Rho mesons in the Skyrme model

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In this article it is shown that the ρ meson can be included in the Skyrme model in a simple, chirally invariant way. Solutions are obtained for the static soliton and time-dependent spinning soliton configurations of the coupled ρ and pion fields. Expressions are derived for the vector and axial-vector currents, and predictions are obtained for static properties of the proton and neutron. These predictions are a few percent better than the corresponding predictions of the massive-pion Skyrme model.

I. INTRODUCTION

Skyrme[1] has shown that the solitons of a chiral theory of mesons have many characteristics of baryons. More recently this correspondence between chiral solitons and baryons has been developed further,[2] and the foundations of the model in the large-N expansion of QCD have been explored.[3] Static properties of nucleons obtained by semiclassical approximation to a $SU(2)\times SU(2)$ theory of pions have been computed.[4,5] Agreement with experiment was found at the 30% level.

One should ask how much these 30% errors depend on the semiclassical approximation, and how much is a consequence of using an incomplete theory of mesons. This question has been discussed by Witten.[6] Attempts to improve the lowest-order semiclassical approximation by including correction terms of order $1/N_c$ have been made,[7] with some success in the case of three flavors. One should also study the solitons of more realistic theories of mesons. These theories should include not only pions but also ρ's, ω's, and higher-mass mesons as well. A model of the pion and ω has already been explored.[8] It was shown that it is possible to use the ω to stabilize the soliton with no quartic pion term. Predictions for physical quantities in this model are improved slightly over the pure Skyrme model, particularly for magnetic moments and the axial-vector coupling constant.

A chiral soliton model including the ρ meson is evidently of great interest. In fact, ρ's have been included in chiral models for many years. One idea is that the ρ is a massive non-Abelian gauge boson of local chiral symmetry.[9] This idea, extended to include the Wess-Zumino term in the Lagrangian, has been used to study anomalous interactions of vector bosons.[10] Other work suggests that the ρ is a dynamical gauge boson of a hidden local symmetry.[11] Recently, ρ's have been incorporated in the Skyrme model in various ways: as "dormant" Goldstone bosons in the same SU(6) multiplet as the pion,[12] and as dynamical gauge bosons.[13]

In the present work I introduce the ρ as an independent field, coupled to the pion in a chirally invariant way. The ρ-π coupling constant is taken from experiment. The advantage of this model is simplicity and computability. The ρ Lagrangian is quadratic with a constraint, the ρ-π coupling is simple, and it is not necessary to deal with an axial-vector (A_1) meson. Using this model I obtain predictions for static properties of nucleons. I view this model as a precursor of a more complete model containing the ω as well as the pion and ρ.

This paper is organized as follows. In Sec. II the model is described. The Lagrangian, the soliton ansatz, and the numerical methods used to obtain the soliton solution are discussed. In Sec. III formulas are given for the vector and axial-vector currents. Numerical predictions for the static properties are presented and the results are discussed.

II. THE MODEL

The Lagrangian of a Skyrme-type model must exhibit global chiral invariance, with the possible exception of noninvariant mass terms. It should also be invariant under the discrete transformations P, C, and T. The following Lagrangian for the coupled ρ-π model has these properties:

$$\mathscr{L}=\frac{F_\pi{}^2}{16}\mathrm{tr}(\partial_\mu U^\dagger\partial^\mu U)+\frac{1}{32e^2}\mathrm{tr}\{[(\partial_\mu U)U^\dagger,(\partial_\nu U)U^\dagger]^2\}+\tfrac{1}{8}F_\pi{}^2 m_\pi{}^2[\mathrm{tr}(U)-2]-\tfrac{1}{8}\mathrm{tr}(R^\dagger_{\mu\nu}R^{\mu\nu})$$
$$+\tfrac{1}{4}m_\rho{}^2\mathrm{tr}(R^\dagger_\mu R^\mu)+\alpha\,\mathrm{tr}(R_{\mu\nu}\partial^\mu U^\dagger U\partial^\nu U^+)\ . \tag{1}$$

