Spontaneous parity violation and Higgs-meson masses in a composite model for leptons, quarks, and Higgs mesons

Tsunehiro Kobayashi* and Shingo Tokitake
Institute of Physics, University of Tsukuba, Ibaraki 305, Japan
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Parity violation in the left-right-symmetric gauge model is naturally understood in a composite model by imposing the consistency of the model, that is, stability of the Higgs potential and positivity of squared masses of observable mesons. Two quite different types of solutions, which are respectively characterized with massless neutrinos and nonzero-mass neutrinos, are left as realistic solutions and the choice of one of them is determined by the sign of one coupling constant in the Higgs potential. The model predicts some Higgs mesons with masses of the order of M_{W_L} ($\sim$100 GeV), one of which will be observed in the charged lepton-antilepton pair decays, such as the $\tau^-\tau^+$ mode.

Higgs mesons are now the last crucial missing particles in the standard model [$SU(2)\otimes U(1)_Y$ gauge theory]. The standard model, however, tells us very little about Higgs mesons; that is, the model does not answer the following types of questions. How many Higgs doublets are there? What mass values do they have? What are the main decay modes? Many models beyond the standard model also do not seem encompassing enough to answer these questions. The composite model for leptons, quarks, and Higgs mesons[1] presented in Ref. 1, however, provides an explicit scheme for Higgs mesons and their couplings, which are written in terms of the left-right-symmetric gauge group $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$. In this paper we shall study the Higgs scheme proposed in the composite model and show that parity violation is naturally understood from the consistency of the model.

The model has preons t^l, t^q, and S^0 with the following representations of the left-right-symmetric gauge group G $\equiv SU(3)_H\otimes SU(3)_c\otimes SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$, where $SU(3)_H$ and $SU(3)_c$, respectively, stand for hypercolor and color interactions:

	$SU(3)_H$	$SU(3)_c$	$SU(2)_L$	$SU(2)_R$	N_{B-L}	J^P
$t^l_{L(R)}$	3	3	2(1)	1(2)	-1	$\frac{1}{2}^+$
$t^q_{L(R)}$	3	$\bar{3}$	2(1)	1(2)	$\frac{1}{3}$	$\frac{1}{2}^+$
S^0	3	3	1	1	0	0^+

(1)

In (1), N_{B-L} and J^P, respectively, represent the $B-L$ number and spin$^{\text{parity}}$ of particles. The preons t^l and t^q are, respectively, described by the charge doublets $(t^{l(0)},t^{l(-1)})$ and $(t^{q(2/3)},t^{q(-1/3)})$, where Q in $t^{a(Q)}$ denotes the charge of t^a. The S^0 boson is introduced in order to generate the generation of leptons and quarks. (For details, see Ref. 1.) Higgs mesons in the model are represented with the bound states written as $\Delta_L^*\equiv t_L^l t_L^l \widetilde{S}^0$, $\Delta_R^*\equiv t_R^l t_R^l \widetilde{S}^0$, $\phi^l\equiv t_L^l \overline{t_R^l}$, and $\phi^q\equiv t_L^q \overline{t_R^q}$, which are described by the following representations[1,2] of the $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$ group:

	$SU(2)_L$	$SU(2)_R$	N_{B-L}
Δ_L	3	1	2
Δ_R	1	3	2
ϕ^l	2	2	0
ϕ^q	2	2	0

(2)

where it is noted that ϕ^l couples with only leptons, while ϕ^q couples with only quarks.[1,3] Note that the mesons presented in (2) are just the same as the Higgs mesons introduced by Mohapatra and Senjanović[4] for spontaneous parity violation in the $SU(2)_L\otimes SU(2)_R\otimes U(1)_{B-L}$ gauge group and the notation $\widetilde{S}^0$ is written in terms of the linear combination of the (3,3) representation of the $(SU(3)_H,SU(3)_c)$ group as

$$\widetilde{S}^0=\frac{1}{\sqrt{N+1}}\{S^0+(S^{0\dagger}S^{0\dagger})_3+[(S^0S^0)_{\bar{3}}(S^0S^0)_{\bar{3}}]_3+\cdots\},$$

where N stands for the number of fermion generations. We should comment on the Vafa-Witten theorem for a vectorlike composite model.[5] Since our model has scalar preons and allows Yukawa couplings among preons,[6] the theorem does not work in our model.

