Mass spectrum of low-lying baryons in the ground state in a relativistic potential model of independent quarks with chiral symmetry

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Under the assumption that baryons are an assembly of independent quarks, confined in a first approximation by an effective potential $U(r)=\frac{1}{2}(1+\gamma^0)(ar^2+V_0)$ which presumably represents the nonperturbative gluon interactions, the mass spectrum of the low-lying ground-state baryons has been calculated by considering perturbatively the contributions of the residual quark-pion coupling arising out of the requirement of chiral symmetry and that of the quark-gluon coupling due to one-gluon exchange over and above the necessary center-of-mass correction. The physical masses of the baryons so obtained agree quite well with the corresponding experimental value. The strong coupling constant $\alpha_c=0.58$ required here to describe the QCD mass splittings is quite consistent with the idea of treating one-gluon-exchange effects in lowest-order perturbation theory.

I. INTRODUCTION

In the study of the mass spectrum of low-lying baryons, several articles[1] based on the nonrelativistic quark model have appeared in the literature. Although the phenomenological picture is quite reasonable at the nonrelativistic level, a relativistic treatment is indispensable on this account in view of the fact that the splitting among the baryon masses is of the same order as the constituent-quark masses. The MIT bag model[2–4] has proved to be quite successful in this respect. In its improved versions, the chiral bag models[5] (CBM's) have included the effect of pion self-energy due to baryon-pion coupling at the vertex to give a better understanding of the physical masses of the baryons. However, such models are not entirely free from any objections due to the assumed static spherical bag boundary to which they owe much of their success and simplicity. This is because of the fact that it is difficult to believe the spherical bag boundary remains static and unperturbed by the creation of a pion. Furthermore, in any bag model, to restore chiral symmetry it is essential to introduce the additional pion field in the region exterior only to the spherical bag boundary. On the other hand, exclusion of pions from the interior of the static bag, for a number of reasons, may not be correct and reasonable for which the CBM does not explicitly exclude the pions from the bag volume. However, the very inclusion of pions in the interior region is rather more or less *ad hoc*. If it is implied by the concept of dynamical symmetry breaking,[6] then instead of treating pions as free particles through all space, it would have been more appropriate to use the expansion of the pion field in terms of the eigenfunctions of some effective potential.

However, the chiral potential models[7] replacing, so to speak, the rigid spherical bag boundary by an effective relativistic confining potential for individual quarks are more straightforward in these respects. The term in the Lagrangian density for quarks corresponding to the effective scalar potential being chirally odd through all space requires the introduction of an additional pionic component everywhere in order to preserve chiral symmetry. The effective potential of individual quarks in such models, which is basically due to the interaction of quarks with the gluon field, may be thought of as being mediated in a self-consistent manner through Nambu—Jona-Lasinio (NJL)-type models,[8] by some kind of instanton-induced effective quark-quark contact interaction with position-dependent coupling strength. The position-dependent coupling strength supposed to be determined by the multigluon mechanism is impossible to be calculated from first principles, although it is believed to be small at the origin and increases rapidly towards the hadron surface. Therefore one needs to introduce the effective potential for individual quarks in a phenomenological manner to seek *a posteriori* justification in finding its conformity with the supposed qualitative behavior of the position-dependent coupling strength in the contact interaction.

However, with no theoretical prejudice in favor of any particular mechanism for generating confinement of individual quarks, we prefer to work in an alternative, but similar scheme with a purely phenomenological individual quark potential in the equally mixed scalar-vector harmonic form. Such a potential model has been used in our earlier works[9] for a reasonable prediction of the core contributions to the magnetic moments of the octet baryons and the charge radius of the proton as well as the weak-electric and -magnetic form factors for semileptonic baryon decays. Then incorporating chiral symmetry in the SU(2)-flavor sector in the usual manner, we have estimated the quark-pion coupling constant[10] in consistency with those extracted from experimental vector-meson decay width ratios by Suzuki and Bhaduri.[11] In the present work we employ such a chiral potential model to study the mass spectrum of the low-lying baryons by taking into account the corrections due to (i) the energy associated with the center-of-mass motion, (ii) the color-electric and -magnetic energy arising out of the residual one-gluon-exchange interaction, and (iii) the pionic self-energy of the baryons arising out of the baryon-pion coupling at the vertex. We treat all these corrections, leading ultimately to the baryon physical masses, independently as though they are of the same order of magnitude.

This model with a harmonic form in particular for the scalar-vector mixed potential, turns out to be quite simple and tractable in these respects, yielding very satisfactory results not only for the physical masses of the low-lying baryons, but also the electromagnetic properties of the nucleons as well as the magnetic moments of octet baryons. In the present work, we focus our attention only on the physical masses of the ground-state baryons, while preferring to report our results on electromagnetic properties of nucleons and the magnetic moments of the octet baryons in our subsequent papers. In Sec. II we have outlined the basic framework of the potential model used with solutions for the relativistic bound states of the individually confined quarks in the ground state of baryons. In a shell-type approach the binding energies of the individually confined quarks contribute additively to the physical mass of the baryon. Such a contribution needs a correction due to the energy associated with the spurious center-of-mass motion. The procedure adopted to account for such a correction is also briefly described in this section. Section III provides an account of a further correction to the baryon mass due to the color-electric and -magnetic interaction energies originating from the hopefully weak residual one-gluon-exchange interaction, treated perturbatively. Then in Sec. IV we outline the usual procedure of incorporating the chiral symmetry in the $(u-d)$-flavor sector only with the quark-pion interaction term in the Lagrangian density taken in the linear form. Obtaining the general baryon-pion vertex function in the (u,d,s) sector, we calculate the pionic self-energy for various baryon intermediate states contributing to the physical mass spectrum. Finally in Sec. V we present the results for the ground-state baryon masses, which come out in very good agreement with the corresponding experimental values with a reasonable choice of the quark-gluon coupling constant α_c which is consistent with the idea of treating the one-gluon exchange in the baryon core in low-order perturbation theory.