The pion is described by the SU(2) matrix U, related to the pion field by

$$U=\exp(2i\boldsymbol{\tau}\cdot\boldsymbol{\pi}/F_\pi)\ . \tag{2}$$

The ρ field is the 2×2 four-vector

$$R^\mu=\rho_0^\mu+i\tau_a\rho_a^\mu\ , \tag{3}$$

where ρ_0^μ and ρ_a^μ are real. I also define

$$R_{\mu\nu}=\partial_\mu R_\nu-\partial_\nu R_\mu\ . \tag{4}$$

Transformations of $SU(2)_L \times SU(2)_R$ act on these matrices according to

$$U \rightarrow LUR^\dagger, \quad R^\mu \rightarrow LR^\mu R^\dagger . \tag{5}$$

The Lagrangian is clearly invariant (except for the pion mass term) under the transformation (5), as well as under P, C, and T (Ref. 14). The chirally invariant constraint

$$0 = \mathrm{tr}(R_\mu^\dagger U) \tag{6}$$

is employed to reduce the number of degrees of freedom in the ρ field to those necessary for unit isospin.[15] The constants appearing in (1) are the pion decay constant $F_\pi = 186$ MeV, the Skyrme coupling constant e (Ref. 16), the pion and ρ masses $m_\pi = 138.0$ MeV, $m_\rho = 769.0$ MeV, and the $\rho\pi\pi$ coupling constant α.

The $\rho\pi\pi$ coupling constant α can be determined by computing the $\rho \rightarrow \pi\pi$ decay width. The interaction term in the Lagrangian is

$$\begin{aligned}\mathscr{L}_{\rho\pi\pi} &= \alpha\, \mathrm{tr}(R_{\mu\nu}\partial^\mu U^\dagger U \partial^\nu U^\dagger) \\ &\rightarrow 8\alpha(m_\rho/F_\pi)^2 \epsilon_{abc}\rho_a^i \pi_b \partial_i \pi_c\end{aligned} \tag{7}$$

for a ρ at rest decaying the two pions. The corresponding width is

$$\Gamma_\rho = \frac{32\alpha^2}{3\pi}\frac{m_\rho{}^2 k^3}{F_\pi{}^4} , \tag{8}$$

where

$$k = [(m_\rho/2)^2 - m_\pi{}^2]^{1/2} \tag{9}$$

is the pion momentum in the center of mass. Using the value[17]

$$\Gamma_\rho = 154(5)\ \mathrm{MeV} , \tag{10}$$

one finds

$$\alpha = 0.0444(10) . \tag{11}$$

The semiclassical analysis of the solitons of this theory is carried out in the same manner as for the original Skyrme model.[4,5] First I make an ansatz for the time-independent soliton configuration

$$\begin{aligned}&U = e^{i\tau\cdot\hat{\mathbf{x}}F(r)} , \\ &R^0 = 0, \quad R^i = i\tau_a \epsilon_{ain}\hat{\mathbf{x}}^n \xi(r) .\end{aligned} \tag{12}$$

This ansatz is consistent with the constraint (6), and with the pseudoscalar nature of the pion and the vector nature of the ρ. The unknown functions $F(r)$ and $\xi(r)$ are obtained by minimizing the static soliton mass

$$M = -\int d^3x\, \mathscr{L} = 4\pi F_\pi \int_0^\infty du\, \mathscr{M}(u) , \tag{13}$$

where

$$\begin{aligned}\mathscr{M}(u) = {}& \frac{u^2}{8}\left[F'^2 + 2\frac{s^2}{u^2}\right] + \frac{s^2}{2e^2}\left[2F'^2 + \frac{s^2}{u^2}\right] \\ &+ \frac{1}{4}\mu_\pi{}^2 u^2 (1-c) + (u^2\xi'^2 + 2u\xi'\xi + 3\xi^2 + \mu_\rho{}^2 u^2 \xi^2) \\ &- 8\alpha s\left[(u\xi' + \xi)F' + \xi\frac{sc}{u}\right] .\end{aligned} \tag{14}$$

In (13) and (14) a change to dimensionless variables has been made, with

$$u = F_\pi r, \quad \tilde{\xi} = \xi/F_\pi, \quad \mu_a = m_a/F_\pi . \tag{15}$$

Primes in (14) refer to u derivatives, $s = \sin(F)$ and $c = \cos(F)$, and I have written ξ instead of $\tilde{\xi}$.

