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Implications of E_6 grand unification

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We study the allowed intermediate symmetries and the charged- and neutral-fermion masses in E_6 grand-unification theories. We isolate an interesting symmetry-breaking chain where a neutral lepton with mass in the GeV range exists. Such a lepton will contribute to the width of the Z boson, which is an interesting feature of the E_6 model.

I. INTRODUCTION

Grand unification of particle interactions based on an exceptional group such as E_6 was originally proposed[1] as an alternative to the SU(5) and SO(10) models and has been discussed from time to time.[2] Before the recent extensive studies of the properties of b quarks, it was hoped that the new quark that exists in the E_6 model could be identified with the b quark, leading to models without top quarks.[3] However, experiments on b quarks as well as the recent discovery of the top quark have made this hypothesis untenable. Then, why consider these models? Our motivations in undertaking this research are the following: (a) the failure of minimal SU(5) to explain the recent results on the proton-decay experiments suggests that if the grand-unification hypothesis is correct some higher unification groups such as SO(10) or E_6 may be relevant; (b) recent developments in string theories appear to point towards exceptional groups as one class of possible grand-unifying groups; (c) with a new generation of machines in the horizon, such as CERN LEP, DESY HERA, and the Superconducting Super Collider (SSC), to probe beyond the standard model sensible grand-unified groups should be analyzed for possible distinctive experimental signatures at these energies; (d) in most previous discussions, the additional quarks and leptons in the fundamental representation were identified with the b quark and the additional charged lepton with τ. We consider the possibility that the additional particles are new particles with new properties [for instance, since some of them do not carry $SU(2)_L$ quantum number, they could be superheavy, etc.].

While there exist several important papers on E_6 grand unification, some aspects of this model have not been thoroughly analyzed; in particular, the analysis of possible intermediate mass scales are mostly semiquantitative and discussion of neutral-fermion masses sketchy (for an exception, see Ref. 3). The latter is of interest for the following reason: a realistic model must contain either an ultralight or massless neutrino with a correct electroweak quantum number. In the SO(10) model, this implies the existence of a massive Majorana right-handed neutrino. Since the E_6 model has five neutral leptons, it will be of interest to know how the rest of the neutral-lepton sector is constrained by the requirement of the ultralight ν_e. We plan to investigate both these issues in the present paper and isolate their experimental signatures.

Our principal results follow.

(i) We present a detailed profile of intermediate mass scales when E_6 breaks down to the standard model via either SO(10) or $SU(3)_C \times SU(3)_L \times SU(3)_R$ groups.

(ii) Neutron-antineutron oscillation is forbidden in the E_6 model in lowest order, for lower-dimensional Higgs multiplets.

(iii) We find phenomenologically acceptable symmetry-breaking chains, where E_6 breaks down to $SU(3)_C \times SU(2)_L \times U(1)_Y$ via $SU(3)_C \times SU(3)_L \times SU(3)_R$, which predicts a new neutral lepton with a mass in the GeV range, having "maximal" coupling to the Z bosons. Such particles are nonexistent in the SO(10) model and therefore their observation will be an indication in favor of the E_6 model. It will not, however, be a decisive test since other extensions of the standard model could lead to such signatures.

This paper is organized as follows. In Sec. II we introduce the basic representations and subgroups of E_6 needed in the model building and study the symmetry-breaking patterns. In Sec. III we present the equations that relate $\sin^2\theta_W$ and α_s to the various intermediate mass scales in the theory, discuss possible restrictions on the mass scales and their phenomenological implications. In Sec. IV we study charged-fermion mass matrices and in Sec. V, the neutral-fermion mass matrices. In Sec. VI we consider proton decay and n-$\bar{n}$ oscillation. We conclude in Sec. VII with some further comments.

