Cosmic strings and the formation of galaxies and clusters of galaxies

Neil Turok*
Department of Physics, University of California, Santa Barbara, California 93106

Robert H. Brandenberger†
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
(Received 30 July 1985)

The evolution of density perturbations around cosmic strings in the matter-dominated era is discussed with attention to the specific nature of string perturbations. The parameter $G\mu$ is calculated in two completely independent ways—from the requirements that Abell clusters are formed and that the galaxy-galaxy correlation function matches the observed one. Both values are consistent, lending support to the cosmic string theory.

I. INTRODUCTION

There has been much interest recently in the idea that topologically stable strings formed at a phase transition in the very early Universe[1] ("cosmic strings") could later provide the density perturbations needed to give rise to galaxies and clusters of galaxies.[2] It has recently been shown that the cosmic string theory predicts a correlation function for Abell's clusters of galaxies which closely matches the observed one.[3]

One of the appealing features of the cosmic string theory is that it possesses only one free parameter: the mass per unit length μ of the string (ultimately of course this is to be determined by the unified theory in question). Some of the predictions of the cosmic string theory, such as the correlations of clusters mentioned above, do not involve gravitational coupling and are thus independent of μ. Many predictions however do depend on the gravitational coupling and in particular on the dimensionless parameter $G\mu\sim(m_{\rm GUT}/m_{\rm Planck})^2$.

In this paper we shall determine $G\mu$ by demanding that the gravitational clustering around cosmic string loops matches that observed today. In particular we shall demand that the loops identified with Abell clusters be sufficiently massive to have collapsed objects with the overdensity of Abell clusters about them. We shall also determine $G\mu$ by the independent requirement that the galaxy-galaxy correlation function matches that observed. The fact that these two calculations give consistent answers for $G\mu$ lends further credibility to the cosmic string theory.

Previous calculations[4,5] of $G\mu$ have used methods which are appropriate for models with initial linear density fluctuations and are thus not obviously applicable to the cosmic string theory of galaxy formation. In this paper we shall show that in many ways the nonlinear evolution of perturbations is simpler to calculate in the cosmic string theory than in other theories.

We shall discuss the accretion of matter onto loops in two scenarios: a baryon-dominated universe with $\Omega=0.1$ ($\Omega_{\rm baryon}\lesssim 0.1$ is required from nucleosynthesis and $\Omega_{\rm total}\gtrsim 0.1$ from dynamical mass measurements[6]) and a cold-dark-matter-dominated universe with $\Omega=1$.

In a future paper we discuss the microwave anisotropies produced by cosmic strings for the value of $G\mu$ determined here.[7,8]

II. DENSITY PERTURBATIONS FROM COSMIC STRINGS

The evolution of a network of cosmic strings in a radiation-dominated universe has been discussed by Albrecht and one of us (N.T.) (Refs. 9 and 10). At any time t, the network consists of lengths of string larger than the horizon and closed loops smaller than the horizon. The long strings are straightened out on the horizon scale $\sim t$, and continually produce loops with radius $\sim t$ by intersecting themselves. These "parent" loops then chop themselves up into several smaller "child" loops which oscillate with fixed physical size and energy until they eventually disappear by gravitational radiation. At any time t, a fixed number of child loops of radius $R=\epsilon t$ with ϵ a small number $\epsilon\simeq 0.2$ (Ref. 10) are produced per expansion time per horizon volume. Thereafter their number density decreases as the cube of the scale factor. In a radiation-dominated universe the number of loops per unit volume with radii between R and $R+dR$ is

$$n(R)dR=\frac{\nu}{R^{5/2}t^{3/2}}dR \tag{1}$$

with $\nu\sim 0.01$ (Ref. 10).

Loops emit gravitational radiation at a rate $\dot{M}\sim-50G\mu^2$ (Refs. 5 and 11). Since their mass[10] is given by $M=\beta\mu R\sim 9\mu R$, we find their lifetime $\tau=|M/\dot{M}|\approx\frac{1}{5}(G\mu)^{-1}R$. At the time t there is therefore a cutoff in the distribution of loops at a radius $R\sim 5G\mu t$.