II. BASIC FRAMEWORK

In this section we outline the framework of the potential model used to describe the individual quark confinement inside the baryon core. The binding energies of the individual constituent quarks contribute additively to the mass of the baryon core. Such a contribution needs a correction due to the center-of-mass motion. A brief account of the procedure adopted for such a correction is also provided here.

A. Potential model

Leaving behind for the moment, the quark-gluon interaction originating from one-gluon exchange at short distances and the quark-pion interaction in the (u,d)-flavor sector required to preserve the chiral symmetry as residual interactions to be treated perturbatively, we start with the confinement part of the interaction which is believed to be dominant in baryonic dimensions. This part of the interaction which is believed to be determined by the multigluon mechanism is impossible to be calculated theoretically from first principles. Therefore from a phenomenological point of view we assume that the quarks in a baryon core are independently confined by an average flavor-independent potential of the form

$$U(r)=\tfrac{1}{2}(1+\gamma^0)V(r)$$

with (2.1)

$$V(r)=(ar^2+V_0),\quad a>0\ .$$

We further assume that these independent quarks obey the Dirac equation with potential $U(r)$, which therefore implies a Lagrangian density in zeroth order as

$$\mathscr{L}_q^0(x)=\bar{q}(x)\left[\frac{i}{2}\gamma^\mu\overleftrightarrow{\partial}_\mu-U(r)-m_q\right]q(x)\ . \tag{2.2}$$

If we consider all the quarks in a baryon core to be in their ground $1S_{1/2}$ state, then the normalized quark wave function $\Psi_q(\mathbf{r})$ satisfying the Dirac equation

$$[\gamma^0E_q-\boldsymbol{\gamma}\cdot\mathbf{P}-m_q-U(r)]\Psi_q(\mathbf{r})=0 \tag{2.3}$$

can be written in the two-component form as

$$\Psi_q(\mathbf{r})=\frac{1}{\sqrt{4\pi}}\begin{bmatrix} ig_q(r)/r \\ \boldsymbol{\sigma}\cdot\hat{\mathbf{r}}f_q(r)/r\end{bmatrix}\chi_\uparrow\ . \tag{2.4}$$

Taking $E_q'=(E_q-V_0/2)$, $m_q'=(m_q+V_0/2)$, $\lambda_q=(E_q'+m_q')$, and $r_{0q}=(a\lambda_q)^{-1/4}$, it can be shown[9] that the reduced radial parts of the upper and lower components of $\Psi_q(\mathbf{r})$ come out as

$$g_q(r)=N_q(r/r_{0q})\exp(-r^2/2r_{0q}{}^2)\ ,$$
$$f_q(r)=-\frac{N_q}{\lambda_q r_{0q}}(r/r_{0q})^2\exp(-r^2/2r_{0q}{}^2)\ , \tag{2.5}$$

when the overall normalization factor N_q satisfies the relation

$$\frac{N_q{}^2\sqrt{\pi}r_{0q}}{8\lambda_q}=1/(3E_q'+m_q')\ . \tag{2.6}$$

The ground-state individual quark binding energy $E_q=(E_q'+V_0/2)$ is obtainable from the energy-eigenvalue condition

$$r_{0q}{}^2(E_q'^2-m_q'^2)=3\ ; \tag{2.7}$$

such a scheme with $V_0=0$ has been used successfully in our earlier work[9] in studying the static electromagnetic properties of the baryon core. However, for the present work we prefer to keep $V_0\neq 0$ in general. The solutions through Eqs. (2.4)–(2.7) provide the quark binding energy E_q which immediately leads to the mass of the baryon core in zeroth order as

$$M_B^0=E_B=\sum_q E_q\ . \tag{2.8}$$

B. Center-of-mass corrections

Clearly in our shell-type relativistic-independent-quark model, the independent motion of quarks inside the baryon core does not lead to a state of definite total momentum as it should to represent the physical state of a baryon. The problem appears in the same way in nuclear physics in case of ^{3}He and also in the bag model, and therefore, has to be resolved accordingly.[12] The energy associated with the spurious center-of-mass motion must provide a correction to the baryon mass obtained from the individual quark binding energy. To account for this we adopt here the prescription followed by Wong[13] and other workers, which has been described in detail in our earlier work.[9] However we briefly outline it here in order to make this paper self-contained.

The static three-quark baryon-core state with the core center at $\mathbf{x}$ is decomposed into components $\phi(\mathbf{P})$ of plane-wave momentum eigenstates as

$$|3q,\mathbf{x}\rangle=\int\frac{d^3\mathbf{P}}{W(P)}e^{i\mathbf{P}\cdot\mathbf{x}}\phi(\mathbf{P})\,|B(\mathbf{P})\rangle\;, \tag{2.9}$$

where the momentum eigenstates $|B(\mathbf{P})\rangle$ of the baryon core are normalized as

$$\langle B(\mathbf{P}')\,|\,B(\mathbf{P})\rangle=(2\pi)^3W(P)\delta(\mathbf{P}-\mathbf{P}')$$

with (2.10)

$$W(P)=(M_B{}^2+\mathbf{P}^2)^{1/2}/M_B\;.$$

The momentum profile function $\phi(\mathbf{P})$ can be obtained from (2.9) and (2.10) as

$$\phi^2(\mathbf{P})=\frac{W(P)}{(2\pi)^3}\tilde{I}(\mathbf{P})\;,$$

where (2.11)

$$\tilde{I}(\mathbf{P})=\frac{1}{(2\pi)^3}\int d^3\mathbf{r}\,e^{-i\mathbf{P}\cdot\mathbf{r}}\langle 3q,0\,|\,3q,\mathbf{r}\rangle$$

is the Fourier transform of the Hill-Wheeler overlap function.[12] Then the expectation value of any $F(\mathbf{P})$ can be obtained as

$$\langle 3q,0\,|\,F(\mathbf{P})\,|\,3q,0\rangle=\int d^3\mathbf{P}\,\tilde{I}(\mathbf{P})F(\mathbf{P})\;. \tag{2.12}$$