II. SYMMETRY BREAKING AND PARTICLE CONTENT

The fundamental and adjoint representations of E_6 are $\{27\}$ and $\{78\}$ dimensional, respectively. Under the two maximal subgroups of E_6, namely, $SU(3)_C \times SU(3)_L \times SU(3)_R$ and $SO(10) \times U(1)_X$, the $\{27\}$-dimensional representation splits as follows:

$$E_6 \rightarrow SU(3)_C \times SU(3)_L \times SU(3)_R ,$$
$$\{27\} = (3,3,1) + (3^*,1,3^*) + (1,3^*,3) , \quad (1)$$

$$E_6 \to SO(10)\times U(1)_X ,$$
$$\{27\}=\{16\}_1+\{10\}_{-2}+\{1\}_{+4} . \tag{2}$$

From Eq. (2), it is clear that $\{27\}$ accommodates all particles per generation, which fit into the $\{16\}$-dimensional representation of SO(10); there are eleven additional fermions, which are unobserved as yet; it would, therefore, be of interest to be able to predict their masses and decay characteristics. We display the fermionic assignment below using Eq. (1):

$$(3,3,1):\ \begin{bmatrix} u \\ d \\ d' \end{bmatrix}_L ,$$
$$(3^*,1,3^*):\ \begin{bmatrix} u^c \\ d^c \\ d'^c \end{bmatrix}_L , \tag{3}$$
$$(1,3,3^*)=\begin{bmatrix} E_1^0 & E^+ & e^+ \\ E^- & E_2^0 & N^c \\ e^- & \nu & n \end{bmatrix}_L .$$

The gauge bosons belong to the $\{78\}$-dimensional representation, which breaks up under the above maximal subgroups as follows:

$$E_6 \to SU(3)_C\times SU(3)_L\times SU(3)_R ,$$
$$\{78\}=(8,1,1)+(1,8,1)+(1,1,8)+(3,3^*,3^*)+(3^*,3,3) , \tag{4}$$
$$E_6 \to SO(10)\times U(1)_X ,$$
$$\{78\}=\{45\}_0+\{16\}_{-3}+\{\overline{16}\}_{+3}+\{1\}_0 . \tag{5}$$

Let us now discuss the Higgs multiplet that will be needed to cause the symmetry breaking from E_6 to the standard model [i.e., $SU(3)_C\times SU(2)_L\times U(1)_Y$] as well as to give masses to the fermions. Since we expect the observed quarks and leptons to pick up mass at the $SU(2)_L\times U(1)_Y$-breaking scale, we would like the $SU(2)_L\times U(1)_Y$ breaking to be caused by a Higgs multiplet belonging to the $\{27\}$-dimensional representation of E_6 since

$$\{27\}\times\{27\}=\{27\}+\{351\}_a+\{351'\}_s . \tag{6}$$

We denote $\{27\}_W$ as the multiplet which causes this part of the symmetry breaking. In the interest of brevity, we list below the various chains of breaking E_6 down to $SU(3)_C\times SU(2)_L\times U(1)_Y$. The Higgs multiplets needed for the purpose are noted as well:

(i) $E_6 \underset{\{650\}\ M_U}{\to} SU(3)_C\times SU(3)_L\times SU(3)_R$

$\{78\}\ \downarrow M_{3R}$

$SU(3)_C\times SU(3)_L\times SU(2)_R\times U(1)_R$

$\{78\}\ \downarrow M_{3L}$

$SU(3)_C\times SU(2_L)\times SU(2)_R\times U(1)_L\times U(1)_R$

$\{351'\}$ or $\{27\}\ \downarrow M_0$

$SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$

$\{351'\}\ \downarrow M_R$

$SU(3)_C\times SU(2)_L\times U(1)_Y ,$

(ii) $E_6 \underset{\{27\}\text{ or }\{351'\}\ M_U}{\to} SO(10)$

$M_{10}\ \downarrow \{351'\}$

$SU(2)_L\times SU(2)_R\times SU(4)_C$

$M_R=M_C\ \downarrow \{351'\}$ or $\{27\}$

$SU(3)_C\times SU(2)_L\times U(1)_Y .$

In the next section, we discuss the mass hierarchies for both the chains allowed by values of $\alpha_s(M_W)\simeq 0.1-0.12$ and $\sin^2\theta_W(M_W)\simeq 0.21-0.23$.

III. INTERMEDIATE MASS SCALES

In order to study the mass hierarchies in the E_6 model, we make use of the evolution equation for coupling constants due to Georgi, Quinn, and Weinberg:[4]

$$\frac{1}{g^2(\mu)}=\frac{1}{g^2(M)}+\frac{B}{8\pi^2}\ln\frac{\mu}{M} , \tag{7}$$

where M and μ are two different mass scales, g denotes the coupling constant of some gauge group, and B is defined by

$$B=\tfrac{11}{3}c_V-\tfrac{4}{3}c_F-\tfrac{1}{6}c_S , \tag{8}$$

where c_V, c_F, and c_S denote the second-order Casimir invariants for the representations containing vectors, fermions, and scalars, respectively.

