Determination of baryon magnetic moments from QCD sum rules

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The magnetic moment μ_B of a baryon B with quark content (aab) is written as $\mu_B = 4e_a(1+\delta_B)e\hbar/2cM_B$, where e_a is the charge of the quark of flavor type a. The experimental values of δ_B have a simple pattern and have a natural explanation within QCD. Using the ratio method, the QCD sum rules are analyzed and the values of δ_B are computed. We find good agreement with data ($\approx 10\%$) for the nucleons and the Σ multiplet while for the cascade the agreement is not as good. In our analysis we have incorporated additional terms in the operator-product expansion as compared to previous authors. We also clarify some points of disagreement between the previous authors. External-field-induced correlations describing the magnetic properties of the vacuum are estimated from the baryon magnetic-moment sum rules themselves as well as by independent spectral representations and the results are contrasted.

I. INTRODUCTION

To explain the experimental value of the magnetic moment of the proton and neutron has been a long-standing challenge to theorists. Theoretical explanations in the recent past have ranged from attributing it to the magnetic moments of the constituent quarks or to the virtual pion cloud, or to a combination of both. Lipkin[1] and Brown and Myhrer[2] have emphasized that a consistent picture of the composite nature of the nucleons and the hyperons should simultaneously explain their magnetic moments. Since QCD is the underlying theory of hadrons, it is the natural starting point for a first-principles calculation. Indeed, using the sum-rule method, Ioffe and Smilga,[3,4] and, independently, Balitsky and Yung[5] have determined the magnetic moments of the nucleons and hyperons. A perusal of their work shows several points of agreement as well as disagreement. The purpose of the present work is to clarify some of the issues involved and make some additional contributions which strengthen the whole framework of the QCD sum-rule approach to this problem.

The experimental value[6] of the proton magnetic moment is 2.793 nuclear magnetons and has at first sight no natural explanation. We shall show that from the point of view of QCD it is natural to regard this number as

$$\mu_p = \tfrac{8}{3}(1+\delta_p)\frac{e\hbar}{2cM_N} , \tag{1.1}$$

where $\delta_p = 0.0473$ is a small calculable correction. Similarly, the hyperon magnetic moments, whose experimental values appear to be a melange of arbitrary numbers, assume a neat pattern when viewed in a similar fashion. For example, writing

$$\mu_{\Sigma^+} = \tfrac{8}{3}(1+\delta_{\Sigma^+})\frac{e\hbar}{2cM_\Sigma} , \tag{1.2}$$

$$\mu_{\Sigma^-} = -\tfrac{4}{3}(1+\delta_{\Sigma^-})\frac{e\hbar}{2cM_\Sigma} , \tag{1.3}$$

the experimental values[7,8] of δ_{Σ^+} and δ_{Σ^-} correspond to

$$\delta_{\Sigma^+} = 0.13 \pm 0.01 , \tag{1.4}$$

$$\delta_{\Sigma^-} = 0.06 \pm 0.04 . \tag{1.5}$$

The experimental values[1,6–9] of six members of the $(\frac{1}{2})^+$ baryon octet which are made from quarks of two flavors only, i.e., of the type (aab), are summarized in Table I. Values are given in units of (i) nuclear magnetons, (ii) the particle's own natural magneton, and (iii) in terms of $1+\delta_B$ where $1+\delta_B$ is obtained from (ii) by multiplying by $1/4e_q$ where e_q is the charge of the quarks of flavor a which occurs twice in the baryon. For the present paper, quark charges e_q's will be given in units of the electronic charge. To be more specific, for p and Σ^+ the quark structure is uud and uus, respectively, and thus we multiply column (ii) by $1/4e_u = \frac{3}{8}$. For all the rest, the n, Σ^-, Ξ^-, and Ξ^0, the doubly occurring quark is either d or s and in either case one multiplies by $1/4e_{d,s} = -\frac{3}{4}$. We shall see in Sec. II that from the point of view of QCD these factors arise naturally. Notice that most of the δ's are small so that $1+\delta$ is close to unity.

In Ref. 3 the magnetic moments of the nucleons are determined by evaluating the baryon current correlation functions in an external magnetic field. There are, in principle, three sum rules for the magnetic moment: one at the odd chiral structure $F^{\mu\nu}(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})$, where $\hat{p}=p_\mu\gamma^\mu$, and the other two at the even chiral structures

TABLE I. Experimental values of baryon octet magnetic moments. Also tabulated are the baryon masses, the effective-pole positions, and the fiducial regions over which the sum rules will be fitted.

Baryon	$e\hbar/2M_N c$	$e\hbar/2M_B c$	$(1+\delta_B)$	M_B	W^2	Region
p	$+2.793$	$+2.793$	1.047	0.94	2.3	$0.9\leq M^2\leq 1.2$
n	-1.913	-1.913	1.435			
Σ^+	$+2.379$	$+3.012$	1.129	1.19	3.4	$1.2\leq M^2\leq 1.6$
Σ^-	-1.12	-1.42	1.063			
Ξ^-	-0.69	-0.97	0.73	1.32	4.1	$1.6\leq M^2\leq 2.0$
Ξ^0	-1.25	-1.76	1.32			

$F^{\mu\nu}(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$ and $F^{\mu\nu}\sigma_{\mu\nu}$.

The $F^{\mu\nu}(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})$ sum rule, after a trivial normalization, has a close resemblance to the mass sum rule derived by Belyaev and Ioffe[10] at the odd chiral structure $\hat{p}$. In particular, the leading terms are identical which results, of course, from the fact that electromagnetism conserves the helicity of the interacting quark. This correspondence enables us to write

$$\tfrac{3}{8}\mu_p=(1+\delta_p)\ , \tag{1.6}$$

in units of the nuclear magneton, $e\hbar/2cM_N$, and to calculate δ_p.

In Sec. III we analyze the sum rule by a procedure which is different from either Refs. 3 or 5. In particular we exploit the close similarity between the magnetic moment sum rule and the mass sum rule and use the ratio of the sum rules to determine the magnetic moments. Structures identical to Eq. (1.6) will emerge naturally in our approach enabling us to compute the various δ's. Our procedure has the advantage that it does not require an explicit knowledge of the coupling strength β_N of the nucleon current to the one-nucleon state. In our analysis we have incorporated additional terms such as $g_c{}^2G^n_{\alpha\beta}G^{n\alpha\beta}F_{\mu\nu}$ and $\bar{q}g_c\sigma\cdot GqF_{\mu\nu}$ in the operator-product expansion (OPE). The external-field-induced correlations like $\langle\bar{q}\sigma_{\mu\nu}q\rangle$, $\langle\bar{q}G_{\mu\nu}q\rangle$, and $\epsilon^{\mu\nu\rho\sigma}\langle\bar{q}\gamma_5G_{\rho\sigma}q\rangle$ are estimated via the magnetic-moment sum rules themselves and in Appendix A we have reanalyzed the sum rules using the procedure of Ioffe and Smilga[3,4] to eliminate these external-field-induced vacuum correlations. In addition we have, in Appendix B, extended previous works[5,11] and computed them using spectral representations.

II. MASS AND MAGNETIC-MOMENT SUM RULES

The nucleon mass was computed by Ioffe[12] by writing sum rules for the current correlation function

$$\Pi(p^2)=i\int d^4x\, e^{ip\cdot x}\langle T\{\eta(x),\bar{\eta}(0)\}\rangle\ , \tag{2.1}$$

where the current $\eta(x)$ with quantum numbers of the proton is taken to be

$$\eta(x)=[u^a(x)C\gamma_\mu u^b(x)]\gamma^\mu\gamma_5 d^c(x)\epsilon_{abc}\ , \tag{2.2}$$

where a,b,c are the color indices and $u(x)$ and $d(x)$ are the up- and down-quark fields. The sum rules are obtained on the one hand by computing $\Pi(p^2)$ for large p^2 using the OPE, and on the other hand by expressing it as a dispersion integral with the absorptive part computed in terms of physical hadron states, viz., the proton and its excited states which have the same quantum numbers apart from parity. The nonperturbative character of the QCD vacuum is incorporated by the fact that operators like $\bar{q}q$, $g_c{}^2G^n_{\alpha\beta}G^{n\alpha\beta},\ldots$ have nonzero vacuum expectation values $\langle\bar{q}q\rangle,\langle g_c{}^2G^n_{\alpha\beta}G^{n\alpha\beta}\rangle,\ldots$. After a Borel transformation to the variable M^2, the sum rules read,[10] for the structure at $\hat{p}$,

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{1}{6}a^2L^{4/9}-\frac{a^2m_0{}^2}{24M^2}=\tilde{\beta}_N{}^2e^{-M_N{}^2/M^2}+\text{e.s.c.} \tag{2.3}$$