For the three $1S_{1/2}$ quarks in this model the Hill-Wheeler overlap with $C=(E_q'-m_q')/6(3E_q'+m_q')$,

$$I(r)=\prod_{q=1}^{3}(1-Cr^2/r_{0q}{}^2)\exp(-r^2/4r_{0q}{}^2) \tag{2.13}$$

permits a ready estimate of the center-of-mass momentum $\mathbf{P}$ through Eqs. (2.11) and (2.12) as

$$\langle\mathbf{P}^2\rangle=\sum_q\langle\mathbf{p}^2\rangle_q\;, \tag{2.14}$$

where $\langle\mathbf{p}^2\rangle_q$ is the average value of the square of the individual quark momentum taken over the $1S_{1/2}$ single-quark states and is given by

$$\langle\mathbf{p}^2\rangle_q=(11E_q'+m_q')(E_q'^2-m_q'^2)/6(3E_q'+m_q')\;. \tag{2.15}$$

In the same manner one can get

$$\langle M_B{}^2/E_B{}^2\rangle=\left[1-\sum_q\langle\mathbf{p}^2\rangle_q/E_B{}^2\right] \tag{2.16}$$

which provides the energy correction to the baryon mass in Eq. (2.8) as

$$(\Delta E_B)_{\text{c.m.}}=\left[\left[E_B{}^2-\sum_q\langle\mathbf{p}^2\rangle_q\right]^{1/2}-E_B\right]\;. \tag{2.17}$$

III. ONE-GLUON-EXCHANGE CORRECTION

The individual quarks in a baryon core are considered so far to be experiencing the only force coming from the average effective potential $U(r)$ in Eq. (2.1) which is assumed to provide a suitable phenomenological description of the nonperturbative gluon interaction including gluon self-coupling. All that remains inside the quark core is the hopefully weak one-gluon-exchange interaction provided by the interaction Lagrangian density $\mathcal{L}_I^g=\sum_a J_i^{\mu a}(x)A_\mu^a(x)$, where $A_\mu^a(x)$ are the eight vector gluon fields and $J_i^{\mu a}(x)$ is the ith quark color current. Since at small distances the quarks should be almost free, it is reasonable to calculate the energy shift in the mass spectrum arising out of the quark-interaction energy due to their coupling to the colored gluons, using a first-order perturbation theory.

If we keep only terms of order α_c, the problem reduces to evaluating the diagrams shown in Figs. 1(a) and 1(b), where Fig. 1(a) corresponds to the one-gluon-exchange part while Fig. 1(b) implies the quark self-energy that normally contributes to the renormalization of quark masses. If $\mathbf{E}_i^a$ and $\mathbf{B}_i^a$ are the color-electric and -magnetic fields, respectively, generated by the ith quark color current

$$J_i^{\mu a}(x)=g_c\bar{q}_i(x)\gamma^\mu\lambda_i^a q_i(x)\;, \tag{3.1}$$

with λ_i^a being the usual Gell-Mann SU(3) matrices and $\alpha_c=(g_c{}^2/4\pi)$, then the contribution to the mass due to the relevant diagrams can be written as a sum of a color-electric and -magnetic part as

$$(\Delta E_B)_g=(\Delta E_B)_g^E+(\Delta E_B)_g^M \tag{3.2}$$

when

$$(\Delta E_B)_g^E=\frac{1}{8\pi}\sum_{i,j}\sum_{a=1}^{8}\int\frac{d^3\mathbf{r}_i d^3\mathbf{r}_j}{|\mathbf{r}_i-\mathbf{r}_j|}\langle B\,|\,J_i^{0a}(\mathbf{r}_i)J_j^{0a}(\mathbf{r}_j)\,|\,B\rangle\;, \tag{3.3}$$

$$(\Delta E_B)_g^M=-\frac{1}{4\pi}\sum_{i<j}\sum_{a=1}^{8}\int\frac{d^3\mathbf{r}_i d^3\mathbf{r}_j}{|\mathbf{r}_i-\mathbf{r}_j|}\langle B\,|\,\mathbf{J}_i^a(\mathbf{r}_i)\cdot\mathbf{J}_j^a(\mathbf{r}_j)\,|\,B\rangle\;. \tag{3.4}$$

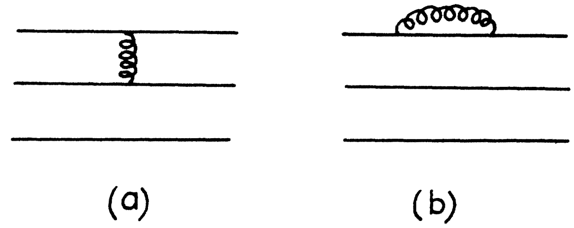

FIG. 1. One-gluon-exchange contributions to the energy.