The density of string in loops with radii greater than R is given by

$$\rho_{>R}=\alpha(\mu/t^2)(t/R)^{1/2} \tag{2}$$

with[9] $\alpha=2\beta\nu\sim0.2$.

From (1) and (2), the smallest loops dominate both the number density and energy density. They have a mean separation

$$d=(3/2\nu)^{1/3}R(t/R)^{1/2}\gg R \quad \text{for } R<t \qquad (3)$$

and the long-range time-averaged field of a loop is simply that of a point mass with mass equal to that of the loop.[12] Thus there is a large region around any loop where a spherical-collapse model (see below) should be valid. Of course larger loops will collect smaller loops as well as matter about them.

In more conventional units, the radius R of loops formed in the radiation-dominated period with mean separation dh^{-1} Mpc today is given by

$$\frac{R}{t_{eq}}=(2/3\nu)^{2/3}(d_{eq}/t_{eq})^2$$
$$=(2/3\nu)^{2/3}(d/2000)^2(2.5\times10^4)(\Omega h)^2 \qquad (4a)$$

using

$$t_{eq}=\tfrac{2}{3}H_{eq}{}^{-1}$$
$$=\tfrac{2}{3}\mathscr{H}_0{}^{-1}(1+z_{eq})^{-1}(1+\Omega z_{eq})^{-1/2},$$
$$d_{eq}=\frac{d}{1+z_{eq}},$$
$$z_{eq}=2.5\times10^4\Omega h^2,\quad \mathscr{H}_0{}^{-1}=3000h^{-1}\ \text{Mpc}.$$

t_{eq} is the time of equal matter and radiation, d_{eq} is the separation at that time, and z_{eq} the red-shift. For galaxies, for example,[13] $d=5h^{-1}$ Mpc and, using $\nu=0.01$,

$$R/t_{eq}=5.5\times10^{-3}(\Omega h)^2, \qquad (4b)$$

while for Abell clusters,[14] $d=55h^{-1}$ Mpc,

$$R/t_{eq}=0.67(\Omega h)^2. \qquad (4c)$$

For $h=0.5$ these loops are formed at $t\simeq R/\epsilon<t_{eq}$.

Matter starts to accrete on loops at red-shift $Z_i\sim Z_{eq}$ in the cold-dark-matter-dominated universe, and at $Z_i\sim Z_{dec}$ (decoupling) in the baryon-dominated universe.[6] The loops we are interested in were formed in the radiation-dominated era so we shall use (1) for their distribution up to the time of equal matter and radiation density. In the baryon-dominated universe, for $\Omega=0.1, h\lesssim1$ (nucleosynthesis demands $\Omega h_{baryon}{}^2\lesssim0.1$ and dynamical mass measurements demand $\Omega\gtrsim0.1$), $Z_{eq}=2.5\times10^4\Omega h^2\approx Z_{dec}$ so again we may use (1) and (2) at the time when perturbations start to grow.

The fraction of mass at the time of equal matter and radiation density t_{eq} in loops of radii greater than R is given by

$$\left[\frac{\delta M}{M}\right]_{>R}=6\pi\alpha G\mu\left[\frac{t_{eq}}{R}\right]^{1/2} \qquad (5)$$

using $\rho=1/6\pi Gt_{eq}{}^2$ for the density of matter at t_{eq}.

According to the spherical model (discussed below) the mass collapsed and virialized[6] around a seed mass δM is given at time t by

$$M_{coll}=\delta M(t/t_i)^{2/3}(3\pi/4)^{-2/3},\quad t_i<t<t_f$$
$$=\text{const},\quad t>t_f, \qquad (6)$$

with t_i the time perturbations start to grow, and t_f the time they stop growing (see below). From (1) and (6) the smaller loops together accrete most matter but the more massive mass concentrations are formed around larger loops. Rare, more massive objects accrete around larger loops. The distribution of collapsed masses is predicted from (1) and (6), in the regime where loops have collapsed only a small fraction of the total mass, so competition between neighboring loops is unimportant. Large loops collect smaller loops as well as matter, so if loops of a certain size form galaxies, larger loops form clusters of galaxies.