The structure at 1

$$\frac{aM^4L^{8/9}}{4}+\cdots=\tilde{\beta}_N{}^2M_Ne^{-M_N{}^2/M^2}+\text{e.s.c.}\ , \tag{2.4}$$

where e.s.c. is an abbreviation for excited-state contributions. Here and in the following

$$a=-(2\pi)^2\langle\bar{q}q\rangle\ , \tag{2.5}$$

$$b=\langle g_c{}^2G^n_{\alpha\beta}G^{n\alpha\beta}\rangle\ , \tag{2.6}$$

$$am_0{}^2=-(2\pi)^2\langle\bar{q}g_c\sigma\cdot Gq\rangle\ , \tag{2.7}$$

$$\alpha_s=g_c{}^2/4\pi\ , \tag{2.8}$$

$$L=\ln(M^2/\Lambda_{\rm QCD}{}^2)/\ln(\mu^2/\Lambda_{\rm QCD}{}^2)\ , \tag{2.9}$$

$$\tilde{\beta}_N{}^2=(2\pi)^4\lambda_N{}^2/4\ , \tag{2.10}$$

where λ_N is defined by

$$\langle 0\,|\,\eta(0)\,|\,p,\sigma\rangle=\lambda_N u(p,\sigma)\ . \tag{2.11}$$

The nucleon spinor normalization is given by

$$\bar{u}(p,\sigma)u(p,\sigma)=2M_N\ .$$

The chiral-symmetry-breaking correlation a is taken to be 0.45 GeV^3, the gluon-field value is to be 0.5 GeV^4 and $m_0{}^2\approx 0.8$ GeV^2 in the following.[10,13] The renormalization scale μ is taken to be 500 MeV and $\Lambda_{\rm QCD}$ is the QCD scale parameter taken to be 100 MeV.

The sum rules for the magnetic moment were derived in Ref. 3 by considering the propagation in the presence of a constant external electromagnetic field

$$A_\mu=-\tfrac{1}{2}x^\nu F_{\mu\nu} \tag{2.12}$$

and picking out the terms in $\Pi(p^2)$ that are linear in $F^{\mu\nu}$.

In the presence of the external field, not only are there correlations which are Lorentz-invariant scalars, as in Eq. (2.5)–(2.7), but also new external-field-induced expectation values

$$\langle \bar{q}\sigma_{\mu\nu}q \rangle = e_q e \chi F_{\mu\nu}\langle \bar{q}q \rangle \ , \qquad (2.13)$$

$$\langle \bar{q}g_c G_{\mu\nu}q \rangle = e_q e \kappa F_{\mu\nu}\langle \bar{q}q \rangle \ , \qquad (2.14)$$

$$\langle \bar{q}\epsilon_{\mu\nu\rho\sigma}g_c G^{\rho\sigma}\gamma_5 q \rangle = i e_q e \xi F_{\mu\nu}\langle \bar{q}q \rangle \ , \qquad (2.15)$$

enter the problem. Recall e_q denotes the quark charge in units of the electric charge e. The susceptibilities χ, κ, and ξ are *a priori* unknown and can be of arbitrary strength. Following Refs. 3 and 5, the sum rules can be written at $(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})$ as

$$\frac{e_u M^4}{8L^{4/9}} + \frac{a^2 L^{4/9}}{72M^2}[-(2e_u+3e_d)+e_u(\kappa-2\xi)] - \frac{e_u \chi a^2}{12M^2 L^{4/27}}\left[M^2 - \frac{m_0^2}{8L^{4/9}}\right] + \frac{b}{48L^{4/9}}(e_u+\tfrac{1}{4}e_d) = \frac{\tilde{\beta}_N^2}{4}\left[\frac{\mu_p}{M^2}+A'\right]e^{-M_N^2/M^2} + \text{e.s.c.} \ , \qquad (2.16)$$

at $(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}$ as

$$\frac{e_d \chi^a}{24L^{16/27}}\left[M^2 + \frac{b}{24M^2}\right] + \frac{a}{8}(e_u+\tfrac{1}{2}e_d)\left[1-\frac{m_0^2}{4M^2L^{4/9}}\right] = \frac{\tilde{\beta}_N^2}{4M_N}\left[\frac{\mu_p^a}{M^2}+B'\right]e^{-M_N^2/M^2} + \text{e.s.c.} \ , \qquad (2.17)$$

and at $\sigma_{\mu\nu}$ as

$$\frac{e_d M^4 \chi a}{48L^{16/27}} + \frac{aM^2}{8}\left[e_u - \frac{e_d}{6}(1+4\kappa-2\xi)\right] + \frac{a e_u m_0^2}{48M^2L^{4/9}}\ln\left[\frac{M^2}{\mu_0^2}\right] = \frac{\tilde{\beta}_N^2}{4}M_N\left[\frac{\mu_p}{M^2} - \frac{\mu_p^a}{2M^2} + A'\right]e^{-M_N^2/M^2} + \text{e.s.c.} \qquad (2.18)$$

Here μ_p and μ_p^a refer to the total and anomalous magnetic moments of the proton in units of $e\hbar/2cM_N$. Notice that in the right-hand side (RHS) the so-called single-pole terms A' and B' appear. These are associated with the fact that in the presence of the external field, for example, the proton created by the current $\eta(x)$ can make a transition to an excited state induced by the external field $F^{\mu\nu}$ before being annihilated by $\eta(0)$. These are of course, *a priori*, of unknown strengths.

In the sum rule (2.16) we have, in addition to the terms computed in Refs. 3 and 5, also included the contribution from the operator $g_c^2 G^n_{\alpha\beta}G^{n\alpha\beta}F^{\mu\nu}$. The calculation of this coefficient is straightforward and in the interest of brevity we omit the details. In the sum rule (2.17), we have incorporated the contribution of the operator $[\bar{q}\sigma^{\alpha\beta}G^n_{\alpha\beta}(\lambda^n/2)q]F^{\mu\nu}$. Our calculation agrees with Ioffe and Smilga[4] but disagrees with Balitsky and Yung.[5]

At first sight, the sum rules involve several unknown coefficients aside from the fact that one must use some approximation to include the excited-state contributions in the right-hand side, and it seems hardly possible to extract the quantity of interest, namely, μ_P. However, let us proceed to multiply Eq. (2.16) by M^2/e_u and write it in the form

$$\frac{M^6}{8L^{4/9}} + \frac{bM^2}{48L^{4/9}}\left[\frac{7}{8}\right] + \frac{a^2L^{4/9}}{72M^2}[-\tfrac{1}{2}+(\kappa-2\xi)] - \frac{\chi a^2}{12M^2L^{4/27}}\left[M^2 - \frac{m_0^2}{8L^{4/9}}\right] = \tilde{\beta}_N^2(\tfrac{3}{8}\mu_p + AM^2)e^{-M_N^2/M^2} + \text{e.s.c.} \ , \qquad (2.19)$$

using the experimental value $\mu_p = 2.793$ one finds

$$\tfrac{3}{8}\mu_p = 1.0475 = (1+\delta_p) \ . \qquad (2.20)$$

It is now seen that Eq. (2.19) bears a close resemblance to the mass sum rule Eq. (2.3). Viewed in this light, the problem of determining the magnetic moment is strikingly close to the problem of determining the axial-vector renormalization constant[13] G_A. In the latter problem a sum rule similar to Eq. (2.19) appears with G_A taking the role of the coefficient $\frac{3}{8}\mu_p$ in the nucleon term and of course both G_A and $\frac{3}{8}\mu_p$ are close to unity.

On the other hand, the even chiral sum rules, Eqs. (2.17) and (2.18), bear no resemblance to the mass sum rule Eq. (2.4). Notice, in particular, in Eqs. (2.17) and (2.18) the leading term arises from interaction of the soft quark with the external field. Therefore most of our discussion will be an analysis of Eq. (2.19).

The sum rules for other baryons n, Σ^+, Σ^-, Ξ^-, and Ξ^0 can be easily obtained from the corresponding currents

$$\eta_n(x) = (d^a C\gamma_\mu d^b)\gamma^\mu\gamma_5 u^c \epsilon_{abc} \ , \qquad (2.21)$$

$$\eta_{\Sigma^+}(x) = (u^a C\gamma_\mu u^b)\gamma^\mu\gamma_5 s^c \epsilon_{abc} \ , \qquad (2.22)$$

$$\eta_{\Sigma^-}(x) = (d^a C\gamma_\mu d^b)\gamma^\mu\gamma_5 s^c \epsilon_{abc} \ , \qquad (2.23)$$

$$\eta_{\Xi^-}(x) = (s^a C\gamma_\mu s^b)\gamma^\mu\gamma_5 u^c \epsilon_{abc} \ , \qquad (2.24)$$

$$\eta_{\Xi^0}(x) = (s^a C\gamma_\mu s^b)\gamma^\mu\gamma_5 d^c \epsilon_{abc} \ . \qquad (2.25)$$

For the hyperons we must also take into account the mass of the strange quark and the difference in the chiral condensate of the strange quark as compared to that of the up or down quarks. Following Ioffe and his collaborators[4,14] we take

$$m_s = 150 \text{ MeV}, \quad f = \langle \bar{s}s \rangle / \langle \bar{u}u \rangle = 0.8 \qquad (2.26)$$

throughout our analysis. The susceptibilities χ, κ, and ξ introduced in Eqs. (2.13)–(2.15) could, in principle, be different for the strange quark as compared to the up or down quarks. We shall use the notation χ_s, κ_s, and ξ_s to denote these susceptibilities.