Here we have not included the self-energy diagram in the calculation of the magnetic part of the interaction, which contributing to the renormalization of the quark masses, can possibly be accounted for in the phenomenological quark masses. The exclusion of this diagram, however, requires that each $\mathbf{B}_i^a$ should satisfy the boundary condition $\hat{\mathbf{r}}\times\mathbf{B}_i^a=0$, separately at the edge of the confining region, which is a possible case. On the other hand, as the electric field $\mathbf{E}_i^a$ is necessarily in the radial direction, it is only possible to satisfy the boundary condition $\hat{\mathbf{r}}\cdot(\sum_i \mathbf{E}_i^a)=0$ for a color-singlet state $|B\rangle$ for which $\langle \sum_i \lambda_i^a \rangle=0$. Therefore, in order to preserve the boundary conditions we are forced to take into account the self-energy diagrams in Fig. 1(b) in the calculation of the electric part only.

Now from Eqs. (2.4), (2.5), and (3.1) one finds

$$J_i^{0a}(r_i)=\frac{g_c\lambda_i^a}{4\pi}\frac{N_i^2}{r_{0i}^2}(1+r_i^2/\lambda_i^2 r_{0i}^4)\exp(-r_i^2/r_{0i}^2)\ ,$$
$$\mathbf{J}_i^a(r_i)=\frac{g_c\lambda_i^a}{2\pi}\frac{N_i^2}{\lambda_i r_{0i}^4}(\sigma_i\times\mathbf{r}_i)\exp(-r_i^2/r_{0i}^2)\ . \tag{3.5}$$

Using Eq. (3.5) together with the identity

$$\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}=\frac{1}{2\pi}\int\frac{d^3\mathbf{k}}{k^2}\exp[i\mathbf{k}\cdot(\mathbf{r}_i-\mathbf{r}_j)]\ ,$$

Eqs. (3.3) and (3.4) become

$$(\Delta E_B)_g^E=\frac{\alpha_c}{64\pi^4}\sum_{i,j}\left\langle\sum_a\lambda_i^a\lambda_j^a\right\rangle\frac{N_i^2N_j^2}{r_{0i}^2r_{0j}^2}\int\frac{d^3\mathbf{k}}{k^2}F_i^E(k)F_j^E(k)\ , \tag{3.6}$$

$$(\Delta E_B)_g^M=-\frac{\alpha_c}{8\pi^4}\sum_{i,j}\left\langle\sum_a\lambda_i^a\lambda_j^a\right\rangle\frac{N_i^2N_j^2}{\lambda_i r_{0i}^4\lambda_j r_{0j}^4}\times\int\frac{d^3\mathbf{k}}{k^2}\mathbf{F}_i^M(k)\cdot\mathbf{F}_j^M(k)\ , \tag{3.7}$$

where

$$F_i^E(k)=\frac{\pi^{3/2}r_{0i}}{\lambda_i^2}\left[(\tfrac{3}{2}+\lambda_i^2r_{0i}^2)-\frac{k^2r_{0i}^2}{4}\right]\exp(-k^2r_{0i}^2/4)\ , \tag{3.8}$$

$$\mathbf{F}_i^M(k)=-\frac{i\pi^{3/2}r_{0i}^5}{2}(\sigma\times\mathbf{k})\exp(-k^2r_{0i}^2/4)\ . \tag{3.9}$$

Then after some straightforward integrations one can find

$$(\Delta E_B)_g^E=\alpha_c\sum_{i,j}\left\langle\sum_a\lambda_i^a\lambda_j^a\right\rangle\frac{1}{\sqrt{\pi}R_{ij}}\times\left[1-\frac{\alpha_i+\alpha_j}{R_{ij}^2}+\frac{3\alpha_i\alpha_j}{R_{ij}^4}\right]\ , \tag{3.10}$$

$$(\Delta E_B)_g^M=\alpha_c\sum_{i<j}\left\langle\sum_a\lambda_i^a\lambda_j^a\sigma_i\cdot\sigma_j\right\rangle\frac{32}{3\sqrt{\pi}}\Big/(3E_i'+m_i')\times(3E_j'+m_j')R_{ij}^3\ , \tag{3.11}$$

where

$$R_{ij}^2=3\left[\frac{1}{(E_i'^2-m_i'^2)}+\frac{1}{(E_j'^2-m_j'^2)}\right]\ ,$$
$$\alpha_i=1/\lambda_i(3E_i'+m_i')\ . \tag{3.12}$$

Finally taking into account the specific quark flavor and spin configurations in various ground-state baryons and using the relations $\langle\sum_a(\lambda_i^a)^2\rangle=\frac{16}{3}$ and $\langle\sum_a\lambda_i^a\lambda_j^a\rangle_{i\neq j}=-\frac{8}{3}$ for baryons, one can write in general the energy correction due to one-gluon exchange as

$$(\Delta E_B)_g^E=\alpha_c(b_{uu}I_{uu}^E+b_{us}I_{us}^E+b_{ss}I_{ss}^E)\ ,$$
$$(\Delta E_B)_g^M=\alpha_c(a_{uu}I_{uu}^M+a_{us}I_{us}^M+a_{ss}I_{ss}^M)\ , \tag{3.13}$$

where a_{ij} and b_{ij} are the numerical coefficients depending on each baryon and are listed in Table I and the quantities $I_{ij}^{E,M}$ are

$$I_{ij}^E=\frac{16}{3\sqrt{\pi}}\frac{1}{R_{ij}}\left[1-\frac{\alpha_i+\alpha_j}{R_{ij}^2}+\frac{3\alpha_i\alpha_j}{R_{ij}^4}\right]\ , \tag{3.14}$$

$$I_{ij}^M=\frac{256}{9\sqrt{\pi}}\frac{1}{R_{ij}^3}\frac{1}{(3E_i'+m_i')}\frac{1}{(3E_j'+m_j')}\ . \tag{3.15}$$

One can note from Table I that the color-electric contribution for the baryon masses vanishes when all the constituent-quark masses in a baryon are equal, whereas it is nonzero otherwise. However, even in the case of strange baryons it would be seen subsequently, that the color-electric contribution is quite small. Therefore the degeneracy among the baryons is essentially removed through the spin-spin interaction energy in the color-magnetic part.