From (5) and (6), the fraction of all matter collapsed onto loops of size $\sim R$ is

$$\left[\frac{M_{coll}}{M}\right]_R=6\pi\alpha G\mu\left[\frac{t_i}{R}\right]^{1/2}\left[\frac{1+z_i}{1+z_f}\right](3\pi/4)^{-2/3}, \qquad (7)$$

where Z_i is the red-shift at which perturbations start to grow and $1+z_f=\Omega^{-1}-1$ gives the red-shift at which perturbations stop growing.

So far we have ignored the spatial correlations of loops which were determined in Ref. 3. In fact as we shall see these play a crucial role in the gravitational clustering of galaxies. Loops with mean separation d are correlated on scales $\gtrsim d$ because their "parent" loops were chopped off a network of Brownian walks. On scales $\lesssim d$, they are correlated because they are produced in clumps as "parent" loops intersect themselves. In Ref. 3 it was shown that, for loops well inside the horizon,

$$\xi_{\text{loop-loop}}(r/d)\sim0.2(d/r)^2,\quad r\lesssim d. \qquad (8)$$

The fact that loops are correlated with loops of similar size means that the perturbations produced by strings are highly "non-Gaussian" and have nonrandom phases.[15] For Gaussian density perturbations, there is only one independent correlation function: the Fourier transform of the power spectrum $\langle\delta_k\delta_{-k}\rangle$. Given an object or any point in space, the average density at a distance r is simply given by $\bar\rho[1+\xi(r)]$. However in the case of strings, things are different. Given a random point in space, the above is still true. Given a loop, however, the average density at a distance r is very different. The former case, rms fluctuations in density around randomly chosen points, is relevant for the microwave background calculations but the latter case is relevant for answering how the loops give rise to galaxies and clusters. In fact if one sits on a loop of radius R_0, the excess mass $\delta M_{<r}$ within a radius r from the center due to these correlations is given by

$$\frac{\delta M_{<r}}{\bar M}=\frac{\int_0^r d^3r\,\xi_{R_0}(r)\rho_{R_0}}{\int_0^r d^3r\,\bar\rho}, \qquad (9)$$

where $\bar M$ is the average mass inside a radius r, $\bar\rho$ the background density, $\rho_{>R_0}$ the density in loops of radius R_0, and ξ_{R_0} their correlation function. Here we have ignored the correlation of loops with widely differing radii. Using

(8) we find

$$\left.\frac{\delta M_{<r}}{\overline{M}}\right|_{\text{correlations}} = 0.6\left[\frac{d}{r}\right]^2 \frac{\rho_{>R_0}}{\bar{\rho}} = 0.6\left[\frac{d}{r}\right]^2 6\pi G\mu\alpha\left[\frac{t}{r_0}\right]^{1/2} . \quad (10)$$

This provides a picture of the typical density profile of a clump of loops.

The excess mass in (10) may be compared to the excess mass inside r due to random fluctuations from uncorrelated loops. The number of loops of radius R inside r is from (1)

$$N \sim (4\pi/3)2/3\nu r^3 R^{-3/2} t^{-3/2}$$

and these give a rms mass fluctuation

$$\left.\frac{\delta M}{\overline{M}}\right|_{\text{rms}} = \sqrt{N}\,\frac{3\beta\mu R}{4\pi\bar{\rho} r^3} = \tfrac{9}{2}\beta G\mu\left[\frac{8\pi}{9}\nu\right]^{1/2} t^{5/4} r^{-3/2} R^{1/4} \lesssim \tfrac{9}{2}\beta G\mu\left[\frac{8\pi}{9}\nu\right]^{1/2}\left[\frac{t}{r}\right]^{5/4} \quad (11)$$

which is dominated by the largest loops in r, with $R \sim r$. In fact, such fluctuations are atypical and a better idea of the typical fluctuation is given[5] by considering loops of number density n such that $n\,4\pi/3r^3 \sim 1$. These give

$$\left.\frac{\delta M}{\overline{M}}\right|_{\text{typical}} = 6\pi\alpha G\mu\left[\frac{8\pi\nu}{9}\right]^{-1/3}\left[\frac{t}{r}\right] \quad (12)$$

which is actually not very different from (11) for r not much smaller than t, the ratio varying as $(t/r)^{1/4}$. Comparing this with (10) we find

$$\frac{\left.\dfrac{\delta M}{\overline{M}}\right|_{\text{correlation}}}{\left.\dfrac{\delta M}{\overline{M}}\right|_{\text{typical}}} = 0.6\left[\frac{4\pi}{3}\right]^{1/3}\frac{d}{r} \sim \frac{d}{r} . \quad (13)$$

So for scales $r < d$ (the scales we shall be most interested in), the mass fluctuations given by (10) are most important with regard to the density profile around a given loop.