The effective couplings among $\Delta_{L,R}$ and ϕ^a are described by line-connected diagrams as shown in Fig. 1 (Refs. 1 and 3). Since ϕ^q does not couple to $\Delta_{L,R}$ and ϕ^l, we shall for simplicity neglect ϕ^q in the following discussions. The effective interaction potential is written as

$$V_{\rm eff}=(\alpha_0+\alpha_\epsilon)[\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)]^2+\alpha_1\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R)+\beta_1\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L\phi\phi^\dagger+\Delta_R^\dagger\Delta_R\phi^\dagger\phi)$$
$$+\beta_2\,\mathrm{Tr}(\Delta_L^\dagger\phi\Delta_R\phi^\dagger+\Delta_R^\dagger\phi^\dagger\Delta_L\phi)+\beta_\epsilon\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)\,\mathrm{Tr}(\phi^\dagger\phi)+\gamma\,\mathrm{Tr}(\phi^\dagger\phi\phi^\dagger\phi)+\gamma_\epsilon[\mathrm{Tr}(\phi^\dagger\phi)]^2$$
$$-\mu^2\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)+m^2\,\mathrm{Tr}(\phi^\dagger\phi)\ , \tag{3}$$

where $\phi\equiv\phi^l$ and the terms represented with the coupling constants α_ϵ, β_ϵ, and γ_ϵ are introduced as the corrections in terms of line-disconnected diagrams[1,3] shown in Fig. 2. Therefore, it should be imposed in the following discussions that

$$|\alpha_\epsilon| \ll |\alpha_0|\ ,\quad |\beta_\epsilon| \ll |\beta_1|\ ,\quad |\gamma_\epsilon| \ll |\gamma|\ . \tag{4}$$

This scheme has essentially 9 parameters, i.e., 7 coupling constants and 2 mass values, which should be compared with 33 parameters of the model given in Ref. 4. In (3) it must be noticed that only the Δ mesons can have a negative squared mass, $-\mu^2$, through the S^0-boson condensation in a vacuum, but the ϕ meson still has a positive squared mass, m^2, in this model.[1] Following the discussion for the S^0-boson condensation given in Ref. 1, we always have to take account of the constraint

$$\mu^2 \gg m^2 > 0 \tag{5}$$

hereafter. The vacuum expectation values appear as

$$V_L\equiv\langle\Delta_L^0\rangle\ ,\quad V_R\equiv\langle\Delta_R^0\rangle\ ,$$
$$a\equiv\langle\phi_1^0\rangle\ ,\quad b\equiv\langle\phi_2^0\rangle\ ,$$

where $\phi_1^0\equiv t_L^{l(0)}\overline{t_R^{l(0)}}$ and $\phi_2^0\equiv t_L^{l(-1)}\overline{t_R^{l(-1)}}$. Now we have the following Higgs potential:

$$V_H=\bar\alpha V^4+\alpha_1(V_L{}^4+V_R{}^4)+\bar\beta V^2a^2+\beta_\epsilon V^2b^2$$
$$+2\beta_2V_LV_Ra^2+\bar\gamma(a^4+b^4)+2\gamma_\epsilon a^2b^2$$
$$-\mu^2V^2+m^2(a^2+b^2)\ , \tag{6}$$

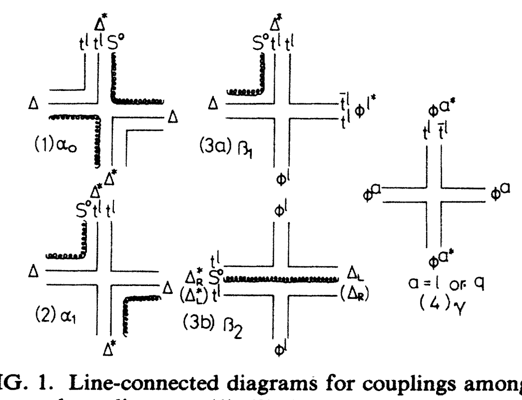

FIG. 1. Line-connected diagrams for couplings among Higgs mesons, where diagrams (1), (2), (3a), (3b), and (4), respectively, represent the interactions

