Creation of relativistic positronium

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The creation of bound electron-positron states, positronium, by photons in the field of an atom is calculated.

The experimental production of relativistic positronium has recently been reported.[1] Positronium was formed in the triplet state (orthopositronium) in the decay of π^0 mesons

$$\pi^0 \to \gamma + (e^+e^-) \tag{1}$$

with an energy of 500 MeV. At this energy orthopositronium is virtually stable in the laboratory system with a decay length of 41.6 km. Close to 200 positronium particles were recorded.

With this successful experiment in mind, it might be of interest to discuss the possibility of creation of (para)positronium by a photon of energy ω in the field of an atom of charge Ze, the ordinary pair-production process,

$$\gamma + Z \to (e^+e^-) + Z \ . \tag{2}$$

Estimates of the cross section for extreme relativistic energies have previously been made by Bilenkii, van Hieu, Nemenov, and Tkebuchava and by Meledin, Serbo, and Slivkov.[2]

The positronium production cross section $d^2\sigma_p$ is obviously closely related to the Bethe-Heitler pair cross section $d^5\sigma_{\text{pair}}$. The difference is essentially due to phase-space and normalization factors since in the positronium production process the electron and positron are created in a bound state, while in pair production the particles are free. The positronium production cross section is, in fact, to lowest order in α most easily obtained by dividing the Bethe-Heitler cross section $d^5\sigma_{\text{pair}}$ for equal electron and positron momenta $\mathbf{p}_+ = \mathbf{p}_- = \mathbf{p}_p/2$, by the two-particle phase-space factor

$$d^3p_+ d^3p_- / dE_- (2\pi)^6 \frac{E_+}{m}\frac{E_-}{m}$$

and multiplying by the one-particle factor

$$\left[d^3p_p / dE_p (2\pi)^3 \frac{E_p}{m_p} \right] N_p^{-2} \ .$$

Here m is the electron mass, $m_p = 2m - B_m \approx 2m$ the positronium mass when we neglect the very small binding energy $B_n = \frac{1}{4}\alpha^2 m/4n^2$, E_+ and E_- the energies of the positron and electron, respectively, and $E_p = \omega$ and $\mathbf{p}_p$ the energy and momentum of the positronium particle. N_p^{-1} is the positronium normalization constant for the nth s level, $N_p^{-2} = \alpha^3 m^3 / 8\pi n^3$. In this way we find the positronium creation cross section,

$$d^2\sigma_p = \frac{Z^2\alpha^6}{n^3} \frac{[1-F(q)]^2}{q^4} \frac{p_p^3 m_p^2 \sin^2\theta \, d\Omega}{2E_p(E_p - p_p\cos\theta)^2} \ , \tag{3}$$

with $F(q)$ the atomic form factor and q the momentum transfer given by

$$q^2 = E_p{}^2 + p_p{}^2 - 2E_p p_p \cos\theta \ , \tag{4}$$

with θ the angle between the momenta of the incoming photon and the outgoing positronium.

Since the decay length of parapositronium ($l_p = c\tau_p^0 p_p / m_p$ with $\tau_p^0 = 1.25 \times 10^{-10}$ sec the parapositronium lifetime) is short except for relativistic energies, the case of relativistic positronium is of particular interest from the point of view of experimental detection. For such energies positronium is emitted at small angles, in what may be thought of as two angular regions $\theta \sim (m_p/E_p)^2$ and $\theta \sim m_p/E_p$ with contributions to the total cross section of the same order of magnitude. The reason for the unusually small angles $\theta \sim (m_p/E_p)^2$ follows from the form of the momentum transfer at high energies,

$$q^2 = E_p{}^2 \left[\frac{m_p{}^4}{4E_p{}^4} + \theta^2 \right] \ , \tag{5}$$

which gives the high-energy, small-angle cross section

$$d\sigma_p = 4\pi \frac{Z^2\alpha^6}{n^3} \frac{m_p{}^2}{E_p{}^4} \frac{[1-F(q)]^2 \theta^3 d\theta}{(m_p{}^4/4E_p{}^4 + \theta^2)^2 (m_p{}^2/E_p{}^2 + \theta^2)^2} \ . \tag{6}$$

