PHYSICAL REVIEW D VOLUME 33, NUMBER 7 1 APRIL 1986

On the interpretation of the European Muon Collaboration effect

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(Received 3 July 1985)

We show that the European Muon Collaboration effect can be understood in terms of the change in the mass scale of a bound nucleon, rather than in terms of a change in confinement scale.

Over the past three years many ideas have been put forward to explain the dependence of the isoscalar structure function of a "nucleon" on its nuclear environment —known as the European Muon Collaboration (EMC) effect.[1,2] Included amongst those proposals are a possible swelling of the nucleon itself, the presence of exotic components (clusters of 6, 12, or more quarks) in the nuclear wave function, color conductivity, and an enhancement of the nuclear pion field.[3] The one common thread in all of this speculation is that in one way or another the distance scale associated with a quark increases with atomic number (A).

On the other hand, Close, Roberts, and Ross (CRR) have observed that the A dependence can be reproduced by simply shifting the momentum scale at which the nucleon structure function is evaluated:[4]

$$F_2^A(x,Q^2)=F_2^N(x,\xi Q^2) \quad , \tag{1}$$

with $\xi=\xi(A,Q^2)$; this is called "rescaling." The exciting link with the models mentioned above came with their further suggestion,[5] motivated by the bag model,[6] that a change in confinement scale from R to R_A would lead to Eq. (1) with

$$\xi_A(Q^2)=\left(\frac{R_A}{R}\right)^{2\alpha_s(\mu_A{}^2)/\alpha_s(Q^2)} \quad . \tag{2}$$

This has led to the widespread belief that while the details remain to be worked out, the essential physics of the EMC effect is an increase in the distances over which quarks move as A goes up.

The contents of this Rapid Communication are as follows. First we discuss what we believe to be an inconsistency in the arguments of CRR relating Eq. (1) to a change in confinement scale. Next we observe that there is another, more obvious scale associated with a bound nucleon, namely, the extent to which it is off mass shell. Perturbative QCD suggests how the structure function changes off shell. The result is again a rescaling, mathematically identical to Eq. (1), but ξ has a completely different interpretation [see Eq. (5) below]. Furthermore, ξ can be calculated without free parameters. While the calculated value is too small to explain the EMC effect by itself, the need to account for off-shell effects suggests a simple modification of the usual Fermi-motion correction. As we shall see, when this is made the SLAC data are quite well reproduced.

Let us begin with the change of confinement scale. Here we rely heavily on the discussion of the valence-quark correlation function $C_-(z,Q^2)$ by Llewellyn Smith.[3,7] It is defined as the cosine transform of the valence structure function $F_3(x,Q^2)$:

$$C_-(z,Q^2)=\int_0^\infty dx\cos(m_N xz)F_3(x,Q^2) \quad . \tag{3}$$

$F_3=(u-\bar{u}+d-\bar{d})$ measures the excess of quarks over antiquarks. Within experimental errors the number of excess (valence) quarks is indeed three.[8]

While this function is not renormalization-group invariant, it varies very slowly with Q^2, as we see in Fig. 1. [Note that because of the ambiguity in the definition of sea and valence quarks at small Bjorken x, the integral in Eq. (3) is smoothly cut off at $x=0.1$, as in Ref. 3(a).] Physically, C_- measures the probability that one can remove a valence quark from a nucleon at one point, put it back a distance z fm away (on the light cone), and still leave a nucleon. It therefore gives a direct indication of the whereabouts of the valence quarks inside the nucleon.