The other equation that we need is a consistency condition for coupling constants at a symmetry-breaking scale:

$$\frac{1}{g^2}=\sum_i \frac{{c_i}^2}{{g_i}^2} , \tag{9}$$

where a number of generators T_i are broken but a combination $T=\sum_i c_i T_i$ survives. For example, when $U(1)_L\times U(1)_R\to U(1)_{B-L}$ we can write

$$\frac{1}{{g_{B-L}}^2}=\frac{1}{{g_{1L}}^2}+\frac{1}{{g_{1R}}^2} \tag{10}$$

with obvious notations. Using this and similar such relations, we can write down the expressions for the low-energy parameters. At this point, we note that for the second symmetry-breaking chain, where $E_6\to SO(10)\to\cdots$, the coupling-constant evolution as well as the intermediate mass scales are exactly the same as in the SO(10) models, which have been extensively studied in recent papers.[5] We, therefore, have only to study the symmetry-breaking chain (i). The equations for $\sin^2\theta_W(M_W)$ and $\alpha(M_W)/\alpha_s(M_W)$ in this case are given by

$$\sin^2\theta_W(M_W)=\frac{3}{8}+\frac{\ln 10}{32\pi}\alpha(M_W)[(8B_{3R}-8B_{3L})(n_U-n_3)+(2B_{1L}+2B_{1R}+6B_{2R}-10B_{2L})(n_3-n_0)+(6B_{2R}+4B_{B-L}-10B_{2L})(n_0-n_R)+(10B_Y-10B_{2L})(n_R-n_W)] \tag{11}$$

and

$$\left.\frac{\alpha}{\alpha_s}\right|_{M_W}=\frac{3}{8}+\frac{\ln 10}{32\pi}\alpha(M_W)[(8B_{3L}+8B_{3R}-16B_{3C})(n_u-n_3)+(6B_{2L}+2B_{1L}+2B_{1R}+6B_{2R}-16B_{3C})(n_3-n_0)+(6B_{2L}+4B_{B-L}+6B_{2R}-16B_{3C})(n_0-n_R)+(6B_{2L}+10B_Y-16B_{3C})(n_R-n_W)]\ , \tag{12}$$

where B_{3L}, for example, is the value of the quantity B defined in Eq. (8) for the group $SU(3)_L$ and

$$n_i=\log_{10}(M_i/1\ \text{GeV}) \tag{13}$$

for any mass scale M_i.

It is interesting to note that the fermion contributions to the B's always cancel in the combination of B's present in Eqs. (11) and (12). In order to estimate the scalar contribution, we need to know what the masses of the scalars are, since in any mass range, the only scalars which contribute to the evolution equations are the ones which have masses within or below that mass range. We use the minimal fine-tuning hypothesis[6] to estimate the scalar masses. In that case, using $\alpha(M_W)=\frac{1}{128}$ and $n_W=2$, we get the following equations from Eqs. (11) and (12) (we have chosen to set $n_{3L}=n_{3R}=n_3$):

$$\sin^2\theta_W(M_W)=0.401-1.988\times10^{-5}(368n_3-29n_0+315n_R)\ , \tag{14}$$

$$\left.\frac{\alpha}{\alpha_s}\right|_{M_W}=0.423-1.988\times10^{-5}(936n_U-16n_3-29n_0+315n_R)\ . \tag{15}$$

A few comments are in order here. First, we see that the evolution equations are very insensitive to the mass scale M_0. This is expected since M_0 appears in the evolution equations through scalar contributions only.[7] However, by our choice of the symmetry-breaking chain $M_0\geq M_R$, and this imposes constraints on the scale M_0, as we shall see below.

