On the mixing and production of exotic fermions in E_6

R. W. Robinett
Department of Physics, University of Massachusetts, Amherst, Massachusetts 01003
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The mixings between exotic fermions in E_6 27-plets (a $Q=-\frac{1}{3}$ isosinglet quark h and charged and neutral leptons E and ν_E) and their familiar counterparts d, e, and ν_e are discussed and constraints from electroweak neutral-current data on such mixings are examined. We find off-diagonal couplings $d\bar{h}Z^0$, $e\bar{E}Z^0$, and $\nu_e\bar{\nu}_E Z^0$ and propose new methods for single production of such new fermions in e^+e^- collisions and Z decay. Such couplings also lead (in general) to flavor-changing neutral currents and we examine constraints on the masses and mixings of the h quarks which can be derived from the K^0-$\bar{K}^0$ system and $K_L\to\mu^+\mu^-$.

I. INTRODUCTION

The current interest in superstring theories,[1] with their promise of a consistent and unified description of gravity, the gauge interactions, and matter, has also led to a resurgence of interest in grand unified theories based on E_6.[2] The only known anomaly-free superstring theory with a chance for an acceptable low-energy phenomenology seems to yield an $N=1$ supergravity model with an E_6 grand-unification group.[3] Investigations into the symmetry breakdown[4] of such theories, in the context of superstrings, imply that the low-energy ($\gtrsim G_F{}^{-1/2}$) electroweak gauge group will necessarily be larger than the standard model, containing at least one extra U(1) symmetry, and will have light fermions in fundamental representations of E_6, i.e., 27-plets. Thus, the additional fields predicted to be present in some versions of such theories seem to be the supersymmetric partners, an extra Z boson (or possibly two), and the additional, exotic fermions in the E_6 **27** beyond those in the usual SU(5) $\mathbf{5}^*+\mathbf{10}$.

The search for supersymmetric particles has been much discussed[5] while electroweak models with additional Z's also have an extensive literature.[6] The additional fermions in the E_6 **27** have recently been discussed, in the context of superstring theories, by Rosner.[7] In the SU(10) decomposition of E_6 one has $\mathbf{27}=\mathbf{16}+\mathbf{10}+\mathbf{1}$ where the 16-plet contains the usual u,d,e,ν_e plus the right-handed partner of the neutrino while the 10-plet contains a $Q=-\frac{1}{3}$ isosinglet quark h and isodoublet charged and neutral leptons, E and ν_E.[8] (The current limits on the masses of such new, charged fermions is $\sim$20 GeV.) From the most general mass matrix possible in such theories, Rosner argues that these new fermions can mix with their ordinary 16-plet counterparts d, e, and ν_e, i.e.,

$$\begin{aligned} d_p &= \cos\alpha\, d + \sin\alpha\, h\ ,\\ h_p &= -\sin\alpha\, d + \cos\alpha\, h \end{aligned} \tag{1.1}$$

with similar expressions (but different mixing angles) for the leptons, the mixings being determined by ratios of various (unknown) $\Delta I=\frac{1}{2},0$ mass terms.[9] The u-type quarks are unmixed. Because their QED and QCD interactions are identical, any mixtures of the familiar (**16**) and exotic (**10**) fermions will have the same $SU(3)_C\times U(1)_{EM}$ couplings. However, because of their different $SU(2)_L\times U(1)$ transformation properties, the weak interactions of the mass eigenstates will be slightly different than those of pure 16- or 10-plets. Because the u-type quarks are unmixed, weak universality (basically all charged-current data) restricts $\sin\alpha\lesssim 0.1$ in Eq. (1.1) which forces a ratio of $\Delta I=0$ to $\Delta I=\frac{1}{2}$ mass terms to be $\gtrsim 10$. In the charged- and/or neutral-lepton sector weak universality only requires that the (e,E) and (ν_e,ν_E) pairs be mixed by the same amount (to the 1% level).

In this paper we will examine further phenomenological implications of such mixings. In the next section we examine the limits placed on (e,E) [and hence on (ν_e,ν_E)] mixing by measurements of the e/μ weak couplings in $e^+e^-\to\mu^+\mu^-$ experiments while in Sec. III we show that such mixings imply off-diagonal neutral-current couplings, i.e., $d\bar{h}Z^0$, $e\bar{E}Z^0$, and $\nu_e\bar{\nu}_E Z^0$ interactions, and we discuss the relevance of these couplings to the single production of new fermions. Finally, in Sec. IV, we discuss the possibility of flavor-changing neutral currents induced by these off-diagonal couplings and derive constraints coming from such processes as K^0-$\bar{K}^0$ mixing and $K_L\to\mu^+\mu^-$.