The derivation of the sum rules is straightforward and they are listed below with the substitutions $e_u = \frac{2}{3}$, $e_d = e_s = -\frac{1}{3}$:

p:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}\left[\frac{7}{2}\right]+\frac{a^2L^{4/9}}{72}\left[-\tfrac{1}{2}+(\kappa-2\xi)\right]$$
$$+\frac{\chi a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]$$
$$=\widetilde{\beta}_N{}^2e^{-M_N{}^2/M^2}[(1+\delta_p)+AM^2]+\text{e.s.c.}\;, \quad (2.27)$$

n:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}(2)+\frac{a^2L^{4/9}}{72}[4+(\kappa-2\xi)]$$
$$+\frac{\chi a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]$$
$$=\widetilde{\beta}_N{}^2e^{-M_N{}^2/M^2}[(1+\delta_n)+AM^2]+\text{e.s.c.}\;, \quad (2.28)$$

Σ^+:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}\left[\frac{7}{2}\right]+\frac{a^2L^{4/9}}{72}\left[-\tfrac{1}{2}+(\kappa-2\xi)\right]$$
$$+\frac{\chi a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]-\frac{m_saf}{8}\left[M^2+\frac{m_0{}^2}{6L^{4/9}}\right]$$
$$=\widetilde{\beta}_\Sigma{}^2e^{-M_\Sigma{}^2/M^2}[(1+\delta_{\Sigma^+})+AM^2]+\text{e.s.c.}\;, \quad (2.29)$$

Σ^-:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}(5)+\frac{a^2L^{4/9}}{72}[-5+(\kappa-2\xi)]$$
$$+\frac{\chi a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]-\frac{m_saf}{8}\left[M^2+\frac{m_0{}^2}{6L^{4/9}}\right]$$
$$=\widetilde{\beta}_\Sigma{}^2e^{-M_\Sigma{}^2/M^2}[(1+\delta_{\Sigma^-})+AM^2]+\text{e.s.c.}\;, \quad (2.30)$$

Ξ^-:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}(5)+\frac{a^2L^{4/9}}{72}[-5+(\kappa_s-2\xi_s)]$$
$$+\frac{\chi_s a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]-\frac{m_saf}{8}\left[3M^2+\frac{m_0{}^2}{6L^{4/9}}\right]$$
$$=\widetilde{\beta}_\Xi{}^2e^{-M_\Xi{}^2/M^2}[(1+\delta_{\Xi^-})+AM^2]+\text{e.s.c.}\;, \quad (2.31)$$

Ξ^0:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}(2)+\frac{a^2L^{4/9}}{72}[4+(\kappa_s-2\xi_s)]$$
$$+\frac{\chi_s a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]+\frac{m_saf}{8}\left[3M^2-\frac{m_0{}^2}{6L^{4/9}}\right]$$
$$=\widetilde{\beta}_\Xi{}^2e^{-M_\Xi{}^2/M^2}[(1+\delta_{\Xi})+AM^2]+\text{e.s.c.} \quad (2.32)$$

It is worth emphasizing the fact that when the ground-state baryon pole term is computed from the term

$$\langle 0\,|\,\eta\,|\,B\rangle\langle B\,|\,j_\mu^{\text{el}}A_{\text{ext}}^\mu\,|\,B\rangle\langle B\,|\,\overline{\eta}\,|\,0\rangle\;,$$

the natural unit in which the magnetic moment term appears is $e\hbar/2cM_B$, although from an experimentalist's point of view it is appropriate to express all magnetic moments in terms of a standard unit, viz., the nuclear magneton $e\hbar/2cM_N$. We also give the corresponding mass sum rules taken from Belyaev and Ioffe:[10,14]

N:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2L^{4/9}}{6}-\frac{a^2m_0{}^2}{24M^2}$$
$$=\widetilde{\beta}_N{}^2e^{-M_N{}^2/M^2}+\text{e.s.c.}\;, \quad (2.33)$$

Σ:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2L^{4/9}}{6}-\frac{a^2m_0{}^2}{24M^2}-\frac{afm_sM^2}{4L^{4/9}}$$
$$-\frac{afm_sm_0{}^2}{24}=\widetilde{\beta}_\Sigma{}^2e^{-M_\Sigma{}^2/M^2}+\text{e.s.c.}\;, \quad (2.34)$$

Ξ:

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2f^2L^{4/9}}{6}-\frac{a^2f^2m_0{}^2}{24M^2}-2\frac{afm_sm_0{}^2}{24}$$
$$=\widetilde{\beta}_\Xi{}^2e^{-M_\Xi{}^2/M^2}+\text{e.s.c.} \quad (2.35)$$

III. ANALYSIS OF THE SUM RULES

It is useful to begin by recapitulating that for any particular baryon, say the proton, the three sum rules, Eqs. (2.16)–(2.18), do not have the same range of validity in the Borel variable M^2. As pointed out by Ioffe and Smilga, the sum rule at the odd chiral structure $(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})$, Eq. (2.16), is the most reliable since more terms in the OPE, viz., from dimension-2 to dimension-8 operators, appear in the left-hand side while at the even structures, Eqs. (2.17) and (2.18), one has incorporated only from dimension-3 to dimension-7 operators. Further the sum rule (2.18) suffers from the fact that at dimension 7 there are operators whose vacuum expectation value cannot be estimated by the factorization hypothesis due to the appearance of infrared singularities. Omission of these in Eq. (2.18) further reduces the reliability of this sum rule.

To be able to use Eqs. (2.16) and (2.17) we need to know the susceptibilities χ and $\kappa-2\xi$. However, because of the assumption that the external-field-induced vacuum expectation values are proportional to the corresponding quark charges [cf. Eqs. (2.13)–(2.15)], Ioffe and Smilga[3] observed that the sum rules for the proton and neutron have the remarkable property, that the linear combinations

$$e_d\times[\text{Eq. (2.16) for proton}]$$
$$-e_u\times[\text{Eq.(2.16) for neutron}]\;,$$
$$e_d\times[\text{Eq. (2.17) for proton}]$$
$$-e_u\times[\text{Eq. (2.17) for neutron}]$$

are independent of χ and $\kappa-2\xi$. Needless to say, this procedure weakens the M^2 range of validity of the sum rules while having the merit of eliminating the unknown susceptibilities χ and $\kappa-2\xi$. The analysis of the sum rules following this procedure, with the inclusion of additional terms in the OPE, is given in Appendix A.

As remarked above the sum rule Eq. (2.19) is expected to be more accurate than the even chiral sum rule Eq. (2.17). In this section we shall focus our analysis on the proton and therefore use only Eq. (2.27). Consider the proton magnetic moment sum rule (2.27) and assume to begin with that the susceptibilities χ and $\kappa-2\xi$ are given at the outset. Let us compare it with the mass sum rule at the structure $\hat{p}$ of Eq. (2.33). Using the narrow resonance approximation for the excited states (only to make the discussion below clearer) we can write

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}\left[\frac{7}{2}\right]+\frac{a^2L^{4/9}}{72}[-\tfrac{1}{2}+(\kappa-2\xi)]+\frac{\chi a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]$$
$$=\tilde{\beta}_N{}^2e^{-M_N{}^2/M^2}[(1+\delta_p)+A_pM^2]+\sum_{i\neq N}^{\infty}\tilde{\beta}_i{}^2e^{-M_i{}^2/M^2}(1+\delta_i+A_iM^2)\ , \qquad (2.27')$$

$$\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2L^{4/9}}{6}-\frac{a^2m_0{}^2}{24M^2}=\tilde{\beta}_N{}^2e^{-M_N{}^2/M^2}+\sum_{i\neq N}^{\infty}\tilde{\beta}_i{}^2e^{-M_i{}^2/M^2}\ . \qquad (2.33')$$

It is seen that the leading asymptotic terms in the LHS of these two equations are identical. Since the OPE is the more accurate the larger is the value of M^2, it is useful to ask how the same asymptotic term $M^6/8L^{4/9}$ is reproduced by the excited-state contributions to the two right-hand sides of Eqs. (2.27′) and (2.33′). Since the spectral density of states and the positive definite coupling strengths $\tilde{\beta}_i{}^2$ are the same in the two sum rules it is reasonable to assume that for the excited states δ_i and $A_i\to 0$ rapidly with increasing mass of those states. Let us therefore introduce the ratio function of the left-hand sides in Eqs. (2.27′) and (2.33′):

$$R(M^2)=\left[\frac{M^6}{8L^{4/9}}+\frac{bM^2}{32L^{4/9}}+\frac{a^2L^{4/9}}{6}-\frac{a^2m_0{}^2}{24M^2}\right]^{-1}\left[\frac{M^6}{8L^{4/9}}+\frac{bM^2}{192L^{4/9}}\left[\frac{7}{2}\right]+\frac{a^2L^{4/9}}{72}[-\tfrac{1}{2}+(\kappa-2\xi)]\right.$$
$$\left.+\frac{\chi a^2}{12L^{4/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right]\right]\ . \qquad (3.1)$$