$$\alpha_0[\mathrm{Tr}(\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R)]^2\ ,\ \alpha_1\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L\Delta_L^\dagger\Delta_L+\Delta_R^\dagger\Delta_R\Delta_R^\dagger\Delta_R)\ ,$$
$$\beta_1\,\mathrm{Tr}(\Delta_L^\dagger\Delta_L\phi^l\phi^{l\dagger}+\Delta_R^\dagger\Delta_R\phi^{l\dagger}\phi^l)\ ,\ \beta_2\,\mathrm{Tr}(\Delta_R^\dagger\phi^{l\dagger}\Delta_L\phi^l+\Delta_L^\dagger\phi\Delta_R\phi^\dagger)\ ,$$

and

$$\gamma\,\mathrm{Tr}(\phi^{l\dagger}\phi^l\phi^{l\dagger}\phi^l+\phi^{q\dagger}\phi^q\phi^{q\dagger}\phi^q)\ .$$

where

$V^2\equiv V_L{}^2+V_R{}^2$, $\bar\alpha\equiv\alpha_0+\alpha_\epsilon$, $\bar\beta\equiv\beta_1+\beta_\epsilon$, and $\bar\gamma\equiv\gamma+\gamma_\epsilon$.

In order that V_H has a stable minimum, the constraints

$$\tilde\alpha\equiv\bar\alpha+\alpha_1>0\ ,\quad \bar\gamma\simeq\gamma>0 \tag{7}$$

are required. Except for the solutions $V_L=V_R=a=b=0$ and one (or more) of $V_L{}^2$, $V_R{}^2$, a^2, and b^2 negative, we can find the extrema satisfying the conditions

$$\frac{\partial V_H}{\partial V_L}=\frac{\partial V_H}{\partial V_R}=\frac{\partial V_H}{\partial a}\frac{\partial V_H}{\partial b}=0$$

at the following 8 points, half of which are symmetric for $V_L{}^2$ and $V_R{}^2$ $(V_L{}^2=V_R{}^2)$ and the other half unsymmetric.

Symmetric solutions:

$(s-1)$ $a=b=0$,

$$V_L^2=V_R^2=\mu^2/[2(2\bar\alpha+\alpha_1)]\ ,V_H^{(s-1)}=-\mu^4/[2(2\bar\alpha+\alpha_1)]\ ;$$

$(s-2)$ $a=0$,

$$b^2=-[\beta_\epsilon\mu^2+(2\bar\alpha+\alpha_1)m^2]/D^{(s-2)}\ ,$$
$$V_L{}^2=V_R{}^2=(\bar\gamma\mu^2+\tfrac{1}{2}\beta_\epsilon m^2)/D^{(s-2)}\ ,$$
$$V_H^{(s-2)}=-[\bar\gamma\mu^4+\beta_\epsilon m^2\mu^2+\tfrac{1}{2}(2\bar\alpha+\alpha_1)m^4]/D^{(s-2)}\ :$$

$(s-3)$ $b=0$,

$$a^2=-[(\bar\beta+\beta_2V_R/V_L)\mu^2+(2\bar\alpha+\alpha_1)m^2]/D^{(s-3)}\ ,$$
$$V_L{}^2=V_R{}^2=[\bar\gamma\mu^2+\tfrac{1}{2}(\bar\beta+\beta_2V_R/V_L)m^2]/D^{(s-3)}\ ,$$
$$V_H^{(s-3)}=-[\bar\gamma\mu^4+(\bar\beta+\beta_2V_R/V_L)m^2\mu^2+\tfrac{1}{2}(2\bar\alpha+\alpha_1)m^4]/D^{(s-3)}\ ;$$

$(s-4)$

$$a^2=\{2[\gamma_\epsilon\beta_\epsilon-\bar\gamma(\bar\beta+\beta_2V_R/V_L)]\mu^2+[-2\gamma(2\bar\alpha+\alpha_1)-\beta_\epsilon(\beta_1+\beta_2V_R/V_L)]m^2\}/D^{(s-4)}\ ,$$
$$b^2=\{2[\gamma_\epsilon(\beta_1+\beta_2V_R/V_L)-\gamma\beta_\epsilon]\mu^2+[-\beta_\epsilon(\bar\beta+\beta_2V_R/V_L)-2\gamma(2\bar\alpha+\alpha_1)+(\bar\beta+\beta_2V_R/V_L)^2]m^2\}/D^{(s-4)}\ ,$$
$$V_L{}^2=V_R{}^2=\{2(\gamma^2-\gamma_\epsilon{}^2)\mu^2+[(\bar\beta+\beta_2V_R/V_L)\gamma+\beta_\epsilon\gamma]m^2\}/D^{(s-4)}\ ,$$
$$V_H^{(s-4)}=-\left[2(\bar\gamma^2-\gamma_\epsilon{}^2)\mu^4+\left[2(2\bar\alpha+\alpha_1)\gamma-\frac{(\beta_1+\beta_2)^2}{2}\right]m^4+2\gamma(\bar\beta+\beta_2+\beta_\epsilon)m^2\mu^2\right]\Big/D^{(s-4)}\ . \tag{8a}$$