Wakes which are formed behind moving strings also contribute to mass fluctuations.[16] A segment of string of length t moving at velocity produces a wake of mass

$$M_{\text{wake}} = \tfrac{1}{2}\theta(vt)^2 t\rho_b = \tfrac{2}{3}v^2\mu t ,$$

where $\theta = 8\pi G\mu$ is the opening angle of the wake. Numerical simulations have shown that $v \sim 0.2$ or so[10] for long strings, so the mass in the wake is less than the mass in the string. Since loops provide even larger mass perturbations than long strings we see that the effects of wakes are smaller than those of loops.

Now let us turn to the question of exactly how the matter collects around loops. We discuss the spherical model in more detail in Ref. 7, along with the question of the initial peculiar velocities of loops, which we show there one can effectively ignore. Here we merely list the formulas we shall need.

In a matter-dominated universe with initial density $\rho_i = \Omega_i\rho_c$, a shell at initial proper radius r_i, within which the mass perturbation is $\delta M/\overline{M} = \delta_i$, evolves in parametrized form as

$$r = \frac{r_i}{2\Delta_i}(1-\cos\theta), \quad t = \frac{3t_i}{4\Delta_i^{3/2}}(\theta - \sin\theta) . \quad (14)$$

r is the proper radius and

$$\Delta_i = \delta_i - (\Omega_i^{-1} - 1) . \quad (15)$$

The shell initially at r_i stops expanding at $\theta = \pi$, yielding (for $\Omega_i \approx 1$) Eq. (6) for the collapsed mass at any time. As one moves out from the seed mass, the fractional mass perturbation δ_i gets smaller and smaller—the above solution is only valid for $\delta_i > \Omega_i^{-1} - 1$. Outside the radius, where $\delta_i = \Omega_i^{-1} - 1$, matter shells are not bound and keep expanding forever. From (6) one sees that the last shells to collapse do so at

$$(t/t_i)^{2/3} \sim \delta_i^{-1} \sim (\Omega_i^{-1} - 1)^{-1} . \quad (16)$$

After this time very little matter accretes onto loops. Ω_i is related to Ω today in a matter-dominated universe by

$$\Omega_i^{-1} - 1 = \frac{\Omega^{-1} - 1}{1 + z_i} . \quad (17)$$

Thus, mass stops accreting at

$$1 + z_f \sim \Omega^{-1} - 1 . \quad (18)$$

This will be important in the baryon-dominated universe.

III. FORMATION OF ABELL CLUSTERS

In this section we will calculate $G\mu$ from the requirement that loops with the mean separation of Abell clusters be massive enough to collapse an object with the overdensity of an Abell cluster. This gives a much cleaner determination of $G\mu$ than previous calculations.[4,5]

The calculation proceeds in two steps. We first determine the red-shift $1 + z_{\max}$ for which Abell clusters start to collapse by comparing the observed overdensity with the overdensity calculated from the spherical collapse model. The shell radius which begins to collapse at red-shift $1 + z_{\max}$ depends on $G\mu$. Demanding it be the Abell radius fixes $G\mu$.

Abell clusters[17] are defined as regions containing more than 50 bright galaxies inside an Abell radius $r_A = 1.5h^{-1}$ Mpc. Their mean separation is $d_A = 55h^{-1}$ Mpc (Ref. 14). Their exact overdensity is uncertain, but may be estimated in two ways. First, 5% of all bright galaxies are thought to be in Abell clusters.[18] If the fraction of all matter in Abell clusters is $0.05f_5$, then their overdensity is

$$\frac{d_A^{\,3}}{\frac{4\pi}{3}r_A^{\,3}}0.05f_5 = 590f_5 . \quad (19)$$

Second, observations of the velocities of galaxies in Abell clusters[19] indicate the density at a distance r from the center is given by