Computing the ratio in terms of physical intermediate states we can write

$$R(M^2)\,|_{\mathrm{RHS}}=\frac{(1+\delta_N+A_NM^2)+\sum_{i\neq N}^{\infty}\frac{\tilde{\beta}_i{}^2}{\tilde{\beta}_N{}^2}(1+\delta_i+A_iM^2)e^{-(M_i{}^2-M_N{}^2)/M^2}}{1+\sum_{i\neq N}^{\infty}\frac{\tilde{\beta}_i{}^2}{\tilde{\beta}_N{}^2}e^{-(M_i{}^2-M_N{}^2)/M^2}}\ . \qquad (3.2)$$

Equation (3.2) can be rewritten as the sum of the ground-state contribution to the numerator function plus a remainder R_c, i.e.,

$$R(M^2)\,|_{\mathrm{RHS}}=(1+\delta_B+A_BM^2)+R_c\ , \qquad (3.3)$$

where

$$R_c=\frac{S}{1+S}\left[\frac{Z+Z_AM^2}{S}-(1+\delta_p+A_pM^2)\right]. \qquad (3.4)$$

Here we denote

$$S=\sum_{i\neq p}^{\infty}\frac{\tilde{\beta}_i{}^2}{\tilde{\beta}_N{}^2}e^{-(M_i{}^2-M_N{}^2)/M^2}\ , \qquad (3.5)$$

$$Z=\sum_{i\neq p}^{\infty}\frac{\tilde{\beta}_i{}^2}{\tilde{\beta}_N{}^2}(1+\delta_i)e^{-(M_i{}^2-M_N{}^2)/M^2}\ , \qquad (3.6)$$

and

$$Z_A=\sum_{i\neq p}^{\infty}\frac{\tilde{\beta}_i{}^2}{\tilde{\beta}_N{}^2}A_ie^{-(M_i{}^2-M_N{}^2)/M^2}\ . \qquad (3.7)$$

Consider the factor $(Z+Z_AM^2)/S$ in Eq. (3.4). If our assumption that δ_i and $A_i\to 0$ for high-mass states is correct, then we expect this factor to be a smoothly varying function of M^2 and stay close to unity. For large M^2 the continuum contribution dominates the right-hand side. By looking at the large-M^2 behavior of the left-hand side we see that S grows like M^6. Taking into account the general structure of S in Eq. (3.5) and the asymptotic behavior of S we assume an effective-pole parametrization for the ratio

$$\frac{S}{1+S}\approx e^{-(W^2-M_N{}^2)/M^2}\ . \qquad (3.8)$$

Notice that as M^2 becomes large, both sides approach unity from below. W is the position of the effective pole. We recall Belyaev and Ioffe[10] have introduced a cutoff parameter which serves as the dividing line separating the ground-state contribution from the continuum contribution. Since as it turns out that our final results are not too sensitive to the precise value of W, for definiteness we identify W here to be their cutoff value.

Combining these points, we are led to our ansatz form for R_c

$$R_c \cong e^{-(W^2-M_N{}^2)/M^2}[\rho+\sigma(W^2-M_N{}^2+M^2)] , \quad (3.9)$$

where ρ and σ are parameters to be determined. With this we can write Eq. (3.3)

$$R(M^2)\,|_{\mathrm{RHS}} = 1+\delta_p + A_p M^2 + [\rho+\sigma(W^2-M_N{}^2+M^2)]e^{-(W^2-M_N{}^2)/M^2} . \quad (3.10)$$

If our ansatz for the effective contribution Eq. (3.10) is good, then we expect the right-hand side and the left-hand side to match over a large region of M^2, for $M^2 \geq M_N{}^2$. In fact, if the two sides matched asymptotically then

$$1+\delta_p+\rho=1 . \quad (3.11)$$

$$\sigma+A_p=0 . \quad (3.12)$$

To find the constants ρ and σ we proceed as follows. We fix σ at an initial value, for example, zero, and start with an arbitrary value of ρ and compute

$$R(M^2)-\rho\exp\left[-\frac{W^2-M_N{}^2}{M^2}\right]=F(M^2) . \quad (3.13)$$

The function $F(M^2)$ is fitted by

$$F(M^2)=\gamma+\delta M^2 \quad (3.14)$$

in the fiducial region

$$0.9 \leq M^2 \leq 1.2 \text{ GeV}^2 . \quad (3.15)$$

If the fitted value γ does not satisfy the condition $\gamma=1+\delta_p=1-\rho$, then a new value $\rho'=(\gamma+\rho)/2$ is chosen. Next the left-hand side of Eq. (3.13) is reevaluated and a new fitted value, γ' is obtained. This process is iterated. After the ith iteration, the acquired value for ρ is given by

$$\rho_{i+1}=(\gamma_i+\rho_i)/2 . \quad (3.16)$$

The convergence of this iteration for the quantity $1+\delta_i=1-\rho_i$ is shown in Fig. 1(a). We note the following points. The iteration in ρ converges rapidly, and more importantly the final value of ρ is independent of its initial value.

We have also tried to satisfy Eq. (3.12) by iterating σ. However, a small nonzero value of $A+\sigma$ persists. It seems proper to us to choose $\sigma=0$ and let the constraint of Eq. (3.12) be mildly violated in the large-M^2 region. Figure 1(b) shows the match between the function $R(M^2)$ and our ansatz Eq. (3.10). It is seen that our failure to match Eq. (3.12) has little effect in the mass region of interest, Eq. (3.15). It is, of course, unreasonable to expect a fit over the entire M^2 region. Therefore, we take our final value of μ_p to be the limit to which ρ converges. Our results are displayed in Table II.

It is straightforward to extend these calculations to the hyperons. The fiducial regions we have chosen in each case is given in the last column of Table I. As remarked

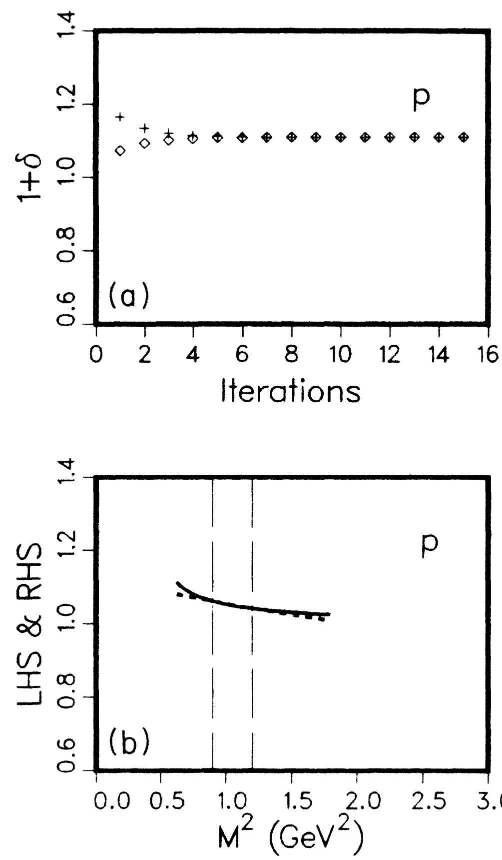

FIG. 1. (a) The convergence of the iterative solution for δ for two different initial guesses. (b) The matching of the left- and right-hand sides. The solid curve is the left-hand ratio while the dashed line is the fitted linear function. The vertical dashed lines indicate the fiducial region in which the fitting is made.

earlier, the value of m_s is taken to be 150 MeV and $f=0.8$.

It is clear from the above discussion that given the values of the susceptibilities $\mathcal{X}$ and $\kappa-2\xi$, the sum rules yield a corresponding set of values for the six baryon magnetic moments. Although the values of $\mathcal{X}$ and $\kappa-2\xi$ can, in principle, be determined by an independent calculation using once again a new set of spectral sum rules (cf. Appendix B for details), we have preferred to treat them as free parameters and searched for the "best values" using the criteria explained below.

The two criteria we use to determine the solution pair $(\mathcal{X},\kappa-2\xi)$ are called, respectively, data fitting and $\mathcal{X}^2$ fitting. In the former we merely ask if it is possible to fit the existing data. That is, we try to find the minimum of

$$\left[\sum_B\left[\frac{\mu_B(\mathrm{expt})-\mu_B(\mathrm{theor})}{\mu_B(\mathrm{expt})}\right]^2\right]^{1/2} . \quad (3.17)$$

The resulting $(\mathcal{X},\kappa-2\xi)$ pair will be called the best fit to data. The second method is the least-$\mathcal{X}^2$ criterion. We

TABLE II. Numerical results for the baryon octet magnetic moments for various susceptibility solution pairs. The magnetic moments are in terms of the nuclear magneton.

