SU(3) breaking and the *H* dibaryon

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(Received 27 November 1985)

SU(3) breaking in quark masses affects hyperfine energy and configuration mixing in the proposed H dibaryon (a $\Lambda\Lambda$ bound state). The hyperfine energy is modified substantially, so that it is much less advantageous than in the SU(3) limit (by about a factor of 3) to recouple the spins of the quarks in two Λ's to form an H. Nonetheless, the H remains an almost pure configuration of the 490-dimensional representation of color-spin SU(6), with a negligible amount of the other possible configuration (189 dimensional) admixed.

The possible existence of multiquark hadrons (beyond $Q\bar{Q}$ or QQQ) has been the object of curiosity for a number of years.[1] The understanding of the crucial role of spin-spin interactions among quarks, and its evaluation using simple ideas of QCD,[2] has permitted one to ask more quantitative questions about such hadrons. Several years ago, Jaffe[3,4] discovered that by recoupling the spins of the quarks in two Λ hyperons one could bind these quarks more deeply. The resulting state, dubbed H, was estimated within a bag model to have a binding energy of about 80 MeV. Searches for the H, while so far negative,[5] may not have been performed yet with the required sensitivity,[6] and further experiments have been proposed.[7–10]

Recently, it has been suggested[11] that the H could be so deeply bound that it lies below threshold for the $\Delta S=1$ weak decay $H\rightarrow \Lambda p e^-\nu$. This would require its binding energy to be greater than $M_\Lambda - M_p - M_e \approx 177$ MeV/c^2. If the H were required to decay via a doubly weak ($\Delta S=2$) process, its lifetime could be long even on galactic scales. The suggestion that unusual particles might be emanating from Cygnus X-3 (Refs. 12 and 13) has thus made the reexamination of the mass of the H a timely problem.

In this Brief Report, we estimate the effects of SU(3) breaking on the quark spin-spin interaction responsible for the original claim that H is a Λ-Λ bound state. To first order in SU(3) breaking, the effect of the increased strange-quark mass is to reduce considerably the advantage gained by recoupling the spins of the quarks in the Λ. As a result, one might have questioned the purity of the six-quark wave function assumed for the H. We find, on the contrary, that this wave function remains nearly unmixed, justifying an assumption made in Ref. 3 and more recently in a calculation made on the basis of lattice gauge theory methods.[14]

We work in a Fermi-Breit approximation to hyperfine interactions, which has proved remarkably accurate for hadrons composed of light quarks.[2,15] The hyperfine splitting due to gluon exchange may be parametrized as

$$\Delta E = -a \sum_{i<j} \boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j / m_i m_j \ , \qquad (1)$$

where a is a constant depending on the square of the S-wave wave function at the origin, $|\Psi(0)|^2$, of a pair of quarks and on the strong fine-structure constant α_s. The sum is performed over all distinct quark pairs. The color matrices $\boldsymbol{\lambda}_i$ of Gell-Mann and the Pauli matrices $\boldsymbol{\sigma}_i$ are summed over eight and three dimensions, respectively. Here we have separated out the quark masses m_i, m_j explicitly. The quantity $\Sigma \equiv \sum_{i<j}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$ has a definite value when the set of quarks in the sum transforms as a specific representation of a color-spin SU(6) [containing $SU(3)_{\rm color}$ and $SU(2)_{\rm spin}$]. Its value may then be expressed in terms of quadratic Casimir operators:[3,4]

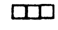
$$\Sigma \equiv \sum_{i<j}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j = 4C^{(6)}(R) - 2C^{(3)}(R) - \tfrac{4}{3}S(S+1) - 8N \ , \qquad (2)$$

where the Casimir operators of some relevant SU(3) and SU(6) representations are given in Table I.

In the limit of SU(3) flavor symmetry, color spin is a convenient technique for calculating hyperfine splittings in three-quark baryons. The ground-state baryons belong to a spin-$\frac{1}{2}$ flavor octet (mixed flavor symmetry) and a spin-$\frac{3}{2}$ flavor decimet (totally symmetric in flavor). Their corresponding color-spin representations must be **70** (mixed) and **20** (totally antisymmetric) in order to satisfy Fermi statistics. The values of Eq. (2) are then found to be $+8$ and -8, respectively, for the ground-state flavor octet and decimet.