IV. CHIRAL SYMMETRY AND PIONIC CORRECTION

Coming back again to the zeroth-order Lagrangian density $\mathcal{L}_q^0$ described in Sec. II, which takes into account the nonperturbative gluon interactions including gluon self-couplings through the phenomenological potential $U(r)$, one can note that under global infinitesimal chiral transformation at least in the (u,d)-flavor sector,

$$q(x)\to q(x)-i\gamma^5\left[\frac{\tau\cdot\epsilon}{2}\right]q(x)\ , \tag{4.1}$$

the axial-vector current of quarks is not conserved as the scalar term proportional to $G(r)=[m_q+V(r)/2]$ in $\mathcal{L}_q^0$ is chirally odd. The vector part of the potential poses no problem in this respect. But in view of the experimental success of the partial conservation of axial-vector current (PCAC) and hence the fact that chiral SU(2)×SU(2) is one of the best symmetries of strong interactions, it is desirable to conserve the total axial-vector current at least in the (u,d)-flavor sector. This is usually done at a phenomenological level[14] by introducing an elementary pion field that also carries an axial-vector current such that the four-divergence of the total axial-vector current satisfies the PCAC condition. Though this consideration can be generalized to include the strange-flavor sector for

TABLE I. Coefficients appearing in the calculation of the color-magnetic and -electric energy corrections due to one-gluon exchange.

Baryons	a_{uu}	a_{us}	a_{ss}	b_{uu}	b_{us}	b_{ss}
N	−3	0	0	0	0	0
Δ	3	0	0	0	0	0
Λ	−3	0	0	1	−2	1
Σ	1	−4	0	1	−2	1
Ξ	0	−4	1	1	−2	1
Σ^*	1	2	0	1	−2	1
Ξ^*	0	2	1	1	−2	1
Ω^-	0	0	3	0	0	0

a chiral SU(3)×SU(3) symmetry, we would ignore it because of the large mass of the kaon involved in the process.

Therefore, we introduce in the usual manner, an elementary pion field $\boldsymbol{\phi}(x)$ of small and finite mass $m_\pi=140$ MeV with the quark-pion interaction Lagrangian density

$$\mathscr{L}_I^\pi=-\frac{i}{f_\pi}G(r)\bar{q}(x)\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\phi})q(x) \tag{4.2}$$

which is linear in the isovector pion field $\boldsymbol{\phi}(x)$. Here $f_\pi=93$ MeV is the phenomenological pion-decay constant. Then the four-divergence of the total axial-vector current becomes $\partial_\mu \mathbf{A}^\mu(x)=-f_\pi m_\pi{}^2\boldsymbol{\phi}(x)$ yielding the PCAC relation. Consequently, the pion coupling of the nonstrange quarks would give rise to pionic self-energy of the baryons which would ultimately contribute to the physical masses of the baryons. This aspect can be studied in the usual perturbative approach,[15] with the Hamiltonian constructed in the subspace of nonexotic color-singlet baryons, a brief account of which is given below.

A. Baryon-pion vertex function

If one considers the color-singlet nonexotic baryon states $|B\rangle$ to be the eigenstates of the Hamiltonian H^0, [obtained from $\mathscr{L}_q^0(x)$ in the canonical way] with masses M_B^0, then in the nonexotic baryon subspace one can write in a conventional second quantized language

$$\tilde{H}^0=\sum_B |B\rangle\langle B|M_B^0=\sum_B \hat{b}_B^\dagger \hat{b}_B M_B^0 , \tag{4.3}$$

where $\hat{b}_B^\dagger$ creates a three-quark baryon state with quantum numbers of N, Δ, etc. Similarly with $\hat{a}_{k_j}$ and $\hat{a}_{k_j}^\dagger$ as the pion destruction and creation operators and $w_k=(\mathbf{k}^2+m_\pi{}^2)^{1/2}$ as the pion energy, the Hamiltonian for quantized free pion field $\phi_j(x)$ becomes

$$\tilde{H}_\pi=\sum_j \int d^3\mathbf{k}\, w_k \hat{a}_{k_j}^\dagger \hat{a}_{k_j} . \tag{4.4}$$

Finally the interaction Hamiltonian corresponding to $\mathscr{L}_I^\pi(x)$ becomes

$$\tilde{H}_I^\pi=-\frac{1}{(2\pi)^{3/2}}\sum_{B,B',j}\int d^3\mathbf{k}[V_j^{BB'}(\mathbf{k})\hat{b}_{B'}^\dagger \hat{b}_B \hat{a}_{k_j}+\text{H.c.}] , \tag{4.5}$$

where j corresponds to the pion-isospin index, and H.c. denotes the Hermitian conjugate. $V_j^{BB'}(\mathbf{k})$, representing the baryon-pion absorption vertex function in the point-pion approximation, is obtained as

$$V_j^{BB'}(\mathbf{k})=-\frac{i}{f_\pi}(2w_k)^{-1/2} \times\int d^3\mathbf{r}\, G(r)\exp(i\mathbf{k}\cdot\mathbf{r}) \times\left\langle B'\left|\sum_q \bar{q}(r)\gamma^5 q(r)\tau_j\right|B\right\rangle . \tag{4.6}$$

Assuming that for the $BB'\pi$ vertex, the spatial orbits of all the quarks in the initial- and final-baryon state are the same $1S_{1/2}$, one can use Eqs. (2.4) and (2.5) to obtain

$$V_j^{BB'}(\mathbf{k})=\frac{i}{f_\pi}(2w_k)^{-1/2}\frac{N_q{}^2\sqrt{\pi}k^{-3/2}}{\sqrt{2}\lambda_q r_{0q}{}^4} \times I(k)\left\langle B'\left|\sum_q(\boldsymbol{\sigma}_q\cdot\mathbf{k})\tau_j\right|B\right\rangle , \tag{4.7}$$

where

$$I(k)=2\int_0^\infty dr\, r^{5/2}G(r)J_{3/2}(kr)\exp(-r^2/r_{0q}{}^2). \tag{4.8}$$

Now using the standard integral result for $I(k)$ and the values obtained for the axial-vector coupling constant $g_A(B)$ in the present model,[9] the expression for $V_j^{BB'}(\mathbf{k})$ can be simplified further.

