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Covariant description of mesons as nontopological solitons

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We provide a fully covariant analysis of a nontopological soliton model of hadron structure and make an application to the structure of various mesons. We study the ρ and ω mesons, charmonium, and the Υ system. The model describes quarks coupled to a scalar field which plays the role of an order parameter of the QCD vacuum. There are a few parameters in this model: a flavor-dependent constituent quark mass, a mass parameter for the scalar field, a coupling constant which determines the strength of the coupling of the quarks to the scalar field, and a cutoff parameter. The mass parameter of the scalar field and the scalar-quark coupling constant are taken from our study of nucleon structure. Therefore, once a value is chosen for the high-momentum cutoff, only a single parameter is varied in this analysis, the flavor-dependent (constituent) quark mass. A reasonably good fit is obtained to a series of mesonic states of quite different mass in this extremely simple model, indicating that a unified approach to hadron structure may be possible. (At this point, we have not attempted to model the confinement mechanism. Further, our Hamiltonian has continuum solutions and, given our method of calculation, these solutions prevent us from studying all but the low-lying states of charmonium and the Υ system, for example.) We have also modified our Lagrangian in order to study gluon-exchange effects; however, the study of such effects requires the introduction of additional parameters. By fitting these new parameters to the mass splitting of the lowest 0^- and 1^- states of the charmonium system, we are able to make a prediction for corresponding splitting in the Υ system.

I. INTRODUCTION

Recently we have constructed a covariant model of nucleon structure by making a fully covariant analysis[1] of a simplified form of the Friedberg-Lee soliton model.[2] While our analysis involved certain simplifying approximations, such as a description of the nucleon as composed of a quark and a "diquark," the fit to nucleon observables was quite remarkable. The radius, electromagnetic form factors, magnetic moments g_A, and the nucleon mass were well reproduced in a simple model.[1] In this model the quarks were coupled to a scalar field which serves as an order parameter of the QCD vacuum. The parameters of the model included the coupling constant of the quarks to the scalar field, a mass parameter for the scalar field, and a quark (constituent) mass parameter. (In addition, a cutoff was introduced to regulate the high-momentum components of the interaction.) In our model of the nucleon we also coupled the quarks to various fields, with the quantum numbers of the σ, π, ρ, and ω mesons, which play an important role in the description of nucleon-nucleon scattering; however, these couplings were related to empirical meson-nucleon coupling constants and we therefore did not require additional parameters in the model. Indeed, the contribution of these fields tended to cancel leaving the overall structure to be governed by the scalar field which we called χ.

One great advantage of a model of this type is the possibility of calculating the modification of the properties of the nucleon when the nucleon is in a nucleus.[3,4] We found that the soliton increased in size when in nuclear matter. In fact, the size increase we found was precisely what was needed to explain the European Muon Collaboration (EMC) effect,[5] if we made use of the rescaling analysis of Jaffe, Close, Roberts, and Ross.[6] In addition, we were able to calculate the modification of nucleon electromagnetic form factors when the nucleon is in a nucleus.[3] These modifications were able to explain the quenching of the longitudinal response observed in (e,e') inclusive reactions near the (nucleon) quasielastic peak.[7] For example, we have recently shown[8] that the longitudinal response function for ^{40}Ca and ^{56}Fe is overestimated by about a factor of 2 in the impulse approximation at $|\mathbf{q}|=550$ MeV/c. This defect is remedied if we calculate the response using the medium-modified form factors calculated previously.[3,8] We have also carried out a detailed study of the longitudinal and transverse response functions for deep-inelastic electron scattering[9] on ^{12}C. Again, the use of our medium-modified form factors leads to a good fit of the longitudinal response, supporting the conclusions we drew from our study of ^{40}Ca and ^{56}Fe. Further, these medium-modified nucleon form factors aid in explaining some long-standing problems in relating (theoretical) matter distributions to the observed charge distributions in nuclei.[10,11]

Because our simple nontopological model appeared quite useful when applied to the study of nucleon structure, we decided to apply this model in a study of meson structure. Our goal was not to make a detailed fit to large numbers of levels and transition rates, but to see if one could find a *unified* approach to hadron structure. Once we obtained an overall qualitative description we could then attempt further refinements. In keeping with this goal we decided to use the fewest possible parameters and the simplest possible dynamical model. We take the coupling constant of the quarks to the χ field, g_χ, and the χ-field mass parameter, m_χ, from our study of nucleon structure. At this point, the only parameters of the model are a flavor-dependent quark mass and a high-momentum cutoff for the interaction, which we will describe at a later point in this discussion.

The Lagrangian of our model is then[12]

$$\mathscr{L}(x)=\bar{q}(x)[i\gamma^\mu\partial_\mu - m_q - g_\chi \chi(x)]q(x) + \tfrac{1}{2}\partial^\mu\chi(x)\partial_\mu\chi(x) - \tfrac{1}{2}m_\chi{}^2\chi^2(x) \ . \tag{1.1}$$

(This model is generalized to include effects due to "gluon exchange" in Appendix D.) The quark mass m_q is a large number in our analysis. [In our study of the ρ and ω mesons, which made use of the Lagrangian of Eq. (1.1), we used $m_q=471$ MeV, for example. This quantity is usually called the "constituent" mass to distinguish it from the "current" mass which is about 5–10 MeV for the up and down quarks.] This large quark mass is thought to have its origin in symmetry breaking associated with the formation of vacuum condensates; however, no theory is presently available that would allow one to calculate the constituent mass with any confidence.

It is sometimes useful[4] to introduce $\phi(x)=\phi_{\text{vac}}+\chi(x)$. One can then write $m_q=m_q^{\text{cur}}+g_\chi\phi_{\text{vac}}$, where m_q^{cur} is a *flavor-dependent* current quark mass. With these substitutions we can write Eq. (1.1) as

$$\mathscr{L}(x)=\bar{q}(x)[i\gamma^\mu\partial_\mu - m_q^{\text{cur}} - g_\chi\phi(x)]q(x) + \tfrac{1}{2}\partial_\mu\phi(x)\partial^\mu\phi(x) - \tfrac{1}{2}m_\chi{}^2[\phi(x)-\phi_{\text{vac}}]^2 \ . \tag{1.2}$$

[If we adopt this scheme we see that $g_\chi\phi_{\text{vac}}\sim 466$ MeV since $m_q^{\text{cur}}\sim 5$ MeV for the up and down quarks, and we have put $m_q\simeq 471$ MeV in our study of the ρ and ω mesons using the Lagrangian of Eq. (1.1). When we included effects of gluon exchange we found $m_q\simeq 619$ MeV for the up and down quarks. In that case we would have $g_\chi\phi_{\text{vac}}\simeq 614$ MeV—see Appendix D.]

We stress that Eq. (1.1) represents one of the simplest models one can use to describe soliton structure. There is clearly no reference to confinement in this simple model. This feature is, of course, unsatisfactory and has the practical consequence of limiting our considerations to only low-lying states in the spectrum of charmonium and in the Υ system. (For example, we achieve a description of the $1S$, $2S$, and $3S$ Υ states, but the $4S$ state is already in the continuum of our model. This problem has its origin in our use of plane-wave states as a basis for the solution of the equations of our model. Satisfactory results for highly excited states may be obtained if we consider an expansion in a different basis set.)

We should also note that there are many competing models of hadron structure. Almost all of these models are *static* models and require difficult and lengthy calculations if one wishes to restore translational invariance to the theory. Among the many models of hadron structure under current study, we can mention potential models,[13] the MIT bag model,[14] chiral bag models,[15] the "cloudy"-bag model,[16] and various models describing topological[17] and nontopological solitons.[2,18] (We assume the reader is reasonably well acquainted with the literature in this field.)

At this point, we can expand upon the concepts underlying our research program. Our aim is to construct a model of hadron structure that is somewhat analogous to the Ginzburg-Landau theory of superconductivity. In that theory the Hamiltonian is written in terms of an order-parameter field. (In a microscopic theory of superconductivity this field can be seen to be proportional to the anomalous expectation value, in the superconducting ground state, of an operator describing a zero-momentum electron pair.) No one has yet identified the appropriate order parameters for quantum chromodynamics. However, we believe that the work reported here, and our earlier work, is suggestive that models of QCD based upon the use of appropriate order-parameter fields may be useful.[12,19] (In Ref. 12 we put forth some conjectures concerning relations between the equations of QCD and models of hadrons of the type considered in this work.)

Before discussing the mathematical details of our analysis it is useful to provide a schematic description of the integral equation which emerges. In Fig. 1 we present such a description. There we see that the object of interest is the amplitude for a meson to decay (virtually) into a $q\bar{q}$ pair. Either the quark or antiquark may be placed on mass shell and the two amplitudes, with either the quark or antiquark on shell, are then related by charge conjugation. In the figure we show an equation for the amplitude where the antiquark is placed on mass shell. In Fig. 1(a) we see that the resulting integral equation contains the scalar form factors of the meson. (The wavy line denotes the propagator for the χ field.) In Fig. 1(b) we have shown the evaluation of the scalar form factors in terms of the amplitudes for meson $\rightarrow(q\bar{q})$ that are the objects of our analysis. Therefore, we see the nonlinear aspect of the problem emerging in a clear fashion. As we will see, the

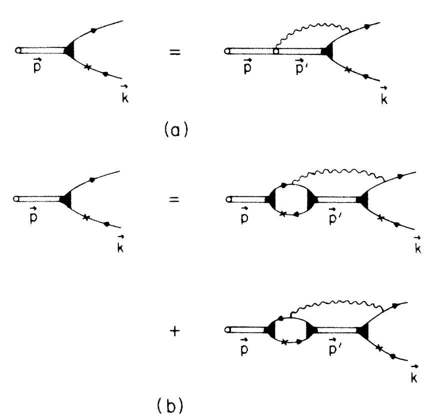

FIG. 1. Schematic representation of the covariant nonlinear equations considered here. (a) An integral equation for the amplitude describing meson decay to a quark-antiquark pair is given. Here the interaction is given in terms of the scalar form factor of the meson. (A cross denotes an on-shell particle and the wavy line represents the scalar field χ.) (b) The form factor is expressed in terms of the amplitudes for meson decay into $\bar{q}q$ pairs. (The nonlinear aspects of this equation become apparent upon inspecting the figure.)

resulting equations are fully covariant and nonlinear; they are solved by a straightforward iteration procedure.

In Sec. II we describe the mathematical techniques used to provide a covariant description of bound-state solutions of our simplified Lagrangian. In Sec. III we study integral equations for various covariantly described amplitudes and in Sec. IV we indicate how one may calculate the mass and size of the soliton (meson). (In Appendix D we extend the model to include effects of "gluon exchange." The modifications of the various equations of the theory required are presented in Appendix D.) Finally in Secs. V and VI we present our numerical results and conclusions.

