Solar System constraints and signatures for dark-matter candidates

Lawrence M. Krauss*
Department of Physics, Harvard University, Cambridge, Massachusetts 02138

Mark Srednicki
Department of Physics, University of California, Santa Barbara, California 93106

Frank Wilczek
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
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We show that if our galactic halo were to consist of scalar or Dirac neutrinos with mass greater than ~ 12 GeV, capture by the Earth and subsequent annihilation would yield a large flux of neutrinos at the surface which could be seen in proton-decay detectors. The luminosity of Uranus provides comparable constraints. Capture in the Sun can yield supplementary information, with detectable signals possible for masses as low as 6 GeV for both Dirac and Majorana neutrinos, scalar neutrinos, and photinos. We discuss in detail the question of evaporation, on which our results and others depend sensitively. We suggest one method of approximating evaporation rates from the Earth and Sun and discuss potential problems with earlier estimates. Finally, we describe how particles which avoid these constraints may still be detectable by bolometric neutrino detectors and isolate a new method to remove backgrounds to this signal in such detectors.

The problem of how to determine what form of matter constitutes the dark halos known to surround galaxies has received much attention recently. Possible detection schemes for various dark-matter candidates have been considered,[1,2] as well as indirect signatures which could be present in cosmic-ray, gamma-ray, and neutrino fluxes at the surface of the Earth.[3,4] In this paper, we discuss other indirect signatures of dark matter that consists of heavy, weakly interacting particles. The neutrino flux coming from annihilation of dark-matter particles trapped inside the Earth is large enough to enable proton-decay detectors to rule out scalar neutrinos or Dirac neutrinos with mass greater than ~ 12 GeV. Similar fluxes could arise from annihilations in the Sun;[4] by comparing the two processes different possible forms of dark matter can be distinguished, and limits extended to ~ 6 GeV. We also discuss other possible effects, such as annihilations in dark-matter atmospheres surrounding the Earth and Sun (which could produce visible gamma rays), and annihilations inside heavy planets (which could produce unacceptable amounts of heat). These effects appear to be less significant. All of these results depend sensitively on estimates of evaporation. Our analysis differs from earlier work[5] in ways which can affect our conclusions. We discuss in detail both our approach, and the potential problems which arise in trying to approximate this nonequilibrium problem by equilibrium or simple dynamical approximations. It would be very worthwhile to attempt a full Monte Carlo simulation of this problem, and we are presently investigating this possibility.

Finally, direct detection of many dark-matter candidates by bolometry appears possible.[2,12] We describe how the limits from indirect detection due to Solar System capture can be extended by these means. In particular we point out an important new method for reducing the background in these detectors for the signal from the scattering of weakly interacting particles off nuclei.

We begin by quoting the rate at which the Sun or a planet will trap dark-matter particles, as computed in Ref. 5:

$$\begin{aligned}
\dot{N} &= (\pi R_{\mathrm{cap}}{}^2 \bar{v} \rho_x m_x{}^{-1})(\tfrac{3}{2}\pi)^{1/2}(a\sigma_N/\sigma_{ON}) \\
&\quad \times \min[1, 3b m_x m_N (m_x{}^2+m_N{}^2)^{-1}(v_{\mathrm{esc}}{}^2/\bar{v}^2)] \,, \\
R_{\mathrm{cap}}{}^2 &= 2GMR/\bar{v}^2 \,, \\
\sigma_{\mathrm{ON}} &= m_N R^2/M \,, \\
v_{\mathrm{esc}}{}^2 &= 2GM/R \,.
\end{aligned} \tag{1}$$