II. CONSTRAINTS FROM NEUTRAL-CURRENT COUPLINGS

As mentioned above, weak universality (charged currents) only constrains the (e,E) and (ν_e,ν_E) mixings to be approximately equal but does not limit their magnitude. Because the E coupling to the Z^0 is different than that of the e, measurements of the charged-lepton weak coupling in $e^+e^-\to\mu^+\mu^-$ experiments will place limits on this mixing. The $f\bar{f}Z^0$ couplings (for weak eigenstates) can be written as

$$-\frac{ie}{\sin(2\theta_W)}\bar{f}\gamma_\mu(g_V+g_A\gamma_5)fZ^0_\mu\ , \tag{2.1}$$

where

$$\begin{aligned} g_V(e)&=-\tfrac{1}{2}+2x,\quad g_A(e)=\tfrac{1}{2}\ ,\\ g_V(E)&=-1+2x,\quad g_A(E)=0\ ,\\ g_V(d)&=-\frac{1}{2}+\frac{2x}{3},\quad g_A(d)=\tfrac{1}{2}\ ,\\ g_V(h)&=\frac{2x}{3},\quad g_A(h)=0\ ,\\ g_V(\nu_e)&=\tfrac{1}{2},\quad g_A(\nu_e)=-\tfrac{1}{2}\ ,\\ g_V(\nu_E)&=1,\quad g_A(\nu_E)=0\ , \end{aligned} \tag{2.2}$$

and $x=\sin^2\theta_W$. Taking mixing into account, the charged-lepton mass eigenstate would have

$$\begin{aligned} g_V&=-\tfrac{1}{2}+2x-\sin^2(\alpha)/2\ ,\\ g_A&=\cos^2(\alpha)/2\ . \end{aligned} \tag{2.3}$$

Measurements[10] of the total cross section and asymmetry in $e^+e^-\to\mu^+\mu^-$ imply that $\cos^2\alpha\gtrsim 0.95-0.98$ (0.9) at the 1σ (2σ) level. Measurements of parity violation in heavy atoms[11] give similar constraints. Future, high-statistics measurements of the $e^+e^-\to\mu^+\mu^-$ angular distribution at the Z^0 pole will further constrain (e,E) mixing by precisely measuring $g_V(e,\mu)$ and $g_A(e,\mu)$. Such analyses will rely on taking electroweak radiative corrections[12] into account very accurately. Such limits in the (e,E) sector are already interesting as they give constraints on various $\Delta I=0$ mass terms which, *a priori*, might have been thought comparable. The mixing in the (d,h) sector depends on ratios of $\Delta I=\frac{1}{2}$ to $\Delta I=0$ mass terms and need not be related to mixings in the lepton sector. The presence of extra U(1) factors can also change the fermion couplings to the Z^0 (as well as its mass). In many such theories[13] these effects change $\bar{f}fZ^0$ couplings by order $(v/V)^2$ where v/V is a ratio of $\Delta I=\frac{1}{2}$ to $\Delta I=0$ vacuum expectation values. Thus, careful measurements of fermion couplings to the Z^0 can constrain the scale of new $\Delta I=0$ physics past the TeV scale. Extra Z's cannot, however, give rise to the off-diagonal fermion-Z^0 couplings which we discuss next.

III. FERMION MIXINGS AND SINGLE PRODUCTION

The new fermions can be produced in many ways, in e^+e^- collisions via virtual γ or Z (or additional neutral Z's), in Z^0 (or Z') decays, or, in the case of the h-type quarks, as a narrow quarkonium resonance in e^+e^- collisions. All of these methods assume that the new fermions must be created in pairs but the existence of fermion mixing allows for the possibility of single production via new, off-diagonal interactions. To see this, we write the weak-interaction Lagrangian in terms of the mass eigenstates (assuming a common mixing angle for simplicity) and we find the terms

$$-\frac{ie}{\sin(2\theta_W)}\sin\alpha\cos\alpha\left[\bar{d}Z^0\left[\frac{1-\gamma_5}{2}\right]h-\bar{e}Z^0\left[\frac{1+\gamma_5}{2}\right]E+\bar{\nu}_eZ^0\left[\frac{1+\gamma_5}{2}\right]\nu_E+\text{H.c.}\right]\ . \tag{3.1}$$

The $V\pm A$ structure of these interactions is a definite prediction of **16-10** mixing and is independent of mixing angles. Thus, the new fermions can be produced singly at the cost of a small mixing angle ($\sin^2\alpha\lesssim 0.01-0.05$). These interactions approximately double the mass range over which it is possible to produce a particular new fermion given a large enough event rate.