Type	χ	$\kappa-2\xi$	μ_p	μ_n	μ_{Σ^+}	μ_{Σ^-}	μ_{Xi^-}	μ_{Xi^0}
χ^2	−1.95	5.73	2.78	−1.68	2.43	−1.08	−0.85	−1.15
best fit	−1.28	8.93	2.96	−1.77	2.45	−1.09	−0.84	−1.14
VDM: 1 pole	−3.3	1.1	2.72	−1.65	2.52	−1.13	−0.89	−1.18
VDM: 2 poles	−4.5	2.0	3.55	−2.06	3.30	−1.38	−0.98	−1.27
Experiment			2.793	−1.913	2.379	−1.12	−0.69	−1.25

demand that the LHS and RHS ratio functions minimize the $\chi_F{}^2$ parameter defined as

$$\chi_F{}^2 = \sum_{\substack{\text{particles}\\ j}} \sum_{\substack{\text{points}\\ i}} [R(M^2)-R_{\text{RHS}}(M^2)]^2/[R(M^2)]^2\,|_{ij}\ , \tag{3.18}$$

where $R(M^2)$ and $R_{\text{RHS}}(M^2)$ are as defined in Eqs. (3.1) and (3.2) and the sum over points i means sum over a sample of points within the fiducial region and measure the deviation of the LHS from the RHS. This method then predicts both the solution pair $(\chi,\kappa-2\xi)$ and the magnetic moments by requiring that $\chi_F{}^2$ be minimized.

To find the solution pairs $(\chi,\kappa-2\xi)$, we numerically search the $\chi-\kappa-2\xi$ plane for the minimum of either Eq. (3.17) or Eq. (3.18) and successively refine the grid on which we search until we have isolated the minimum to a desired degree of accuracy. We note that it is possible to exactly fit a single baryon's magnetic moment by varying χ and $\kappa-2\xi$. We have chosen to exactly fit the proton magnetic moment and then minimize its corresponding $\chi_F{}^2$. The solution pairs for the proton as well as the two methods outlined above are plotted in Fig. 2. Also plotted are values given by Ioffe and Smilga[3] as well as values obtained by various model calculations. (See Appendix B.)

The band we have put around our values is meant to imply a region in which to look for solution pairs. Its existence is obvious if one notes that in each sum rule for the various baryons the factor

$$\kappa-2\xi-\frac{6\chi}{L^{16/27}}\left[M^2-\frac{m_0{}^2}{8L^{4/9}}\right] \tag{3.19}$$

appears. If one ignores the M^2 dependence for a moment and just take $M\approx 1$ GeV then this factor is just

$$\kappa-2\xi-\text{const}\times\chi\ . \tag{3.20}$$

Obviously there is some value of this constant which will best fit the sum rules in any one of the schemes given above. Thus, we see that χ and $\kappa-2\xi$ are (approximately) linearly related with a negative slope and this is the origin of the band drawn on Fig. 2.

It would perhaps be more satisfying if χ and $\kappa-2\xi$ were determined "from the outside," for then the sum rules would make a prediction for the magnetic moments without qualifications. Such calculations are possible, again based on a new set of sum rules, and the use of vector dominance to saturate them. Balitsky and Yung[5] used a one-pole approximation to predict a value of $\chi=-3.3$ GeV^{-2}. Belyaev and Kogan[11] used a two-pole approximation to obtain a value of $\chi=-5.7$ GeV^{-2} which seems to be in rough agreement with the value favored by Ioffe and Smilga[3] of $\chi=-8.0$ GeV^{-2} as giving best agreement with their method of analyzing the sum rules. We have followed these ideas and have calculated χ, κ, and ξ in a similar manner in Appendix B. The values obtained, $\chi=-4.5$ GeV^{-2} and $\kappa-2\xi=2.0$ do not lead to values of μ_B in agreement with data (cf. Table II). It is curious that the values of χ and $\kappa-2\xi$ obtained by one-pole approximation yield μ_B close to experiment and consequently are close to our values as well.

In Table II all the solutions presented are for $\chi_s=\chi$, $\kappa_s=\kappa$, and $\xi_s=\xi$. Notice here our numerical solutions for the cascade magnetic moments are not in as good agreement with the data as for the other baryons. In our numerical analysis we have also varied the strange-quark

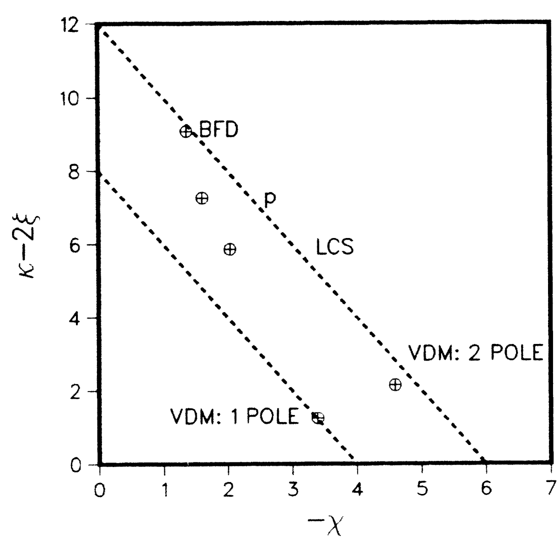

FIG. 2. This plot shows the various solution pairs $(\chi,\kappa-2\xi)$ obtained from various criteria. BFD stands for "best fit to data," LCS stands for "least chi square," P indicates the best χ^2 fit for the proton and the other two points are the results of model calculations using the vector-dominance model (VDM) in a one- and two-pole approximation.

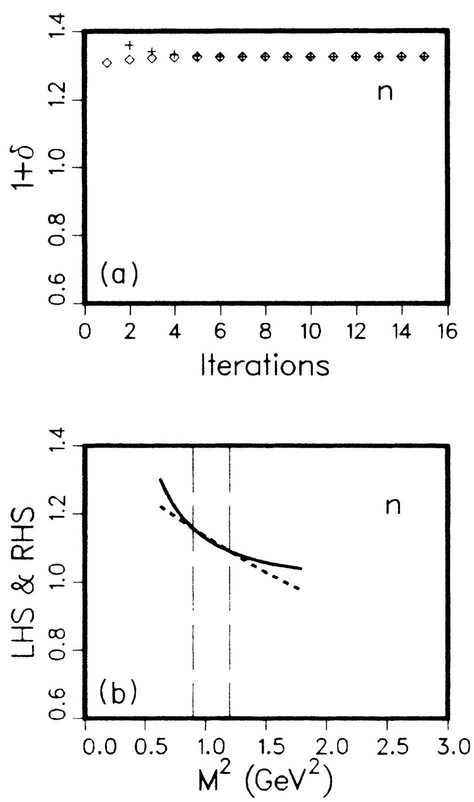

FIG. 3. (a) Convergence of iterations for the neutron for two initial values. (b) The fit of the sum rule. The solid curve is the left-hand ratio function and the dashed line is the fitted linear function. Vertical dashed lines indicate the fiducial region.

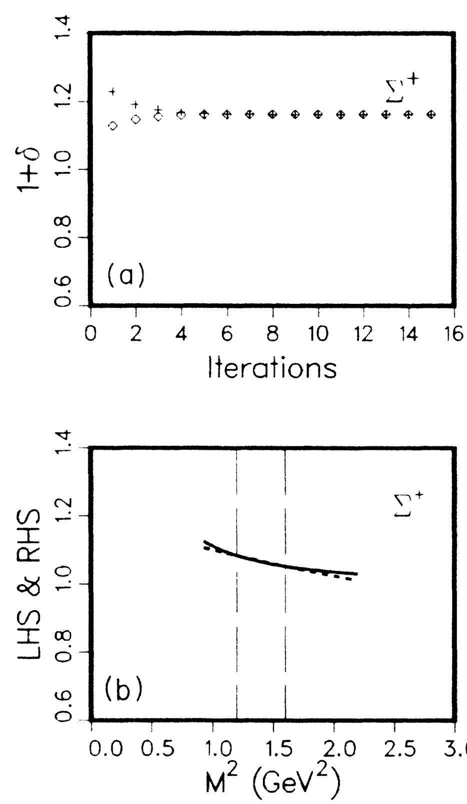

FIG. 4. (a) Convergence of iterations for the Σ^+ for two initial values. (b) The fit of the sum rule. The solid curve is the left-hand ratio function and the dashed line is the fitted linear function. Vertical dashed lines indicate the fiducial region.

susceptibilities by writing $\chi_s=\phi\chi$, $\kappa_s=\phi\kappa$, and $\xi_s=\phi\xi$. In the context of the one-pole approximation of the vector-dominance model, ϕ is given by $m_\rho{}^2/m_\phi{}^2$ although the situation is more complicated in two-pole approximation. We have also considered the possibility where ϕ is less than unity. This extra freedom, however, does not lessen the cascade discrepancy.