$$\rho(r)=2.4\times10^{-27}h^2\ \mathrm{g\,cm^{-3}}r^{-2}\ , \tag{20}$$

where r is in h^{-1} Mpc. Thus the overdensity inside an Abell radius is

$$\frac{M_{<r_A}}{M}=\frac{3\rho(r_A)}{\bar\rho}=170\Omega^{-1}\ , \tag{21}$$

using $\bar\rho=1.9\times10^{-24}\Omega h^2$ g cm^{-3}. For an $\Omega=1$ universe, (19) and (21) agree if we take $f_5\sim0.3$, interpreting this as the fact that bright galaxies form preferentially in overdense regions. A baryon-dominated universe with $\Omega=0.1$ has trouble explaining the existence of Abell clusters at all, as we shall see.

In the spherical model, the overdensity inside a shell which is at its maximum radius $r_{\max}$ is given by $(3\pi/4)^2=5.6$. Thereafter the shell collapses and according to numerical simulations,[20] virializes at a time $t_{\rm coll}\sim1.8t_{\max}$ and a radius $\sim\frac{1}{2}r_{\max}$. At this time, its overdensity is

$$\left[\frac{3\pi}{4}\right]^2 8\left[\frac{t_{\rm coll}}{t_{\max}}\right]^2\approx150 \tag{22}$$

(the last factor being due to the background density decreasing like t^{-2} during the collapse process). Thereafter the overdensity grows as the cube of the scale factor. For the $\Omega=1$ universe, we conclude that Abell clusters formed very recently, and that the mass inside an Abell radius is virialized. From the spherical model it follows that the Abell radius turned around at $1+z_{\max}\sim1.5$ and the radius just turning around now is

$$(1+z_{\max})^{4/3}3h^{-1}\ \mathrm{Mpc}\sim5h^{-1}\ \mathrm{Mpc}\ .$$

In the $\Omega=0.1$ universe, shells stop collapsing at $1+z_f\sim\Omega^{-1}\sim10$. Thus one should not see many objects with an overdensity less than

$$(3\pi/4)^2 8(1+z_f)^3\sim4\times10^4\ .$$

To be more precise, one can show in this case that the final overdensity inside an unbounded shell is a constant, given by

$$\left[\frac{\Omega_i^{-1}-1}{\Omega_i^{-1}-1-\delta_i}\right]^{3/2}\ . \tag{23}$$

In order to get an overdensity much less than 4×10^4 therefore one has to fine-tune the radius r_i around any particular loop. This would make Abell clusters very rare objects. The baryon-dominated universe only works if we take

$$\frac{M_{<r_A}}{\bar M}\gtrsim4\times10^4\ .$$

$G\mu$ is determined by equating the collapse radius from the spherical model with the Abell radius. For bound shells, $\delta_i>\Omega_i^{-1}-1$ and from (6) the initial perturbation δ_i inside the shell collapsing at $Z_{\max}$ is given by

$$\delta_i\left[\frac{1+Z_{\rm eq}}{1+Z_{\max}}\right]=\left[\frac{3\pi}{4}\right]^{2/3}\ . \tag{24}$$

We also know the initial radius r_i of this shell since its maximum radius is $r_{\max}=2r_A=3h^{-1}$ Mpc. By (4)

$$\begin{aligned} r_i&=\delta_i r_{\max}\\ &=\delta_i\tfrac{3}{2000}t_{\rm eq}z_{\rm eq}{}^{3/2}\Omega^{1/2} \end{aligned} \tag{25}$$

and

$$\delta_i=\frac{\beta\mu R}{\dfrac{4\pi r_i^3}{3}\dfrac{1}{6\pi G t_{\rm eq}^2}}=\frac{9\beta}{2}G\mu\left[\frac{R}{t_{\rm eq}}\right]\left[\frac{t_{\rm eq}}{r_i}\right]^3 \tag{26}$$