Baryon spin and isospin degrees of freedom are described through collective coordinates describing time-dependent SU(2) rotations of the soliton. The time-dependent ansatz is

$$\begin{aligned}&U = A(t)e^{i\tau\cdot\hat{\mathbf{x}}F(r)}A^\dagger(t) , \\ &R^\mu = A(t)(\rho_0^\mu + i\tau_a\rho_a^\mu)A^\dagger(t) ,\end{aligned} \tag{16}$$

where

$$\begin{aligned}&\rho_a^0 = K^a\zeta_1(r) + \hat{\mathbf{x}}^a(\mathbf{K}\cdot\hat{\mathbf{x}})\zeta_2(r) , \\ &\rho_a^i = \epsilon_{ain}\hat{\mathbf{x}}^n\xi(r) + O(K) ,\end{aligned} \tag{17}$$

with ρ_0^μ determined by the constraint. In (17) new pieces of the ρ field have been excited by the time-dependent rotation. The new pieces are dependent on the quantity K^a, which is defined by

$$A^\dagger(t)\partial_0 A(t) = i\tau_a K^a . \tag{18}$$

In the $1/N_c$ expansion time derivatives are small, so terms of $O(K)$ are small, of order $1/N_c$. The terms of $O(K)$ in ρ_a^i are in fact not excited, and I ignore them. The time-dependent rotations introduce two new functions $\zeta_1(r)$ and $\zeta_2(r)$ into the picture, just as a nonzero spatial component of the ω field was induced in the ω model.[8]

In the presence of the time-dependent soliton the Lagrangian takes the form

$$\int dx^3\, \mathscr{L} = -M + I\, \mathrm{tr}(\partial_0 A^\dagger \partial_0 A) + \cdots , \tag{19}$$

where the terms left out in (19) are of higher order in the time derivatives. The static mass M is given in (13), and the soliton moment of inertia I is

$$\begin{aligned}I = \frac{4\pi}{F_\pi}\int_0^\infty du\, u^2 \Bigg\{ & \frac{s^2}{6}\left[1 + \frac{4}{e^2}\left[F'^2 + \frac{s^2}{u^2}\right]\right] \\ &+ \frac{4}{3}\xi^2 - \frac{16}{3}\alpha\xi\frac{s^2 c}{u}\Bigg\} + I' ,\end{aligned} \tag{20}$$

where

$$\begin{aligned}I' = \frac{4\pi}{6F_\pi}\int_0^\infty du\, u^2 \Bigg[& \frac{1}{2}\left[2\zeta_1'^2 + \frac{1}{c^2}\Phi'^2 + 2\frac{sF'}{c^3}\Phi\Phi' \right. \\ &\left. + \frac{F'^2}{c^4}\Phi^2 + \frac{2}{u^2}\frac{s^2}{c^2}\Phi^2 + \frac{4}{u^2}\zeta_2{}^2\right] \\ &+ \frac{1}{2}\mu_\rho{}^2\left[2\zeta_1{}^2 + \frac{1}{c^2}\Phi^2\right] \\ &+ 16\alpha\left[s\zeta_1' F' + \frac{s^2 c}{u^2}\zeta_1 - \frac{s^2}{cu^2}\Phi\right]\Bigg]\end{aligned} \tag{21}$$

is the contribution to I from ζ_1 and ζ_2, and where $\Phi=\zeta_1+\zeta_2$.