Experimental data indicate that $0.21<\sin^2\theta_W(M_W)<0.23$ and $0.10<\alpha_s<0.12$. Roughly speaking, this implies, via Eq. (14), that

$$26.1>n_3+\tfrac{5}{6}n_R>23.4 \tag{16}$$

and via Eq. (15), that

$$19.2>n_U+\tfrac{1}{3}n_R>18.5\ . \tag{17}$$

Obviously, (16) and (17) impose stringent constraints of the scale of the right-handed breaking. We see that even if we push M_3 to 10^{18} GeV or so, we cannot have a right-handed scale M_R less than about 10^5 GeV. The scale M_0, as we remarked before, must be bigger than that but is otherwise unconstrained in the model. The detailed implications for the unification scale and various intermediate scales are depicted in Figs. 1 and 2. For case I if we choose $n_U=n_3=n_0=n_R$ we find $n_U=14.6$ for $\sin^2\theta_W=0.21$ and $\alpha_S=0.11$ as is to be expected in the case of no intermediate scale unified theories such as SU(5). (This case is not shown in the figure.) In Fig. 1 we have plotted $\sin^2\theta_W$ against $\ln M_i$. For each $\sin^2\theta_W$ we have two masses. The lower one is M_R and the higher

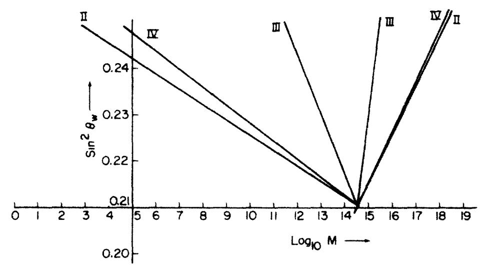

FIG. 1. Predictions for $\sin^2\theta_W$ corresponding to various values of M_U and intermediate mass scales in the E_6 model for the three symmetry-breaking chains. Case II:

$$E_6\xrightarrow[M_U]{}SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R\xrightarrow[M_R]{}SU(3)_C\times SU(2)_L\times U(1)_Y\xrightarrow[M_W]{}SU(3)_C\times U(1)_{em}\ .$$

Case III:

$$E_6\xrightarrow[M_U]{}SU(3)_C\times SU(3)_L\times SU(3)_R\xrightarrow[M_R]{}SU(3)_C\times SU(2)_L\times U(1)_Y\xrightarrow[M_W]{}SU(3)_C\times U(1)_{em}\ .$$

Case IV:

$$E_6\xrightarrow[M_U]{}SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\xrightarrow[M_R]{}SU(3)_C\times SU(2)_L\times U(1)_Y\xrightarrow[M_W]{}SU(3)_C\times U(1)_{em}\ .$$

For a particular $\sin^2\theta_W$ we have two values of M, the lower one corresponds to M_R and the higher one to M_U. Case I is represented by the point where all the lines meet corresponding to the chain

$$E_6\rightarrow SU(3)_C\times SU(2)_L\times U(1)_Y\rightarrow SU(3)_C\times U(1)_{em}\ .$$

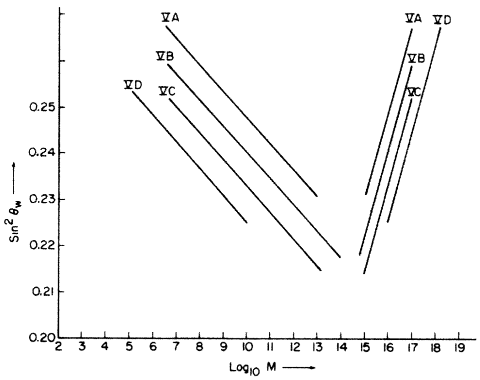

FIG. 2. The plot of $\sin^2\theta_W$ against $\log_{10}M$ for different values of M_3 for the case

$$E_6 \underset{M_U}{\rightarrow} SU(3)_C\times SU(3)_L\times SU(3)_R$$

$$\underset{M_3}{\rightarrow} SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$$

$$\underset{M_R}{\rightarrow} SU(3)_C\times SU(2)_L\times U(1)_Y$$

$$\underset{M_W}{\rightarrow} SU(3)_C\times U(1)_{em}\ .$$

The plots VA, VB, VC, and VD correspond to $n_3=13$, 14, 15, 16, respectively. Here again the lower M corresponds to M_R and the higher M corresponds to M_U for a particular value of $\sin^2\theta_W$. The disjoint nature comes from the fact that in those regions either $n_R>n_3$ or $n_3>n_U$ which are not allowed in the symmetry-breaking chain considered.

one is M_U. The cases are (II) $n_U=n_3$ and $n_0=n_R$, (III) $n_3=n_0=n_R$, and (IV) $n_U=n_3=n_0$. In all these we have taken $\alpha_s=0.11$. From all these three separate cases we see that for allowed values of $\sin^2\theta_W$, $8.7\le n_R\le 14.6$ and $14.6\le n_U\le 16.4$. In Fig. 2 we have chosen $n_0=n_R$ and plotted $\sin^2\theta_W$ against n_R and n_U for four different values of n_3. The discontinuities are due to the fact that in those regions either $M_3<M_R$ or $M_U<M_3$, which is not allowed. These sets of plots give us the bounds on n_3 as $13\le n_3\le 17$, which in turn shows that $8\le n_R\le 13$ and $14.8\le n_0\le 16.4$.