which with (25) and (4c) gives $G\mu$ in terms of δ_i determined from (24). Thus

$$\begin{aligned} G\mu&=\frac{2}{9\beta}\frac{1}{0.67}\left[\frac{3}{2000}\right]^3\delta_i{}^4Z_{\rm eq}{}^{9/2}\Omega^{3/2}\frac{1}{(\Omega h)^2}\\ &=\frac{2}{9\beta}\frac{1}{0.67}\left[\frac{3}{2000}\right]^3\left[\frac{3\pi}{4}\right]^{8/3}Z_{\rm eq}{}^{1/2}(1+Z_{\max})^4\frac{\Omega^{3/2}}{(\Omega h)^2}\\ &=\frac{2}{9\beta}\frac{1}{0.67}\left[\frac{3}{2000}\right]^3\left[\frac{3\pi}{4}\right]^{8/3}(2.5\times10^4)^{1/2}(1+Z_{\max})^4h^{-1}\simeq1.10^{-6}h^{-1}\ , \end{aligned} \tag{27}$$

using $(1+Z_{\max})=1.5$ and $\beta=9$. For $\Omega=1$ one must[6] take $h\sim0.5$. Thus

$$G\mu\,\Big|_{\substack{\text{cold dark matter}\\ \Omega=1\\ h=0.5}}\simeq2\times10^{-6}\ . \tag{28}$$

For $\Omega=0.1$, we must have $1+Z_{\max}\sim10$, so

$$G\mu\,\Big|_{\substack{\text{baryons}\\ \Omega=0.1\\ h=1}}\simeq2.10^{-3}\ , \tag{29}$$

although as stated earlier, in this case Abell clusters would have an overdensity far larger than the observations indicate.

IV. CLUSTERING OF GALAXIES

We shall now calculate $G\mu$ from a completely independent requirement—that the correlation function for galaxies matches that observed. As mentioned in Ref. 3, the primordial correlations of loops (i.e., those before gravitational clustering has occurred) are four times smaller than the observed correlations of galaxies.

In this section we shall construct a simple model for the gravitational enhancement of correlations. This involves several extra assumptions about the gravitational clustering of loops which make it a less clear determination of $G\mu$, one that will certainly need to be checked by N-body simulations before it can be reliably believed. Our picture of the distribution of loops is that they occur in clumps, and that around any given loop the excess mass is given (for scales $r<d$, the mean separation of the loops) by (10). We will model the evolution of a clump by a spherical model. Since the number of loops in a clump is ~ 10 this assumption is dubious but nevertheless should give a rough idea at least of what happens.

Now in the spherical model, one assumes each shell evolves separately, without crossing other shells. Since the number of loops in a given shell is constant, we should have, in some average sense,

$$\bar{n}[1+\xi(r)]r^2dr=\bar{n}_i[1+\xi_i(r_i)]r_i^2dr_i \;, \tag{30}$$

where $\bar{n}$ is the mean number density of loops, $\xi(r)$ the correlation function, and r the radius. The subscript i refers to the initial time.

We shall follow the evolution of a shell with initial radius r_i using (10) for the overdensity inside the shell at t_{eq}, when perturbations start to grow. Equation (30) yields

$$1+\xi(r/d)=\frac{\partial(r_i/d_i)^3}{\partial(r/d)^3}\left[1+\xi\left[\frac{r_i}{d_i}\right]\right] , \tag{31}$$

where d is the mean separation of the loops.

From (4) and (5) it follows that for collapsed shells

$$r/d=\tfrac{1}{2}r_i/d_i\delta_i^{-1}\left[\frac{t_{eq}}{t}\right]^{2/3} . \tag{32}$$

From the spherical model it follows that once a shell has collapsed

$$\frac{r}{d}\leq 0.2\frac{r_i}{d_i} . \tag{33}$$

Thus, (32) is applicable only provided (33) is satisfied. Using (10) for the initial mass excess δ_i we obtain

$$r/d=\frac{1}{1.2}\left[\frac{r_i}{d_i}\right]^3\left[6\pi\alpha G\mu\left[\frac{t_{eq}}{R}\right]^{1/2}\right]^{-1}\left[\frac{t_{eq}}{t}\right]^{2/3} . \tag{34}$$

