\partial_\mu(g^{\mu\nu}\sqrt{-g}\,\partial_\nu) ,$$

and a semicolon denotes the covariant derivative.

Identification of the expression (2.4) as EGCC is not for convenience but it is very natural. This is justified by the form of the Lagrangian. Notice that the coefficient of R is exactly this expression and in the spirit of the standard Einstein-Hilbert Lagrangian it would be right to recognize expression (2.4) as the coupling coefficient of the gravitational field. As a result of this recognition, the energy-momentum tensor for the scalar field is devoid of the quantities R and $R_{\mu\nu}$. In this sense the expression for $\tilde{\theta}_{\mu\nu}$ differs from the standard ones found in the literature.[10–12] Note that due to this modification, the scalar-field energy-momentum tensor is not traceless anymore though Eq. (2.2) are conformally invariant. Taking the trace of Eq. (2.3) we get $R = \kappa T^\mu_\mu$. Using this in the scalar-field equations we get

$$\Box\phi + 2\sigma\phi^*\phi^2 + \tfrac{1}{6}\kappa T^\mu_\mu\phi = 0 . \tag{2.7}$$

Observe that though we had assumed the scalar field and the rest of the matter to be noninteracting, the coupling indeed shows up through geometry. This, as is well known, is the reassurance of the fact that matter and geometry are tied up.

In general T^μ_μ is a space-time-dependent quantity. Hence, apart from $\phi = 0$, Eq. (2.7) does not admit constant solutions (unless when T^μ_μ itself is a constant). If $\phi = 0$ happens to be an unstable solution then the stable vacuum solution (VS) has to be a space-time-dependent function and a new type of symmetry breaking occurs. This immediately implies that EGCC does not remain constant, and can undergo a change in sign indicating the onset of repulsive gravity. As our aim is to demonstrate this feature we shall be interested in obtaining VS of Eq. (2.2) with the help of *a priori* specified metric and consider particular forms for $T_{\mu\nu}$ in Sec. IV.

III. SPONTANEOUS SYMMETRY BREAKDOWN AND REPULSIVE GRAVITY

Let the space-time be described by a Friedmann-Robertson-Walker metric. We consider, in what follows, both the open and closed isotropic models. For the open isotropic model (three-space of negative curvature) the metric is given by

$$ds^2 = a^2(t)[dt^2 - d\chi^2 - \sinh^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)] , \tag{3.1}$$

where the function $a(t)$ is the cosmic scale factor. The scalar curvature corresponding to the above metric is

$$R = \frac{6}{a^3}(a - \ddot{a}) . \tag{3.2}$$

Let the vacuum state of the scalar field be denoted by ξ. In general, ξ can depend on space and time. But the choice of the homogeneous and isotropic metric restricts it to be at most time dependent. Using relation (3.2) in Eq. (2.2) we get the following equation for ξ:

$$\ddot{\xi} + 2\frac{\dot{a}}{a}\dot{\xi} - \left[1 - \frac{\ddot{a}}{a} \right]\xi + 2\sigma\xi^*\xi^2 a^2 = 0 . \tag{3.3}$$

Here, a dot denotes differentiation with respect to t. Solution to the above equation has been obtained by several authors.[10–12] Guided by the fact that the scalar-field equations are conformally invariant we try $\xi = \pm\gamma f(t)/a(t)$, where $\gamma = 1/(2\sigma)^{1/2}$ is a constant. Substituting this in Eq. (3.3) we get an equation in $f(t)$ only:

$$\ddot{f} - f + f^* f^2 = 0 . \tag{3.4}$$

As can easily be seen the stable solutions of this equation are $f = \pm 1$ and not $f = 0$. Thus, the stable solutions of Eq. (3.3) are

$$\xi = \pm\gamma/a(t) . \tag{3.5}$$

Incidentally, these solutions are energetically more favorable than the $\xi = 0$ solution. The spontaneous symmetry breakdown which occurs due to the nonvanishing VS has the following novel features. Since ξ depends inversely on $a(t)$, symmetry breakdown is more and more important for the early Universe and symmetry is restored only in the limit $a(t)\rightarrow\infty$. It is this feature, which is in contrast with the usual grand unified theories where symmetry is restored as $a(t)$ decreases or T (temperature) increases, that enables the singularity avoidance. A similar result has been obtained in Ref. 7 for a slightly different model. Note that there is no mass term in the Lagrangian whose sign has to be properly chosen to facilitate the symmetry breakdown. This is not surprising. The nonminimal coupling in a sense behaves like a mass term. Indeed, one can see by writing down the effective Lagrangian that the physical field has a mass term which is time dependent. For the VS (3.5), Eq. (2.4) implies