As for example, we consider the NN π-vertex function $V_j^{NN}(\mathbf{k})$ with the axial-vector coupling constant $g_A(N)$ obtained in this model[9] as

$$g_A(N)=\frac{5}{9}\frac{5E_u'+7m_u'}{3E_u'+m_u'} \tag{4.9}$$

to get

$$V_j^{NN}(\mathbf{k})=\frac{i}{2f_\pi}(2w_k)^{-1/2}g_A(N)ku(k)(\sigma^{NN}\cdot\hat{\mathbf{k}})\tau_j^{NN} \tag{4.10}$$

Here the form factor $u(k)$ with

$$A=\left[\frac{E_u'-m_u'}{2(5E_u'+m_u')}\right]$$

comes out as

$$u(k)=(1-Ar_{0u}{}^2\mathbf{k}^2)\exp(-r_{0u}{}^2\mathbf{k}^2/4) \tag{4.11}$$

which reduces to one for $k \to 0$ as expected. Finally using the familiar Goldberger-Treiman relation which establishes a connection between the pseudovector nucleon-pion coupling $f_{NN\pi}$ and the axial-vector coupling g_A in the form

$$\sqrt{4\pi}\frac{f_{NN\pi}}{m_\pi}=\frac{g_A(N)}{2f_\pi} \tag{4.12}$$

one gets

$$V_j^{NN}(\mathbf{k})=i\sqrt{4\pi}\frac{f_{NN\pi}}{m_\pi}\frac{ku(k)}{\sqrt{2w_k}}(\sigma^{NN}\cdot\hat{\mathbf{k}})\tau_j^{NN} . \tag{4.13}$$

In the same manner the general baryon-pion vertex function can be written as

$$V_j^{BB'}(\mathbf{k})=i\sqrt{4\pi}\left[\frac{f_{BB'\pi}}{m_\pi}\right]\frac{ku(k)}{\sqrt{2w_k}}(\sigma^{BB'}\cdot\hat{\mathbf{k}})\tau_j^{BB'} . \tag{4.14}$$

The pseudovector baryon-pion coupling constants $f_{BB'\pi}$ are summarized in Table II in relation to $f_{NN\pi}$. Now with the vertex function $V_j^{BB'}(\mathbf{k})$ on hand, it is possible to calculate the pionic self-energy for various baryons with appropriate baryon intermediate states contributing to the process.

B. Pionic self-energy

The coupling of the pion field to the nonstrange quarks in a minimal way, as given by the single-loop self-energy diagram shown in Fig. 2, causes a shift in the energy of the baryon core. From the second-order perturbation theory, the pionic self-energy is usually given by

$$\Sigma_B(E_B)=\sum_k\sum_{B'}\frac{V^{\dagger BB'}V^{BB'}}{E_B-w_k-M_{B'}^0} , \tag{4.15}$$

when $\sum_k \equiv \sum_j \int d^3\mathbf{k}/(2\pi)^3$ and B' is the intermediate baryon state. For degenerate intermediate states on mass shell with $M_B^0=M_{B'}^0$, the self-energy correction becomes

$$\delta M_B=\Sigma_B(E_B=M_B^0=M_{B'}^0)=-\sum_{k,B'}\frac{V^{\dagger BB'}V^{BB'}}{w_k} . \tag{4.16}$$

Now using the explicit expressions for $V^{BB'}(\mathbf{k})$ as in Eq. (4.14), one gets

$$\delta M_B=-\tfrac{1}{3}I_\pi\sum_{B'}C_{BB'}f_{BB'\pi}{}^2 ,$$

when (4.17)

$$C_{BB'}=(\sigma^{BB'}\cdot\sigma^{BB'})(\tau^{BB'}\cdot\tau^{BB'})$$

and

$$I_\pi=\frac{1}{\pi m_\pi{}^2}\int_0^\infty dk\frac{k^4u^2(k)}{w_k{}^2} . \tag{4.18}$$

Using Eq. (4.11) for $u(k)$ and substituting $z=\frac{1}{2}m_\pi{}^2r_{0u}{}^2$, the integral I_π can be written in the form

$$I_\pi=\frac{1}{\pi m_\pi{}^2}(I_4-2Ar_{0u}{}^2I_6+A^2r_{0u}{}^4I_8) , \tag{4.19}$$

when the reduced integrals

$$\begin{aligned}I_{2n}&=\int_0^\infty\frac{dk\,k^{2n}}{k^2+m_\pi{}^2}\exp(-zk^2/m_\pi{}^2)\\&=\left[\frac{2z}{r_{0u}{}^2}\right]^{n-1/2}\left[(-1)^n\frac{\pi}{2}\exp(z)\right.\\&\qquad\left.+\frac{\Gamma(n+\frac{1}{2})}{2n-1}F(1,\tfrac{3}{2}-n,z)\right] .\end{aligned} \tag{4.20}$$