In the discussion which follows the states of pseudoscalar mesons of isospin T and projection M_T will be denoted as $|\mathbf{p},TM_T\rangle$. These states are normalized such that

$$\langle \mathbf{p}',TM_T' \,|\, \mathbf{p},TM_T\rangle=\delta(\mathbf{p}-\mathbf{p}')\delta_{M_T'M_T} . \tag{1.3}$$

States of vector mesons will be denoted as $|\mathbf{p},S\lambda TM_T\rangle$ where S is the meson spin and λ is the helicity. These states are normalized such that

$$\langle \mathbf{p}',S\lambda' TM_T' \,|\, \mathbf{p},S\lambda TM_T\rangle=\delta(\mathbf{p}-\mathbf{p}')\delta_{\lambda\lambda'}\delta_{M_TM_T'} . \tag{1.4}$$

(We use the Dirac matrices and metric defined in the texts of Bjorken and Drell.[20])

II. A COVARIANT DESCRIPTION OF MESONS AS NONTOPOLOGICAL SOLITONS

We now consider the Lagrangian of Eq. (1.1) which describes the interaction of quarks with the χ field. (Here m_q, m_χ, and g_χ are parameters of our model.) From the Lagrangian density of Eq. (1.1) we obtain the following field equations:

$$(i\gamma^\mu\partial_\mu-m_q)q(x)=g_\chi q(x)\chi(x) , \tag{2.1}$$

$$(\Box+m_\chi{}^2)\chi(x)=-g_\chi\bar{q}(x)q(x) . \tag{2.2}$$

We may now form *matrix elements* of Eq. (2.2) between meson states. We are then led to define various form factors for pseudoscalar and vector particles as follows. For a pseudoscalar meson we write

$$\langle \mathbf{p}',TM_T' \,|\, \chi(0) \,|\, \mathbf{p},TM_T\rangle=-g_\chi\frac{1}{m_\chi{}^2-q^2}\frac{\delta_{M_T'M_T}}{(2\pi)^3}\frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}}F_S(q^2) . \tag{2.3}$$

For vector mesons we denote the polarization vector (with helicity parameter λ) by ξ_λ, and use the notation $\xi_\lambda^\mu=\xi_\lambda^\mu(\hat{\mathbf{p}})$ and $\xi_\lambda'^\mu=\xi_\lambda^\mu(\hat{\mathbf{p}}')$. Then we have

$$\langle \mathbf{p}',S\lambda' TM_T' \,|\, \chi(0) \,|\, \mathbf{p},S\lambda TM_T\rangle=-g_\chi\frac{1}{m_\chi{}^2-q^2}\frac{\delta_{M_TM_T'}}{(2\pi)^3}\frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}}\left[\frac{\xi_{\lambda'}'^{*}\cdot p\,\xi_\lambda\cdot p'}{m^2}F_1(q^2)+\xi_{\lambda'}'^{*}\cdot\xi_\lambda F_2(q^2)\right] , \tag{2.4}$$

where m and $\omega(\mathbf{p})$ are the mass and energy of the meson under consideration. (The isospin indices will, of course, be absent if we are considering isoscalar particles.)

We now turn to a description of certain meson-quark amplitudes. Specifically, we consider the decay amplitude for a meson of momentum $\mathbf{p}$ to go into an off-shell quark and an on-shell antiquark of momentum $\mathbf{k}$. The amplitude for a meson going into an off-shell antiquark and an on-shell quark is related to the first amplitude introduced here by charge conjugation. We consider the pseudoscalar and vector mesons separately.

A. Pseudoscalar mesons

For a pseudoscalar meson of mass m and isospin projection M_T decaying into quark with momentum $\mathbf{k}$ and isospin projection t we define, using the notation of Bjorken and Drell, and noting that α is a Dirac index,

$$\langle \mathbf{k}st \,|\, \bar{q}_{\alpha i}(0) \,|\, \mathbf{p},TM_T\rangle=\frac{1}{[2\omega(\mathbf{p})]^{1/2}}\left[\frac{m_q}{E_q(\mathbf{k})}\right]^{1/2}\frac{1}{(2\pi)^3}\left[\bar{u}_s(\mathbf{k})\left[A+\frac{B\not{p}}{m}\right]\gamma_5\right]_\alpha(\chi_t^\dagger\tau\cdot\hat{\mathbf{e}}_{M_T})_i . \tag{2.5}$$

Here A and B are Lorentz scalars and $\hat{\mathbf{e}}_{M_T}$ is a vector of unit norm. Using charge conjugation we have (see Appendix A), for decay into an on-shell antiquark,

$$\langle \overline{\mathbf{k}st} \,|\, q_{\alpha i}(0) \,|\, \mathbf{p},TM_T\rangle=\frac{1}{[2\omega(\mathbf{p})]^{1/2}}\left[\frac{m_q}{E_q(\mathbf{k})}\right]^{1/2}\frac{1}{(2\pi)^3}\left[\gamma_5\left[E+\frac{F\not{p}}{m}\right]v_s(\mathbf{k})\right]_\alpha[(\tau\cdot\hat{\mathbf{e}}_{M_T}^{*})^T\chi_{-t}\eta_{-t}]_i , \tag{2.6}$$

where $E=A$, $F=-B$, and η is a phase factor.

The scalar invariants E and F can be taken to be functions of Lorentz invariants $[(p\cdot k/m_q)^2-m^2]^{1/2}$. We denote the scalar invariants which are functions of $[(p'\cdot k/m_q)^2-m^2]^{1/2}$ by E' and F'—see Fig. 1. In the meson rest frame, $\mathbf{p}=0$, the amplitude defined in Eq. (2.6) becomes

$$\langle \overline{\mathbf{k}st} \,|\, q_\alpha(0) \,|\, \mathbf{p}=0,TM_T\rangle=\frac{1}{(2\pi)^3}\left[\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\frac{1}{4m}\right]^{1/2}\begin{bmatrix}(E-F)\chi_{-s}\\ (E+F)\dfrac{k}{\epsilon_q(\mathbf{k})}\sigma\cdot\hat{\mathbf{k}}\chi_{-s}\end{bmatrix}(\tau\cdot\hat{\mathbf{e}}_{M_T}^{*})^T\chi_{-t}\eta_{-t} , \tag{2.7}$$

where $\epsilon_q(\mathbf{k})=(\mathbf{k}^2+m_q{}^2)^{1/2}+m_q$. We recall that the scalar invariants E and F are functions of

$$[(p\cdot k/m_q)^2-m^2]^{1/2} .$$

Thus in Eq. (2.7)

$$E=E\left[\frac{mk}{m_q}\right]=E(k') , \tag{2.8}$$

$$F=F\left[\frac{mk}{m_q}\right]=F(k') . \tag{2.9}$$

Here $\mathbf{k}'$ is the momentum of the meson in the Lorentz frame where $\mathbf{k}=0$. We define wave functions as follows:

$$\hat{\mathbf{R}}_u(k)=\left[\frac{4\pi}{(2\pi)^3}\frac{1}{m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}[E(k')-F(k')] , \tag{2.10}$$

$$\hat{\mathbf{R}}_l(k)=\left[\frac{4\pi}{(2\pi)^3}\frac{1}{m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}[E(k')+F(k')]\frac{k}{\epsilon_q(\mathbf{k})} . \tag{2.11}$$

We also choose the normalization

$$\int_0^\infty k^2dk[\hat{\mathbf{R}}_u{}^2(k)+\hat{\mathbf{R}}_l{}^2(k)]=1 . \tag{2.12}$$

B. Vector meson

We consider the decay of a vector meson into an on-shell quark and an off-shell antiquark. The description of this amplitude is somewhat more complicated than in the pseudoscalar case described above. In general, there are four scalar invariants needed to describe this amplitude. We organize these invariants such that, in the rest frame of the meson, only s-wave decay is allowed. This leads to

$$\langle \mathbf{k}st\,|\,\bar{q}_{\alpha i}(0)\,|\,\mathbf{p},S\lambda TM_T\rangle=\frac{1}{[2\omega(\mathbf{p})]^{1/2}}\left[\frac{m_q}{E_q(\mathbf{k})}\right]^{1/2}\frac{1}{(2\pi)^3}\times\left\{\bar{u}_s(\mathbf{k})\left[\frac{\xi_\lambda\cdot k}{m_q}A_1\left[1+\frac{\not p}{m}\right]+\not\xi_\lambda\left[\tilde{A}+\tilde{B}\frac{\not p}{m}\right]\right]\right\}_\alpha(\chi_t^\dagger\tau\cdot\hat{\mathbf{e}}_{M_T})_i . \tag{2.13}$$

Using charge conjugation (Appendix A) we have

$$\langle \overline{\mathbf{k}st}\,|\,q_\alpha(0)\,|\,\mathbf{p},S\lambda TM_T\rangle=-\frac{1}{[2\omega(\mathbf{p})]^{1/2}}\left[\frac{m_q}{E_q(\mathbf{k})}\right]^{1/2}\frac{1}{(2\pi)^3}\times\left\{\left[\frac{\xi_\lambda\cdot k}{m_q}A_1\left[1-\frac{\not p}{m}\right]-\not\xi_\lambda\left[\tilde{A}+\tilde{B}\frac{\not p}{m}\right]\right]v_s(\mathbf{k})\right\}_\alpha(\tau\cdot\hat{\mathbf{e}}^*_{M_T})^T\chi_{-t}\eta_{-t} . \tag{2.14}$$

These equations describe s-wave decay if

$$A_1=-\frac{\tilde{A}+\tilde{B}}{\left[1+\dfrac{p\cdot k}{m_q m}\right]} . \tag{2.15}$$

The functional dependence of A_1, A, and B is the same as in the case of scalar mesons. That is, they can be taken as functions of $[(p\cdot k/m_q)^2-m^2]^{1/2}$. In the meson rest frame, $\mathbf{p}=0$, we find

$$\langle \overline{\mathbf{k}st}\,|\,q(0)\,|\,\mathbf{p}=0,S\lambda TM_T\rangle=-\frac{1}{(2\pi)^3}\left[\frac{1}{4m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}\begin{bmatrix}(\tilde{A}-\tilde{B})\hat{\xi}_\lambda\cdot\sigma\chi_{-s}\\ \dfrac{(\tilde{A}+\tilde{B})k}{\epsilon_q(\mathbf{k})}\sigma\cdot\hat{\mathbf{k}}\hat{\xi}_\lambda\cdot\sigma\chi_{-s}\end{bmatrix}[(\tau\cdot\hat{\mathbf{e}}^*_{M_T})^T\chi_{-t}\eta_{-t}] . \tag{2.16}$$

From this we can again identify wave functions,

$$\hat{\mathbf{R}}_u(k)=\left[\frac{4\pi}{(2\pi)^3}\frac{1}{m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}[\tilde{A}(k')-\tilde{B}(k')] , \tag{2.17}$$

$$\hat{\mathbf{R}}_l(k)=\left[\frac{4\pi}{(2\pi)^3}\frac{1}{m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}[\tilde{A}(k')+\tilde{B}(k')]\frac{k}{\epsilon_q(\mathbf{k})} , \tag{2.18}$$

and choose the normalization

$$\int_0^\infty k^2dk[\hat{\mathbf{R}}_u{}^2(k)+\hat{\mathbf{R}}_l{}^2(k)]=1 . \tag{2.19}$$