The soliton equations were solved numerically by minimizing first M and then I'. Physical values were assigned to the parameters m_π, m_ρ, and α, while F_π and e were chosen so that

$$M_N=M+\frac{3}{8I}$$

and (22)

$$M_\Delta=M+\frac{15}{8I}$$

gave the correct nucleon and Δ masses. This procedure results in an F_π that is below its physical value, but it seems to give the best fit to the physical quantities. There is in fact some justification to taking F_π below its physical value in this model.[18] Boundary conditions were chosen so that the soliton is bounded in space, has unit baryon number, and so that U and R^μ are continuous functions. The boundary conditions are

$$F(0)=\pi,\quad \xi(0)=0\ ,$$
$$\zeta_1'(0)=0,\quad \zeta_2(0)=0\ , \tag{23}$$

with all functions vanishing at $r=\infty$.

The baryon masses M_N and M_Δ are obtained correctly when $F_\pi=104.8$ MeV and $e=4.648$. Graphs of F, ξ, ζ_1, and ζ_2 are shown in Figs. 1 and 2. The only numerical difficulty occurs in the minimization of I' for ζ_1 and ζ_2. The problem is that I' involves inverse powers of $c=\cos F$, and F passes through $\pi/2$ in the region of interest. The minimization technique for ζ_1 and ζ_2 works, but requires several times the number of iterations that were necessary to obtain F and ξ to similar accuracy.

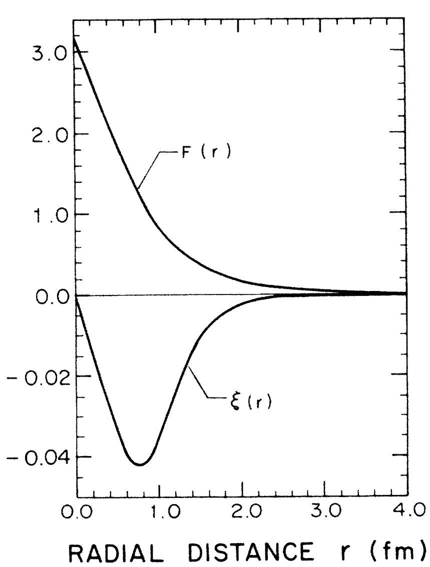

FIG. 1. The static-soliton ansatz involves two functions of the radial distance, F and ξ. These functions are obtained by minimizing the static soliton mass M. This is a plot of the chiral angle $F(r)$, which is related to the pion field, and the dimensionless version of $\xi(r)$, which is related to the spatial component of the ρ field.

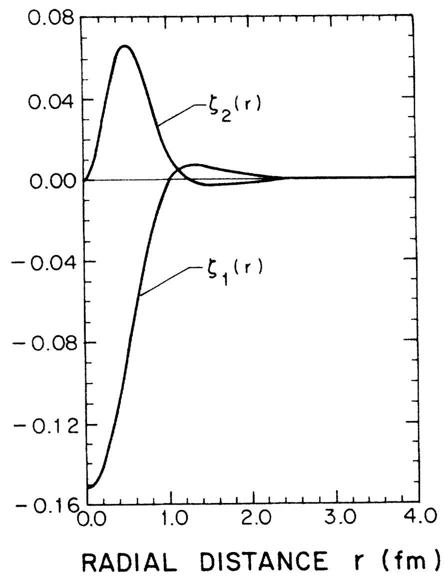

FIG. 2. Time dependence excites two additional soliton functions, $\zeta_1(r)$ and $\zeta_2(r)$, related to the time component of the ρ field. These dimensionless functions are found by minimizing the functional I'.