A crucial element in the argument of Close and collaborators[4,5] is the supposition that there may be a scale ($\mu_A{}^2$) at which the twist-two piece of the structure function of a nucleus can be approximated by a "valence-quark distribution, with no radiated gluons." The same point for a nucleon need not occur at the same scale (μ^2). Equation (1) can be

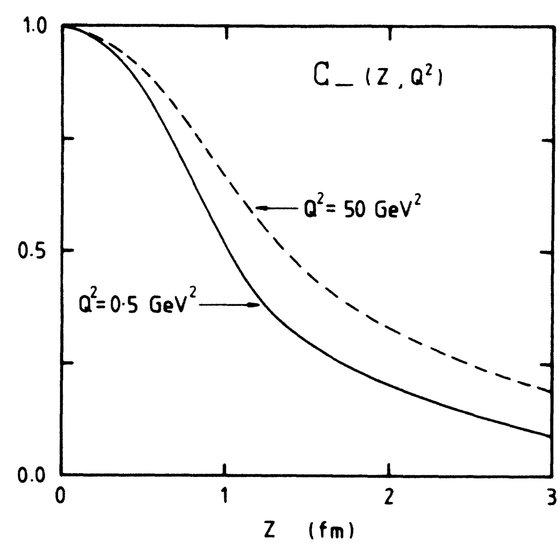

FIG. 1. The correlation function $C_-(z,Q^2)$, which is a measure of the spatial distribution of the valence quarks (arbitrarily normalized to unity at $z=0$).

derived by assuming that, at their respective scales, the structure functions of the targets are equal [i.e., $F_3^A(x,\mu_A^2)=F_3^N(x,\mu^2)$].

Now the argument that the EMC effect represents a change of confinement scale requires a further, rather natural step. The valence-only approximation is so much like the bag model that one is tempted to identify μ^2 (μ_A^2) as the scale at which a nucleon (nucleus) looks like a bag (collection of bags) of valence quarks. Since the bag has only one scale, its radius, it is tempting to guess that $\mu^2/\mu_A^2=R_A^2/R^2$, in an obvious notation. Since $R\neq R_A$ there is a change of confinement scale.

The difficulty with this procedure is that because F_3^A at μ_A^2 is identical to F_3 at μ^2, $C_-^A(z,\mu_A^2)$ is equal to $C_-(z,\mu^2)$. That is, contrary to the assumption that $R_A\neq R$, the spatial distribution of valence quarks in the two targets is identical at their respective mass scales. Thus, it would appear that there is an inconsistency in relating the difference in momentum scales in two targets to a change in confinement scale. [Of course if one were to compare the size of the valence-quark distributions at the same Q^2 ($\geq\mu^2$), they would be different. However, as we can see from Fig. 1 the QCD evolution of C_- with Q^2 is so slow[7] that the difference could be no more than 1–2%, nothing like the 15% of Jaffe, Close, Roberts, and Ross.[5]]

Faced with this realization, we were led to reconsider the meaning of the renormalization scale μ. In perturbative QCD the moments $M_n(Q^2)$ of any nonsinglet structure function (e.g., F_3) depend on μ^2 as[9]

$$M_n(Q^2)=M_n(\mu^2)\left(\frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)}\right)^{d_n} . \tag{4}$$

However, $M_n(Q^2)$ is an observable, which should not depend on an *ad hoc* choice of renormalization point. In order to avoid this problem *the moments must depend on another mass scale in just such a way that the unphysical dependence on μ^2 is eliminated.*

The other obvious mass scale is the invariant mass of the target. Thus, if we were to make a very simple model of a nucleus as a collection of off-mass-shell nucleons, we would again be led to Eq. (1), but ξ is now given by

$$\xi=\left(\frac{m_N^2}{p_A^2}\right)^{\alpha_s(p_A^2)/\alpha_s(Q^2)} , \tag{5}$$

where p_A^2 is the invariant mass of the bound nucleon. As we shall see below, this rescaling is significantly smaller than that needed to fit the experimental data. Nevertheless, having been led to the realization of the importance of the nucleon being off shell, we were motivated to include this simple effect in the treatment of Fermi motion (see also Ref. 10).