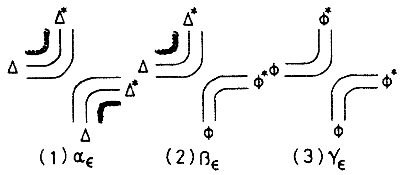

FIG. 2. Line-disconnected diagrams for couplings among Higgs mesons.

Unsymmetric solutions ($V_R{}^2 > V_L{}^2$) :

$(u-1)$ $a=b=V_L=0$,

$V_R{}^2=\mu^2/2\tilde{\alpha}$, $V_H^{(u-1)}=-\mu^4/4\tilde{\alpha}$,

$(u-2)$ $a=V_L=0$,

$b^2=-(\beta_\epsilon\mu^2+2\tilde{\alpha}m^2)/D^{(u-2)}$,

$V_R{}^2=(2\bar{\gamma}\mu^2+\beta_\epsilon m^2)/D^{(u-2)}$,

$V_H^{(u-2)}=-(\bar{\gamma}\mu^4+\beta_\epsilon m^2\mu^2+\tilde{\alpha}m^4)/D^{(u-2)}$,

$(u-3)$ $b=0$,

$a^2=-(\bar{\beta}\mu^2+2\tilde{\alpha}m^2)/D^{(u-3)}$,

$2\alpha_1 V_L V_R=\beta_2 a^2$,

$V^2=[(\bar{\gamma}+\beta_2{}^2/2\alpha_1)\mu^2+\tfrac{1}{2}\bar{\beta}m^2]/D^{(u-3)}$,

$V_H^{(u-3)}=-[(\bar{\gamma}+\beta_2{}^2/2\alpha_1)\mu^4+\bar{\beta}m^2\mu^2+\tilde{\alpha}m^4]/D^{(u-3)}$,

$(u-4)$

$a^2=-[2(\beta_\epsilon\gamma+\beta_1\bar{\gamma})\mu^2+(4\tilde{\alpha}\gamma+\beta_1\beta_\epsilon)m^2]/D^{(u-4)}$,

$b^2=[(2\beta_1\gamma_\epsilon-2\gamma\beta_\epsilon-\beta_2{}^2\beta_\epsilon/\alpha_1)\mu^2$
$\quad -(4\tilde{\alpha}\gamma-\beta_1\bar{\beta}+2\tilde{\alpha}\beta_2{}^2/\alpha_1)m^2]/D^{(u-4)}$,

$V^2=\{[4(\bar{\gamma}^2-\gamma_\epsilon{}^2)+2\beta_2{}^2\bar{\gamma}/\alpha_1]\mu^2$
$\quad +[2\gamma(\bar{\beta}+\beta_\epsilon)+\beta_\epsilon\beta_2{}^2/\alpha_1]m^2\}/D^{(u-4)}$,

$2\alpha_1 V_L V_R=\beta_2 a^2$,

$V_H^{(u-4)}=-\{[2(\bar{\gamma}^2-\gamma_\epsilon{}^2)+\beta_2{}^2\bar{\gamma}/\alpha_1]\mu^4$
$\quad +[4\tilde{\alpha}\gamma-\beta_1{}^2/2+\beta_2{}^2\tilde{\alpha}/\alpha_1]m^4$
$\quad +[2\gamma(\bar{\beta}+\beta_\epsilon)+\beta_\epsilon\beta_2{}^2/\alpha_1]m^2\mu^2\}/D^{(u-4)}$, (8b)

where $V_H^{(A)}$ stands for the value of the Higgs potential for solution(A) and