III. INTEGRAL EQUATIONS FOR INVARIANT AMPLITUDES

We now analyze Eq. (2.1) using the fact that

$$\langle \overline{\mathbf{k}st} \,|\, q(x) \,|\, \mathbf{p},TM_T \rangle = e^{-i(p-k)\cdot x} \langle \overline{\mathbf{k}st} \,|\, q(0) \,|\, \mathbf{p},TM_T \rangle \ . \tag{3.1}$$

We obtain, for pseudoscalar mesons, upon inserting a set of mesonic states, $|\,\mathbf{p}',TM_T'\rangle$, between the operators $q(0)$ and $\chi(0)$:

$$(\not p - \not k - m_q)\langle \overline{\mathbf{k}st} \,|\, q(0) \,|\, \mathbf{p},TM_T \rangle = g_\chi \sum_{M_T'} \int d\mathbf{p}' \langle \overline{\mathbf{k}st} \,|\, q(0) \,|\, \mathbf{p}',TM_T' \rangle \langle \mathbf{p}',TM_T' \,|\, \chi(0) \,|\, \mathbf{p},TM_T \rangle \ . \tag{3.2}$$

Similarly for vector mesons, we obtain

$$(\not p - \not k - m_q)\langle \overline{\mathbf{k}st} \,|\, q(0) \,|\, \mathbf{p},S\lambda TM_T \rangle = g_\chi \sum_{\lambda' M_T'} \int d\mathbf{p}' \langle \overline{\mathbf{k}st} \,|\, q(0) \,|\, \mathbf{p}',S\lambda' TM_T' \rangle \langle \mathbf{p}',S\lambda' TM_T' \,|\, \chi(0) \,|\, \mathbf{p},S\lambda TM_T \rangle \ . \tag{3.3}$$

We analyze Eqs. (3.1) and (3.2) in the antiquark rest frame where $\mathbf{k}=0$. For that analysis it is useful to define, for pseudoscalar mesons

$$\overline{R}_u \equiv \overline{R}_u(p) = \left[E(p) - F(p)\frac{\omega(\mathbf{p})}{m} \right] \frac{1}{[\omega(\mathbf{p})]^{1/2}} \ , \tag{3.4}$$

$$\overline{R}_l \equiv \overline{R}_l(p) = -F(p)\frac{p}{m}\frac{1}{[\omega(\mathbf{p})]^{1/2}} \ , \tag{3.5}$$

$$\overline{R}\,'_u \equiv \overline{R}_u(p') = \left[E(p') - F(p')\frac{\omega(\mathbf{p}')}{m} \right] \frac{1}{[\omega(\mathbf{p}')]^{1/2}} \ , \tag{3.6}$$

$$\overline{R}\,'_l \equiv \overline{R}_l(p') = -F(p')\frac{p'}{m}\frac{1}{[\omega(\mathbf{p}')]^{1/2}} \ . \tag{3.7}$$

The isospin factors may be canceled in Eq. (3.2) and we obtain equations for the amplitudes defined above,

$$\omega(\mathbf{p}) \begin{bmatrix} \overline{R}_u(p) \\ \overline{R}_l(p) \end{bmatrix} = \begin{bmatrix} 2m_q & p \\ p & 0 \end{bmatrix} \begin{bmatrix} \overline{R}_u(p) \\ \overline{R}_l(p) \end{bmatrix}$$
$$= -g_\chi{}^2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}} \times \frac{F_S(q^2)}{m_\chi{}^2 - q^2} \begin{bmatrix} 1 & 0 \\ 0 & -\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}} \end{bmatrix} \begin{bmatrix} \overline{R}_u(p') \\ \overline{R}_l(p') \end{bmatrix} . \tag{3.8}$$

Similarly for the study of vector mesons we define

$$\overline{R}_u(p) = \frac{1}{[\omega(\mathbf{p})]^{1/2}} \left[\widetilde{A}(p) - \widetilde{B}(p)\frac{\omega(\mathbf{p})}{m} \right] , \tag{3.9}$$

$$\overline{R}_l(p) = -\frac{1}{[\omega(\mathbf{p})]^{1/2}} \widetilde{B}(p)\frac{p}{m} \ , \tag{3.10}$$

$$\overline{R}_u(p') = \frac{1}{[\omega(\mathbf{p}')]^{1/2}} \left[\widetilde{A}(p') - \widetilde{B}(p')\frac{\omega(\mathbf{p}')}{m} \right] , \tag{3.11}$$

$$\overline{R}_l(p') = -\frac{1}{[\omega(\mathbf{p}')]^{1/2}} \widetilde{B}(p')\frac{p'}{m} \ . \tag{3.12}$$

Again we can remove the isospin factors from Eq. (3.3), and using the fact that

$$\sum_\lambda (\xi_\lambda^\mu)^* \xi_\lambda^\nu = -g^{\mu\nu} + \frac{p^\mu p^\nu}{m^2} \ , \tag{3.13}$$

we obtain

$$\omega(\mathbf{p}) \begin{bmatrix} \overline{R}_u(p) \\ \overline{R}_l(p) \end{bmatrix} = \begin{bmatrix} 2m_q & p \\ p & 0 \end{bmatrix} \begin{bmatrix} \overline{R}_u(p) \\ \overline{R}_l(p) \end{bmatrix} - g_\chi{}^2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{1}{3} \frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}} \frac{1}{m_\chi{}^2 - q^2}$$
$$\times \left[\begin{bmatrix} V_{11}{}^1 & V_{12}{}^1 \\ V_{21}{}^1 & V_{22}{}^1 \end{bmatrix} F_1(q^2) + \begin{bmatrix} V_{11}{}^2 & V_{12}{}^2 \\ V_{21}{}^2 & V_{22}{}^2 \end{bmatrix} F_2(q^2) \right] \begin{bmatrix} \overline{R}_u(p') \\ \overline{R}_l(p') \end{bmatrix} , \tag{3.14}$$

where

$$V_{11}{}^1=-\frac{\omega\mathbf{p}'^2}{m^3}-\frac{\omega'\mathbf{p}^2}{m^3}+\frac{\mathbf{p}\cdot\mathbf{p}'}{m^2}\left[\frac{\omega}{m}+\frac{\omega'}{m}+\frac{\omega'\omega}{m^2}-1\right]-(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')^2\frac{\mathbf{p}^2\mathbf{p}'^2}{m^4}\left[\frac{\omega}{\omega+m}+\frac{\omega'}{\omega'+m}\right]+(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')^3\frac{|\mathbf{p}||\mathbf{p}'|}{m^2}\left[1-\frac{\omega'}{m}-\frac{\omega}{m}+\frac{\omega\omega'}{m^2}\right], \quad (3.15)$$

$$V_{22}{}^1=\frac{|\mathbf{p}'||\mathbf{p}|}{m^2}\left[1-\frac{\omega\omega'}{m^2}\right]+(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')\left[\frac{\mathbf{p}'^2\omega^2+\mathbf{p}^2\omega'^2}{m^4}\right]-(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')^2\frac{|\mathbf{p}'||\mathbf{p}|}{m^2}\left[\frac{\omega'\omega}{m^2}+1\right], \quad (3.16)$$

$$V_{11}{}^2=-\frac{\omega'}{m}-\frac{\omega}{m}-1+\frac{\mathbf{p}'\cdot\mathbf{p}}{m^2}+(\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}')^2\left[\frac{\omega'}{m}+\frac{\omega}{m}-\frac{\omega\omega'}{m^2}-1\right], \quad (3.17)$$

$$V_{22}{}^2=-\frac{|\mathbf{p}||\mathbf{p}'|}{m^2}+\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}'\left[\frac{\omega'\omega}{m^2}+2\right], \quad (3.18)$$

$$V_{12}{}^1=V_{21}{}^1=V_{21}{}^2=V_{12}{}^2=0 . \quad (3.19)$$

Here

$$\omega=\omega(\mathbf{p})=(m^2+\mathbf{p}^2)^{1/2} , \quad (3.20)$$

and

$$\omega'=\omega(\mathbf{p}')=(m^2+\mathbf{p}'^2)^{1/2} . \quad (3.21)$$

Equations (3.8) and (3.14) can be solved for $\overline{R}_u(p)$ and $\overline{R}_l(p)$ for pseudoscalar and vector mesons. From the solution of these equations we can construct $E(p)$, $F(p)$, $A_1(p)$, $\tilde{A}(p)$, and $\tilde{B}(p)$.

Some questions of normalization are discussed in Appendix B, while the evaluation of various form factors is described in Appendix C. Finally, we note that we have introduced a high-momentum cutoff in these calculations. Effectively we replace the propagator $(m_\chi{}^2-q^2)^{-1}$ by

$$(m_\chi{}^2-q^2)^{-1}\left[\frac{\Lambda^2-q^2}{\Lambda^2}\right]^{-3} ,$$

where $\Lambda^2=100\ \text{fm}^{-2}$. This is somewhat larger than the cutoff used in our calculation of nucleon properties. In the latter case we put $\Lambda^2\simeq 40\ \text{fm}^{-2}$. The larger cutoff is needed in particular for the description of $\Upsilon(1S)$ which is a quite small object. A value of $\Lambda\sim 2$ GeV seems not unreasonable since at these large momentum transfers noncollective aspects of QCD are expected to play a role. That is, one expects that a description in terms of QCD order parameters should break down at some large momentum scale.

IV. EVALUATION OF THE MESON MASSES AND RADII

We recall that

$$\langle \mathbf{p}',TM_T' \,|\, \mathbf{p},TM_T\rangle=\delta(\mathbf{p}'-\mathbf{p})\delta_{M_T'M_T} , \quad (4.1)$$

and also note that

$$\langle \mathbf{p}',TM_T \,|\, H \,|\, \mathbf{p},TM_T\rangle=\left\langle \mathbf{p}',TM_T \,\middle|\, \int \mathcal{H}(x)d\mathbf{x} \,\middle|\, \mathbf{p},TM_T\right\rangle=(2\pi)^3\delta(\mathbf{p}'-\mathbf{p})\langle \mathbf{p},TM_T \,|\, \mathcal{H}(0) \,|\, \mathbf{p},TM_T\rangle . \quad (4.2)$$

Here $\mathcal{H}(x)$ is the Hamiltonian density and is given by

$$\mathcal{H}(x)=\bar{q}(x)\left[\frac{1}{i}\gamma\cdot\nabla+m_q+g_\chi\chi(x)\right]q(x)+\frac{1}{2}\left[\left[\frac{\partial\chi}{\partial t}(x)\right]^2+|\nabla\chi(x)|^2+m_\chi^2\chi^2(x)\right] . \quad (4.3)$$

We now define m_H as follows:

$$\langle \mathbf{p}',TM_T \,|\, H \,|\, \mathbf{p},TM_T\rangle=\delta(\mathbf{p}-\mathbf{p}')(\mathbf{p}^2+m_H{}^2)^{1/2} . \quad (4.4)$$