A simpler calculation allows one to evaluate the effects of symmetry breaking. In a $\Lambda = uds$, the u and d are in a state

TABLE I. Quadratic Casimir operators for SU(3) and SU(6) representations.

	Young tableau	Dimension	$C^{(3)}(R)$
SU(3)			
		3*	$\frac{4}{3}$
		6	$\frac{10}{3}$
SU(6)			$C^{(6)}(R)$
		15	$\frac{14}{3}$
		21	$\frac{20}{3}$
		56	$\frac{45}{4}$
		70	$\frac{33}{4}$
		20	$\frac{21}{4}$
		105	$\frac{32}{3}$
		490	18

of $I=0$ (antisymmetric), S wave (symmetric), color 3^* (antisymmetric), and hence spin 0 (antisymmetric). Since the ud pair must remain spinless, it has no hyperfine interaction with the strange quark. All the hyperfine energy in the Λ thus comes from the ud pair. Since it is flavor antisymmetric, it is color-spin symmetric (a 21-plet), so

$$\Sigma=4(\tfrac{20}{3})-2(\tfrac{4}{3})-(\tfrac{4}{3})(0)-8(2)=8 \ ,$$

and

$$\Delta E_\Lambda=-8a/m_u^2 \ . \tag{3}$$

Like the Λ, the N is an SU(3)-octet member, and is composed entirely of nonstrange quarks. Thus,

$$\Delta E_N=-8a/m_u^2 \tag{4}$$

as well. In fact, for all members of the baryon octet and decimet, it suffices to know the values of the Σ for pairs of quarks in each color and spin state. These values are given in Table II. Thus, for example, the quark pairs in a Δ (spin $\frac{3}{2}$) all are necessarily in relative spin-1 (color 3^*) states. Thus,

$$\Delta E_\Delta=-3(-8a/3m_u^2)=8a/m_u^2 \ . \tag{5}$$

The masses of the baryon octet and decimet are described by the simple expression

$$M(qqq)=\sum_i m_i - a\sum_{i<j}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j/m_i m_j \tag{6}$$

to better than 20 MeV. Here we neglect quark kinetic energies altogether, replacing them by effective masses. A fit[15] gives

$$m_u=m_d=363\ \text{MeV}/c^2;\quad m_s=538\ \text{MeV}/c^2 \ . \tag{7}$$

The hyperfine terms in (2)–(4) may be evaluated in terms of physical masses:

$$M_\Delta-M_N=16a/m_u^2=-2\Delta E_\Lambda=300\ \text{MeV}/c^2 \ . \tag{8}$$

In all dibaryons, one stands the best chance of avoiding Pauli-principle constraints with two u's, two d's, and two s's. The state must then be a flavor singlet, or it would be related to one with more than two of at least one quark flavor. Jaffe[3] searched for the color-spin state compatible with a flavor singlet and having the largest value of Σ. This occurs when $C^{(6)}$ is as large as possible and $C^{(3)}$ and $S(S+1)$ are as small as possible, compatible with overall Fermi statistics. The corresponding dibaryon, the H, belongs to a 490-dimensional representation of color spin (see Table I), and has no color or spin.[16] Its value of Σ is thus 24, so

$$\Delta E_H=-24a/m_u^2=-450\ \text{MeV} \tag{9}$$

TABLE II. Hyperfine interaction $\sum_{i<j}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\ \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$ between a pair of quarks in color 3^* or 6 and spin $S=0$ or 1 states.

Spin \ Color	3^*	6
0	$+8$	-4
1	$-\frac{8}{3}$	$+\frac{4}{3}$

in the limit $m_u=m_d=m_s$. Thus, in this limit,

$$\Delta E_H-2\Delta E_\Lambda=-150\ \text{MeV} \ , \tag{10}$$

so the H is bound by $(M_\Delta-M_N)/2=150$ MeV, if we neglect all other dynamical effects, including variations of $|\Psi(0)|^2$, confinement radius, kinetic energies, etc.