$$D^{(s-2)}=2(2\bar{\alpha}+\alpha_1)\bar{\gamma}-\beta_\epsilon{}^2 ,$$

$$D^{(s-3)}=2(2\bar{\alpha}+\alpha_1)\bar{\gamma}-(\bar{\beta}+\beta_2 V_R/V_L)^2 ,$$

$$D^{(s-4)}=4(\bar{\gamma}^2-\gamma_\epsilon{}^2)(2\bar{\alpha}+\alpha_1)+4(\bar{\beta}+\beta_2 V_R/V_L)\gamma_\epsilon\beta_\epsilon -2\bar{\gamma}[\beta_\epsilon{}^2+(\bar{\beta}+\beta_2 V_R/V_L)^2] ,$$

$$D^{(u-2)}=4\tilde{\alpha}\bar{\gamma}-\beta_\epsilon{}^2 ,$$

$$D^{(u-3)}=4\tilde{\alpha}(\bar{\gamma}+\beta_2{}^2/2\alpha_1)-\bar{\beta}^2 ,$$

$$D^{(u-4)}=8\tilde{\alpha}(\bar{\gamma}^2-\gamma_\epsilon{}^2+\beta_2{}^2\bar{\gamma}/2\alpha_1) -4\beta_1\beta_\epsilon\gamma-2\beta_1{}^2\bar{\gamma}-\left[4\gamma+\frac{\beta_2{}^2}{\alpha_1}\right]\beta_\epsilon{}^2 ,$$

and the unsymmetric solutions with $V_L{}^2 > V_R{}^2$ corresponding to the solutions $(u-1)$ and $(u-3)$ are not written here. We have also omitted the solutions with $a=0$ and $a=b=0$ for $\alpha_1=0$ and those with $V_L=0$ and $b=V_L=0$ for $\beta_2=0$, because those solutions are easily shown to be local minimums.

We easily see that in the limit of $\beta_\epsilon=\gamma_\epsilon=0$ the solutions $(s-4)$ and $(u-4)$ are, respectively, equal to the solutions $(s-3)$ and $(u-3)$ because of the positivity of γ and m^2. Taking account of the inequality relation (5) ($\mu^2 \gg m^2$), the following conditions are derived from the constraints (7) and the positivity of a^2, b^2, $V_L{}^2$ and $V_R{}^2$:

$$\begin{aligned} &(s-2)\ \ D^{(s-2)}>0,\ \ \beta_\epsilon<0 ,\\ &(s-3)\ \ D^{(s-3)}>0,\ \ \bar{\beta}+\beta_2 V_R/V_L<0 ,\\ &(u-2)\ \ D^{(u-2)}>0,\ \ \beta_\epsilon<0 ,\\ &(u-3)\ \ D^{(u-3)}>0 ,\ \ \bar{\beta}<0 . \end{aligned} \tag{9}$$

From the smallness condition of β_ϵ and γ_ϵ given in (4), we may also impose the same constraints for $(s-3)$ and $(u-3)$ on $(s-4)$ and $(u-4)$, respectively. The constraints for β 's in (9) can easily be understood from the form of the Higgs potential. That is to say, since the terms written by $(\bar{\beta}V^2+2\beta_2 V_L V_R)a^2$ are reduced to $(\bar{\beta}+\beta_2 V_R/V_L)V^2a^2$ for the symmetric solutions and to $\bar{\beta}V^2a^2+(\beta_2{}^2/\alpha_1)a^4$ for the unsymmetric ones, the solutions with $a^2>0$ have the constraint $(\bar{\beta}+\beta_2 V_R/V_L)<0$ for the symmetric ones and $\bar{\beta}<0$ for the unsymmetric ones. The constraints

$$\begin{aligned} \Gamma^{(s-4)}\equiv{}&2[\gamma_\epsilon(\bar{\beta}+\beta_2 V_R/V_L)-\bar{\gamma}\beta_\epsilon]\mu^2\\ &-[2(2\bar{\alpha}+\alpha_1)\gamma+(\bar{\beta}+\beta_2 V_R/V_L)\beta_\epsilon\\ &-(\bar{\beta}+\beta_2 V_R/V_L)^2]m^2>0\ \ \text{for } (s-4) ,\\ \Gamma^{(u-4)}\equiv{}&2(\beta_1\gamma_\epsilon-\beta_\epsilon\gamma-\beta_2{}^2\beta_\epsilon/2\alpha_1)\mu^2\\ &-[2\tilde{\alpha}(2\gamma+\beta_2{}^2/\alpha_1)-\bar{\beta}\beta_1]m^2>0\\ &\text{for } (u-4) \end{aligned} \tag{10}$$

must be satisfied because of $b^2>0$. We find a constraint for β_1 for the solution $(u-2)$ from the positivity of squared masses given in Table I:

$$\beta_1>2\gamma b^2/V_R{}^2\simeq 2\left[\frac{\gamma}{\bar{\gamma}}\right]|\beta_\epsilon|\gtrsim 0\ \ \text{for } (u-2) . \tag{11}$$