For completeness, the illustration for the iteration of ρ and the matching of the RHS and LHS of the sum rule for n, Σ^+, Σ^-, Ξ^-, and Ξ^0 for the best fit to data solution are presented in Figs. 3, 4, 5, 6, and 7, respectively. As in Fig. 1, again one finds a rapid convergence of the iteration and the matching of the two sides of the sum rules is good even beyond the fiducial region indicating the relative insensitivity to the fiducial regions chosen.

IV. DISCUSSION

We have shown that from the point of view of QCD it is natural to study the baryon magnetic moments in terms of δ_B defined through the equation

$$\mu_B=4e_a(1+\delta_B)\frac{e\hbar}{2cM_B} . \tag{4.1}$$

This is to be contrasted with historical approaches which regard for example the entire neutron magnetic moment as anomalous or the nonrelativistic quark-model approaches. While the latter does take into account the baryon substructure it is to be borne in mind that a nonrelativistic potential model description of up and down quarks remains somewhat *ad hoc* with the introduction of constituent-quark masses. The QCD sum-rule approach stays close to the basic premises of the theory and involves no arbitrary assumptions. The sum-rule approach has its own limitations, viz., the need to have an approximate expression for the sum over the excited-state contributions. Further with hyperons we must contend with the definition of the fiducial region over which the LHS and RHS of the sum rules are to be matched. We have chosen this region to be centered around the mass of the hyperon in question.

We have for the most part used only the sum rules at the odd chiral structure [Eqs. (2.27) and (2.33)] basically because these, by virtue of incorporating more terms in the OPE, are expected to be more accurate. The similarity between the sum rules Eqs. (2.27) and (2.33) is traced to the fact that electromagnetism respects chirality of the quarks and accounts for the partly kinematical origin of the structure Eq. (4.1). It is useful to note that the qualitative nature of the pattern of δ_B which is largest for n

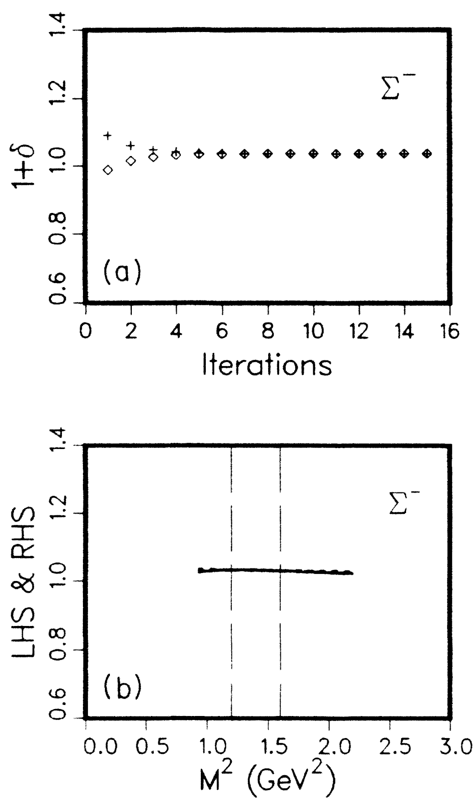

FIG. 5. Convergence of iterations for the Σ^- for two initial values. (b) The fit of the sum rule. The solid curve is the left-hand ratio function and the dashed line is the fitted linear function. Vertical dashed lines indicate the fiducial region.

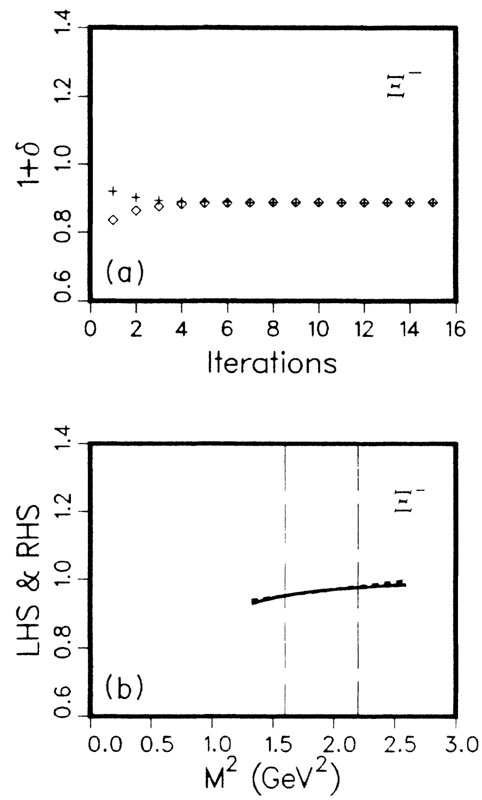

FIG. 6. (a) Convergence of iterations for the Ξ^- for two initial values. (b) The fit of the sum rule. The solid curve is the left-hand ratio function and the dashed line is the fitted linear function. Vertical dashed lines indicate the fiducial region.

and Ξ^0 (with the singly occurring quarks having the large charge) is already apparent in the non-leading terms in the LHS of these sum rules, Eqs. (2.28) and (2.32). We have used the ratio method to compute the various δ_B. Our procedure has the advantage that the value of the coupling strengths $\tilde{\beta}_B$ is not needed to determine δ_B. Our approximation to the continuum contribution in the ratio function $R(M^2)$ is different from that of Ioffe and Smilga.[3,4] We wish to stress that our iteration procedure for determining δ_B is independent of the initial value at the start of the iteration and our ansatz Eq. (3.10) fits $R(M^2)$ not only over the fiducial region but over a wider range of M^2 as well. We have checked that the output values of δ_B are only mildly sensitive to variations in the value of W^2, the effective mass used for approximating the excited state contributions.

The susceptibilities χ and $\kappa-2\xi$ which determine the magnetic properties of the QCD vacuum have been evaluated by two independent methods. In one, we have looked for the best fit to experimental values of μ_B and in the other by requiring the best fit for the sum rules. The two methods yield results in reasonable agreement with each other as well as with experiment. Our results for the cascade magnetic moments are not as good as for other baryons. In this connection it is worth recalling that the mass sum rules do not work as well[10,14] for cascade as they do for the nucleon.

We have not computed the Λ magnetic moment in this paper. It offers no conceptual difficulty; however, since all its three quarks are dissimilar it is algebraically more complicated. The Σ-Λ transition moment offers an additional level of complexity due to Σ,Λ mass splitting. We hope to return to these questions elsewhere.

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APPENDIX A: MAGNETIC MOMENTS BY IOFFE AND SMILGA METHOD

In this appendix we repeat the determination of the magnetic moments of the baryons based on essentially the

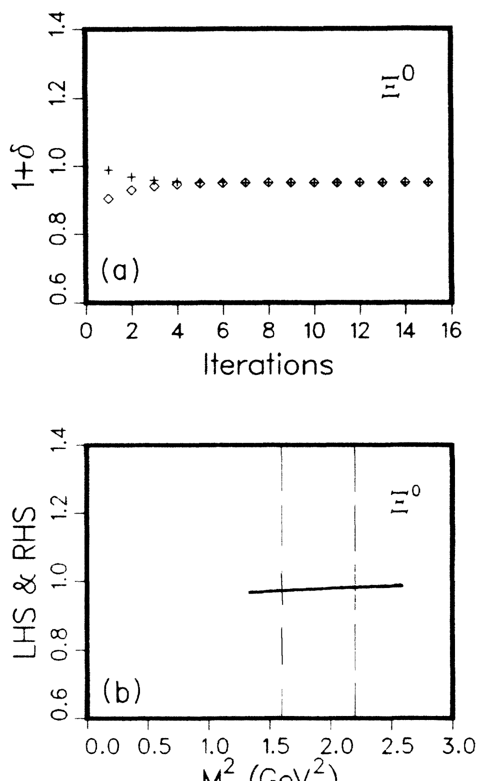

FIG. 7. (a) Convergence of iterations for the Ξ^0 for two initial values. (b) The fit of the sum rule. The solid curve is the left-hand ratio function and the dashed line is the fitted linear function. Vertical dashed lines indicate the fiducial region.

approach of Ioffe and Smilga.[3,4] As remarked earlier in the text we have incorporated in Eq. (2.16) and (2.17) additional terms in the LHS for the OPE and we have also taken into account the anomalous dimensions. First we briefly outline the method.

For the $(\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p})$ structure, taking the linear combination of the sum rule Eq. (2.16) and an analogous one for the neutron, i.e., exchanging subscripts $u \leftrightarrow d$, and dividing both sides by a factor $\tilde{\beta}_N^2 e^{-M_N^2/M^2}$ one obtains

$$\frac{1}{18}\left[a^2L^{4/9}-\frac{bM^2}{8L^{4/9}}\right]e^{M_N^2/M^2}\Big/\tilde{\beta}_N^2 = (e_{\mu_p}-e_\mu\mu_n)+M^2(e_dA_p-e_uA_n) . \quad \text{(A1)}$$

For the $(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$ structure, a similar approach leads to

$$\frac{aM_N}{6}\left[M^2-\frac{m_0^2}{4L^{4/9}}\right]e^{M_N^2/M^2}\Big/\tilde{\beta}_N^2 = (e_u\mu_p{}^a-e_d\mu_n)+M^2(e_uB_p+e_dB_n) , \quad \text{(A2)}$$

where μ_p^a is the anamolous magnetic moment of the proton with $\mu_p=\mu_p^a+1$.