IV. CHARGED-FERMION MASSES

Of all the Higgs scalars, only the $\{27\}$, the $\{351'\}$, and the $\{27_W\}$ couple with fermion bilinears. From Eq. (3) it is obvious that the $\{27\}$ has five singlets of the unbroken gauge group $SU(3)_C\times U(1)_Y$. Among them, only one (which has the same quantum numbers as the fermion n_L) is a singlet of the group $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$. Let us say that this component develops a vacuum expectation value (VEV) V which breaks $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ to $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}$. The VEV's of the other neutral components of the $\{27\}$ must be zero or at most of the order of the scales of lower stages of symmetry breaking. We assume that they are negligible.

The $\{351'\}$ contains two components that can perform the breaking $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_{B-L}\rightarrow SU(3)_C\times SU(2)_L\times U(1)_Y$. One of them comes from the $(1,1,3,-\frac{2}{3},-\frac{1}{3})$ multiplet of $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$, the other one from the $(1,2,3,-\frac{2}{3},\frac{1}{6})$ one. We call these two VEV's y_1 and y_2, respectively. Finally, the VEV's of different components of $\{27_W\}$ are shown in terms of their $SU(3)_C\times SU(2)_L\times SU(2)_R\times U(1)_L\times U(1)_R$ representations:

$$\langle(1,2,2,-\tfrac{1}{6},\tfrac{1}{6})\rangle=\begin{bmatrix}x_1 & 0\\ 0 & x_2\end{bmatrix},$$

$$\langle(1,1,2,\tfrac{1}{3},\tfrac{1}{6})\rangle=\begin{bmatrix}0\\ x_3\end{bmatrix},$$

$$\langle(1,2,1,-\tfrac{1}{6},-\tfrac{1}{3})\rangle=\begin{bmatrix}x_4\\ 0\end{bmatrix},$$

$$\langle(1,1,1,\tfrac{1}{3},-\tfrac{1}{3})\rangle=x_5\ .$$

The mass terms are generated through the Yukawa couplings given by

$$F[27\times 27]\times\{27\}+a[27\times 27]\times\{351'\}+f[27\times 27]\times\{27_W\}\ , \quad (18)$$

where the multiplets in square brackets denote fermions and those in curly brackets are scalars. Using (18) and the VEV's, we can now easily write down the masses of different fermions:

(i) charge $\frac{2}{3}$ quarks: $$(u_L\ u_L^c)\begin{bmatrix}0 & fx_1\\ fx_1 & 0\end{bmatrix}\begin{bmatrix}u_L\\ u_L^c\end{bmatrix}, \quad (19)$$

(ii) charge $-\frac{1}{3}$ quarks:

$$(d_L\ d_L^c\ d_L'\ d_L'^c)\begin{bmatrix}0 & fx_2 & 0 & fx_4\\ fx_2 & 0 & fx_3 & 0\\ 0 & fx_3 & 0 & FV\\ fx_4 & 0 & FV & 0\end{bmatrix}\begin{bmatrix}d_L\\ d_L^c\\ d_L'\\ d_L'^c\end{bmatrix}, \quad (20)$$

(iii) charged leptons: same form as charge $-\frac{1}{3}$ quarks.

Before writing down the neutral-lepton mass matrix, let us discuss the mass eigenvalues of the charged fermions. For a single generation, the up-quark mass is obviously just fx_1. What then makes the up-quark mass so small compared to M_W—is it the smallness of the Yukawa coupling or of the VEV in question? If it is the latter, then we will have trouble explaining the top-quark mass when we take three generations into account. So we must say $f\sim 10^{-4}$ for the first generation in order that we get $m_u\sim 5-10$ MeV. The mass of the d quark comes out of the diagonalization of the matrix in Eq. (20).