From a similar consideration, we may put the similar constraint

$$\beta_1>O(m^2/\mu^2)\simeq 0\ \ \text{for}(s-1)\text{ and }(u-1) . \tag{12}$$

Further constraints can also be derived from the positivity of squared mass values of observable components of

TABLE I. Squared masses of Higgs mesons where the relations $V_R{}^2 \gg a^2+b^2 \gg V_L{}^2$ are used when we need the approximation.

	$(u-2)$	$(u-3)$	$(u-4)$
$\Delta_L^{++},\Delta_R^{++}$	$-2\alpha_1 V_R{}^2+(\beta_1-\lvert\beta_2\rvert)b^2$	$-2\alpha_1 V^2-\beta_1 a^2$	$-2\alpha_1 V^2+\beta_1(b^2-a^2)+\lvert\beta_2\rvert b^2$
	$-2\alpha_1 V_R{}^2+(\beta_1+\lvert\beta_2\rvert)b^2$	$-2\alpha_1 V^2-\beta_1 a^2$	$-2\alpha_1 V^2+\beta_1(b^2-a^2)+\lvert\beta_2\rvert b^2$
$\Delta_L^{+},\Delta_R^{+}$	$-2\alpha_1 V_R{}^2+\frac{1}{2}\beta_1 b^2$	$-2\alpha_1 V_R{}^2-\beta_1(V_L{}^2+a^2/2)-\beta_2^2 a^2/\alpha_1$	$-2\alpha_1 V_R{}^2+2\gamma a^2+\beta_1(V_L{}^2-a^2/2)$
ϕ_1^{+},ϕ_2^{+}	$\beta_1 V_R{}^2+\frac{1}{2}\beta_1 b^2$	$-2\alpha_1 V_L{}^2-\beta_1(V_R{}^2+a^2/2)-\beta_2^2 a^2/\alpha_1$	$-2\alpha_1 V_L{}^2+2\gamma a^2+\beta_1(V_R{}^2-a^2/2)$
	0	0	0
	0	0	0
$(m_0)_i$	$-2\alpha_1 V_R{}^2$	$-2\alpha_1 V^2-2\beta_2^2 a^2/\alpha_1$	$-2\alpha_1[V^2+(\beta_2^2/\alpha_1^2)a^2]$
	$\beta_1 V_R{}^2-2\gamma b^2$	$-\beta_1 V^2-(2\gamma+\beta_2^2/\alpha_1)a^2$	0
	0	0	0
	0	0	0
$(m_0)_r$	$-2\alpha_1 V_R{}^2$	$-2\alpha_1 V_R{}^2$	$-2\alpha_1 V_R{}^2$
	$4\tilde{\alpha} V_R{}^2$	$4\tilde{\alpha} V_R{}^2$	$4\tilde{\alpha} V_R{}^2$
	$\beta_1 V_R{}^2-2\gamma b^2$	$-\beta_1 V^2-(2\gamma+\beta_2^2/\alpha_1)a^2$	$4\bar{\gamma} b^2$
	$(4\tilde{\alpha}\bar{\gamma}-\beta_\epsilon^2)b^2/\tilde{\alpha}$	$\left[4\tilde{\alpha}\gamma-\bar{\beta}^2+\frac{2\beta_2\tilde{\alpha}}{\alpha_1}\right]\frac{a^2}{\tilde{\alpha}}$	$\left[4\tilde{\alpha}\bar{\gamma}-\bar{\beta}^2+\frac{2\beta_2\tilde{\alpha}}{\alpha_1}\right]\frac{a^2}{\tilde{\alpha}}$