Using the meson-quark decay amplitudes determined earlier and inserting a complete set of quark and antiquark states between the various operators in Eq. (4.3) we find

$$m_H=2(m-\langle E_q\rangle)+\mathcal{E}_\chi^S , \quad (4.5)$$

where

$$\langle E_q\rangle=\int_0^\infty k^2dk\, E_q(\mathbf{k})[\hat{\mathbf{R}}_u{}^2(k)+\hat{\mathbf{R}}_l{}^2(k)] \quad (4.6)$$

and

$$\mathcal{E}_\chi^S=g_\chi{}^2\frac{4m}{(2\pi)^2}\int_0^\infty \frac{p'^2dp'}{\omega(\mathbf{p}')}\left[\frac{2(q^0)^2}{(m_\chi{}^2-q^2)^2}+\frac{1}{m_\chi{}^2-q^2}\right]\times[F_S(q^2)]^2 . \quad (4.7)$$

Here q^μ is given by

$$q^0=m-\omega(\mathbf{p}') \quad (4.8)$$

and

$$\mathbf{q}=-\mathbf{p}' . \quad (4.9)$$

Similarly for vector mesons we find

$$m_H=2(m-\langle E_q\rangle)+\mathcal{E}_\chi^V , \quad (4.10)$$

where again we have

$$\langle E_q\rangle=\int_0^\infty k^2dk\, E_q(\mathbf{k})[\hat{\mathbf{R}}_u{}^2(k)+\hat{\mathbf{R}}_l{}^2(k)] \quad (4.11)$$

and also define

$$\mathscr{E}_\chi^V = g_\chi{}^2 \frac{4m}{(2\pi)^2} \int_0^\infty \frac{p'^2 dp'}{\omega(\mathbf{p}')} \left[\frac{2(q^0)^2}{(m_\chi{}^2 - q^2)^2} + \frac{1}{m_\chi{}^2 - q^2} \right] \left\{ \frac{\mathbf{p}'^2}{3m^2} \left[\left[\frac{\omega(\mathbf{p}')}{m} F_1(q^2) + F_2(q^2) \right]^2 - [F_1(q^2)]^2 \right] + [F_2(q^2)]^2 \right\} . \tag{4.12}$$

For systems with isospin zero (charmonium and Υ) we find the same expression for the mass if we use the normalization conventions described in Appendix B.

We have also calculated the size of the solitons. We define

$$\hat{\mathbf{R}}_u(r) = \frac{4\pi}{(2\pi)^{3/2}} \int_0^\infty k^2 dk\, j_0(kr) \hat{\mathbf{R}}_u(k) , \tag{4.13}$$

$$\hat{\mathbf{R}}_l(r) = \frac{4\pi}{(2\pi)^{3/2}} \int_0^\infty k^2 dk\, j_1(kr) \hat{\mathbf{R}}_l(k) , \tag{4.14}$$

and calculate the mean-square radii of the baryon and scalar densities:

$$\langle r^2 \rangle_B = \int_0^\infty r^2 dr\, r^2 [\hat{\mathbf{R}}_u{}^2(r) + \hat{\mathbf{R}}_l{}^2(r)] , \tag{4.15}$$

$$\langle r^2 \rangle_S = \int_0^\infty r^2 dr\, r^2 [\hat{\mathbf{R}}_u{}^2(r) - \hat{\mathbf{R}}_l{}^2(r)] . \tag{4.16}$$

Values for the root-mean-square radii are presented in Tables I and III.

V. CALCULATIONAL PROCEDURES AND RESULTS

In this section we will describe the procedure used in our calculations. The discussion is somewhat complicated since several different masses are described. To clarify this presentation we can define the quantities m_H, m_{expt}, and m_λ. We will first describe m_{expt} and m_λ. We use m_{expt} to denote the experimental meson mass. (This quantity is known for most of the states we consider here.) Now consider the meson energy $\omega(\mathbf{p})$ which appears on the left-hand side of Eqs. (3.8) and (3.14). We write $\omega(\mathbf{p}) = (\mathbf{p}^2 + m_\lambda{}^2)^{1/2}$ and determine m_λ as the eigenvalue of the nonlinear equations, Eqs. (3.8) and (3.14). The meson mass is also required in the calculation of the form factors [see Eqs. (C4), (C9), and (C10), for example] and in the potentials [see Eqs. (3.15)–(3.18)]. The equations are solved so the mass m, which appears in many places in our equations, is equal to m_λ, the eigenvalue of our equations, as described above. After the equality between the eigenvalue m_λ and the meson mass used to construct the form factors and potentials is achieved, we have only a single mass parameter in our equations. We can denote this mass as m. (See Tables I–IV.)

We proceed as follows. We guess a value for the form factors [$F_S(q^2)$ in the case of pseudoscalar mesons, or $F_1(q^2)$ and $F_2(q^2)$ in the case of vector mesons]. We then construct the potential terms appearing in the right-hand side of Eqs. (3.8) and (3.14). If we are treating the ρ or ω mesons, or the lowest 1^- state of charmonium and of the Υ system, we use m_{expt} in Eqs. (3.8) and (3.14). We then determine m_q such that $m = m_{\text{expt}}$. This procedure provides a value for the constituent mass of the up and down quarks, the charmed quark, and the bottom quark. With these quark masses fixed, we can then study the 0^- states of charmonium and of the Υ system, as well as the excited 1^- states of both systems. Since m_q is fixed, we now have no parameters at our disposal. We again proceed to solve Eqs. (3.8) and (3.14) so that m_λ, the eigenvalue, is the same as the mass used to generate the form factors and potentials. Therefore, at the end of our analysis there is only a single mass m which appears in our equations for each mesonic state considered (see Tables I–IV).

The notation m_H is used for the mass of the meson which is calculated by forming the expectation value of the Hamiltonian. This expectation value is calculated us-

TABLE I. Results of calculations based upon the Lagrangian of Eq. (1.1). The quark masses m_q are fixed so the corresponding underlined masses are equal. The baryon and scalar radii are defined in Sec. IV. The electromagnetic radius is defined in terms of the slope of the appropriate form factor.

Meson	J^π	m (expt) (MeV)	m (theory) (MeV)	m_q (MeV)	Baryon-density radius (rms) (fm)	Scalar density radius (rms) (fm)	Electromagnetic radius (fm)
ρ,ω	1^-	<u>775</u>	<u>775</u>	471	1.38	1.21	0.949
π	0^-	140	782	471	1.39	1.21	0.936
$J/\psi(1S)$	1^-	<u>3100</u>	<u>3100</u>	2025	0.484	0.434	
$J/\psi(2S)$	1^-	3685	3795	2025	1.21	1.19	
$\chi_c(1S)$	0^-	2980	3101	2025	0.483	0.431	
$\chi_c(2S)$	0^-	3590	3794	2025	1.21	1.19	
$\Upsilon(1S)$	1^-	<u>9460</u>	<u>9460</u>	5700	0.272	0.263	
$\Upsilon(2S)$	1^-	10025	10355	5700	0.531	0.524	
$\Upsilon(3S)$	1^-	10355	10900	5700	0.893	0.886	
$\Upsilon(1S)$	0^-		9460	5700	0.271	0.262	
$\Upsilon(2S)$	0^-		10356	5700	0.534	0.527	
$\Upsilon(3S)$	0^-		10900	5700	0.889	0.883	

TABLE II. Results of calculations based upon the Lagrangian of Eq. (1.1). The various quantities $\langle E_q \rangle$, $\mathcal{E}_\chi^S$, $\mathcal{E}_\chi^V$, and m_H are defined in Sec. IV.

Meson	J^π	m (expt) (MeV)	m (theory) (MeV)	$\langle E_q \rangle$ (MeV)	$\mathcal{E}_\chi^S$ (MeV)	$\mathcal{E}_\chi^V$ (MeV)	$\langle H \rangle = m_H$ (MeV)
ρ,ω	1^-	775	775	524		240	742
π	0^-	140	782	527	224		734
$J/\psi(1S)$	1^-	3 100	3 100	2125		735	2 685
$J/\psi(2S)$	1^-	3 685	3 795	2103		339	3 721
$\chi_c(1S)$	0^-	2 980	3 101	2126	726		2 676
$\chi_c(2S)$	0^-	3 590	3 794	2105	341		3 719
$\Upsilon(1S)$	1^-	9 460	9 460	5806		1236	8 545
$\Upsilon(2S)$	1^-	10 025	10 355	5847		905	9 922
$\Upsilon(3S)$	1^-	10 355	10 900	5831		588	10 726
$\Upsilon(1S)$	0^-		9 460	5806	1234		8 542
$\Upsilon(2S)$	0^-		10 356	5847	906		9 924
$\Upsilon(3S)$	0^-		10 900	5830	590		10 710

ing the self-consistent invariant amplitudes, wave functions and form factors, and the equations developed in Sec. IV. (As we will see, m_H and m differ significantly in some cases. We will comment on this aspect of the calculation at a later point in the discussion.)

In our calculations we did not attempt to vary m_χ and g_χ. We choose $m_\chi = 500$ MeV, which is the value we used for the study of the properties of the nucleon.[1,3] We took $g_\chi = 7.0$, which is approximately the average value of this parameter used in Ref. 1 ($g_\chi = 6.3$) and in Ref. 3 ($g_\chi = 7.5$). Thus, once a high-momentum cutoff is chosen, there is only a single free parameter, the flavor-dependent constituent quark mass in the analysis which is fixed by the procedure described above.

The results of some of our calculations are given in Tables I and II. We note that the choice $m_q = 471$ MeV for the up and down quarks leads to a fit to the experimental ρ and ω mass. A value of $m_q = 2025$ MeV for the charmed-quark mass leads to a fit to the $J/\psi(3100)$ state, while the value $m_q = 5700$ MeV for the bottom quark leads to a fit to the (vector) Υ meson, $\Upsilon(1S)$. The other masses, and various radii given in Table I, represent predictions of the model. In general, we see that the model leads to too large a separation between the various states of the charmonium and Υ systems.

In Table II we present values calculated for the expectation value of H (denoted as m_H) and the quantities $\langle E_q \rangle$, $\mathcal{E}_\chi^S$, and $\mathcal{E}_\chi^V$, defined previously. [See Eqs. (4.5)–(4.7) and Eqs. (4.10)–(4.12).] It is interesting to note that the largest discrepancies between m and m_H appear for the mesons with the smallest size. The discrepancy becomes smaller as the meson size increases in a fairly systematic fashion indicating that for quite large objects one could achieve consistency for m and m_H. For example, the mass difference $m_H - m$ is quite small for ρ, ω, $J/\psi(2S)$, and $\chi_c(2S)$, which are large objects with (scalar density) radii of about 1.2 fm. For even larger objects we anticipate that m and m_H will be almost equal. This feature of the calculation deserves further study.

We note from Tables I and II that we obtain no split-

TABLE III. Results of calculations including the gluon-exchange potential (see Appendix D). The quark masses are fixed so that the underlined theoretical and experimental masses are equal.