Thus, we modify the Fermi averaging procedure of Llewellyn Smith[3] by allowing a nucleon in state i to be off shell. Its energy is $p_i^0=m_N-|\epsilon_i|$, with ϵ_i the single-particle binding energy, and its four-momentum $(p_i^0,\mathbf{k})$. Then, including the effect of rescaling as in Eq. (5), we find

$$F^A(x,Q^2)=A^{-1}\sum_i n_i\int d^3k\,\rho_i(\mathbf{k}^2)\times F^N\left(\frac{m_N x}{p_i^0+k_z},\xi_i(Q^2)Q^2\right) . \tag{6}$$

Here n_i is the occupation number of the shell-model state and ρ_i the square of the momentum-space wave function. In order to make Eq. (6) numerically tractable we use the harmonic-oscillator model for the states i, with $\hbar\omega$ and the well depth fitted to the separation energies of the major shells.[11] Furthermore, we evaluate ξ_i using an average value for the invariant mass of the bound nucleon:

$$p_i^2=(m_N-|\epsilon_i|)^2-\langle\mathbf{k}^2\rangle_i . \tag{7}$$

Then, defining y as $(p_i^0+k_z)/m_N$, we find

$$F^A(x,Q^2)=\sum_i\int_x^A dy\,f_i(y)F^N\left(\frac{x}{y},\xi_i(Q^2)Q^2\right) , \tag{8}$$

where

$$f_i(y)=N_i\int d^2k_T\rho_i[k_T^2+(m_Ny-p_i^0)^2] , \tag{9}$$

and N_i is the normalization constant. The nucleon structure function is calculated from (a modification of) the parametrization given in Ref. 12. We stress that Eqs. (5), (8), and (9) involve no free parameters.

In Fig. 2 we show the predictions of this approach in comparison with recent SLAC data.[2] The agreement with the data is rather good, once the off-mass-shell rescaling effect is included (dashed curves). In order to show the relative importance of our rescaling as opposed to Fermi motion, the solid curves are calculated with all ξ_i set arbitrarily to one. With regard to the Fermi-motion correction, we stress that the small (2–4%) difference between p_i^0 and m_N in Eq. (6) is responsible for the dip near $x=0.6$ shown in the solid curves. This is the most interesting piece of the EMC effect, and it is clearly associated with the nucleon being off the mass shell. [While this observation has also been made by Akulinichev *et al.*,[10] these authors neglected the off-mass-shell effect in D. When recoil of the spectator nucleon is included (as we do), one finds a 3% EMC effect in D, which must be included.]

In conclusion, we have demonstrated that the major part of the EMC effect arises because bound nucleons are off the mass shell. While the off-mass-shell Fermi motion is the biggest contributor, our rescaling correction does seem to be also necessary in the heavier nuclei. Of course, the present experimental uncertainties, plus our use of harmonic-oscillator wave functions, do not permit us to conclude that our ansatz (that the characteristic mass scale of the target is its invariant mass) is quantitatively correct. Nevertheless, a small scale change consistent with this ansatz does seem to be required by the data. Further theoretical work on this matter seems very worthwhile. It would also be worthwhile to make the calculations using a Woods-Saxon potential, but we believe this will only make a significant change near $x=1$.

In this connection we note that whereas the integral of $f_i(y)$ is one (in order to satisfy the Gross–Llewellyn Smith sum rule), the integral of $yf_i(y)$ is $p_i^0/m_N<1$. That is, after accounting for binding, the nucleons do not carry all of the momentum of the nucleus. However, this was exactly what was postulated in those models which predicted an enhancement of the nuclear pion field.[13–15] Thus, our conclusions appear to match the pionic models rather nicely —even though our starting point was quite different.