Δ_L, Δ_R, and ϕ. In particular, from the squared mass of the observable component of the imaginary parts of neutral Higgs mesons given by

$$(m_{0,i})^2=-\alpha_1 V^2 \quad \text{for}\,(s-1)\text{ and }(s-2)$$
$$=-2\alpha_1\left[V^2+\frac{\beta_2^2}{\alpha_1^2}a^2\right]$$

for all unsymmetric solutions, the constraint

$$\alpha_1<0 \tag{13}$$

is derived. For $(s-3)$ and $(s-4)$ we obtain

$$\alpha_1<-(\beta_1-\lvert\beta_2\rvert)(a^2+b^2)/V^2 \tag{14}$$

from the positivity of $(m_{0,i})^2$ and $m^2(\Delta^{++})$, where the relation $\beta_2 V_R/V_L=-\lvert\beta_2\rvert$ is also derived in the evaluation. Now we see that all interesting solutions with $(a^2+b^2)\sim m_{W_L}{}^2 \ll V_R{}^2 \sim m_{W_R}{}^2$ are in the region with $\alpha_1<0$, where m_{W_L} and m_{W_R} are, respectively, the masses of the left-handed and right-handed weak bosons.[1,4]

Let us search out the real minimum of Higgs potential under the constraints derived in (9) — (13). We can easily get inequality relations

$$V_H^{(s-1)}>V_H^{(u-1)}\,,\quad V_H^{(s-2)}>V_H^{(u-2)}\,. \tag{15}$$

The relations

$$\begin{gathered} V_H^{(s-1)}>V_H^{(s-2)}>V_H^{(u-1)}>V_H^{(u-2)}\,,\\ V_H^{(s-3)}\geq V_H^{(s-4)}\,,\quad V_H^{(u-3)}\geq V_H^{(u-4)}\,,\\ V_H^{(s-3)}>V_H^{(u-3)}\,,\quad V_H^{(s-4)}>V_H^{(u-4)}\,, \end{gathered} \tag{16}$$

are also derived in the parameter region where the solutions appearing in each relation coexist. Instead of the complicated form of $V_H^{(s-4)}-V_H^{(u-1)}$, we study the relation in the limit of $\beta_\epsilon=\gamma_\epsilon=0$, where it is evaluated as

$$V_H^{(s-4)}-V_H^{(u-1)}\cong\left[\frac{1}{4\tilde{\alpha}}-\frac{1}{4\tilde{\alpha}-2\alpha_1-(\beta_1-\lvert\beta_2\rvert)^2/\gamma}\right]\mu^4 + O(m^2\mu^2)\,. \tag{17}$$

It is easily seen that

$$\begin{aligned} V_H^{(s-4)}>V_H^{(u-1)} &\quad \text{for } 2\lvert\alpha_1\rvert\gamma>(\beta_1-\lvert\beta_2\rvert)^2\,,\\ V_H^{(s-4)}<V_H^{(u-1)} &\quad \text{for } 2\lvert\alpha_1\rvert\gamma<(\beta_1-\lvert\beta_2\rvert)^2\,. \end{aligned} \tag{18}$$

The flow chart for the relation among the real minimun of V_H and constraints for the coupling constants are illustrated in Fig. 3. We may now conclude that the spontane-

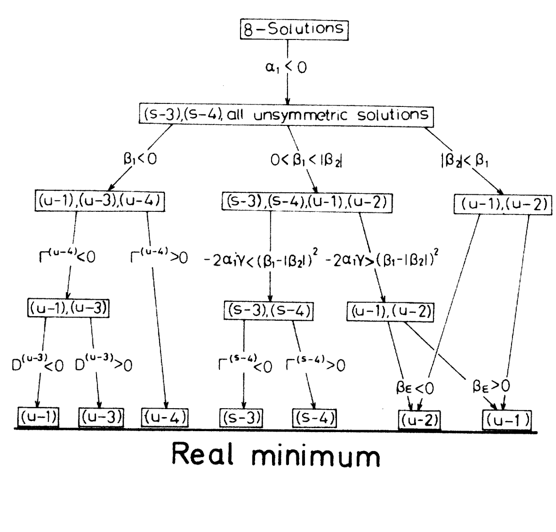

FIG. 3. Flow chart for the real minimum of the Higgs potential and the constraints for the couplings.

ous parity violation with the satisfactory relation $(a^2+b^2) \ll V_R{}^2$ is naturally realized in the region with $\alpha_1<0$ except for the choice of β_1 such that the constraints $|\beta_2|>\beta_1>0$ and $2|\alpha_1|\gamma<(\beta_1-|\beta_2|)^2$ are satisfied. It should be stressed that the parity-conserving case is realized only for the very special choice of parameters in this model.