In these formulas, M and R are the mass and radius of the Sun or planet; m_x is the mass of the dark-matter particle, ρ_x its mean mass density in the Solar System, and $\bar{v}$ its rms velocity; m_N is the mass of a typical nucleus off which the particle elastically scatters with cross section σ_N; and a and b are numerical factors of order unity which depend on the density profile of the Sun or planet. For the Sun,[5] $a_\odot \simeq 0.89$ and $b_\odot \simeq 3.4$. For the Earth, we estimate $ab \simeq 0.34$ (see below), and the trapping rate is

$$\begin{aligned}
\dot{N}_\oplus &= (4.7\times 10^{17}\ \mathrm{sec}^{-1})(3ab) \\
&\quad \times (\rho_{0.3}\bar{v}_{300}{}^{-3}\sigma_{N,32})(1+m_x{}^2/m_N{}^2)^{-1} \,,
\end{aligned} \tag{2}$$

where m_x and m_N are in GeV, $\sigma_N = (10^{-32}\ \mathrm{cm}^2)\sigma_{N,32}$, $\rho_x = (0.3\ \mathrm{GeV/cm}^3)\rho_{0.3}$, and $\bar{v} = (300\ \mathrm{km/sec})\bar{v}_{300}$. If we assume that the trapped particles all disappear via annihilations at the center of the Earth, and not by evaporation (see below), we find a flux at the Earth of neutrinos of species i:

$$\phi_{\oplus i}=\tfrac{1}{2}N_i\dot{N}_{\oplus}/4\pi R^2 , \quad (3)$$

where N_i is the number of type-i neutrinos produced per annihilation. Most of the neutrinos in this flux will be moving directly upward; using the numerical methods described below, we find that the fraction of neutrinos with trajectories at the surface within five degrees of vertical is approximately $[1-\exp(-0.032m_x)]$, and within ten degrees of vertical is approximately $[1-\exp(-0.125m_x)]$. For a cone of half-angle five degrees (solid angle 0.024 sr), this gives a flux

$$\begin{aligned}\phi_{\oplus i}=&(1.9\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1})(3ab)\\ &\times N_i\rho_{0.3}\sigma_{N,32}\bar{v}_{300}{}^{-3}(1+m_x{}^2/m_N{}^2)^{-1}\\ &\times[1-\exp(-0.032m_x)] .\end{aligned} \quad (4)$$

This should be compared to atmospheric background, which for $\nu_\mu+\bar{\nu}_\mu$ with $E_\nu>1$ GeV is predicted[6] to be about

$$\phi_{\mathrm{atm}}\simeq 0.03\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1} . \quad (5)$$

Dark-matter candidate particles can have σ_N as large as 10^{-33} cm^2 if they weigh $\gtrsim 5-10$ GeV and they couple to nuclei with a vector-vector interaction. This is the case for both Dirac and scalar neutrinos; each has been suggested as a dark-matter candidate.[7,8] If the particle is a Dirac neutrino whose left- (right-) handed part has weak hypercharge Y_L (Y_R), let $\bar{Y}=\frac{1}{2}(Y_L+Y_R)$; if the particle is a scalar neutrino with weak hypercharge Y, let $\bar{Y}=Y$. Then the cross section for elastic scattering of a nonrelativistic dark-matter particle off a nucleus with Z protons, N neutrons, $\bar{N}=N-(1-4\sin^2\theta_w)Z$ is (for $\bar{N}\gg 1$)[2]

$$\sigma_N=\frac{1}{2\pi}G_F{}^2\bar{Y}{}^2\bar{N}{}^2m_x{}^2m_N{}^2(m_x+m_N)^{-2} . \quad (6)$$

For iron, which accounts for about 40% of the mass of the Earth, the cross section is

$$\sigma_{\mathrm{Fe}}=(1.6\times 10^{-32}\ \mathrm{cm}^2)\bar{Y}{}^2m_x{}^2(m_x+m_{\mathrm{Fe}})^{-2} . \quad (7)$$