Similarly, one may obtain susceptibility independent combinations for the hyperons.[4] One must include the effects of the nonzero mass of the strange quark in the sum rule at the structure $(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$. One obtains the following pairs of sum rules:

$$\frac{e^{M_\Sigma^2/M^2}}{6\tilde{\beta}_\Sigma^2}\left[a^2/L^{4/9}-\frac{bM^2}{8L^{4/9}}\right]=\mu_{\Sigma^+}+2\mu_{\Sigma^-} +\text{const}\times M^2 , \quad \text{(A3)}$$

$$\frac{e^{M_\Sigma^2/M^2}}{2\tilde{\beta}_\Sigma^2}M_\Sigma\left[af\left[M^2-\frac{m_0^2}{4L^{4/9}}\right]+\frac{4}{9}\frac{m_sa^2}{M^2}+m_sM^4E_0(M^2)L^{-8/9}\right] = -2+\mu_{\Sigma^+}-\mu_{\Sigma^-}+\text{const}\times M^2 , \quad \text{(A4)}$$

$$\frac{e^{M_\Xi^2/M^2}}{6\tilde{\beta}_\Xi^2}\left[a^2f^2L^{4/9}-\frac{bM^2}{8L^{4/9}}+6m_safM^2L^{-4/9}\right] =\mu_{\Xi^-}-\mu_{\Xi^0}+\text{const}\times M^2 , \quad \text{(A5)}$$

$$-\frac{e^{M_\Xi^2/M^2}}{2\tilde{\beta}_\Xi^2}M_\Xi\left[a\left[M^2-\frac{m_0^2}{4L^{4/9}}\right]-\frac{m_saf}{9M^2}\right] =2+2\mu_{\Xi^-}+\mu_{\Xi^0}+\text{const}\times M^2 , \quad \text{(A6)}$$

where we have used $\mu_{\Sigma^+}=\mu_{\Sigma^+}^a+1$, $\mu_{\Sigma^-}=\mu_{\Sigma^-}^a-1$, etc. The function $E_0(M^2)$ is defined by

$$E_0(M^2)=1-e^{-W_\Sigma^2/M^2} .$$

The $e^{-W_\Sigma^2/M^2}$ part represents the continuum contribution in Ioffe and Smilga's method of analyzing the sum rules. We take their value $W_\Sigma^2=3.2$ GeV.[2] It is seen that in Eqs. (A1)–(A6) the RHS's are linear in M^2. The coefficients of the constant term (the term of interest) and the linear term are easily obtained by fitting the LHS of these equations to a straight line in the fiducial regions given in Table I. (Ioffe and Smilga applied the differential operator $1-M^2\partial/\partial M^2$ to the right- and left-hand sides of the sum rules. This has the effect of removing the unknown single-pole terms. In any event, the two methods are essentially equivalent.)

We use here the values[3,4] $a=0.55$ GeV^3 and $b=0.48$ GeV.[4] The values of the coupling constants for these values are taken from Ioffe and Smilga[4] and are

$$\begin{aligned}\tilde{\beta}_N^2&=0.26\ \text{GeV}^6 ,\\ \tilde{\beta}_\Sigma^2&=0.46\ \text{GeV}^6 ,\\ \tilde{\beta}_\Xi^2&=0.62\ \text{GeV}^6 .\end{aligned} \quad \text{(A7)}$$

Note that $\tilde{\beta}^2=\tilde{\lambda}^2/8$ to convert Ioffe and Smilga's notation to our own. The OPE expressions for the individual magnetic moments are shown as solid curves in Fig. 8. The dotted lines are the linear fits to the sum rule in the vicinity of the individual baryon's mass. The intercept on the y axis is then the magnetic moment in units of its

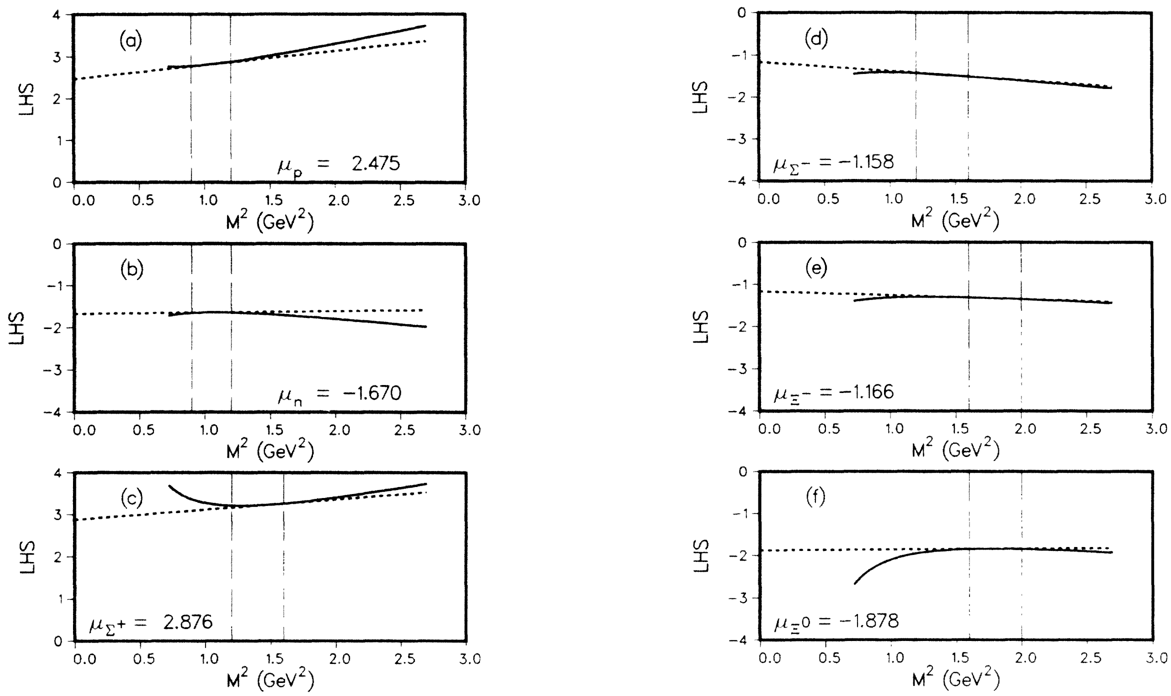

FIG. 8. Plots of the susceptibility independent sum rules. The solid curve shows the left-hand side of the sum rule while the dashed line shows the fitted linear function. The intercept with the vertical axis gives the magnetic moment in the baryon's natural magneton. Notice that in Table III the magnetic moments are converted to nuclear magnetons for comparison with experiment.

natural magneton $e\hbar/2cM_B$. The corresponding moments in terms of the proton magneton as well as the experimental values for comparison are given in Table III.

Our results disagree with those of Ioffe and Smilga.[4] The two sources of disagreement between us are first, we define the anomalous magnetic moment of a baryon B of charge $+1$ to be $\mu_B = 1+\mu_B^a$ in units of the baryon's own natural magneton. To convert to nuclear magnetons we then multiply the entire magnetic moment by m_N/m_B. In Ioffe and Smilga's sum rules, only the anomalous magnetic moment was converted. Technically speaking, this is incorrect. The difference between their treatment and the exact one, however, is small since it is of the order of SU(3)-flavor-symmetry breaking. The second source of disagreement affects only the Ξ's. Instead of Ioffe and Smilga's value of $\frac{17}{36} m_s a^2 f/M^2$ for the contribution of the nine-dimensional operators contributing to the mass correction term, we get $-\frac{1}{9} m_s a^2 f/M^2$. This correction term is small and therefore should not seriously change Ioffe and Smilga's results. [We should mention here, to avoid confusion, Ioffe and Smilga define $f = (\langle \bar{s}s \rangle / \langle \bar{u}u \rangle) - 1$ while we use $f = \langle \bar{s}s \rangle / \langle \bar{u}u \rangle$. Thus, wherever we have an f one should replace it by $1+f$ in Ioffe and Smilga's notation.]

TABLE III. Sum-rule predictions for the susceptibility-independent determination of magnetic moments. Magnetic moments in units of the nuclear magneton.

Baryon	μ_B	Experiment
μ_p	$+2.48$	$+2.793$
μ_n	-1.67	-1.913
μ_{Σ^+}	$+2.27$	$+2.38$
μ_{Σ^-}	-0.92	-1.12
μ_{Ξ^-}	-0.92	-1.12
μ_{Ξ^-}	-0.83	-0.69
μ_{Ξ^0}	-1.34	-1.25

APPENDIX B: SPECTRAL SUM-RULE CALCULATIONS FOR THE SUSCEPTIBILITIES χ, κ, AND ξ

In this appendix we turn to the estimate of the susceptibilities based on vector-dominance model. We recall that the quantitative estimates on the baryon susceptibilities relevant to the baryon magnetic moment problem were first carried out by Balitsky and Yung.[5] Recently, Belyaev and Kogan[11] extended their calculation and gave a presumably improved estimate for the value of χ. Below we review briefly the approach of Belyaev and Kogan and present our calculations for the susceptibilities κ and ξ in addition to χ.