Let us here compare three unsymmetric solutions $(u-2)$, $(u-3)$, and $(u-4)$ with each other under the phenomenological constraint $V_L{}^2 \ll a^2+b^2 \ll V_R{}^2$. We shall ignore the solution $(u-1)$, because it derives $m_{W_L}=0$. A crucial difference appears in neutrino masses. Nonzero neutrino masses, as was discussed in Refs. 1 and 4, are derived for $(u-3)$ and $(u-4)$, while in the solution $(u-2)$ the left-handed components of neutrinos are still massless because of $a=V_L=0$ and only their right-handed components can acquire masses through the Majorana-type couplings with Δ_R. A similar difference can also be seen in the relation between the masses of W_L and Z_L^0, that is, the same relation with that of the standard model is derived for $(u-2)$, whereas a little difference[1,4] arises for $(u-3)$ and $(u-4)$ at the tree level. We see that the solution $(u-2)$ completely reproduces the standard model at low energies. It may be noted that for $(u-3)$ and $(u-4)$ the further constraint

$$|\bar{\beta}| \sim m^2/\mu^2 \tag{19}$$

is required in order to reproduce $a^2+b^2 \ll V^2 \sim \mu^2$, while only $|\beta_\epsilon| \sim m^2/\mu^2$ is imposed but $|\bar{\beta}|$ can be taken arbitrarily for $(u-2)$. We may, therefore, say that the solution $(u-2)$ is most favorable in this model if we impose that all coupling constants for the connected diagrams are in same order of magnitude, i.e.,

$$|\alpha_0| \sim |\alpha_1| \sim |\beta_1| \sim |\beta_2| \sim |\gamma| \ . \tag{20}$$

From Table I we can see that for three unsymmetric solutions the lightest mass of neutral Higgs mesons are order of $m_{W_L} \sim (a^2+b^2)^{1/2} \sim 100$ GeV, of which the main decay mode is always a pair of a charged lepton-antilepton pair (l^+l^-). We propose that the lightest Higgs meson should be searched for in $\tau^+\tau^-$ decays. For $(u-3)$ and $(u-4)$ one more neutral Higgs meson (the mixing state of Δ_L^+, Δ_R^+, and ϕ^+) is also estimated to have masses of order of m_{W_L}, of which the main decay modes are, respectively, $\nu_L\bar{\nu}_R$ decay and $\nu_L l^+$ (or $\bar{\nu}_L l^-$) decay. The charged Higgs mesons should be searched for in modes such as a charged lepton plus large missing energy.

Up until now we neglected the ϕ^q mesons. Since the ϕ^q does not couple to the Δ^0 mesons via the connected diagrams the same as $\phi_2^{l(0)}$, the introduction of ϕ^q does not change the above argument at all. We have, however, to take account of ϕ^q when we discuss the quark masses. We easily see that the Higgs couplings given in (3) do not lead to any mass difference between the u- and d-quark series. In order to explain it, we have to consider the couplings among mesons introduced by Chang, Mohapatra, Pal, and Pati.[7] Fortunately we can easily show that all mesons introduced in Ref. 7 can be expressed in terms of the bound states $\Delta_{L(R)}$, ϕ^l, ϕ^q, $\phi^{lq}=t^l\overline{t^q}$, and $\Delta^{lq}=t^lt^q\widetilde{S}^0$. We will discuss the point in a future paper.

In this model the exotic particles, such as ϕ^{lq}, Δ^{lq}, etc., appear. The exotic meson ϕ^{lq} should be payed particular attention to, because $\phi^{l(0)q}=t^{l(0)}\overline{t^q}$ has a decay mode such as ν_L+ a quark jet, which might be seen as the monojet events reported by UA (1) group at the CERN $p\bar{p}$ collider.[8] In order to estimate the masses of such exotic particles, we must study the contribution of ϕ^q, which has been ignored in this paper. We shall investigate them in a future paper.

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*Present address: Max-Planck Institute, 8000 Munich 40, Federal Republic of Germany.

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