III. PREDICTIONS FOR PHYSICAL QUANTITIES

Currents are the keys to obtaining many of the predictions for physical properties of nucleons from Skyrme-type models. In the present model the isoscalar current is

$$B^\mu=\frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\mathrm{tr}[(U^\dagger\partial_\nu U)(U^\dagger\partial_\alpha U)(U^\dagger\partial_\beta U)]\ , \tag{24}$$

where $\epsilon_{0123}=+1$. With the soliton ansatz this becomes

$$B^0=\frac{-1}{2\pi^2}\frac{s^2}{r^2}F'\ ,$$
$$B^i=\frac{-1}{2\pi^2}\frac{s^2}{r^2}F'\epsilon_{ijk}x^j\mathrm{tr}(iA^\dagger\partial_0 A\,\tau_k)\ . \tag{25}$$

The vector and axial-vector currents are obtained from the left- and right-handed currents as

$$J_V^{\mu s}=J_L^{\mu s}+J_R^{\mu s},\quad J_A^{\mu s}=J_L^{\mu s}-J_R^{\mu s}\ , \tag{26}$$

where, for example,

$$\mathscr{L}\rightarrow\mathscr{L}+\partial_\mu\alpha^s(x)J_L^{\mu s}(x) \tag{27}$$

under the left-handed transformation

$$U\rightarrow LU,\quad R_\mu\rightarrow LR_\mu\ ,$$
$$L=1-i\alpha^s(x)(\tau^s/2)\ . \tag{28}$$

The currents are

$$J_V^{0s}=K^a\left[(\delta_{ar}-\hat{x}_a\hat{x}_r)\left\{-\frac{s^2}{4}\left[F_\pi{}^2+\frac{4}{e^2}\left[F'^2+\frac{s^2}{r^2}\right]\right]+2\alpha\left[4\xi\frac{s^2c}{r}+\left[\frac{s}{c}\Phi\right]\frac{s^3}{r^2}+\zeta_2\frac{s^2c}{r^2}-\zeta_1' sF'\right]\right\}\right.$$
$$\left.-\xi^2(\delta_{ar}+\hat{x}^a\hat{x}^r)-\frac{1}{2}\zeta_2\left[\frac{\xi}{r}\right](\delta_{ar}-3\hat{x}^a\hat{x}^r)\right]\mathrm{tr}(\tau_r A^\dagger\tau_s A)\,, \tag{29a}$$

$$J_V^{is}=\left\{\frac{1}{8}\frac{s^2}{r}\left[F_\pi{}^2+\frac{4}{e^2}\left[F'^2+\frac{s^2}{r^2}\right]\right]+\frac{1}{r}\xi^2+2\alpha\left[-3\xi\frac{s^2c}{r^2}-\left[\xi'+\frac{\xi}{r}\right]sF'\right]\right\}\epsilon_{inr}\hat{x}^n\mathrm{tr}(\tau_r A^\dagger\tau_s A)\,, \tag{29b}$$

$$J_A^{0s}=K^a\left\{\frac{sc}{4r^2}\left[F_\pi{}^2+\frac{4}{e^2}\left[F'^2+\frac{s^2}{r^2}\right]\right]-\frac{1}{2r}\left[\frac{s}{c}\Phi\right]\xi+2\alpha\left[2\xi\frac{s}{r}(s^2-c^2)-\left[\frac{s}{c}\Phi\right]\frac{s^2c}{r^2}-\zeta_2\frac{sc^2}{r^2}+\zeta_1' cF'\right]\right\}$$
$$\times\epsilon_{anr}\hat{\mathbf{x}}^n\mathrm{tr}(\tau_r A^\dagger\tau_s A)\,, \tag{29c}$$

$$J_A^{is}=\left[\delta_{ir}\left\{\frac{sc}{8r}\left[F_\pi{}^2+\frac{4}{e^2}\left[F'^2+\frac{s^2}{r^2}\right]\right]+2\alpha\left[\xi\frac{s}{r^2}(3s^2-2)-\left[\xi'+\frac{\xi}{r}\right]cF'\right]\right\}\right.$$
$$+\hat{\mathbf{x}}^i\hat{\mathbf{x}}^r\left\{-\frac{sc}{8r}\left[F_\pi{}^2+\frac{4}{e^2}\left[F'^2+\frac{s^2}{r^2}\right]\right]+\frac{F'}{8}\left[F_\pi{}^2+\frac{8}{e^2}\frac{s^2}{r^2}\right]\right.$$
$$\left.\left.+\,2\alpha\left[-3\xi\frac{s^3}{r^2}-2\xi'\frac{s}{r}+\left[\xi'+\frac{\xi}{r}\right]cF'\right]\right\}\right]\mathrm{tr}(\tau_r A^\dagger\tau_s A)\,. \tag{29d}$$