Now we write the H wave function in a manner suitable for examining SU(3)-breaking effects.[3,14] Each pair of quarks (uu, dd, and ss) must belong to an antisymmetric representation of color spin, the 15-dimensional one noted in Table II. By examining the Young tableau for the **490**, we see that the configuration of its nonstrange quarks must involve exclusively the 105-dimensional color-spin representation (also depicted in Table I). Thus, the nonstrange quarks in the H should belong to a **105**, and the strange quarks should belong to a **15**. This should continue to be so in the presence of SU(3) breaking.

The product of the color-spin representations for nonstrange and strange quarks is

$$\mathbf{105}\times\mathbf{15}=\mathbf{490}+\mathbf{896}+\mathbf{189} \ . \tag{11}$$

Of these, only **490** and **189** contain a spin singlet. Thus, our problem reduces to estimating the effects of SU(3) breaking on **490**-**189** mixing in the H.

We have found it more convenient to work in a two-dimensional basis labeled by the total spin S_s of the strange quarks, which can be 0 (for a color sextet) or 1 (for a color 3^*). The corresponding total spin S_n of the nonstrange quarks must then be 0 (for a color 6^*) or 1 (for a color 3). In the SU(3) limit, the hyperfine Hamiltonian is the 2×2 matrix

$$H_{\text{HF}}=\begin{pmatrix}0 & \mu\\ \mu & \lambda\end{pmatrix} \ , \tag{12}$$

where we shall find μ and λ presently. The zero in Eq. (12) arises because the hyperfine energies of the nonstrange and strange quarks are found to exactly cancel one another in an explicit color-spin calculation for $S_n=S_s=0$, and because all $\boldsymbol{\sigma}_n\cdot\boldsymbol{\sigma}_s$ interactions cancel out for these spinless systems.

The eigenvalues of Eq. (12) are[3]

$$\Delta E_{490}=-24a/m^2 \ , \tag{13}$$

$$\Delta E_{189}=-8a/m^2 \ , \tag{14}$$

where m is the quark mass, so that

$$\text{Tr}H_{\text{HF}}=\lambda=-16a/m^2 \tag{15}$$

and

$$\text{Det}H_{\text{HF}}=-\mu^2=-192(a/m^2)^2 \ . \tag{16}$$

Making an arbitrary sign choice for μ, we then can write

$$H_{\text{HF}}=-\frac{8a}{m^2}\begin{pmatrix}0 & \sqrt{3}\\ \sqrt{3} & 2\end{pmatrix} \ . \tag{17}$$

The eigenstates are thus

$$|490\rangle=\frac{1}{2}|0\rangle+\frac{\sqrt{3}}{2}|1\rangle \ , \tag{18}$$

$$|189\rangle=\frac{\sqrt{3}}{2}|0\rangle-\frac{1}{2}|1\rangle \ , \tag{19}$$

where $|0\rangle$ and $|1\rangle$ label $S_s=S_n=0,1$, respectively.

In the presence of SU(3) breaking, hyperfine terms are of three types: $\sim m_u^{-2}$, $\sim m_u^{-1}m_s^{-1}$, and $\sim m_s^{-2}$, where $m_u = m_d$. We label terms in $H_{\rm HF}$ proportional to m_u^{-2} and m_s^{-2} as H_n and H_s, respectively. Such terms are easily evaluated in states of definite $S_n = S_s$, and we find, using Eqs. (1), (2), and Table II, that

$$\langle 0|H_n|0\rangle = -4a/m_u^2 \ , \tag{20}$$

$$\langle 0|H_s|0\rangle = 4a/m_s^2 \ , \tag{21}$$

$$\langle 1|H_n|1\rangle = -16a/3m_u^2 \ , \tag{22}$$

$$\langle 1|H_s|1\rangle = 8a/3m_s^2 \ . \tag{23}$$

All additional contributions to $H_{\rm HF}$ are proportional to $m_u^{-1}\ m_s^{-1}$, so we can immediately write

$$H_{\rm HF} = 4a\begin{pmatrix} \frac{1}{m_s^2} - \frac{1}{m_u^2} & -\frac{2\sqrt{3}}{m_u m_s} \\ -\frac{2\sqrt{3}}{m_u m_s} & -\frac{4}{3m_u^2} + \frac{2}{3m_s^2} - \frac{10}{3m_u m_s} \end{pmatrix} \tag{24}$$