Meson	J^π	m (expt) (MeV)	m (theory) (MeV)	m_q (MeV)	Baryon-density radius (rms) (fm)	Scalar density radius (rms) (fm)
ρ,ω	1^-	775	775	619	0.781	0.641
π	0^-	140				
$J/\psi(1S)$	1^-	3 100	3 100	2299	0.337	0.288
$J/\psi(2S)$	1^-	3 685	3 961	2299	0.763	0.741
$\chi_c(1S)$	0^-	2 980	2 980	2299	0.310	0.264
$\chi_c(2S)$	0^-	3 590	3 910	2299	0.713	0.692
$\Upsilon(1S)$	1^-	9 460	9 460	6037	0.210	0.200
$\Upsilon(2S)$	1^-	10 025	10 505	6037	0.419	0.412
$\Upsilon(3S)$	1^-	10 355	11 152	6037	0.672	0.665
$\Upsilon(1S)$	0^-		9 425	6037	0.208	0.199
$\Upsilon(2S)$	0^-		10 503	6037	0.415	0.408
$\Upsilon(3S)$	0^-		11 157	6037	0.674	0.668

TABLE IV. Results of calculations including the gluon-exchange potential. (See Appendix D for the definitions of m_H and $\mathscr{E}_G$.) The quantities $\mathscr{E}_\chi$ and $\langle E_q \rangle$ are defined in Sec. IV for pseudoscalar and vector mesons.

Meson	J^π	m (expt) (MeV)	m (theory) (MeV)	$\langle E_q \rangle$ (MeV)	$\mathscr{E}_\chi$ (MeV)	$\mathscr{E}_G$ (MeV)	$\langle H \rangle = m_H$ (MeV)
ρ,ω	1^-	775	775	755	362	306	708
π	0^-	140					
$J/\psi(1S)$	1^-	3 100	3 100	2485	758	570	2 556
$J/\psi(2S)$	1^-	3 685	3 961	2469	509	318	3 811
$\chi_c(1S)$	0^-	2 980	2 980	2544	770	768	2 409
$\chi_c(2S)$	0^-	3 590	3 910	2508	533	298	3 736
$\Upsilon(1S)$	1^-	9 460	9 460	6208	1264	664	8 431
$\Upsilon(2S)$	1^-	10 025	10 505	6263	1023	442	9 948
$\Upsilon(3S)$	1^-	10 355	11 152	6252	771	308	10 980
$\Upsilon(1S)$	0^-		9 425	6216	1272	698	8 388
$\Upsilon(2S)$	0^-		10 503	6266	1009	454	9 937
$\Upsilon(3S)$	0^-		11 157	6251	750	314	10 874

ting between vector and pseudoscalar states with the same number of nodes. Therefore, we have investigated the effects of adding a potential which represents the exchange of a massless colored vector field. The modification of our equations required to study such "gluon-exchange effects" are given in Appendix D, where the additional coupling constant and cutoff required are specified. The results of this study are given in Tables III and IV. We note that we now require larger constituent quark masses and we also find smaller values for the baryon-density and scalar-density radii. Again we see a discrepancy between m and m_H. For objects of the *same size*, however, the difference between m and m_H is essentially the same in the calculations reported in Tables II and IV, that is, for those results with and without the gluon-exchange potential.

Note that the strength of the gluon-exchange force is fixed such that the mass difference between $J/\psi(1S)$ and $\chi_c(1S)$ is given correctly (see Table III). Once this strength is fixed we have the following prediction for mass splitting in the Υ system:

$$m[\Upsilon(1S)]_{J=1^-} - [\Upsilon(1S)]_{J=0^-} = 35 \text{ MeV} .$$

The splittings of the $\Upsilon(2S)$ are $\Upsilon(3S)$ states given in Table III are quite small and probably not significant, given the numerical accuracy of our calculation.

In Figs. 2 and 3 we show some typical coordinate-space wave functions obtained for ρ, ω, $\chi_c(2980)$, $J/\psi(3100)$, $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$ in these calculations. These

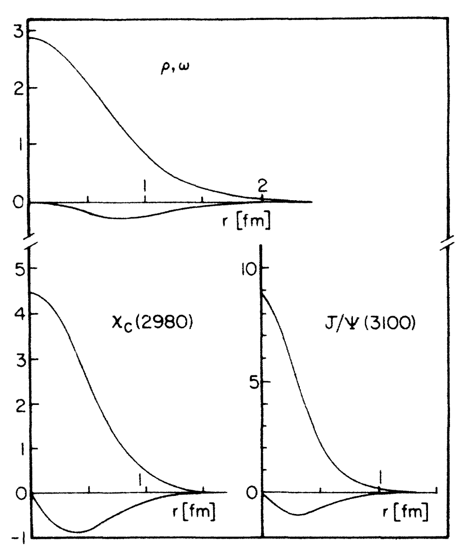

FIG. 2. Coordinate-space wave function for the ρ meson, $\chi_c(2980)$, and $J/\psi(3100)$. Both upper and lower components are shown.

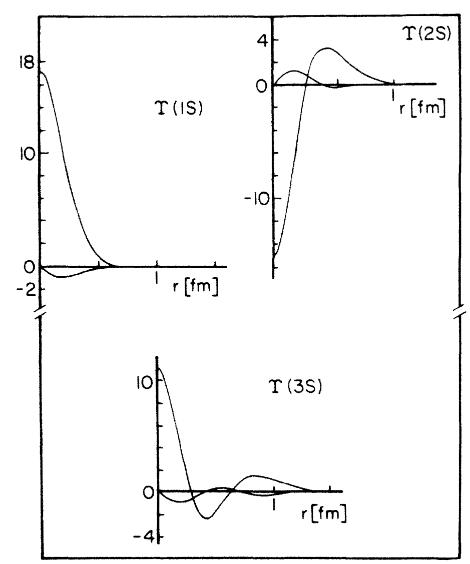

FIG. 3. Coordinate-space wave functions obtained here for $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(3S)$. Both upper and lower components are shown.

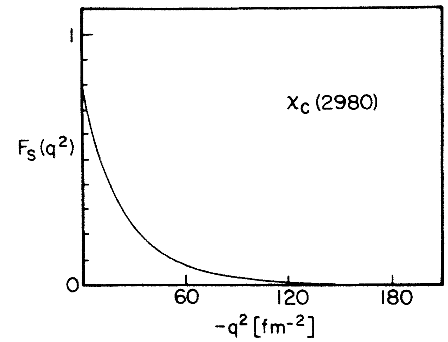

FIG. 4. The scalar form factor for the ground state of charmonium [$\chi_c(2980)$].

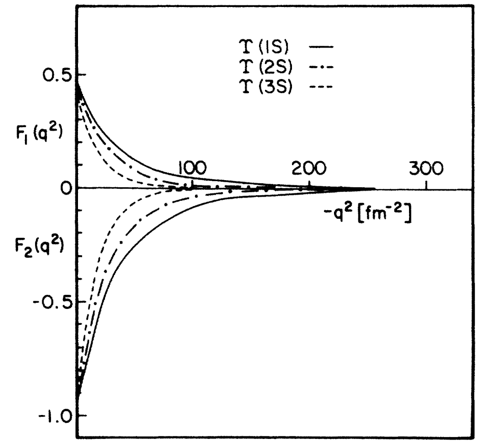

FIG. 6. Vector form factors, $F_1(q^2)$ and $F_2(q^2)$, for states of the Υ system.

wave functions are obtained by Fourier transformation of the momentum-space wave functions [see Eqs. (4.13) and (4.14)] in the meson rest frame. We may note the decreasing importance of the lower component as we move to the description of the more massive mesons.

In Figs. 4–6 we exhibit some typical values for various form factors: $F_S(q^2)$ is obtained in the calculation of the properties of χ_c (2980) and $F_1(q^2)$ and $F_2(q^2)$ are obtained when studying the dynamics of vector mesons.

Finally, we note that the model, extended to include gluon-exchange effects, leads to quite a strong attraction in the pion channel of the $q\bar{q}$ system. Indeed, we were not able to find a stable solution of our equations when we studied the pion channel with the gluon-exchange potential included. (The pion state did move down in energy in our calculations and the pion became very small, but the numerical results were not stable.) It is not clear whether our formalism, in its present form, can describe the pion which is (most likely) the Goldstone boson associated with the breaking of chiral symmetry.

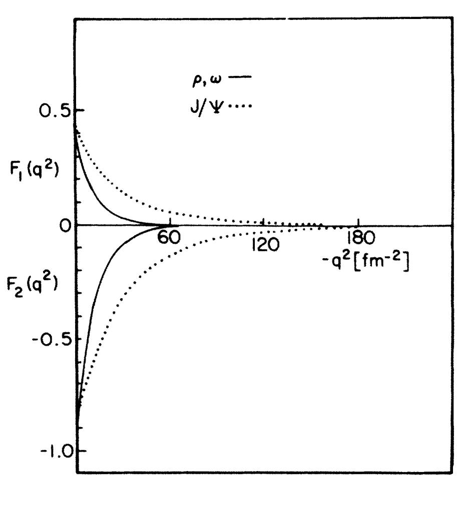

FIG. 5. Vector form factors, $F_1(q^2)$ and $F_2(q^2)$, for the ρ meson and for $J/\psi(3100)$.

VI. DISCUSSION

Clearly there are two major limitations of this analysis. We have not as yet extended this theory to include a model of confinement. The second major limitation is that we do not have a good model for the structure of the pion. Clearly, a satisfactory model of pion structure requires some understanding of the breaking of chiral symmetry in QCD. Indeed, the effective Lagrangian of Eq. (1.1) does not exhibit chiral symmetry. Therefore our model is based upon the fact that chiral symmetry is broken and the assumption that we may write an effective Lagrangian, which does not exhibit chiral symmetry, but which is useful in the broken-symmetry configuration. If we were to understand how chiral symmetry is broken we would, of course, have a better understanding of pion structure, as noted above.

In a future publication we will extend our analysis to include a theory of solitons with relative p-wave motion of the quark and antiquark. We will discuss electromagnetic decays of mesons, described as nontopological solitons.

ACKNOWLEDGMENTS

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APPENDIX A: CHARGE CONJUGATION

In the following we use the notation of Bjorken and Drell. In deriving Eq. (2.6) from Eq. (2.5) and Eq. (2.14) from Eq. (2.13) we have used the following relations:

$$Cq_{\alpha a}(0)C^{-1}=C_{\alpha\beta}\bar{q}_{\beta a}(0)\,, \tag{A1}$$

$$\langle \mathbf{k}st \mid C^{-1} = \langle \overline{\mathbf{k}s-t} \mid \eta_t \;, \tag{A2}$$

$$C \mid \mathbf{p}, TM_T \rangle = (-1)^{M_T} \mid \mathbf{p}, T - M_T \rangle \;, \tag{A3}$$

$$C \mid \mathbf{p}, S'\lambda TM_T \rangle = (-1)^{1+M_T} \mid \mathbf{p}, S'\lambda T - M_T \rangle \;. \tag{A4}$$

Here C is the charge-conjugation operator and $\mid \overline{\mathbf{k}st} \rangle$ is the antiquark state. The difference between Eqs. (A3) and (A4) is due to the fact that the neutral pseudoscalar mesons and vector mesons are eigenstates of the charge-conjugation operator with eigenvalues $+1$ and -1, respectively.