In general, the diagonalization is quite involved. If, as a special case, we assume $x_2\simeq 0$ and $x_3\simeq x_4$, the matrix in (20) leads to $m_d\simeq(fx_3)^2/FV$. In this case one could hope to understand the smallness of m_d as resulting from the existence of a higher-mass scale. However, since $V \gtrsim 10^6$ GeV, this can imply a disastrously small value for the coupling F ($\lesssim 10^{-9}$) since x_3 cannot be much larger than x_1. This destroys the hope and leaves the only other simple scenario $x_3 \simeq x_4 \simeq 0$ and $m_d \simeq f x_2$ as plausible. The other quark d' then has a mass of the order FV and is therefore much heavier. The same comments apply for the charged leptons. This procedure is repeated for higher generations.

V. NEUTRAL-LEPTON MASSES

In this section we wish to analyze the problem of neutrino mass. The point is that in the E_6 model, there are five neutral leptons in a single generation. They will in general mix with each other subsequent to the gauge symmetry breaking. It is, therefore, *a priori* not clear whether the tiny ($\sim$eV range) mass of the ν_e would be a natural consequence (i.e., without unnatural fine-tuning of parameters) in general symmetry-breaking patterns implemented by arbitrary Higgs multiplets. We will now argue that the small neutrino mass does severely restrict the kind of Higgs multiplet that can be used to implement the gauge symmetry breaking. We will assume throughout this section that the mass hierarchy is of the following type.

In symmetry-breaking chain (i)

$$M_U \gg M_{3L,R} \gtrsim M_0 \gtrsim M_R \gg M_W \,, \tag{21}$$

and in chain (ii) it is

$$M_U \gg M_{10} \gg M_{C,R} \gg M_W \,. \tag{22}$$

The strategy is to look at the Higgs multiplets needed to break the symmetry at a given stage and see if it mixes any two neutral leptons. It is of course important to know which submultiplet of the E_6 multiplet caused the symmetry breaking. In addition to these contributions at the tree level, there may be additional ones at the two-loop level.[8] However, within the framework of the mass hierarchy we are talking about, such contributions are negligible compared to the tree-level values and therefore we will ignore them.

In general, we find two classes of mass matrices.

Class I:

$$\begin{bmatrix} M_{\rm I} & 0 \\ 0 & M_{\rm II} \end{bmatrix} , \tag{23}$$

where $M_{\rm I}$ is a 2×2 matrix and $M_{\rm II}$ is a 3×3 matrix.

Class II:

$$\begin{bmatrix} M'_{\rm I} & N \\ N^T & M'_{\rm II} \end{bmatrix} , \tag{24}$$

where N is a 2×3 matrix with at least one nonzero element. Here the mass matrices are written in a basis in which the successive rows and columns correspond to ν, N^C, E_1^0, E_2^0, and n defined in (3). We then find that only for mass matrices of class-I type can one have an understanding of the neutrino mass following recent works in various models.[9] On the other hand, type-II matrices lead to unacceptable neutrino properties (i.e., either universality of β and μ decay gets affected or neutrino mass is not reasonable).

Below we list the symmetry-breaking chains with associated Higgs multiplets that lead to class-I mass matrices.

To start with, we consider chain (i) with $M_{3L} = M_{3R}$ and denote it as (ia) if M_0 arises from a $\{\mathbf{27}\}$, and (ib) if it arises from a $\{\mathbf{351'}\}$. The mass matrix is then of class-I type. Any other choice of Higgs multiplets leads to type-II mass matrices and is therefore unacceptable.

(ia) Here

$$M_{\rm I} = \begin{bmatrix} 0 & M_W \\ M_W & M_R \end{bmatrix} \tag{25}$$

and

$$M_{\rm II} = \begin{bmatrix} 0 & 0 & M_W \\ 0 & M_0 & M_W \\ M_W & M_W & 0 \end{bmatrix} . \tag{26}$$

The $M_{\rm I}$ yields the conventional light ν_e and the heavy right-handed N_R, whereas the approximate eigenvalues (orders of magnitude) and eigenvectors of $M_{\rm II}$ are

$$\begin{aligned} &N_1 \simeq E_1^0, \quad m_{N_1} \simeq M_W \,, \\ &N_2 \simeq E_2^0, \quad m_{N_2} \approx M_0 \,, \\ &N_3 \simeq n, \quad m_{N_3} \approx M_W \,. \end{aligned} \tag{27}$$

N_1 and N_3 together form a Dirac neutrino of mass of order M_W. This case is of enormous experimental interest because this Dirac neutrino could be in the tens of GeV range and then will contribute to the Z width.