Terms of higher order in K have been neglected in these expressions for the currents. The currents satisfy certain consistency conditions, such as vector-current conservation

$$\partial_i J_V^{is}=0\,. \tag{30}$$

Note that to lowest order in K one need not consider the time-derivative contribution. Also the isospin operator is obtained by integrating the isovector density

$$I^s=\int d^3x\,J_V^{0s}=\frac{i}{2}\left[a_0\frac{\partial}{\partial a^s}-a_s\frac{\partial}{\partial a^0}-\epsilon_{sij}a_i\frac{\partial}{\partial a^j}\right], \tag{31}$$

TABLE I. Predictions of the present model compared against the predictions of the massive-pion Skyrme model (Ref. 5) and against experiment.

Physical quantity	Prediction (This paper)	Prediction [Skyrme model (Ref. 5)]	Measured value
	Results		
M_N (MeV)	Input	Input	938.9
M_Δ (MeV)	Input	Input	1232
m_π (MeV)	Input	Input	138.0
m_ρ (MeV)	Input		796.0
F_π (MeV)	104.8	108	186
e	4.648	4.84	
α	Input		0.0444(10)
$(\langle r^2\rangle^{1/2})_{E,I=0}$ (fm)	0.70	0.68	0.72
$(\langle r^2\rangle^{1/2})_{E,I=1}$ (fm)	1.08	1.04	0.88
$(\langle r^2\rangle^{1/2})_{M,I=0}$ (fm)	0.98	0.95	0.81
$(\langle r^2\rangle^{1/2})_{M,I=1}$ (fm)	1.06	1.04	0.80
μ_p (nuclear magnetons)	2.16	1.97	2.79
μ_n (nuclear magnetons)	−1.38	−1.24	−1.91
$g_{\pi NN}$	13.1	11.9	13.5
$g_{\pi N\Delta}$	19.7	17.8	20.3
$\mu_{N\Delta}$	2.50	2.3	3.3
g_A	0.65	0.65	1.23
σ (MeV)	38	38	36(20)

where use has been made of the canonical quantization relation

$$-i\frac{\partial}{\partial a^i}=\pi_i=4I\dot{a}_i \ . \tag{32}$$

Finally, the axial-vector current satisfies

$$\partial_i J_A^{is}=\frac{1}{8}m_\pi{}^2 sF_\pi{}^2\hat{\mathbf{x}}^r \mathrm{tr}(\tau_r A^\dagger \tau_s A) \ , \tag{33}$$

which to lowest order in the pion field is the PCAC (partial conservation of axial-vector current) relation

$$\partial_i J_A^{is}=m_\pi{}^2(F_\pi/2)\pi^s(x) \ . \tag{34}$$

The static nucleon properties are obtained from the currents in the same manner as in previous work.[4,5,8] The results are displayed in Table I. The agreement of the predictions with measured values in the present model is somewhat better than in the massive-pion Skyrme model.[5] Predictions for the magnetic moments are improved by about 7%, and the πNN and $\pi N\Delta$ coupling constants are improved by about 9%. The charge radii, however, are 3–5 % worse in the present model, although the equality of the electric and magnetic isovector charge radii is broken in the physically correct way. The axial-vector coupling constant and the σ terms are unchanged.

In this work I have shown that the ρ can be included in the Skyrme model in a consistent and simple way. The predictions for static properties are only slightly changed from those of the original Skyrme model, mostly for the better. I believe that the techniques used in this model can be extended to include the ω as well as the ρ. Such a model could be much more realistic than the original Skyrme model or the model considered here, and could well give much improved values for the static properties.