Following Ioffe and Smilga[3] we define susceptibilities induced by the electromagnetic field by

$$\langle \bar{q}\sigma_{\mu\nu}q \rangle = e_q e \chi F_{\mu\nu} \langle \bar{q}q \rangle \,, \tag{B1}$$

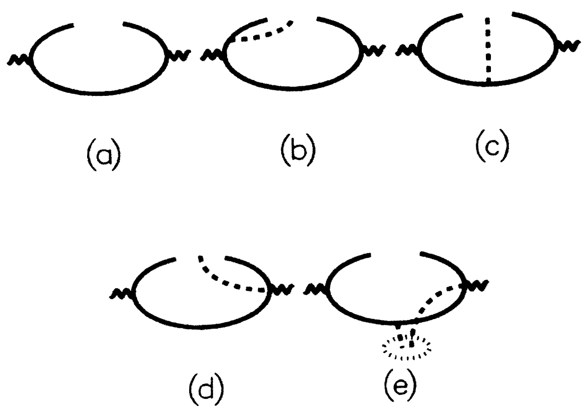

FIG. 9. Diagrams used in model calculations for the susceptibilities. (a), (b), and (c) are used in evaluating $\mathcal{X}$ while (d) and (e) are used to obtain κ and ξ.

$$\langle \bar{q} g_c G_{\mu\nu} q \rangle = e_q e \kappa F_{\mu\nu} \langle \bar{q} q \rangle \ , \tag{B2}$$

$$\langle \bar{q} \epsilon_{\mu\nu\rho\sigma} g_c G^{\rho\sigma} \gamma_5 q \rangle = i e_q e \xi F_{\mu\nu} \langle \bar{q} q \rangle \ . \tag{B3}$$

To evaluate $\mathcal{X}$, κ, and ξ one first introduces a set of susceptibility functions $\mathcal{X}(q^2)$, $\kappa(q^2)$, and $\xi(q^2)$ defined as the Fourier transforms of various induced vacuum expectation values. More specifically, to lowest nontrivial order in the perturbation expansion one has,[5,11] letting $j^\lambda(x) = \bar{q}(x)\gamma^\lambda q(x)$,

$$\mathcal{X}(q^2)\langle \bar{q} q \rangle (g^{\mu\lambda} q^\nu - g^{\nu\lambda} q^\mu) = \int d^4x \, e^{iq\cdot x} \langle T\{ j^\lambda(x), \bar{q}\sigma^{\mu\nu} q \} \rangle \ , \tag{B4}$$

$$\kappa(q^2)\langle \bar{q} q \rangle (g^{\mu\lambda} q^\nu - g^{\nu\lambda} q^\mu) = \int d^4x \, e^{iq\cdot x} \langle T\{ j^\lambda(x), \bar{q} g_c G^{\mu\nu} q \} \rangle \ , \tag{B5}$$

$$i\xi(q^2)\langle \bar{q} q \rangle (g^{\mu\lambda} q^\nu - g^{\nu\lambda} q^\mu) = \int d^4x \, e^{iq\cdot x} \langle T\{ j^\lambda(x), \bar{q} g_c G_{\rho\sigma} \gamma_5 q \} \rangle \epsilon^{\mu\nu\rho\sigma} \ . \tag{B6}$$

Notice here if one contracts both sides of Eqs. (B4)–(B6) by A_λ and take the limit $q_\mu \to 0$, one recovers the corresponding expressions of Eqs. (B1)–(B3) with the identifications

$$\mathcal{X}(0) = \mathcal{X} \ , \tag{B7}$$

$$\kappa(0) = \kappa \ , \tag{B8}$$

$$\xi(0) = \xi \ . \tag{B9}$$

The asymptotic behavior of $\mathcal{X}(q^2)$, $\kappa(q^2)$, and $\xi(q^2)$ can be found as in Ref. 11 using the corresponding diagrams presented in Fig. 10. From Figs. 9(a)–9(c) we get, for $-q^2 = Q^2 \to \infty$,

$$\mathcal{X}(Q^2) \to \frac{-2}{Q^2} + \frac{2m_0^2}{3Q^4} \ , \tag{B10}$$

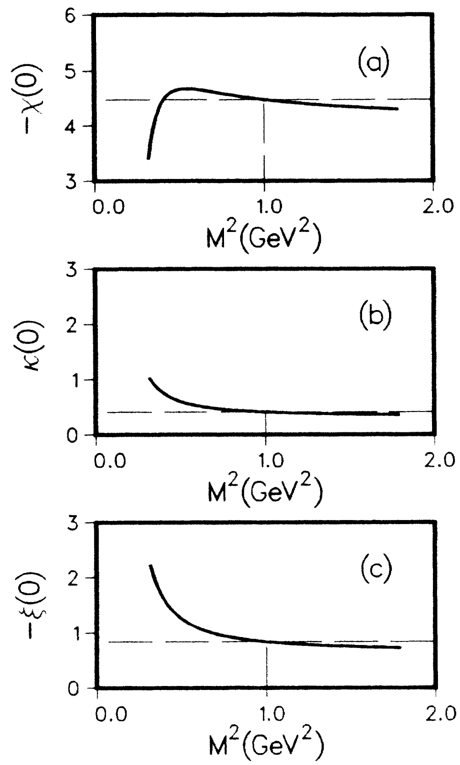

FIG. 10. Plots of the susceptibilities as a function of the Borel mass parameter M^2.

while Figs. 9(d) and 9(e) for $Q^2 \to \infty$ give

$$\kappa(Q^2) \to \frac{m_0^2}{6Q^2} \ , \tag{B11}$$

$$\xi(Q^2) \to -\frac{m_0^2}{3Q^2} - \frac{\langle g_c^2 G^2 \rangle}{72 Q^4} \ . \tag{B12}$$

Notice that the coefficient of the $1/Q^4$ term for $\kappa(Q^2)$ is identically zero. This is due to the vanishing of the trace of Dirac matrices. All three of these susceptibilities have an operator-product expansion of the form $A/Q^2 + B/Q^4$. On the other hand, computing Eqs. (B4)–(B6) in terms of the physical intermediate states keeping only the lowest and first excited state (the two-pole approximation) we can write approximately

$$\frac{A}{Q^2} + \frac{B}{Q^4} \approx \frac{f_\rho}{Q^2 + M_\rho^2 - i\epsilon} + \frac{f_{\rho'}}{Q^2 + M_{\rho'}^2 - i\epsilon} \ . \tag{B13}$$

Demanding the correct asymptotic behavior for the susceptibility leads to the condition

$$f_\rho + f_{\rho'} \approx A \ . \tag{B14}$$

On the other hand, a Borel transform on (B13) gives

$$A + B\frac{1}{M^2} \approx f_\rho e^{-M_\rho^2/M^2} + f_{\rho'} e^{-M_{\rho'}^2/M^2} \ . \tag{B15}$$

From Eqs. (B14) and (B15) one gets

$$f_\rho = \left[A\,(1-e^{-M_{\rho'}{}^2/M^2}) + B\frac{1}{M^2} \right] \Big/ (e^{-M_\rho{}^2/M^2} - e^{-M_{\rho'}{}^2/M^2}) \,. \tag{B16}$$

from Eq. (B7), (B10), and (B13)

$$\chi(0) = \frac{f_\rho}{M_\rho{}^2} + \frac{A - f_\rho}{M_{\rho'}{}^2} \,, \tag{B17}$$

with $A=-2$ and $B=2m_0{}^2/3$. A plot of $-\chi(0)$, $\kappa(0)$, and $-\xi(0)$ as a function of the Borel mass parameter M^2 is shown in Fig. 10. The expressions for $\kappa(0)$ and $\xi(0)$ are with the replacements of $A=m_0{}^2/6$ and $B=0$ for $\kappa(0)$, and $A=-m_0{}^2/3$ and $B=-\langle g_c^2 G^2\rangle/72$ for $\xi(0)$. The values for vector masses are $M_\rho=0.77$ GeV and $M_{\rho'}=1.25$ GeV.

Balitsky and Yung kept only the ground-state ρ-pole term in the right-hand side of Eq. (B13) as well as only the Q^{-2} term in the left-hand side. Their one-pole approximation gives

$$\chi(0)=-3.3\ \text{GeV}^{-2},\quad \kappa(0)=0.22,\quad \xi(0)=-0.44\,, \tag{B18}$$

while the two-pole approximation gives

$$\chi(0)=-4.4\ \text{GeV}^{-2},\quad \kappa(0)=0.4,\quad \xi(0)=-0.8 \tag{B19}$$

at $M^2=1$ GeV^2. Clearly, other choices of values for M^2 in Eq. (B15) will lead to slightly different values for χ, κ, and ξ.

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