(ib) Here

$$M_{\rm II} = \begin{bmatrix} 0 & 0 & M_W \\ 0 & 0 & M_W \\ M_W & M_W & M_0 \end{bmatrix} . \tag{28}$$

In this case, we have

$$\begin{aligned} &N_1 \approx (E_1^0 - E_2^0)/\sqrt{2}, \quad m_{N_1} \simeq 0 \,, \\ &N_2 \approx (E_1^0 + E_2^0)/\sqrt{2}, \quad m_{N_2} \simeq M_W{}^2/M_0 \,, \\ &N_3 \approx n, \quad m_{N_3} \simeq M_0 \,. \end{aligned} \tag{29}$$

This case predicts two additional ultralight neutrinolike particles N_1 and N_2 which interact weakly with strength G_F. This case will be in conflict with nucleosynthesis requirements[10] that there be not more than four light neutrinos, and is therefore ruled out.

Coming to the chain (ii), we find type-I matrices by using the $\{\mathbf{351'}\}$ multiplet for the breaking at the scale $M_C = M_R$. As for the breaking at the M_U scale, we can have two choices as indicated in Sec. II. We denote as (iia) if we use a $\{\mathbf{351'}\}$ multiplet to break E_6 to SO(10) and as (iib) if we use a $\{\mathbf{27}\}$ for that purpose.

For chain (iia), we find

$$M_{\rm II} = \begin{bmatrix} 0 & M_{10} & M_W \\ M_{10} & 0 & M_W \\ M_W & M_W & M_U \end{bmatrix} . \tag{30}$$

The eigenvalues of this matrix are $\pm M_{10}$ and M_U which therefore lead to superheavy neutral leptons with no observable consequences at low energies.

For the chain (iib), on the other hand, we obtain

$$M_{\rm II}=\begin{bmatrix} 0 & M_U & M_W \\ M_U & 0 & M_W \\ M_W & M_W & 0 \end{bmatrix} . \tag{31}$$

This leads to two superheavy and one nearly massless neutral lepton:

$$\begin{aligned} &N_1 \simeq (E_1^0+E_2^0)/\sqrt{2}, \quad m_{N_1} \simeq M_U \ , \\ &N_2 \simeq (E_1^0-E_2^0)/\sqrt{2}, \quad m_{N_2} \simeq M_U \ , \\ &N_3 \simeq n, \quad m_{N_3} \simeq \frac{{M_W}^2}{M_U} \approx 0 \ . \end{aligned} \tag{32}$$

But the N_3 is neutral with respect to electroweak quantum numbers and will therefore have no observable consequence at low energies.

In summary, we find one particular symmetry-breaking chain (ia) which has an additional neutral lepton with distinct properties from the SO(10) model.

VI. PROTON DECAY VERSUS NEUTRON-ANTINEUTRON OSCILLATION IN THE E_6 MODEL

As is well known, baryon nonconservation is one of the important signatures of grand unification and different selection rules can be used to distinguish between different models. An important selection rule is the $\Delta B=2$ process such as n-$\bar{n}$ oscillation.[11] As has been discussed in the literature,[12] the SU(5) model does not allow an observable rate for n-$\bar{n}$ oscillation. The SO(10) model with D-parity breaking,[5] on the other hand, allows it at an observable rate. To study the situation in the E_6 model, we note that two requirements must be satisfied to have n-$\bar{n}$ oscillation: (i) there must be a low-mass scale for the partial unification $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($M_C \simeq 10^6-10^7$ GeV) and (ii) an effective four-Higgs-boson coupling of the form $\lambda'\Sigma^4$ where Σ transforms as $(1,\bar{3},10)$ under the $SU(2)_L \times SU(2)_R \times SU(4)_C$ group. The second requirement becomes nontrivial in the context of grand-unification groups G since $(1,\bar{3},10)$ must come from an irreducible representation Σ_G of G, and G invariance must allow Σ_G^4 coupling. In the case of the SO(10) group, $\Sigma_G \equiv \{126\}$ and there exists a unique $(\Sigma_G)^4$ coupling. Therefore, if there is a low-M_C scale, we can expect observable n-$\bar{n}$ oscillation in the SO(10) model.