in order that (24) reduce to (17) in the SU(3) limit. Equation (24) applies to the basis $S_s = S_n = (0,1)$. It is convenient to reexpress $H_{\rm HF}$ in the (**490, 189**) basis, since then one can see the effects of SU(3) breaking. The result is

$$H_{\rm HF} = a\begin{pmatrix} \frac{3}{m_s^2} - \frac{5}{m_u^2} - \frac{22}{m_u m_s} & \frac{1}{\sqrt{3}}\left(\frac{1}{m_u} - \frac{1}{m_s}\right)^2 \\ \frac{1}{\sqrt{3}}\left(\frac{1}{m_u} - \frac{1}{m_s}\right) & \frac{11}{3m_s^2} - \frac{13}{3m_u^2} + \frac{26}{3m_u m_s} \end{pmatrix} . \tag{25}$$

The **490-490** term was obtained previously by Aerts and Dover[6] and Mackenzie and Thacker.[14]

To simplify $H_{\rm HF}$ further, we define δ by

$$m_s^{-1} = m_u^{-1}(1-\delta) \ , \tag{26}$$

so that

$$H_{\rm HF} = \frac{a}{m_u^2}\begin{pmatrix} -24 + 16\delta + 3\delta^2 & \delta^2/\sqrt{3} \\ \delta^2/\sqrt{3} & 8 - 16\delta + \frac{11}{3}\delta^2 \end{pmatrix} . \tag{27}$$

The shifts due to $\delta \neq 0$ are far more important than the mixings of states. For the masses (7), we find $\delta = 0.325$, so for the H we obtain

$$\Delta E_H \approx -18.8\frac{a}{m_u^2} = -353 \text{ MeV} \ , \tag{28}$$

to be compared with $-24a/m_u^2 = -450$ MeV [Eq. (9)] in the SU(3) limit and $16a/m_u^2 = -300$ MeV [Eq. (8)] for two noninteracting Λ's. Thus, the H is bound by only about 53 MeV instead of the 150 MeV obtained in the SU(3) limit. The value of 53 MeV is the number to be compared with Jaffe's 80 MeV, obtained in the bag model. More recent estimates have led to possible variations of ±100 MeV from Jaffe's value,[17,18] and to the recent conclusion[14] that the H is highly unbound.

The mixture of states is of order δ^2 *in amplitude*, with a small coefficient. We find

$$|H\rangle \approx |490\rangle - 0.003|189\rangle \tag{29}$$

for $\delta \approx \frac{1}{3}$.

We conclude with a brief remark about the applicability of a simple Fermi-Breit Hamiltonian to an estimate of the H mass. The advantage of recoupling spins was estimated in the bag model for zero quark mass[3] to be $2m_N - m_H(m_q = 0) = 2m_N - 1760 \text{ MeV} = 116$ MeV. This value is changed to 80 MeV when the strange-quark mass is set at a value which gives correct hyperon masses. Thus, the bag model appears much less sensitive to SU(3)-breaking effects than the term (1). In the bag model, a fully relativistic calculation, *all* effects of $m_s \neq m_u$ are taken into account; these include shifts in kinetic energies and in average separations, and changes in quark-quark overlaps. These are unlikely to affect mixings, however, so that we continue to expect the H to be relatively pure if only such effects are taken into account. Much more important are coupled-channel effects due to SU(3) breaking in final dibaryon states. These have been discussed in Ref. 18.

One can describe the masses of eight 0^- mesons, nine 1^- mesons, eight $\frac{1}{2}^+$ baryons, and ten $\frac{3}{2}^+$ baryons to better than 20 MeV by adding constituent-quark masses to a term of the form (1). If a similar description is valid for multiquark systems, our result indicates that the H indeed is bound, but its binding energy is unlikely to exceed Jaffe's estimate.

I am grateful to Bill Bardeen, Val Fitch, Ken Heller, Bob Jaffe, Paul Mackenzie, and Hank Thacker for interesting discussions, and to Chris Quigg and the Theory Group at Fermilab for their hospitality during the inception of this study. This work was supported in part by the U.S. Department of Energy under Contract No. DE-AC02-82ER 40073.

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