Hence with (31)

$$\left[1+\xi\left[\frac{r}{d}\right]\right]=1.2\times 6\pi\alpha G\mu\left[\frac{t_{eq}}{R}\right]^{1/2}\left[\frac{t}{t_{eq}}\right]^{2/3}\frac{1}{3}\left[\frac{d}{r}\right]^2\left[1+\xi_i\left[\frac{r_i}{d_i}\right]\right] \tag{35}$$

which produces the correct power law for the galaxy-galaxy correlation function if ξ_i can be neglected. The observed galaxy-galaxy correlation function is[21]

$$\xi_{gg}(r/d)\approx 1.1(d/r)^2,\;\; 0.02\lesssim r/d\lesssim 1 \;, \tag{36}$$

and comparing this with (35) we find, using (4b),

$$\begin{aligned} G\mu &= \frac{3.3}{1.2}\frac{1}{6\pi\alpha}(5.5\times 10^{-3})^{1/2}\frac{1+z_f}{1+z_{eq}}\Omega h \\ &= 2\times 10^{-6}h^{-1}\;\text{ for }\Omega=1 \\ &= 2.5\times 10^{-5}h^{-1}\;\text{ for }\Omega=0.1 \;, \end{aligned} \tag{37}$$

where the range of r/d for which (35) reproduces (36) is determined by the condition from (12) that $r_i/d_i<1$ and from ignoring ξ_i which requires $r_i/d_i\gtrsim 0.4$. r/d is related to $(r_i/d_i)^3$ by (34). Hence the range is

$$0.03\lesssim\frac{r}{d}=\frac{1}{3.3}\left[\frac{r_i}{d_i}\right]^3\lesssim 0.3 . \tag{38}$$

On larger scales the fluctuations due to uncorrelated loops are important. In fact the spectrum $\delta M/M\propto 1/r$ from (11) also predicts $\xi\propto 1/r^2$.

The value of $G\mu$ found here is entirely consistent with the previous value (27) for the $\Omega=1$ universe. The baryon-dominated universe has problems forming Abell clusters and the two values are not consistent.

We can perform a further test by asking when a certain region around a "galaxy" loop virialized. Using the overdensity $\delta M/M\sim 1.5\times 10^4\Omega^{-1}h^{-2}$ for the central parts of a galaxy ($r\lesssim 30h^{-1}$ kpc) derived from rotation curves,[6] one finds $1+z_{max}\sim 10$ for the red-shift when this radius turned around. Just as in (25)—(27), using (4b) instead of (4c), we find

$$G\mu=\frac{2}{9\beta}\frac{1}{5.5\times10^{-3}}\left[\frac{60\times10^{-3}}{2000}\right]^3\left[\frac{3\pi}{4}\right]^{8/3}$$
$$\times(2.5\times10^4)^{1/2}(1+z_{\rm max})^6h^{-1}$$
$$=2\times10^{-6}h^{-1} \qquad (39)$$

which is again consistent with our previous values. Galaxies are far in the nonlinear regime and thus this criterion is not as reliable as the previous two.

Using these values of $G\mu$, we can now determine which loops have collapsed all the matter upon them—from (7), their radius is given by

$$\left[\frac{R}{t_{\rm eq}}\right]^{1/2}=6\pi\alpha G\mu\frac{1+z_{\rm eq}}{1+z_f}\left[\frac{3\pi}{4}\right]^{-2/3}$$
$$\sim3\times10^{-2} \qquad (40)$$

for $\Omega=1$, $h=0.5$ and from (4a) their separation today is

$$d\sim3.5h^{-1}\ {\rm Mpc}\ . \qquad (41)$$

For $\Omega=0.1$, $h=1$, $(R/t_{t_{\rm eq}})^{1/2}\sim10^{-2}$ and

$$d\sim10h^{-1}\ {\rm Mpc}\ . \qquad (42)$$

Thus all matter has accreted on galaxies in the $\Omega=1$ universe, almost all in the $\Omega=0.1$ universe. In this picture, the galaxy scale emerges as the scale below which loops have accreted all the matter and so competition between the loops is important. For this reason the distribution of loop masses (1) cannot be naively applied to give a distribution of masses of objects smaller than galaxies.