Now substituting Eq. (4.20) in (4.19) and using the identity that

$$F(1,b,z)=1+\frac{z}{b}F(1,b+1,z) \tag{4.21}$$

one can finally get

$$\begin{aligned}I_\pi=\left[\frac{m_\pi}{\pi}\right]&\left[(1+4Az+4A^2z^2)\left[\frac{\pi}{2}\exp(z)+\frac{\sqrt{\pi}}{4}z^{-3/2}-\frac{\sqrt{\pi}}{2}z^{-1/2}F(1,\tfrac{1}{2},z)\right]\right.\\&\left.+A^2\left[\frac{15\sqrt{\pi}}{4}z^{-3/2}-\frac{3\sqrt{\pi}}{2}z^{-1/2}\right]-A\frac{3\sqrt{\pi}}{2}z^{-3/2}\right] .\end{aligned} \tag{4.22}$$

For the intermediate baryon states B', we consider only the octet and decouplet ground states. Using the values of $f_{BB'\pi}$ and $C_{BB'}$ summarized in Table II according to Ref. 1, the pionic self-energy for different baryons can be computed as

$$\begin{aligned}\delta M_N&=-\tfrac{171}{25}I_\pi f_{NN\pi}{}^2 ,\\ \delta M_\Delta&=-\tfrac{99}{25}I_\pi f_{NN\pi}{}^2 ,\\ \delta M_\Lambda&=-\tfrac{108}{25}I_\pi f_{NN\pi}{}^2 ,\\ \delta M_\Sigma&=\delta M_{\Sigma^*}=-\tfrac{12}{5}I_\pi f_{NN\pi}{}^2 ,\\ \delta M_\Xi&=\delta M_{\Xi^*}=-\tfrac{27}{5}I_\pi f_{NN\pi}{}^2 ,\end{aligned} \tag{4.23}$$

and of course, $\delta M_{\Omega^-}=0$, since the strange quarks in Ω^- have no interaction with the pion. The self-energy δM_B calculated here contains both the quark self-energy [Fig. 3(a)] and the one-pion-exchange contributions [Fig. 3(b)].

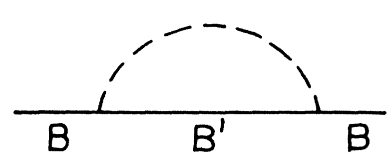

FIG. 2. Baryon self-energy due to coupling with pion.

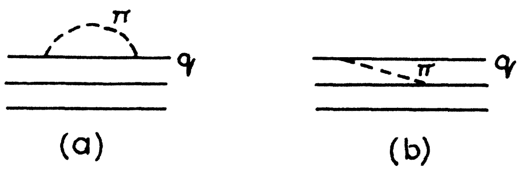

FIG. 3. One-pion-exchange contributions to the energy.

V. RESULTS AND CONCLUSION

We have shown in the previous sections that the zeroth-order mass $M_B^0 = E_B$ of a ground-state baryon, arising out of the binding energies of the constituent quarks confined independently by a phenomenological average potential $U(r)$ that presumably represents the dominant nonperturbative gluon interactions, must be subjected to certain corrections due to the residual quark-gluon and quark-pion interactions together with that due to the spurious center-of-mass motion. Treating all these corrections independently as though they are of the same order of magnitude, one can obtain the physical mass of a low-lying baryon in its ground state as

$$M_B = E_B + (\Delta E_B)_{\text{c.m.}} + (\Delta E_B)_g^E + (\Delta E_B)_g^M + \delta M_B \quad (5.1)$$

when (ΔE_B) is the energy associated with the spurious c.m. motion [Eq. (2.17)], $(\Delta E_B)_g^E + (\Delta E_B)_g^M$ is the color-electric and -magnetic interaction energies arising out of the residual one-gluon-exchange processes [Eqs. (3.12)–(3.14)], and δM_B is the pionic self-energy of the baryon due to pion-coupling of nonstrange quarks in the baryon.

For quantitative evaluations of these terms in Eq. (5.1) we have first of all considered the potential parameters a and V_0 in Eq. (2.1) as flavor independent and taken the quark masses as $m_u = m_d \neq m_s$. Then for convenience, absorbing V_0 appropriately in E_q and m_q of Eq. (2.3), we have obtained the solutions leading to individual quark bound states in terms of $E_q' = (E_q - V_0/2)$ and $m_q' = (m_q + V_0/2)$ through Eqs. (2.4)–(2.7). Then for a suitable choice like $a = 0.017\,166$ GeV^3, $m_u' = m_d' = 10$ MeV and $m_s' = 247$ MeV, the eigenvalue condition in Eq.

TABLE II. Baryon-pion coupling constant and spin-isospin reduced matrix elements for various baryon states.

Baryon B	Baryon intermediate baryon states BB'	$\frac{f_{BB'\pi}}{f_{NN\pi}}$	$(\sigma^{BB'} \cdot \sigma^{BB'})$	$(\tau^{BB'} \cdot \tau^{BB'})$	$C_{BB'}$
N	NN	1	3	3	9
	$N\Delta$	$\frac{6\sqrt{2}}{5}$	2	2	4
Δ	$\Delta\Delta$	$\frac{1}{5}$	15	15	225
	ΔN	$6\sqrt{2}/5$	1	1	1
Λ	$\Lambda\Lambda$	0	3	0	0
	$\Lambda\Sigma$	$-2\sqrt{3}/5$	3	3	9
	$\Lambda\Sigma^*$	$-\frac{6}{5}$	2	3	6
Σ	$\Sigma\Sigma$	$\frac{4}{5}$	3	2	6
	$\Sigma\Lambda$	$-2\sqrt{3}/5$	3	1	3
	$\Sigma\Sigma^*$	$-2\sqrt{3}/5$	2	2	4
Σ^*	$\Sigma^*\Sigma^*$	$\frac{2}{5}$	15	2	30
	$\Sigma^*\Lambda$	$-\frac{6}{5}$	1	1	1
	$\Sigma^*\Sigma$	$-2\sqrt{3}/5$	1	1	2
Ξ	$\Xi\Xi$	$-\frac{1}{5}$	3	3	9
	$\Xi\Xi^*$	$-2\sqrt{3}/5$	2	3	6
Ξ^*	$\Xi^*\Xi^*$	$\frac{1}{5}$	15	3	45
	$\Xi^*\Xi$	$-2\sqrt{3}/5$	1	3	3

TABLE III. Energy corrections and physical masses of ground-state baryons (in MeV).