APPENDIX B: NORMALIZATION CONSTRAINTS

We may check our choice of normalization by calculating the charge of a meson with $M_T = +1$ or -1:

$$\langle \mathbf{p}, TM_T' \mid Q \mid \mathbf{p}, TM_T \rangle = (2\pi)^3 \delta(\mathbf{p}' - \mathbf{p}) \left\langle \mathbf{p}', TM_T' \left| :\bar{q}_{\alpha a}(0) \gamma^0_{\alpha\beta} \left[\frac{1}{6} + \frac{\tau_3}{2} \right]_{ab} q_{\beta b}(0): \right| \mathbf{p}, TM_T \right\rangle$$
$$= M_T \delta_{M_T' M_T} \delta(\mathbf{p}' - \mathbf{p}) \;. \tag{B1}$$

We find

$$M_T \delta_{M_T' M_T} = (2\pi)^3 \sum_{st} \int d\mathbf{k} \left[\langle \mathbf{p}TM_T' \mid \bar{q}(0) \mid \overline{\mathbf{k}st} \rangle \gamma^0 \left[\frac{1}{6} + \frac{\tau_3}{2} \right] \langle \overline{\mathbf{k}st} \mid q(0) \mid \mathbf{p}TM_T \rangle \right.$$
$$\left. - \langle \mathbf{p}TM_T' \mid q(0) \mid \mathbf{k}st \rangle \gamma^0 \left[\frac{1}{6} + \frac{\tau_3}{2} \right] \langle \mathbf{k}st \mid \bar{q}(0) \mid \mathbf{p}TM_T \rangle \right] \;. \tag{B2}$$

We evaluate this expression in the frame where $\mathbf{p} = 0$ and find

$$M_T \delta_{M_T' M_T} = M_T \delta_{M_T' M_T} \frac{4}{(2\pi)^3} \int d\mathbf{k} \left[[E^2(k') + F^2(k')] \frac{E_q(\mathbf{k})}{m_q} - 2E(k')F(k') \right] \frac{m_q}{2mE_q(\mathbf{k})} \;, \tag{B3}$$

or

$$M_T = M_T \int_0^\infty k^2 dk [\hat{\mathbf{R}}_l{}^2(k) + \hat{\mathbf{R}}_u{}^2(k)] \;.$$

In Eq. (B3) the quantity $k' = |\mathbf{k}'|$ is the momentum of the meson or of the off-shell quark in the frame where $\mathbf{k} = 0$. We have $k' = mk/m_q$, a relation which follows from evaluating the invariant

$$\left[\frac{(p \cdot k)^2}{m_q{}^2} - m^2 \right]^{1/2}$$

in the frame where $\mathbf{p} = 0$ and in the frame where $\mathbf{k} = 0$. [In the frame where $\mathbf{p} = 0$ we have $p \cdot k = mE_q(\mathbf{k})$, while in the frame where $\mathbf{k} = 0$ we have $(p \cdot k) = \omega(\mathbf{p}) m_q$.] This analysis is appropriate for the vector-meson normalization also.

For a pseudoscalar meson and quark of zero isospin we define

$$\langle \mathbf{k}s \mid \bar{q}(0) \mid \mathbf{p} \rangle = \frac{\sqrt{2}}{[2\omega(\mathbf{p})]^{1/2}} \left[\frac{m_q}{E_q(\mathbf{k})} \right]^{1/2} \frac{1}{(2\pi)^3} \times \left[\bar{u}_s(\mathbf{k}) \left[E - F \frac{\not{p}}{m} \right] \gamma_5 \right] \tag{B4}$$

and in the case of vector mesons, we have

$$\langle \mathbf{k}, s \mid \bar{q}(0) \mid \mathbf{p}, S\lambda \rangle = \frac{\sqrt{2}}{[2\omega(\mathbf{p})]^{1/2}} \left[\frac{m_q}{E_q(\mathbf{k})} \right]^{1/2} \frac{1}{(2\pi)^3} \times \left\{ \bar{u}_s(\mathbf{k}) \left[\frac{\xi_\lambda \cdot k}{m_q} A_1 \left[1 + \frac{\not{p}}{m} \right] + \not{\xi}_\lambda \left[\tilde{A} + \tilde{B} \frac{\not{p}}{m} \right] \right] \right\} \;. \tag{B5}$$

For pseudoscalar mesons we define

$$\hat{\mathbf{R}}_u(k) = \left[\frac{4\pi}{(2\pi)^3} \frac{1}{m} \frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})} \right]^{1/2} [E(k') - F(k')] \;,$$
$$\hat{\mathbf{R}}_l(k) = \left[\frac{4\pi}{(2\pi)^3} \frac{1}{m} \frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})} \right]^{1/2} [E(k') + F(k')] \frac{k}{\epsilon_q(\mathbf{k})} \;, \tag{B6}$$

while for vector mesons we have

$$\hat{\mathbf{R}}_u(k)=\left[\frac{4\pi}{(2\pi)^3}\frac{1}{m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}[\tilde{A}(k')-\tilde{B}(k')] ,$$

$$\hat{\mathbf{R}}_l(k)=\left[\frac{4\pi}{(2\pi)^3}\frac{1}{m}\frac{\epsilon_q(\mathbf{k})}{E_q(\mathbf{k})}\right]^{1/2}[\tilde{A}(k')+\tilde{B}(k')]\frac{k}{\epsilon_q(\mathbf{k})} . \quad \text{(B7)}$$

Again we use the normalization condition

$$\int_0^\infty dk\, k^2[\hat{\mathbf{R}}_u{}^2(k)+\hat{\mathbf{R}}_l{}^2(k)]=1 . \quad \text{(B8)}$$

The expression for the form factors and for m_H remain the same as in the case of mesons of isospin unity.

APPENDIX C: CALCULATION OF FORM FACTORS

In this appendix we discuss the calculation of various form factors. For a pseudoscalar meson we have,

$$\langle \mathbf{p}',TM_T' \mid :\bar{q}(0)q(0): \mid \mathbf{p},TM_T\rangle$$

$$=\sum_{st}\int d\mathbf{k}[\langle \mathbf{p}',TM_T' \mid \bar{q}(0)\mid \overline{\mathbf{k}st}\rangle\langle \overline{\mathbf{k}st}\mid q(0)\mid \mathbf{p},TM_T\rangle-\langle \mathbf{p}'TM_T'\mid q(0)\mid \mathbf{k}st\rangle\langle \mathbf{k}st\mid \bar{q}(0)\mid \mathbf{p},TM_T\rangle] \quad \text{(C1)}$$

$$=2\delta_{M_TM_T'}\frac{1}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}}\frac{1}{(2\pi)^3}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{m_q}{E_q(\mathbf{k})}\mathrm{Tr}\left[\left[E'+F'\frac{\not p'}{m}\right]\left[E+F\frac{\not p}{m}\right]\left[\frac{-\not k+m_q}{2m_q}\right]+\left[A+B\frac{\not p}{m}\right]\left[A'+\frac{B'\not p'}{m}\right]\left[\frac{\not k+m_q}{2m_q}\right]\right] \quad \text{(C2)}$$

$$=\delta_{M_TM_T'}\frac{1}{(2\pi)^3}\frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}}F_S(q^2) . \quad \text{(C3)}$$

We obtain from Eqs. (C3) and (C2),

$$F_S(q^2)=\frac{4}{2m}\int\frac{d\mathbf{k}}{(2\pi)^3}\frac{m_q}{E_q(\mathbf{k})}\left[EE'+FF'\frac{p'\cdot p}{m^2}-E'F\frac{p\cdot k}{mm_q}-EF'\frac{p'\cdot k}{mm_q}\right] , \quad \text{(C4)}$$

where

$$E=E\left[\left[\left[\frac{p\cdot k}{m_q}\right]^2-m^2\right]^{1/2}\right] ,$$

$$E'=E\left[\left[\left[\frac{p'\cdot k}{m_q}\right]^2-m^2\right]^{1/2}\right] , \quad \text{(C5)}$$

with similar expressions for F and F'. (One can understand the appearance of the different arguments in E and F and in E' and F' by making reference to Fig. 1. We see that E and F parametrize the vertex where the meson has momentum $p=[\omega(\mathbf{p}),\mathbf{p}]$ and (E',F') parametrize the vertex where the meson has momentum $p'=[\omega(\mathbf{p}')\ \mathbf{p}']$.)

For vector mesons the form factors are somewhat more complicated. Using the same procedure as used to obtain Eq. (C3), we find

$$\langle \mathbf{p}',S\lambda'TM_T' \mid :\bar{q}(0)q(0): \mid \mathbf{p},S\lambda TM_T\rangle=\delta_{M_TM_T'}\frac{1}{(2\pi)^3}\frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}}\left[\frac{\xi_{\lambda'}'^{*}\cdot p\,\xi_\lambda\cdot p'}{m^2}F_1(q^2)+\xi_{\lambda'}'^{*}\cdot\xi_\lambda F_2(q^2)\right] . \quad \text{(C6)}$$

Now if we define

$$\tilde{q}^\mu=(p'-p)^\mu/2m \quad \text{(C7)}$$

and

$$\tilde{\pi}^\mu=(p+p')^\mu/2m , \quad \text{(C8)}$$

we have

$$F_1(q^2)=\frac{2}{m}\frac{1}{(2\pi)^3}\int d\mathbf{k}\frac{m_q}{E_q(\mathbf{k})}\left[\frac{f_1}{4}\left[\frac{(\tilde{\pi}\cdot k)^2}{m_q{}^2(\tilde{\pi}^2)^2}-\frac{(\tilde{q}\cdot k)^2}{m_q{}^2(\tilde{q}^2)^2}\right]+\frac{f_1}{8}\left[1-\frac{(\tilde{\pi}\cdot k)^2}{m_q{}^2(\tilde{\pi}^2)}-\frac{(\tilde{q}\cdot k)^2}{m_q{}^2\tilde{q}^2}\right]\left[\frac{1}{\tilde{q}^2}-\frac{1}{\tilde{\pi}^2}\right]+\frac{f_2}{2}\left[\frac{\tilde{\pi}\cdot k}{m_q\tilde{\pi}^2}-\frac{\tilde{q}\cdot k}{m_q\tilde{q}^2}\right]+\frac{f_3}{2}\left[\frac{\tilde{\pi}\cdot k}{m_q\tilde{\pi}^2}+\frac{\tilde{q}\cdot k}{m_q\tilde{q}^2}\right]+f_5\right] , \quad \text{(C9)}$$

$$F_2(q^2)=\frac{2}{m}\frac{1}{(2\pi)^3}\int\frac{d\mathbf{k}\,m_q}{E_q(\mathbf{k})}\left[f_4+\left[1-\frac{(\widetilde{\pi}\cdot k)^2}{m_q{}^2\widetilde{\pi}^2}-\frac{(\widetilde{q}\cdot k)^2}{m_q{}^2\widetilde{q}^2}\right]\frac{f_1}{2}\right]. \quad \text{(C10)}$$

Here the invariant functions f_1, f_2, f_3, f_4, and f_5 are functions of $p\cdot k$ and $p'\cdot k$. They are given as

$$f_1=-A_1A_1'\left[1+\frac{p\cdot k}{m_q m}+\frac{p'\cdot k}{mm_q}+\frac{p'\cdot p}{m^2}\right]-A_1'\widetilde{A}-A_1\widetilde{A}'$$
$$-\frac{p\cdot p'}{m^2}(A_1'\widetilde{B}+A_1\widetilde{B}'), \quad \text{(C11)}$$

$$f_2=A_1'\widetilde{B}\frac{k\cdot p}{m_q m}-\widetilde{B}'\widetilde{A}-A_1'\widetilde{A}, \quad \text{(C12)}$$

$$f_3=A_1\widetilde{B}'\frac{k\cdot p'}{mm_q}-\widetilde{B}\widetilde{A}'-A_1\widetilde{A}', \quad \text{(C13)}$$

$$f_4=-\widetilde{A}\widetilde{A}'-\widetilde{B}\widetilde{B}'\frac{p'\cdot p}{m^2}+\widetilde{A}'\widetilde{B}\frac{k\cdot p}{mm_q}+\widetilde{B}'\widetilde{A}\frac{k\cdot p'}{mm_q}, \quad \text{(C14)}$$

$$f_5=\widetilde{B}\widetilde{B}'. \quad \text{(C15)}$$

Further details concerning calculations of the type reported here may be found in Ref. 1.