$$\tilde{\kappa}=\kappa\left[1-\frac{a_c{}^2}{a^2}\right]^{-1}, \tag{3.6}$$

where $a_c=(\kappa/12\sigma)^{1/2}$ has the dimensions of length. Observe that a_c is the "critical radius" at which EGCC undergoes a change in sign, indicating that gravity becomes repulsive below this value of $a(t)$. This slows down the implosion of the Universe towards the singularity and might halt the collapse before the singular point is reached. The critical radius depends inversely on σ and therefore a weaker quartic self-coupling of the scalar field would bring about a sign change of $\tilde{\kappa}$ at a larger value of $a(t)$. This does not mean that if $\sigma=0$ gravity has to be repulsive always as none of the above arguments regarding symmetry breaking would then hold good. It is only a nonzero, sufficiently small value of σ that would give the desired features. If repulsive gravity occurs at a suitable value of $a(t)$ then singularity can eventually be avoided. When the Universe is dominated by radiation, $a(t)$ and temperature T are related by $a(t)T=\text{const}$. Using this, expressions (3.5) and (3.6) can be written as $\xi=\gamma' T$ and $\tilde{\kappa}=\kappa(1-T^2/T_c{}^2)^{-1}$. The behavior of the vacuum state and EGCC can now be equivalently given in terms of temperature instead of $a(t)$. EGCC undergoes a change in sign at a critical temperature T_c above which gravity becomes repulsive. Symmetry breaking due to the nonzero VS is more important at high temperatures and symmetry is restored only as $T\rightarrow 0$.

Let us now consider the closed isotropic model for which the metric is

$$ds^2=a^2(t)[dt^2-d\chi^2-\sin^2\chi(d\theta^2+\sin^2\theta\, d\phi^2)] \tag{3.7}$$

and the scalar curvature is

$$R=\frac{-6}{a^3}(a+\ddot{a}) . \tag{3.8}$$

By following the steps of the previous case we get an equation for f:

$$\ddot{f}-\frac{2\ddot{a}}{a}f-f+f^*f^2=0 . \tag{3.9}$$

Unfortunately, a solution to this equation could not be obtained for a general $T_{\mu\nu}$. When $T_{\mu\nu}$ is traceless, Eq. (2.3) implies $R=0$ and from expression (3.8) $\ddot{a}=-a$. Thus, when the scalar field and a traceless $T_{\mu\nu}$ form the source of geometry, Eq. (3.9) reduces to

$$\ddot{f}+f+f^*f^2=0 . \tag{3.10}$$

In this case the stable solutions are not $f=\pm i$, but the solution $f=0$. Thus, for the closed-universe model, when $T_{\mu\nu}$ is traceless the symmetry is unbroken, the repulsive gravity cannot occur and the singularity is not removed. In this case $\tilde{\kappa}=\kappa$ is a constant.

IV. SINGULARITY AVOIDANCE

Our next task is to obtain solutions to the gravitational field equations and demonstrate that singularity can be avoided at least in the open isotropic model. In this section we rewrite the gravitational field equations in the form

$$G_{\mu\nu}=-\kappa(\theta_{\mu\nu}+T_{\mu\nu}) , \tag{4.1}$$

where $\theta_{\mu\nu}$ is now the total energy-momentum tensor of the scalar field. Explicitly

$$\theta_{\mu\nu}=\tilde{\theta}_{\mu\nu}-\tfrac{1}{6}\phi^*\phi G_{\mu\nu} . \tag{4.2}$$

Let us examine the solutions of the field equations (4.1) for the metric of the open universe. For the scalar field VS given by Eq. (3.5) $\theta_{\mu\nu}$ has the following nonzero components:

$$\theta_0^0=-1/8\sigma a^4 , \tag{4.3}$$

$$\theta_i^j=(1/24\sigma a^4)\delta_i^j . \tag{4.4}$$

Observe that θ_0^0, which represents the energy density of the scalar field, is negative. Its dependence on $a(t)$ is the same as that of radiation energy density. Identifying θ_i^i (no summation over i implied) as the pressure we find that the equation of state is again the same as that of radiation. Thus, for VS given by Eq. (3.5), $\theta_{\mu\nu}$ has just the same form as that of radiation energy-momentum tensor but the opposite sign. It is this negative energy density θ_0^0 which violates one of the assumptions of the singularity theorem. Thus, when the only source of geometry is the scalar field, singularity certainly does not occur. However, if we have a nonzero $T_{\mu\nu}$ whether or not the singularity is removed depends on the behavior of T_0^0. Suppose $T_0^0\propto a^{-n}(t)$. If $n<4$ singularity is surely avoided; if $n=4$ the magnitude of the total energy corresponding to the scalar field has to exceed that of $T_{\mu\nu}$ to overcome the singularity problem. This is the reason why as small a value of σ as is possible is needed. Since the experiments set an upper limit on the present value of the cosmological constant, a lower limit on σ can be obtained if we interpret the vacuum energy (4.3) of the scalar field as the cosmological term.

Using expressions (4.3) and (4.4) in Eq. (4.1) we get

$$\frac{3}{a^4}(a^2-\dot{a}^2)=\frac{3}{2}\frac{a_c{}^2}{a^4}-\kappa T_0^0 , \tag{4.5}$$

$$\frac{1}{a^4}(2a\ddot{a}-a^2-\dot{a}^2)=\frac{1}{2}\frac{a_c{}^2}{a^4}+\kappa T_1{}^1 . \tag{4.6}$$

Since we have assumed spatial isotropy the other space-space component equations are the same as Eq. (4.6). We consider a few simple cases to exhibit singularity avoidance.

(i) The simplest case would be to assume that the only source of curvature is the scalar field which means $T_{\mu\nu}=0$. This has been dealt with in Ref. 12, and we mention the results below.

When $T_{\mu\nu}=0$ Eqs. (4.5) and (4.6) take the form

$$a^2-\dot{a}^2=\frac{a_c{}^2}{2} \tag{4.7}$$

and

$$2a\ddot{a}-a^2-\dot{a}^2=\frac{a_c{}^2}{2} . \tag{4.8}$$

Solving the above equations self-consistently (which necessitates fixing one constant of integration) we get

$$a(t)=\frac{a_c}{\sqrt{2}}\cosh(t+\lambda) . \tag{4.9}$$

Here λ is a constant of integration which can be used to fix the origin of time. The choice $\lambda=0$ renders $a(t)$ to be minimum at $t=0$ given by

$$a_{\min}=\frac{a_c}{\sqrt{2}} . \tag{4.10}$$

Though very simple, the above example has given elegant results. The minimum of $a(t)$ is less than the critical radius. This means that the implosion would not halt as soon as EGCC changes sign but continues till the repulsive field becomes insurmountable. Incidentally, the minimum of $a(t)$ is inversely proportional to the coupling constant σ. Thus the weaker the self-coupling of the scalar field the larger would be the initial "radius" of the Universe. As should be expected, the value of $a_{\min}$ decreases when we have a nonzero $T_{\mu\nu}$. This we shall consider in the next example.

(ii) Let $T_{\mu\nu}$ to be that of a perfect fluid:

$$T_{\mu\nu}=(p+\rho)u_\mu u_\nu-pg_{\mu\nu} , \tag{4.11}$$

where ρ, p, and u_μ are the energy density, pressure and four-velocity of the fluid, respectively. Now Eq. (4.1) takes the form

$$\frac{3}{a^2}(a^2-\dot{a}^2)=\frac{3}{2}\frac{a_c{}^2}{a^2}-\kappa\rho a^2 \tag{4.12}$$

and

$$\frac{1}{a^2}(2a\ddot{a}-a^2-\dot{a}^2)=\frac{a_c{}^2}{2a^2}-\kappa p a^2 . \tag{4.13}$$

With the help of these equations we arrive at the conservation law

$$\frac{d}{da}(\rho a^3)=-3pa^2 . \tag{4.14}$$

Note that the presence of the scalar field, whose VS is nonzero, has not altered the conservation law in any way. This is due to the fact that $\theta^{\mu\nu}{}_{;\mu}$ is identically equal to zero.