Scattering cross sections for the other most common elements in the earth—silicon, magnesium, and oxygen—are much lower, even for $m_x=12$ GeV. (As we will see, $m_x<12$ GeV results in evaporation rather than annihilation, and hence is not interesting.) The resulting flux of neutrinos within five degrees of vertical is

$$\begin{aligned}\phi_{\oplus i}=&(0.38\ \mathrm{cm}^{-2}\mathrm{sec}^{-1}\mathrm{sr}^{-1})(3ab)\bar{Y}{}^2\\ &\times N_i\rho_{0.3}\bar{v}_{300}{}^{-3}[8m_x{}^2m_{\mathrm{Fe}}{}^2(m_x+m_{\mathrm{Fe}})^{-2}\\ &\times(m_x{}^2+m_{\mathrm{Fe}}{}^2)^{-1}]\\ &\times[1-\exp(-0.032m_x)] .\end{aligned} \quad (8)$$

This flux is plotted as a function of m_x in Fig. 1, including evaporation effect for $m_x\lesssim 12$ GeV, to be described later. The major uncertainty concerns N_i. For scalar neutrinos, it is possible to have each annihilation event produce a pair of neutrinos of the same species as the scalar neutrinos.[8] Thus if the dark matter consisted of scalar electron neutrinos, $N_{\nu_e}=2$, all other $N_i=0$. Alternatively, if all gauge fermions were very heavy, it is possible to have suppressed direct neutrino pair production; secondary neutrinos would still be produced by weak decays of other annihilation products. For Dirac neutrinos, annihilation occurs through an intermediate Z^0, which gives $N_\nu=N_{\bar{\nu}}\simeq 0.07$ for each species of neutrino. Obviously our predicted fluxes place strong constraints on scalar and Dirac neutrinos as dark matter, but it is necessary to analyze the data for each alternative in order to make definite statements, since the experimental signatures of different neutrino species are very different.

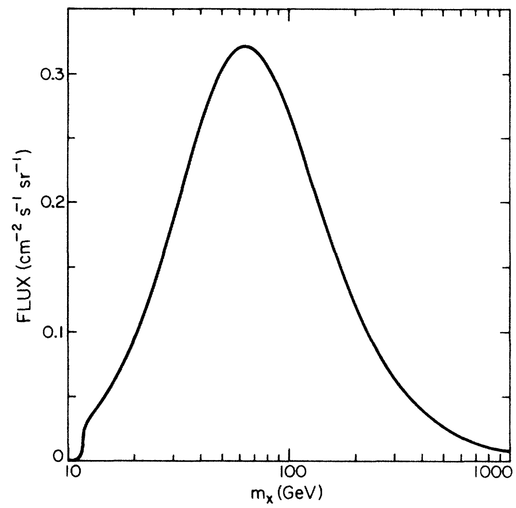

FIG. 1. The flux of neutrinos in a vertical cone of half-angle 5° at the Earth's surface in as a function of m_x with $(3ab)\bar{Y}{}^2N_i\rho_{0.3}\bar{v}_{300}{}^{-3}=1$.

Scalar or Dirac neutrinos, and also Majorana neutrinos and Higgs fermions with masses $\gtrsim 6$ GeV, are likely to be trapped and annihilate in the Sun, resulting in potentially observable fluxes similar to or greater than those for photinos.[4] Majorana fermions (i.e., neutrinos with a Majorana mass, massive photinos, massive Higgs fermions) all have $Y_L=-Y_R$ and hence do not scatter coherently off nuclei. Instead, nonrelativistic Majorana fermions couple to the spin of the nucleus. Since all common nuclei in the Earth are spinless, the Earth will not trap significant numbers of Majorana fermions, and thus, unlike the Sun, cannot be used to constrain them as dark-matter candidates. We also note that neutrino fluxes from the Sun due to trapped photinos can be much larger than those quoted in Ref. 4 if we are willing to consider smaller scalar-quark masses (~ 30 GeV), and hence larger cross sections. This would also result in a density of photinos below critical density.

Notice that heavy ($\gtrsim 12$ GeV) coherently scattering particles tend to give rise to a large terrestrial flux, whereas the solar flux is most significant for lighter particles down to 6 GeV, whether or not they interact coherently. Thus the Earth and Sun together give complementary bounds.