APPENDIX D: SINGLE-GLUON-EXCHANGE POTENTIALS

In this appendix we consider the modification of our analysis required to include the effects of gluon exchange. If we add this effect to our model we can fit the observed splitting of the $\chi_c(2980)$ and $J/\psi(3100)$ states of charmonium, for example. We neglect the self-interaction of the gluons and consider the following Lagrangian, with

$$\mathcal{L}(\chi)=\bar{q}(x)[i\partial\!\!\!/-g_\chi\chi(x)-m_q]q(x)$$
$$+\tfrac{1}{2}[\partial_\mu\chi(x)\partial^\mu\chi(x)-m_\chi{}^2\chi^2(x)]$$
$$-g\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)A^a_\mu(x)-\tfrac{1}{4}F^{\mu\nu}_a(x)F^a_{\mu\nu}(x). \quad \text{(D1)}$$

Here a is a color index ($a=1,\ldots,8$) and g is a (new) parameter of the model.

We obtain the following equations of motion:

$$(i\partial\!\!\!/-m_q)q(x)=g_\chi q(x)\chi(x)+g\gamma^\mu\frac{\lambda^a}{2}q(x)A^a_\mu(x), \quad \text{(D2)}$$

$$\Box A^a_\mu(x)=g\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x), \quad \text{(D3)}$$

$$(\Box+m_\chi{}^2)\chi(x)=-g_\chi\bar{q}(x)q(x). \quad \text{(D4)}$$

We can now introduce the amplitude

$$\langle\overline{\mathbf{k},s,t,a}\,|\,q_{\alpha ib}(0)\,|\,\mathbf{p},TM_T\rangle,$$

where a and b are color labels, i is an isospin (flavor) label, and α is a Dirac index. We now form matrix elements of Eq. (D2) between quark and meson states and use the procedures described in the main text.

We can write, upon using Eq. (D3),

$$\langle\overline{\mathbf{k},s,t,b}\,|\,A^a_\mu(x)\,|\,\overline{\mathbf{k}',s',t',b'}\rangle$$
$$=g\langle\overline{\mathbf{k},s,t,b}\,|\,\bar{q}(x)\gamma^\mu\frac{\lambda^a}{2}q(x)\,|\,\overline{\mathbf{k}',s',t',b'}\rangle\left[\frac{1}{-q^2}\right], \quad \text{(D5)}$$

where $q^2=(k-k')^2$. We can also write an expression for the quark color current taken between on-shell antiquark states,

$$\left\langle\overline{\mathbf{k},s,t,b}\left|\bar{q}(0)\gamma^\mu\frac{\lambda^a}{2}q(0)\right|\overline{\mathbf{k}',s',t',b'}\right\rangle$$
$$=-\delta_{tt'}\left[\frac{\lambda^a}{2}\right]_{b'b}\frac{f_G(q^2)}{(2\pi)^3}\left[\frac{m_q}{E_q(\mathbf{k})}\right]^{1/2}\left[\frac{m_q}{E_q(\mathbf{k}')}\right]^{1/2}$$
$$\times[\bar{v}(\mathbf{k}',s')\gamma^\mu v(\mathbf{k},s)]. \quad \text{(D6)}$$

Here $f_G(q^2)$ is a new cutoff function needed to regulate the high-momentum behavior of the model.

The analysis of Eq. (D2) then yields

$$(p\!\!\!/-k\!\!\!/-m_q)\langle\overline{\mathbf{k},s,t}\,|\,q(0)\,|\,\mathbf{p},TM_T\rangle=g_\chi\sum_{M_T'}d\mathbf{p}'\langle\overline{\mathbf{k},s,t}\,|\,q(0)\,|\,\mathbf{p}',TM_T'\rangle\langle\mathbf{p}',TM_T'\,|\,\chi(0)\,|\,\mathbf{p},TM_T\rangle$$
$$-\tfrac{4}{3}g^2\sum_{s'}\int d\mathbf{k}\frac{m_q{}^2}{[E_q(\mathbf{k})E_q(\mathbf{k}')]^{1/2}}\frac{f_G(q^2)}{(2\pi)^3}\bar{v}(\mathbf{k}',s')\gamma_\mu v(\mathbf{k},s)\gamma^\mu$$
$$\times\langle\overline{\mathbf{k}',s',t}\,|\,q(0)\,|\,\mathbf{p},TM_T\rangle, \quad \text{(D7)}$$

where we have performed the color-index sums to obtain a factor of $(-\frac{4}{3})$.

Once we have obtained the $(-\frac{4}{3})$ color factor, there is no need to make further reference to color dynamics, except for a minor modification of the normalization of the amplitudes noted below.

1. Pseudoscalar mesons

Again we consider the amplitude $\langle \mathbf{k},s,t,a \mid \bar{q}_{aib}(0) \mid \mathbf{p},TM_T \rangle$ and write

$$\langle \mathbf{k},s,t,a \mid \bar{q}_{aib}(0) \mid \mathbf{p},TM_T \rangle = \frac{\delta_{ab}}{\sqrt{3}} \frac{1}{[2\omega(\mathbf{p})]^{1/2}} \frac{1}{(2\pi)^3} \left[\bar{u}(\mathbf{k},s) \left[A + \frac{B\not{p}}{m} \right] \gamma_5 \right]_{\alpha} (\chi_t^\dagger \tau \cdot \hat{\mathbf{e}}_{M_T})_i \ . \tag{D8}$$

This expansion is the same as that used previously except for the $(\delta_{ab}/\sqrt{3})$ factor. This factor appears here because we need to keep track of the color indices in the extended model which includes "gluon-exchange" effects.

Again, it is useful to develop our equations in the antiquark rest frame where $\mathbf{k}=0$, we obtain

$$\omega(\mathbf{p}) \begin{bmatrix} \bar{R}_u(p) \\ \bar{R}_l(p) \end{bmatrix} = -g_\chi{}^2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{4m}{[4\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}} \frac{F_S(q^2)}{m_\chi{}^2 - q^2} \begin{bmatrix} 1 & 0 \\ 0 & -\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}} \end{bmatrix} \begin{bmatrix} \bar{R}_u(p') \\ \bar{R}_l(p') \end{bmatrix}$$
$$-\tfrac{4}{3} g^2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{f_G(q^2)}{[\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}} \left[\frac{1}{-q^2} \right] \frac{m_q{}^3(\omega+\omega')^2}{(\omega\omega'+m^2+\mathbf{p}\cdot\mathbf{p}')^2} \begin{bmatrix} 2 - \frac{E_q'}{m_q} & -\frac{\hat{\mathbf{p}}'\cdot\mathbf{k}'}{m_q} \\ -\frac{\mathbf{k}'\cdot\hat{\mathbf{p}}}{m_q} & \hat{\mathbf{p}}'\cdot\hat{\mathbf{p}} + \frac{\hat{\mathbf{p}}\cdot\mathbf{k}'\hat{\mathbf{p}}'\cdot\mathbf{k}'}{m_q \epsilon_q(\mathbf{k}')} \end{bmatrix} \begin{bmatrix} \bar{R}_u(p') \\ \bar{R}_l(p') \end{bmatrix} . \tag{D9}$$

The first term on the right-hand side of Eq. (D9) is identical to that given previously for the case of pseudoscalar mesons. The new term in Eq. (D9) corresponds to an attractive potential since $-q^2>0$. The appearance of the momentum $\mathbf{k}'$ in this equation can be understood by recalling the structure of Eq. (D7). We see that $\mathbf{k}'$ is an *intermediate* momentum of the on-shell antiquark in the frame where $\mathbf{k}=0$. The structure of the second term in Eq. (D9) follows upon expressing the meson decay amplitude in the frame where the intermediate antiquark has momentum $\mathbf{k}=0$. In that frame the meson momentum is $\mathbf{p}'$. We have made a change of variables in Eq. (D9) in order to perform our integration over $\mathbf{p}'$ rather than over $\mathbf{k}$ as in Eq. (D7).

2. Vector mesons

We now consider the amplitude

$$\langle \mathbf{k},s,t,b \mid \bar{q}_{aia}(0) \mid \mathbf{p},S\lambda TM_T \rangle = \frac{\delta_{ab}}{\sqrt{3}} \frac{1}{[2\omega(\mathbf{p})]^{1/2}} \left[\frac{m_q}{E_q(\mathbf{k})} \right]^{1/2} \frac{1}{(2\pi)^3}$$
$$\times \left\{ \bar{u}(\mathbf{k},s) \left[\frac{\xi_\lambda \cdot k}{m_q} A_1 \left[1 + \frac{\not{p}}{m} \right] + \not{\xi}_\lambda \left[\tilde{A} + \tilde{B}\frac{\not{p}}{m} \right] \right] \right\}_{\alpha} (\chi_t^\dagger \tau \cdot \hat{\mathbf{e}}_{M_T})_i \ , \tag{D10}$$

where we have again made the color indices explicit and therefore have included a factor of $(1/\sqrt{3})$ on the right-hand side of Eq. (D10). We again obtain an equation in the antiquark rest frame:

$$\omega(\mathbf{p}) \begin{bmatrix} \bar{R}_u(p) \\ \bar{R}_l(p) \end{bmatrix} = -g_\chi{}^2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{4m}{[2\omega(\mathbf{p})2\omega(\mathbf{p}')]^{1/2}} \frac{1}{3} \left[\frac{1}{m_\chi{}^2 - q^2} \right]$$
$$\times \left[\begin{bmatrix} V_{11}{}^1 & V_{12}{}^1 \\ V_{21}{}^1 & V_{22}{}^2 \end{bmatrix} F_1(q^2) + \begin{bmatrix} V_{11}{}^2 & V_{12}{}^2 \\ V_{21}{}^2 & V_{22}{}^2 \end{bmatrix} F_2(q^2) \right] \begin{bmatrix} \bar{R}_u(p') \\ \bar{R}_l(p') \end{bmatrix}$$
$$-\tfrac{4}{3} g^2 \int \frac{d\mathbf{p}'}{(2\pi)^3} \frac{1}{[\omega(\mathbf{p})\omega(\mathbf{p}')]^{1/2}} \frac{1}{3} \left[\frac{f_G(q^2)}{-q^2} \right] \frac{m_q{}^3(\omega+\omega')^2}{(\omega\omega'+m^2+\mathbf{p}\cdot\mathbf{p}')^2} \begin{bmatrix} V_{11}{}^G & V_{12}{}^G \\ V_{21}{}^G & V_{22}{}^G \end{bmatrix} \begin{bmatrix} \bar{R}_u(p') \\ \bar{R}_l(p') \end{bmatrix} . \tag{D11}$$