Coming to the E_6 model, we observe that the relevant irreducible representation is $\{351'\}$ dimensional and there is no E_6-invariant coupling of the form $\{351'\}^4$. Thus, the n-$\bar{n}$ oscillation is forbidden to lowest order in the E_6 model.

As far as proton decay is concerned, its observability of course depends on the value of the unification scale M_U. We find that (Figs. 1 and 2) there exist cases where it may be observable. A particularly intriguing situation arises where E_6 descends to the standard model via the trinification[13,14] group, i.e.,

$$\begin{aligned} E_6 &\xrightarrow[M_U]{} SU(3)_C \times SU(3)_L \times SU(3)_R \\ &\xrightarrow[M_3]{} SU(3)_C \times SU(2)_L \times U(1)_Y \ . \end{aligned}$$

From Fig. 1, case (iii), we see that allowed values of $\sin^2\theta_W$ restrict M_U and M_3 considerably in this case, i.e., for $\sin^2\theta_W \simeq 0.22$, $M_U \simeq 10^{14.8}$ GeV and $M_3 \simeq 10^{14}$ GeV. More generally if $\sin^2\theta_W \leq 0.24$, $M_U \leq 10^{15.2}$ GeV and $M_3 \geq 10^{12.2}$ GeV. In the latter case, we expect $\tau_p \leq 6.5 \times 10^{31.8}\ (\Lambda_{\overline{\rm MS}}/160\ {\rm MeV})^4$ yr, where $\overline{\rm MS}$ stands for the modified minimal-subtraction scheme. This is within the reach of the current generation of experiments and therefore, this single step breaking could be ruled out in the near future.

VII. FURTHER COMMENTS AND CONCLUSION

In summary, in this paper, we have taken another look at the E_6 grand-unified theory with a view to identifying any possible experimental signatures at low energies which is characteristic of the E_6 model rather than of the SO(10) group contained in E_6. This means we have to focus on the symmetry-breaking chain (i) in the text. In that case, we predict the existence of one extra light neutrinolike object, which couples to the Z boson with semiweak strength and can contribute to the Z width. However, its charged current couplings will involve the heavy ($\simeq M_W$) charged lepton $E^{\pm}$ and will, therefore, not be of interest at low energies.

The second point we note is that $\Delta B=2$ transitions are forbidden to leading order in E_6. Thus observation of $\Delta B=2$ transitions will rule out the minimal E_6 model.

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[1]F. Gursey, P. Sikivie, and P. Ramond, Phys. Lett. **60B**, 177 (1976); F. Gursey and M. Serdaroglu, Nuovo Cimento **65A**, 337 (1981); Y. Achiman and B. Stech, Phys. Lett. **77B**, 389 (1978).

[2]Q. Shafi, Phys. Lett. **79B**, 301 (1978); J. A. Harvey, Nucl. Phys. **163B**, 254 (1979).

[3]H. Ruegg and T. Schucker, Nucl. Phys. **B161**, 388 (1979).

[4]H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).

[5]D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. D **30**, 1052 (1984); D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak, and M. K. Parida, *ibid.* 31, 1718 (1985).

[6]R. N. Mohapatra and G. Senjanović, Phys. Rev. D **27**, 1601 (1983); F. del Aguila and L. Ibañez, Nucl. Phys. **B177**, 60 (1981).

[7]Y. Tosa, G. C. Branco, and R. E. Marshak, Phys. Rev. D **28**, 1731 (1983).

[8]E. Witten, Phys. Lett. **91B**, 81 (1980).

[9]T. Yanagida, in *Proceedings of the Workshop on the Baryon Number of the Universe and Unified Theories, Tsukuba, Japan, 1979,* edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,* edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).

[10]For a recent survey, see G. Steigman, Bartol report, 1984 (unpublished).

[11]R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).

[12]For a review, see R. N. Mohapatra, *in Proceedings of the Harvard Workshop on N-$\bar{N}$ Oscillation,* edited by M. Goodman *et al.* (Harvard University Press, Cámbridge, 1982).

[13]S. L. Glashow, in *Proceedings of the Vth Workshop on Grand Unification, Brown University, 1984,* edited by K. Kang, H. Fried, and P. Frampton (World Scientific, Singapore, 1984).

[14]S. M. Barr, Phys. Lett. **112B**, 219 (1982).