In order to look more closely at the two regions of θ, we define an angle θ_0 by

$$\left[\frac{m_p}{E_p} \right]^2 << \theta_0 << \frac{m_p}{E_p} \ ,$$

which gives the cross section in the region of extremely small angles, $\theta \leqslant \theta_0$,

$$d\sigma_p = 4\pi \frac{Z^2\alpha^6}{n^3} [1-F(q)]^2 \frac{\theta^3 d\theta}{m_p{}^2 (m_p{}^4/4E_p{}^4 + \theta^2)^2} \ , \tag{7}$$

which was also obtained by Meledin, Serbo, and Slivkov,[2] for the case of no screening. In addition, there is, however, an angular tail for $\theta \geqslant \theta_0$,

$$d\sigma_p = 4\pi \frac{Z^2\alpha^6}{n^3} [1-F(q)]^2 \frac{m_p{}^2 d\theta}{E_p{}^4 \theta (m_p{}^2/E_p{}^2 + \theta^2)^2} \ , \tag{8}$$

extending out to angles of the order m_p/E_p. This observation may be important from the point of view of the detection of relativistic positronium, since it may be essential to avoid the high-energy photon beam. It is easy to see that the number of positronium particles in the two angular regions is of the same order of magnitude.

The total cross section is obtained by integration of Eq. (3) over all angles and by summing over all s states. We use a simplified Molière screening potential, exponential screening,[3]

$$\frac{1-F(q)}{q^2}=\frac{1}{\Lambda^2+q^2} ,$$

where $\Lambda^{-1}=121Z^{-1/3}m^{-1}$ is the screening radius in this simple model. We find for lower energies ($E_p+p_p << 242Z^{-1/3}m_p$, no screening)

$$\sigma_p = 4\pi\frac{Z^2\alpha^6}{m_p^2}\zeta(3)\left[\ln\frac{E_p+p_p}{m_p}-\frac{p_p}{E_p}\right] , \tag{9}$$

while for extremely high energies we obtain ($E_p+p_p >> 242Z^{-1/3}m_p$, complete screening)

$$\sigma_p = 4\pi\frac{Z^2\alpha^6}{m_p^2}\zeta(3)[\ln(242Z^{-1/3})-1] . \tag{10}$$

Here $\zeta(p)=\sum^{\infty}n^{-p}$ is the Riemann ζ function, with $\zeta(3)=1.20205$.

Independent of the amount of screening, the relation to the total pair cross section $\sigma_{\rm pair}$ is approximately for high energies,

$$\sigma_p \approx \pi\frac{9}{28}\alpha^3\zeta(3)\sigma_{\rm pair} \approx \alpha^3 1.2\sigma_{\rm pair} . \tag{11}$$

A useful estimate of the magnitude of the high-energy cross section is

$$\sigma_p \approx 12\alpha^4\left[\frac{e^2}{m}\right]^2 Z^2 = 2.6\times10^{-33}Z^2\ \mathrm{cm}^2 , \tag{12}$$

where we have used[4] $\sigma_{\rm pair}\approx 10\alpha(e^2/m)^2Z^2$ for $\omega \gtrsim 100$ MeV.

The cross section Eq. (12) is small but not forbiddingly small. What would make the detection of relativistic positronium in pair production a challenge is the fact that the positronium particle is large, with a radius twice that of hydrogen and loosely bound with binding energy only half of the hydrogen-atom ionization energy. Thus positronium produced by a photon in a fixed target breaks up easily when colliding with atoms in the target.[5]

A simple estimate shows that the breakup process $(e^+e^-)+Z \rightarrow e^+ + e^- + Z$ for relativistic positronium is closely related to the scattering of the electron or positron by the atom. Positronium is broken up if the energy transfer to the electron or positron in the collision is larger than the binding energy,

$$\frac{q^2}{2m} > B .$$

The breakup cross section σ_b is then simply the scattering cross section $d\sigma_{sc}/dq$, for the electron and positron integrated over all momentum transfers larger than $\sqrt{2mB}$,

$$\sigma_b = 2\int_{q>\sqrt{2mB}}\frac{d\sigma}{dq}dq . \tag{13}$$

Since, however, the scattering is weak for impact parameters much larger than the screening radius Λ^{-1}, the cross section is small for $q \lesssim \Lambda$. With $\Lambda=(m/121)Z^{1/3} \gtrsim \sqrt{2mB}$, we conclude that the contribution to the scattering cross section for $q < 2mB$ is negligible in our approximation. We thus find the simple relation

$$\sigma_b = 2\sigma_{sc} . \tag{14}$$

With the Molière screening potential[3] we find

$$\sigma_b = Z^{4/3}0.45\times10^{-19}\ \mathrm{cm}^2 , \tag{15}$$

which is indeed fairly close to the exact result obtained by Dulyan, Kotsinyan, and Faustov.[5] Thus only a small part of the target with thickness $L\approx(N\sigma_b)^{-1}$ is effective in producing positronium. Here N is the number of atoms per cm^3. The effective target thickness is very small; one finds for Al, $L\approx 1.2\times10^{-5}$ cm and for Pb, 0.2×10^{-5} cm.

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