Again, the first term on the right-hand side has been given previously [see Eq. (3.14)]. Further, $V_{11}{}^1$, $V_{12}{}^1$, $V_{21}{}^1$, and $V_{22}{}^1$ were defined in Eqs. (3.15)–(3.18). The new terms, $V_{11}{}^G$, $V_{12}{}^G$, $V_{21}{}^G$, and $V_{22}{}^G$ are given by

$$V_{11}{}^G=2+\frac{\omega}{m}-\frac{1}{1+\frac{\omega'}{m}}\left[-\frac{\mathbf{k}'^2}{m_q{}^2}+\frac{(\mathbf{k}'\cdot\hat{\mathbf{p}})^2}{m_q{}^2}+\left[\frac{|\mathbf{p}|}{m}\frac{E_q}{m_q}-\frac{\hat{\mathbf{p}}\cdot\mathbf{k}'}{m_q}\frac{\omega}{m}\right]\left[\frac{\hat{\mathbf{p}}\cdot\mathbf{k}'}{m_q}+\frac{|\mathbf{p}|}{m}\right]\right], \quad \text{(D12)}$$

$$V_{12}{}^G=\frac{1}{\left[1+\frac{\omega}{m}\right]}\left[\frac{|\mathbf{p}'|}{m}\frac{E_q}{m_q}+\frac{\hat{\mathbf{p}}'\cdot\mathbf{k}'}{m_q}\frac{\omega'}{m}\right]\left[\frac{E_q}{m_q}+\frac{\omega'}{m}\right]-\frac{|\mathbf{p}'|}{m}, \quad \text{(D13)}$$

$$V_{21}{}^G=\frac{1}{\left[1+\frac{\omega'}{m}\right]}\left[\frac{|\mathbf{p}|}{m}\frac{E_q}{m_q}-\frac{\hat{\mathbf{p}}\cdot\mathbf{k}'}{m_q}\frac{\omega}{m}\right]\left[\frac{E_q}{m_q}+\frac{\omega}{m}\right]-\frac{|\mathbf{p}|}{m}, \quad \text{(D14)}$$

$$V_{22}{}^G=-\frac{m}{|\mathbf{p}'|}\left[\left[\frac{|\mathbf{p}|}{m}\frac{E_q}{m_q}-\frac{\hat{\mathbf{p}}\cdot\mathbf{k}'}{m_q}\frac{\omega}{m}\right]\left[\frac{E_q}{m_q}+\frac{\omega}{m}-4\right]-\frac{\omega'}{m}\frac{|\mathbf{p}|}{m}+\frac{\hat{\mathbf{p}}\cdot\mathbf{k}'}{m_q}\right], \quad \text{(D15)}$$

with $\omega=\omega(\mathbf{p})$ and $\omega'=\omega(\mathbf{p}')$.

Again, we solve Eq. (D11) by iteration and obtain a covariant self-consistent solution. Note that if *only* the gluon term were present, Eq. (D11) would be a *linear* equation, which could be solved directly, without any need for iteration of the solution.

At this point we turn to a specification of the new cutoff introduced into the formalism. We set

$$f_G(q^2)=\left[\frac{\lambda^2}{\lambda^2-q^2}\right]^n, \quad \text{(D16)}$$

with $n=6$ and $\lambda=30\ \text{fm}^{-1}$. We then fixed g by fitting the splitting of the $\chi_c(2980)$ and $J/\psi(3100)$ states, and adjusting the charmed-quark mass so that the mass of the $J/\psi(3100)$ was given correctly. We found $g^2=6.08$ or $\alpha_c=g^2/4\pi=0.48$. The results of these calculations are presented in Tables III and IV.

As noted in the main text, the gluon-exchange potential is very attractive in the $q\bar{q}$ channel with the quantum numbers of the pion. We were not able to find a stable solution describing the pion using a value of the quark mass ($m_q=619$ MeV) which led to a satisfactory description of the ρ and ω mesons.

3. The Hamiltonian

The addition of gluon exchange to the model leads to an additional term in the Hamiltonian:

$$H_G=\int\left[\frac{F_a^{\mu\nu}(x)F_{\mu\nu}^a(x)}{4}-F_a^{0\rho}(x)\dot{A}\,{}^a_\rho(x)\right]d\mathbf{x}\,. \quad \text{(D17)}$$

We can write $H=H_0+H_G$, where H_0 is the Hamiltonian in the absence of gluon terms. We may obtain the expectation value of H_G between states of a meson using the techniques introduced previously. In performing that calculation, it is useful to eliminate the gluon field $A_a^\mu(x)$ in favor of the color current source. Thus we write

$$A_\mu^a(x)=g\int D(x-x')j_\mu^a(x')d^4x'\,, \quad \text{(D18)}$$

with

$$\Box D(x-x')=\delta^{(4)}(x-x')\,. \quad \text{(D19)}$$

(We recall that we have dropped all gluon self-interaction terms in this analysis.)

a. Pseudoscalar mesons

We find, with $q=(k-k')^2$

$$\langle\,\mathbf{p}',TM_T'\,|\,H_G\,|\,\mathbf{p},TM_T\,\rangle$$

$$=(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')\,\frac{g^2}{2}\int d\mathbf{k}\,d\mathbf{k}'\frac{q^2-2(q^0)^2}{(-q^2)^2}\frac{-4}{(2\pi)^3}\left[\frac{m_q}{E_q(\mathbf{k})}\right]^{1/2}\left[\frac{m_q}{E_q(\mathbf{k}')}\right]^{1/2}f_G(q^2)$$

$$\times(2)\sum_{ss't}\langle\,\mathbf{p},TM_T'\,|\,\bar{q}(0)\,|\,\overline{\mathbf{k},s,t}\,\rangle\gamma^\mu\bar{v}(\mathbf{k}',s')\gamma_\mu v(\mathbf{k},s)\langle\,\overline{\mathbf{k}',s',t}\,|\,q(0)\,|\,\mathbf{p},TM_T\,\rangle \quad \text{(D20)}$$

$$=(2)\delta(\mathbf{p}-\mathbf{p}')\delta_{M_TM_T'}\frac{4}{3m}\frac{g^2}{(2\pi)^6}\int d\mathbf{k}\,d\mathbf{k}'\frac{m_q{}^2}{E_q(\mathbf{k})E_q(\mathbf{k}')}f_G(q^2)\frac{-q^2+2(q^0)^2}{q^4}$$

$$\times\left[EE'\left[2-\frac{k\cdot k'}{m_q{}^2}\right]+E'F\left[-\frac{2E_q}{m_q}+\frac{E_q'}{m_q}\right]+EF'\left[\frac{-2E_q'}{m_q}+\frac{E_q}{m_q}\right]+FF'\left[\frac{2E_qE_q'}{m_q{}^2}-1\right]\right] \quad \text{(D21)}$$

$$=\delta_{M_TM_T'}\delta(\mathbf{p}-\mathbf{p}')\mathscr{E}_G^S\,, \quad \text{(D22)}$$

where $E'=E(p\cdot k')$ and $E=E(p\cdot k)$, etc. Further, $E=E(mk/m_q)$ and $E'=E(mk'/m_q)$. The factors of 2 in Eqs. (D21) and (D22) are included since there is a similar contribution from the quark intermediate states which are not written explicitly. The quantity $\mathscr{E}_G^S$ is given in Table IV for the various pseudoscalar mesons considered here.

b. Vector mesons

In this case we find

$$\langle \mathbf{p}',S\lambda'TM_T' \mid H_G \mid \mathbf{p},S\lambda TM_T\rangle$$

$$=(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')\frac{g^2}{2}\int d\mathbf{k}\,d\mathbf{k}'\frac{[q^2-2(q^0)^2]^2}{q^4}\left[\frac{-8}{(2\pi)^3}\right]f_G(q^2)$$

$$\times\left[\frac{m_q}{E_q}\right]^{1/2}\left[\frac{m_q}{E_q'}\right]^{1/2}\sum_{ss't}\langle \mathbf{p}',S\lambda'TM_T' \mid \bar{q}(0)\mid \overline{\mathbf{k},s,t}\rangle$$

$$\times\gamma^\mu\bar{v}(\mathbf{k}',s')\gamma_\mu v(\mathbf{k},s)\langle\overline{\mathbf{k}',s',t}\mid q(0)\mid \mathbf{p},S\lambda TM_T\rangle \quad \text{(D23)}$$

$$=\delta(\mathbf{p}-\mathbf{p}')\delta_{M_TM_T'}\delta_{\lambda\lambda'}\frac{g^2}{9}\frac{1}{(2\pi)^4}\frac{16}{m}\int k'^2dk\int k^2dk\,d(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')\frac{m_q{}^2}{E_qE_q'}f_G(q^2)\frac{2(q^0)^2-q^2}{q^4}$$

$$\times\Bigg[(4\tilde{B}\tilde{B}'-\tilde{A}\tilde{A}')kk'\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}'+(\tilde{A}+\tilde{B})(\tilde{A}'+\tilde{B}')\left[1-\frac{E_q'}{m_q}\right]\left[1-\frac{E_q}{m_q}\right](\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}')^2$$

$$+\left[\frac{E_q}{m_q}-1\right](\tilde{A}+\tilde{B})\left[\tilde{A}'-\tilde{B}'\frac{E_q'}{m_q}\right]+\left[\frac{E_q'}{m_q}-1\right](\tilde{A}'+\tilde{B}')\left[\tilde{A}-\tilde{B}\frac{E_q}{m_q}\right]$$

$$+3\left[\tilde{A}-\tilde{B}\frac{E_q}{m_q}\right]\left[\tilde{A}'-\tilde{B}'\frac{E_q'}{m_q}\right]\Bigg] \quad \text{(D24)}$$

$$=\delta_{M_TM_T'}\delta_{\lambda\lambda'}\delta(\mathbf{p}-\mathbf{p}')\mathscr{E}_G^V\,. \quad \text{(D25)}$$

The quantity $\mathscr{E}_G^V$ is given in Table IV for the various vector mesons studied in this work.

In Table IV, the quantity m_H is given by

$$m_H=2(m-\langle E_q\rangle)+\mathscr{E}_\chi^S+\mathscr{E}_G^S \quad \text{(D26)}$$

for pseudoscalar mesons, and by

$$m_H=2(m-\langle E_q\rangle)+\mathscr{E}_\chi^V+\mathscr{E}_G^V \quad \text{(D27)}$$

for vector mesons.

We remark that the gluon-exchange potential is attractive for *both* pseudoscalar and vector mesons. It is somewhat more attractive for pseudoscalar states, leading to the mass splitting of 0^- and 1^- states with the same number of nodes, as shown in Table III.

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