Baryon	E_B	$(\Delta E_B)_{\text{c.m.}}$	$(\Delta E_B)_g$ $(\Delta E_B)_g^M$	$(\Delta E_B)_g^E$	δM_B	M_B Present work	M_B Experiment
N			-112.5		-159.5	939.056	940
	1413.75	-202.694		0			
Δ			$+112.5$		-92.34	1231.216	1232
Λ			-112.5		-100.7	1131.965	1116
Σ	1533.5	-196.835	-93.48	8.50	-56.0	1195.685	1193
Σ^*			$+103.0$		-56.0	1392.165	1385
Ξ			-101.722		-25.2	1334.328	1321
	1653.25	-192		8.5			
Ξ^*			94.748		-25.2	1539.298	1533
Ω^-	1773	-187.835	87.774	0	0	1672.939	1672

(2.7) yields $E_u' = E_d' = 540$ MeV and $E_s' = 659.75$ MeV.

Now it is straightforward to calculate the various quantities $I_{ij}^{E,M}$ from Eqs. (3.13) and (3.14) which are necessary for evaluating $(\Delta E_B)_g^{E,M}$. We find that

$$(I_{uu}^E, I_{us}^E, I_{ss}^E) \equiv (597.194, 644.852, 707.22 \text{ MeV}) ,$$
$$(I_{uu}^M, I_{us}^M, I_{ss}^M) \equiv (64.6812, 56.4575, 50.317 \text{ MeV}) . \quad (5.2)$$

Then with $z = \frac{1}{2} m_\pi{}^2 r_{0u}^2 \simeq 0.1$, the integral expression I_π in Eq. (4.22) is calculated as $I_\pi = 291.493$ MeV which enables one to obtain the pionic self-energies of various baryons through Eqs. (4.23). The values of δM_B so obtained with $f_{NN\pi}{}^2 = 0.08$, for various baryons are provided in Table III. Now referring to the physical masses of N and Δ, which are

$$M_\Delta = \left[E_N{}^2 - \sum_q \langle \mathbf{p}^2 \rangle_q \right]^{1/2} + 3\alpha_c I_{uu}^M + \delta M_\Delta ,$$
$$M_N = \left[E_N{}^2 - \sum_q \langle \mathbf{p}^2 \rangle_q \right]^{1/2} - 3\alpha_c I_{uu}^M + \delta M_N , \quad (5.3)$$

we find that

$$6\alpha_c I_{uu}^M = (M_\Delta - M_N) - (\delta M_\Delta - \delta M_N) . \quad (5.4)$$

Since $(M_\Delta - M_N) \simeq 292$ MeV and $(\delta M_\Delta - \delta M_N) \simeq 67$ MeV as seen from Table III, we observe that the QCD splitting among the N and Δ masses (i.e., $6\alpha_c I_{uu}^M$) is only 225 MeV. Therefore, one does not need anywhere near as large a value of α_c as in the original MIT work, where, without including pionic corrections the QCD splitting was equated with $(M_\Delta - M_N) \simeq 292$ MeV. In fact, α_c of the order 0.5–0.6 is sufficient here. We take $\alpha_c = 0.58$ which is comparable with 0.55 found by DeGrand *et al.*[4] and is not too much different from the value 0.3–0.4 obtained in CBM.[16] Again from Eqs. (5.3) one can fix the potential parameter V_0 independent of α_c using the combination

$$(M_\Delta - \delta M_\Delta) + (M_N - \delta M_N) = 2 \left[E_N{}^2 - \sum_q \langle \mathbf{p}^2 \rangle_q \right]^{1/2} . \quad (5.5)$$

Finding $E_N = 3(E_u' - V_0/2)$ from Eq. (5.5) we fix the potential parameter as a value $V_0 = -137.5$ MeV. Now using all these results one can calculate all the individual terms leading to the physical masses of various ground-state baryons. These quantities for various baryons considered here are provided in Table III. Consequently we find the physical masses of baryons like N, Δ, Λ, Σ, Ξ, and Ξ^* in very good agreement with the corresponding experimental masses. The strong-coupling constant $\alpha_c = 0.58$ used in our calculation is quite consistent with the idea of treating one-gluon-exchange effects in lowest-order perturbation theory.

So we found that in the first step the SU(3)-breaking effect due to the quark masses $m_u = m_d \neq m_s$, lifts the degeneracy in baryon masses through the energy term $[E_B + (\Delta E_B)_{\text{c.m.}}]$ among the groups (N,Δ), $(\Lambda,\Sigma,\Sigma^*)$, (Ξ,Ξ^*), and Ω^-. Then in the second step, the constraint of chiral symmetry imposed on the baryon core removes the degeneracy partially through the spin-isospin interaction energy δM_B between N and Δ, Λ and Σ, whereas Σ^* still remains degenerate with Σ and Ξ^* with Ξ. However, the color-electric and -magnetic interaction energy arising out of the one-gluon exchange with the dominant color-magnetic part giving a spin-spin contribution removes the mass degeneracy completely among these baryons.

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