We now turn to the question of the distribution of dark-matter particles inside the Earth. We assume that they are described by a Boltzmann distribution inside, with energy given by the gravitational potential energy, and a temperature T which is some average of the interior temperature of the Earth:[9]

$$n(r)=n_c\exp[-m_x\xi(r)/T] ,\\ \xi(r)=G\int_0^r dz\, M(z)/z^2 . \tag{9}$$

Here, r is the distance from the center of the Earth, $n(r)$ is the number density of dark-matter particles, n_c is the number density at the center of the Earth, and $M(r)$ is the mass of the Earth contained within r. We use a schematic density profile for the Earth of the form

$$\rho(r)=(11.7\ \mathrm{g\,cm^{-3}})(1-r/6r_0),\quad r<r_0\\ =(5.84\ \mathrm{g\,cm^{-3}})[1-(r-r_0)/3(R-r_0)] ,\quad r>r_0 , \tag{10}$$

$$r_0=3.4\times 10^8\ \mathrm{cm} ,$$

$$R=6.37\times 10^8\ \mathrm{cm} .$$

The assumption of a Boltzmann distribution inside the Earth is made both for reasons of simplicity and because the average lifetime of particles in the Earth is long compared to collision times. In fact as we later discuss, both because of capture and evaporation effects, the high energy tail of the distribution may deviate from this form. Unfortunately, it is this region which is important for estimating evaporation rates. Outside the Earth, we assume that the dark-matter particles form a collisionless gas which we divide into two components: ballistic particles, which move on orbits which will eventually intersect the Earth's surface, and escaping particles, which will eventually escape to infinity, without intersecting the Earth's surface. We ignore here a third possibility, namely, satellite particles (those which orbit the Earth without intersecting it), because of the lack of a physical mechanism to populate this component. Simple analytical formulas are known[10] for the number densities of both ballistic and escaping particles. The number density is continuous at the Earth's surface. The flux of escaping particles, under these assumptions, is[10]

$$F=n_s(T/2\pi m_x)^{1/2}(1+GMm_x/RT)\exp(-GMm_x/RT) , \tag{11}$$

where n_s is the number density of dark-matter particles at the surface of the Earth obtained using Eq. (9). The number density of particles escaping per unit time is this flux times the surface area of the Earth. It is worthwhile to pause at this time and reflect upon the validity of Eq. (11). As mentioned above, the use of a Boltzmann distribution for dark-matter particles inside the Earth implies that one is assuming thermal equilibrium conditions and relying on statistical, not dynamical, arguments to derive the velocity distribution. In this regard our results for evaporation differ from earlier estimates of Press and Spergel. They essentially multiplied the velocity prefactor in the first set of parentheses in Eq. (11) by an additional suppression factor obtained by estimating the mean collision time for a particle in highly eccentric orbit, which can be long compared to the mean crossing time characterized by the thermal velocity prefactor. They argued that this time would be typical of that required to replenish the Boltzmann tail. We have doubts about the validity and consistency of this analysis. In the first place, use of the exponential distribution implies that one is assuming thermal equilibrium conditions in which case Eq. (11) is valid as it stands without any additional factors. Any nonequilibrium effects which might require (11) to be altered would require changes in the whole formula *including the exponential factor* which is far more important than the prefactor. In addition, dynamical arguments appropriate to collisions of particles in the mean of the distribution are not likely to be appropriate to those which are responsible for populating the exponentially small Boltzmann tail, which can in fact be populated by rare events. For example, an exponentially small number of particles will undergo more than one collision on a single pass through the Earth or Sun, or receive much more than the mean energy transfer in a single collision. If one is going to go beyond the equilibrium approach and attempt to dynamically determine the flux from the surface one must consider such processes. Finally, in this case evaporation is not taking place in a vacuum, but rather amidst a constant flux of energetic incoming particles, which can preferentially populate the region near the Boltzmann tail immediately upon their capture. This could result in evaporation rates somewhat larger than our estimates. It is also worthwhile pointing out that our results are very sensitive to the assumed temperature in the core of the Earth, and to a lesser extent the Sun. Changing the temperature from our assumed value has a much more significant effect than changing the prefactor discussed above.