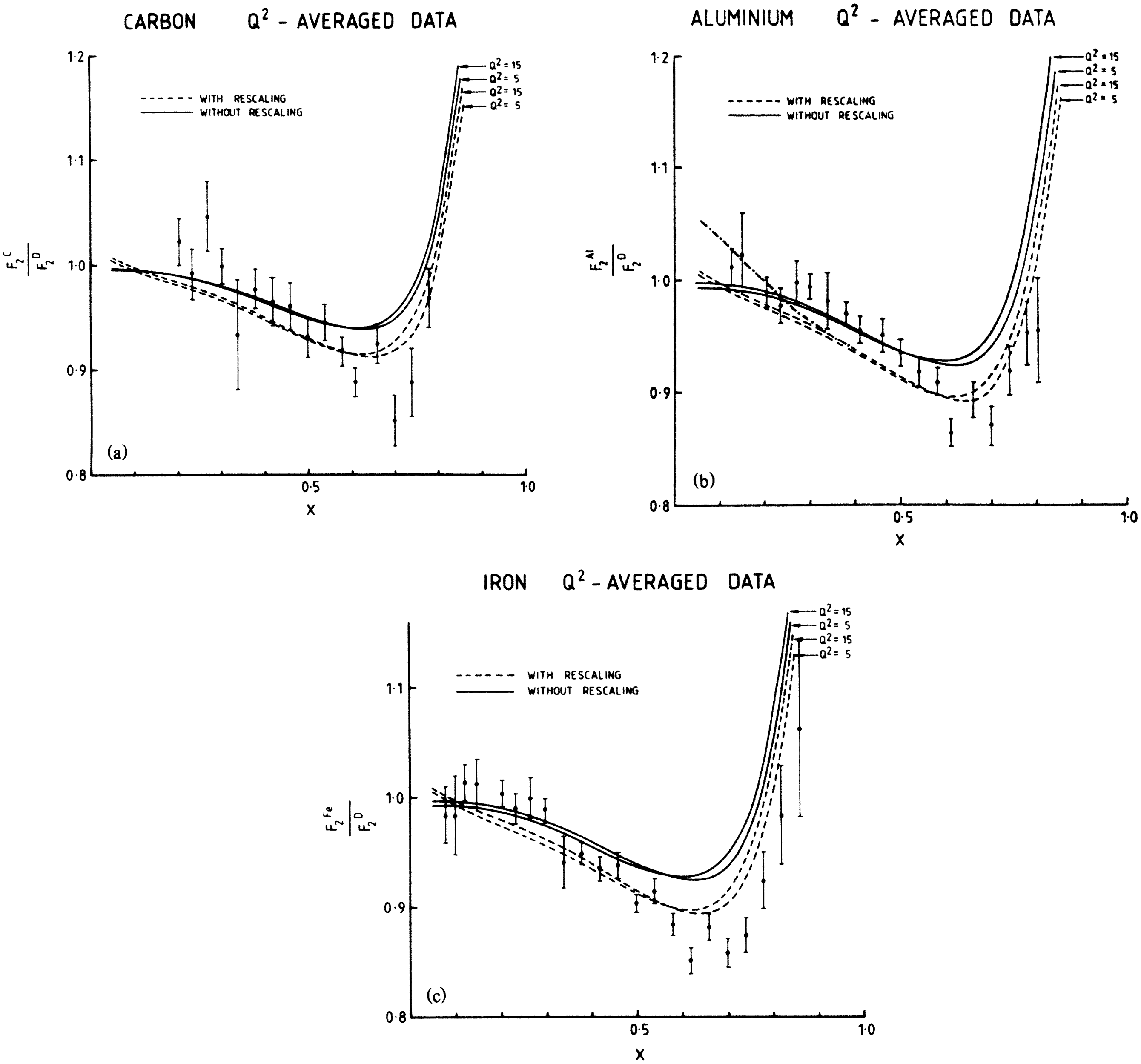

FIG. 2. Predictions for the ratio of the nuclear structure functions for C, Al, and Fe compared with D. The calculations are shown at 5 and 15 GeV (Ref. 2) and compared with the Q^2-averaged data from SLAC (Ref. 2). The solid curve shows the effect of Fermi averaging alone (with an off-mass-shell nucleon), while the dashed curve also includes rescaling, i.e., $\xi_i \neq 1$ in Eq. (8). For Al we also show the effect of balancing momentum by adding a pionic contribution (Ref. 14).

We would like to thank C. Llewellyn Smith and A. Williams for useful discussions. We would also like to acknowledge the assistance of S. Barry, R. Hartley, and J. Knispel with the numerical work. This work was supported by the Australian Research Grant Scheme, and the University of Adelaide.

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