V. CONCLUSION

We have determined the parameter $G\mu$ from two completely independent requirements—that the cosmic string loops are massive enough to form Abell clusters, and that they cluster strongly enough to fit the galaxy-galaxy correlation function (the correct power law emerges naturally from our simplified model). For an $\Omega=1$ cold-dark-matter-dominated universe, we find consistent values for $G\mu$, for which the microwave background anisotropy (discussed in Refs. 7 and 8) is well below the observational limits. For the $\Omega=0.1$ baryon-dominated universe, it is difficult to form Abell clusters at all. If we ignore this, the value for $G\mu$ obtained from galaxy clustering is also (but not so far) below the observational bounds. We will discuss the bounds on $G\mu$ from gravitational radiation elsewhere.[22]

Our determinations of $G\mu$ give consistent results, but nevertheless, because of the approximations in the analyses, should only be viewed as order-of-magnitude determinations. The spherical model is certainly not exact, and our calculations involve average quantities like β, ν, and the overdensity of Abell clusters, so at this stage one can hardly do better than an order-of-magnitude estimate.

Note added. We have also considered the case of an $\Omega=1$ neutrino-dominated universe with cosmic strings. Remarkably enough, the two main calculations of this paper go through *unchanged.* The formation of Abell clusters is unaltered since the cluster mass $M_{\rm cl}$ is in fact very similar to the neutrino Jeans mass $M_\nu\sim2\times10^{15}M_0$. The clustering of "galaxy" loops is also unaffected since the motion of loops is not affected by the matter around them.

Note added in proof. We recently received a paper from H. Sato reporting similar calculations.

ACKNOWLEDGMENTS

For useful discussions we thank A. Albrecht, D. Schramm, A. Stebbins, J. Traschen, M. Turner, and especially W. Zurek, who has performed similar calculations independently. The work of N.T. was supported by National Science Foundation (NSF) Grant No. PHY83-13324, the work of R.B. by NSF Grant No. PHY82-17853, supplemented by funds from the National Aeronautics and Space Administration.

*Present address: Department of Theoretical Physics, Imperial College, London SW7, England.

†Present address: Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England.

[1]T. W. B. Kibble, J. Phys. A 9, 1387 (1976).

[2]Ya. Zeldovich, Mon. Not. R. Astron. Soc. **192**, 663 (1980); A. Vilenkin, Phys. Rev. Lett. **46**, 1169 (1981); **46**, 1496(E) (1981); Phys. Rep. **121**, 263 (1985).

[3]N. Turok, Phys. Rev. Lett. **55**, 1801 (1985).

[4]A. Vilenkin and Q. Shafi, Phys. Rev. Lett. **51**, 1716 (1983).

[5]N. Turok, Nucl. Phys. **B242**, 520 (1984).

[6]For a review of the relevant standard cosmology, see J. Primack, Report No. SLAC-PUB-3387 (unpublished); lectures presented at the International School of Physics, "Enrico Fermi," Varenna, Italy, 1984 (unpublished).

[7]R. Brandenberger and N. Turok, following paper, Phys. Rev. D **33**, 2182 (1986).

[8]J. Traschen, N. Turok, and R. Brandenberger, ITP Report No. NSF-ITP-85-110, 1985 (unpublished).

[9]A. Albrecht and N. Turok, Phys. Rev. Lett. **54**, 1868 (1985).

[10]A. Albrecht and N. Turok (in preparation).

[11]T. Vachaspati and A. Vilenkin, Phys. Rev. D **31**, 3052 (1985).

[12]N. Turok, Phys. Lett. **123B**, 387 (1983).

[13]M. Davis and J. Huchra, Astrophys. J. **254**, 437 (1982).

[14]N. A. Bahcall and R. M. Soneira, Astrophys. J. **270**, 20 (1983).

[15]N. Turok and D. N. Schramm, Nature (London) **312**, 598 (1984).

[16]J. Silk and A. Vilenkin, Phys. Rev. Lett. **53**, 1700 (1984).

[17]G. Abell, Astrophys. J. Suppl. 3, 211 (1958).

[18]N. A. Bahcall, Astrophys. J. **232**, 689 (1979).

[19]P. J. E. Peebles, in *Physical Cosmology,* 1979 Les Houches Lectures, edited by R. Balian, J. Audouze, and D. N. Schramm (North-Holland, Amsterdam, 1980), p. 213.
[20]P. J. E. Peebles, Astrophys. J. **75**, 13 (1970).
[21]M. Davis and P. J. E. Peebles, Astrophys. J. **267**, 465 (1983).
[22]R. Brandenberger, A. Albrecht, and N. Turok, ITP report, 1985 (unpublished).