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[1] T. H. R. Skyrme, Proc. R. Soc. London **A260**, 127 (1961).

[2] N. K. Pak and H. C. Tze, Ann. Phys. (N.Y.) **117**, 164 (1979); J. M. Gipson and H. C. Tze, Nucl. Phys. **B183**, 524 (1981); A. P. Balachandran, V. P. Nair, S. G. Rajeev, and A. Stern, Phys. Rev. Lett. **49**, 1124 (1982); Phys. Rev. D **27**, 1153 (1983).

[3] E. Witten, Nucl. Phys. **B160**, 57 (1979); **B223**, 422 (1983); **B223**, 433 (1983).

[4] G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. **B228**, 552 (1983).

[5] G. S. Adkins and C. R. Nappi, Nucl. Phys. **B233**, 109 (1984).

[6] E. Witten, in *Solitons in Nuclear and Elementary Particle Physics*, edited by A. Chodos, E. Hadjimichael, and C. Tze (World Scientific, Singapore, 1984), pp. 306–312.

[7] E. Guadagnini, Nucl. Phys. **B236**, 35 (1984); G. S. Adkins and C. R. Nappi, *ibid.* **B249**, 507 (1985).

[8] G. S. Adkins and C. R. Nappi, Phys. Lett. **137B**, 251 (1984).

[9] J. J. Sakurai, *Currents and Mesons* (University of Chicago Press, Chicago, Illinois, 1969); J. Schwinger, Phys. Lett. **24B**, 473 (1967); J. Wess and B. Zumino, Phys. Rev. **163**, 1727 (1967); S. Weinberg, *ibid.* **166**, 1568 (1968). For a review see S. Gasiorowicz and D. A. Geffen, Rev. Mod. Phys. **41**, 531 (1969).

[10] Ö. Kaymakcalan, S. Rajeev, and J. Schechter, Phys. Rev. D **30**, 594 (1984); Y. Brihaye, N. K. Pak, and P. Rossi, Nucl. Phys. **B254**, 71 (1985); Ö. Kaymakcalan and J. Schechter, Phys. Rev. D 31, 1109 (1985).

[11] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985).

[12] D. G. Caldi, in *Solitons in Nuclear and Elementary Particle Physics* (Ref. 6), pp. 257–280.

[13] T. Fujiwara, Y. Igarashi, A. Kobayashi, H. Otsu, T. Sato, and S. Sawada, Prog. Theor. Phys. **74**, 128 (1985); Y. Igarashi, M. Johmura, A. Kobayashi, H. Otsu, T. Sato, and S. Sawada, Nucl. Phys. **B259**, 721 (1985).

[14] The field transformations are $U\to U^\dagger$, $R^\mu\to -R_\mu^\dagger$ under P, $U\to U^T$, $R^\mu\to -R^{\mu T}$ under C, and $U\to U^*$, $R^\mu\to -R_\mu^*$ under T.

[15] The ρ theory considered here, a constrained ρ connected to pions by the interaction term in Eq. (1), is inequivalent to the usual model in which the ρ is taken to be a massive non-Abelian gauge field. One way to see this is to note that the effective pion Lagrangian obtained by eliminating the ρ field is of sixth order in pion field derivatives. The usual model results in a Skyrme term which is of fourth order in the derivatives [see K. Iketani, Kyushi Report No. KYUSHU-84-HE-2, 1984 (unpublished); T. N. Pham and T. N. Truong, Phys. Rev. D 31, 3027 (1985); I. J.R. Aitchison, C. M. Fraser, and P. J. Miron, Oxford report, 1985 (unpublished)].

[16] The Skyrme coupling constant e can be obtained from the $\pi\pi$ scattering data using the formulas given in J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Rev. Lett. **53**, 747 (1984). The result is 7.4 (1.6).

[17] Particle Data Group, Rev. Mod. Phys. **56**, S1 (1984).

[18] See the paper of Ref. 16.