For these reasons it would be worthwhile to do a detailed numerical simulation to verify our results. We are now investigating this possibility. This is particularly important since (a) several of the potential effects which we will estimate to be subdominant in the Earth and Sun could be significant if evaporation rates, and thus also the masses susceptible to our analysis, are smaller, and (b) smaller evaporation rates mean better mass limits. In any case, we feel, in the absence of a detailed dynamical simulation, that the thermal evaporation rate given in (11) is the best approximation one can make.

The number of particles disappearing through annihilation is much less controversial; it is given by

$$\dot{N}_{\mathrm{ann}}=\langle\sigma_a v\rangle\int d^3r\, n^2(\mathbf{r}) , \tag{12}$$

where $\langle\sigma_a v\rangle$ is the thermally averaged annihilation cross section times relative velocity. For weakly interacting particles with mass $\sim 1-100$ GeV, $\langle\sigma_a v\rangle$ must be about $10^{-26}\ \mathrm{cm^3\,sec^{-1}}$ in order to end up with a critical density of such particles left over from the big bang.[7,3] We set the annihilation rate plus the escape rate equal to the capture rate, and solve for n_s. We find that there is a sharp crossover from escape to annihilation as the dominant disappearance mechanism as a function of m_x. The crossover point is relatively independent of the capture rate and annihilation cross section; for $T=4000$ K, it occurs at about 12 GeV. (We actually determine only

m_x/T in this way; for $T=7000$ K, which is a good upper limit, the crossover is at 21 GeV.) Figure 2 shows the annihilation and escape rates as a function of m_x using the following parameters: $T=4000$ K; a capture rate given by Eq. (2) using $\sigma_{\rm Fe}$ as given in Eq. (7); $\langle\sigma_a v\rangle=10^{-26}$ cm^3 sec^{-1}; and $\rho_{0.3}=\bar{v}_{300}=3ab=\bar{Y}=1$.

We computed the flux of neutrinos reaching a point $\mathbf{R}$ on the surface of the Earth by integrating $n^2(\mathbf{r})/4\pi|\mathbf{r}-\mathbf{R}|^2$ over all space. The result reproduces the approximation of Eq. (3) to within a few percent. Integrating only over an appropriate cone gives the last factor in Eq. (4). We also considered the flux of particles produced in annihilations which occur above the surface. Any gamma rays produced in these annihilations would be visible. This flux has a sharp peak as a function of m_x at the crossover point; for the parameters given previously, the maximum value was $(4\times10^{-17}\ \text{cm}^{-2}\text{sec}^{-1})N_\gamma$. This is well below the isotropic gamma-ray background. We did a similar calculation for the Sun, using an $n=3$ polytropic model for the interior; the flux due to external annihilations peaked at $(5\times10^{-21}\ \text{cm}^{-2}\text{sec}^{-1})N_\gamma$ at a crossover point of $m_x\simeq6$ GeV. It is easy to understand why external annihilation rates peak at the crossover mass. For smaller masses, particles escape as fast as they are captured, and a large number density excess never accumulates. For larger masses, all the particles sink to the center, and once again there is no large number density at the surface.

In fact, present proton-decay detectors provide more sensitive probes of annihilations in the region of the Earth's surface. Such annihilations, if they occur inside a proton-decay detector, will produce spectacular back-to-back events. Since the volume of such detectors is about 10^9 cm^3, an event rate of one per year would correspond to an annihilation rate of 3×10^{-17} cm^{-3}sec^{-1} at the Earth's surface. Unfortunately, this is seven orders of magnitude larger than the actual annihilation rate for m_x at the crossover point.

If the actual crossover point between evaporation and annihilation is lower than that determined using the thermal equilibrium approximations described above, then these latter effects could be significantly increased, and be more important than the indirect neutrino signal from annihilations.