Since we are interested in the consequences of the field equations for the early Universe we shall take $T_{\mu\nu}$ to be that of radiation, for which Eq. (4.14) implies $\rho=\epsilon/a^4$, where ϵ is a constant. Then Eqs. (4.12) and (4.13) reduce to the form

$$a^2-\dot{a}^2=\frac{\omega^2}{2} \tag{4.15}$$

and

$$2a\ddot{a}-a^2-\dot{a}^2=\frac{\omega^2}{2} . \tag{4.16}$$

Here $\omega^2=a_c{}^2-\frac{2}{3}\kappa\epsilon$, is a constant. Note that the above equations are of the same form as Eqs. (4.7) and (4.8); thus the solution is

$$a(t)=\frac{\omega}{\sqrt{2}}\cosh t . \tag{4.17}$$

The essential features of $T_{\mu\nu}=0$ case are retained here. The minimum $a(t)$ is given by

$$a'_{\min}=\frac{1}{\sqrt{2}}(a_c{}^2-\tfrac{2}{3}\kappa\epsilon)^{1/2} . \tag{4.18}$$

Since $a'_{\min}$ has to be greater than or equal to zero, we get $\sigma\le\frac{1}{8}\epsilon$. As remarked earlier $a'_{\min}<a_{\min}$. Thus, the presence of matter other than the scalar field reduces the minimum of $a(t)$ and the singularity can occur when σ and ϵ are perfectly matched.

V. SOLUTION TO THE HORIZON PROBLEM

To show that the horizon problem disappears whenever the singularity is removed, we consider the Friedmann-Robertson-Walker metric for the open universe to be of the form

$$ds^2=dt^2-a^2(t)\left[\frac{dr^2}{1+r^2}+r^2(d\theta^2+\sin^2\theta\, d\phi^2)\right] . \tag{5.1}$$

The above metric is completely equivalent to the metric given by expression (3.1). Carrying out the analysis of Sec. III for the above metric, we find that ξ has the same solution as in the case of metric (3.1). We assume that solutions $\xi=\pm\gamma/a(t)$ are stable here also, as the metric (3.1) and (5.1) are physically equivalent. The form of the solutions for $a(t)$ obtained using metric (5.1) differs from those obtained using metric (3.1) and is given by

$$a^2(t)=t^2+\frac{a_c{}^2}{2} \tag{5.2}$$

for $T_{\mu\nu}=0$ and,

$$a^2(t)=t^2+\frac{\omega^2}{2} \tag{5.3}$$

for the radiation case. Notice that the minimum of $a(t)$ is the same as that obtained in Sec. IV.

The horizon distance is defined by

$$l(t,t_0)=a(t)\int_{t_0}^{t}a^{-1}(t')dt' . \tag{5.4}$$

The horizon problem would not occur if $l(t,t_0)$ diverges, as $t_0 \rightarrow 0$ in singular cosmological models or as $t_0 \rightarrow -\infty$ in singularity-free models (see, for example, Weinberg[13]). Substituting for $a(t)$ in the integral (5.1) from Eq. (5.2) or Eq. (5.3) and integrating we get

$$l(t,t_0)=(t^2+a_{\min}{}^2)^{1/2}\ln\left[\frac{t+(t^2+a_{\min}{}^2)^{1/2}}{t_0+(t_0{}^2+a_{\min}{}^2)^{1/2}}\right]. \tag{5.5}$$

Here $a_{\min}$ could be either $a_c/\sqrt{2}$, in which case it is nonzero, or $\omega/\sqrt{2}$, which can be zero for a special choice of σ. Note that $l(t,t_0)$ diverges as $t_0 \rightarrow -\infty$ and the model is free of the horizon problem. Even if the singularity occurs, the horizon distance blows up as $t_0 \rightarrow 0$, implying that the horizon problem disappears whether or not the singularity is removed.

VI. CONCLUSIONS

We have shown in this paper that a nonminimal coupling of the scalar field with gravity leads to a spontaneous symmetry breaking which has many novel features. The most important characteristic is that the symmetry breakdown is permanent and it is more relevant at the early stages of the Universe. This is unlike the usual grand unified theories where symmetry is restored at a high temperature. As a result of the symmetry breakdown, gravity becomes repulsive, (or equivalently the scalar-field energy density has a negative value). This makes possible a singularity-free cosmological model in which the horizon problem also disappears. We believe that the nonminimal coupling of the matter fields with gravity is the answer to a lot of problems posed in the standard model. Work is in progress to see whether a nonminimal coupling of other matter fields to gravity leads to a singularity avoidance.

ACKNOWLEDGMENTS

We should like to thank Professor T. W. B. Kibble and Professor P. C. Vaidya for helpful discussions during their visit to Bangalore. One of us (B.S.S.) is grateful to the University Grants Commission for financial support.

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