We can use the density profile of Eq. (10) to estimate the parameters a and b. Following Ref. 5 we define a as the integrated column density of the Earth seen by a typical dark-matter particle coming in from infinity, times R^2/M. Using straight trajectories and counting only the Earth's iron core yields $a\simeq0.20$. We define b as $[-\xi(r)+GM/R]/(GM/R)$ with r as a typical scattering location.[5] For $r=1.7\times10^8$ cm, half the core radius, we get $b\simeq1.7$.

We also considered trapping of dark-matter particles by giant planets. Sufficiently heavy dark-matter particles will disappear via annihilation, produing heat. For $\sigma_N=10^{-38}$ cm^2, the power produced by the annihilations is about four orders of magnitude below the actual thermal power outputs[11] of Jupiter (10^{17} W) and Uranus (10^{15} W). It is likely that Uranus has an iron core of radius ~8000 km accounting for $\sim25\%$ of the planet's mass.[11] In this

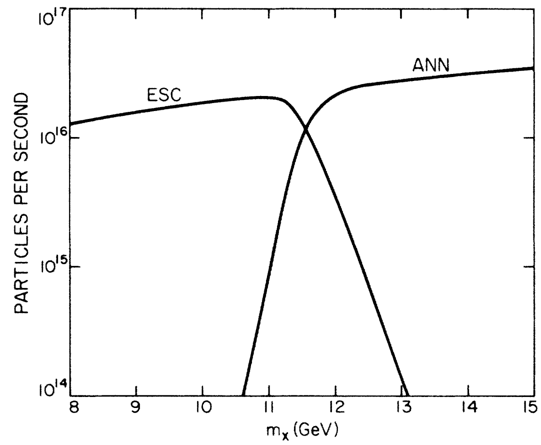

FIG. 2. The escape and annihilation rates as a function of m_x with the parameters $\langle\sigma_a v\rangle=10^{-26}$ cm^{3}sec^{-1}, $\rho_{0.3}=\bar{v}_{300}=3ab=\bar{Y}=1$, $T=4000$ K.

case, multiple coherent scattering in the core becomes possible for scalar or Dirac neutrinos with mass $\gtrsim30$ GeV, and a very large fraction of the incident flux will be captured, producing too much heat.

For the parameters not excluded by the arguments present here, direct detection of heavy weakly interacting particles may soon be possible. Recently, along with Cabrera, two of us have proposed a new method of detecting low energy solar and reactor neutrinos bolometrically.[12] It appears feasible to detect energy deposits as small as about 1 keV in ultracold silicon blocks. The maximum energy deposit in a single scattering of a halo particle of mass m off a nucleus of mass M is $2(mv)^2/M$, where v is the velocity of the particle. Assuming virial velocities of $\sim10^{-3}c$, we thus expect such a bolometric detector will be sensitive to particles of mass greater than about 2—3 GeV. As Goodman and Witten have shown,[2] the rates for energy deposition in materials due to scattering of coherently interacting dark-matter candidates in this mass range with halo densities of about 0.3 GeV/cm^3 is greater than 50/kg day. This signal is large enough that it may be measured in prototype versions of bolometric detectors.[12] Majorana neutrinos and photinos, because of their purely axial couplings, scatter only off nuclei with spin. Approximately 5% of silicon is ^{29}Si, which has spin $\frac{1}{2}$, and thus should provide a good target for such Majorana fermions. The expected rates for such scatterings are[2] 0.005—0.05 events/kg day in a silicon detector. Such event rates could in principle be probed by the 10—100 kg bolometric detectors proposed for reactor neutrino measurements.

An alternative target is GaAs, which has thermal properties similar to silicon. GaAs consists entirely of nuclei with spin, so the photino cross section is much larger than for Si ($\approx\times50$ for the same total mass).

We would like to emphasize that the characteristic signal for dark-matter interactions, like that for coherent neutrino scattering, in low-temperature semiconductors is *energy deposit into phonons with very little accompanying ionization (particle-hole creation). A hybrid detector monitoring both phonons and charged particles would be especially powerful in distinguishing these signals.*

Note added. After this paper was completed, we learned that related work has been done by Freese.[13]

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*Present address: Yale University, New Haven, CT 06511.

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