If the exotic fermions are lighter than $M_Z/2$ they can be produced in pairs in Z^0 decay at essentially full strength but if they satisfy $M_Z/2<M_f<M_Z$ they can still be produced singly via Eq. (3.1). We find the decay rates (normalized to Z^0 decay into one neutrino species)

$$\begin{aligned} \frac{\Gamma(Z^0\to\bar{e}E+e\bar{E})}{\Gamma(Z^0\to\nu\bar{\nu})}&=\frac{\Gamma(Z^0\to\nu_e\bar{\nu}_E+\bar{\nu}_e\nu_E)}{\Gamma(Z^0\to\nu\bar{\nu})}\\ &=\frac{1}{3}\frac{\Gamma(Z^0\to d\bar{h}+\bar{d}h)}{\Gamma(Z^0\to\nu\bar{\nu})}\\ &=2\sin^2\alpha\cos^2\alpha\,K_1\ , \end{aligned} \tag{3.2}$$

where K_1 is the relevant phase-space factor:

$$K_1=(1-M_f{}^2/M_Z{}^2)^2(1+M_f{}^2/2M_Z{}^2)\ . \tag{3.3}$$

The new fermions decay weakly via either Z^0 or W exchange, e.g., $E\to e(l\bar{l},\nu\bar{\nu},q\bar{q})$ or $\nu_e(l\bar{\nu}_l,q\bar{q}')$, with a rate suppressed by $\sin^2\alpha$. The signal in Z^0 decay would then be a jet or hard lepton (or missing energy) with $E=(M_Z{}^2-M_f{}^2)/2M_Z$ and such a decaying fermion opposite in momentum. An interesting possibility for $\nu_e\bar{\nu}_E$ decays is when $\nu_E\to\nu_e(q\bar{q})$ which gives rise to monojet-type events. Asymmetries in $e^+e^-\to Z^0\to e\bar{E},d\bar{h}$ can test the $V\pm A$ nature of the $e\bar{E}Z^0,d\bar{h}Z^0$ couplings.

The additional U(1) present in seemingly all superstring-motivated E_6 electroweak models will couple to all $f\bar{f}$ pairs via a new charge Q'. Besides producing exotic fermions in pairs via Z' decays (if kinematically allowed) one can also produce them singly using similar fermion mixing arguments. Using the specific choice of U(1) charges discussed in Ref. 7 we find the (selected) relative decay rates for Z' decay modes:

$$d\bar{d}:d\bar{h}+\bar{d}h:h\bar{h}:e\bar{e}:e\bar{E}+\bar{e}E:E\bar{E}$$
$$=15:216\sin^2\alpha\cos^2\alpha:51:5:7\sin^2\alpha\cos^2\alpha:17\ ,$$

where we have assumed all fermions to be massless for simplicity.

Another possibility is the production of a $d\bar{h}$ quarkonium in e^+e^- collisions via a virtual Z^0. Such a bound state would have its mass determined by the heavy quark ($M=M_h+m_d-\epsilon_b$) while its dynamics (wave function) would be determined by the light, d-type quark. Such a state would be much closer to the hydrogen atom in structure than the positroniumlike configuration of ordinary quarkonia. Its decays would also be different as decays into three gluon final states would not be allowed; decays into $f\bar{f}$ pairs via the Z^0 or single-quark decays of the heavy quark would be the dominant modes. Because all such modes are suppressed by a factor of $\sin^2\alpha \lesssim 0.01$, such a quarkonium would be much narrower than expected for a heavy $q\bar{q}$ bound state, likely much narrower than the beam width, and so would not be observable as a narrow state. (This is similar to the case of scalar quarkonium where production of a $^3S_1$ bound state forces the scalar quarks to be in a relative P wave thereby suppressing its e^+e^- width.)

Other reactions can also produce exotic fermions but will likely have lower rates and much larger backgrounds. The process $ed \to eh$ via t-channel Z^0 exchange can singly produce h quarks at ep colliders where $ep \to EX$ is also possible. Heavy quarks can be produced by gluon fusion at $pp, p\bar{p}$ colliders and Z^0's (or Z''s) produced in such reactions can have exotic fermions in their decay products as discussed already.

IV. CONSTRAINTS FROM FLAVOR-CHANGING NEUTRAL CURRENTS

We have, so far, considered only one generation of fermions. If one includes mixings among the various generations of d-type quarks (and h-type quarks) we have the interactions

$$-\frac{ie}{\sin(2\theta_W)}[\bar{D}^u Z^0(g_V+g_A\gamma_5)D^u+\sin\alpha\cos\alpha\,\bar{D}^u Z^0 H_L^v+\text{H.c.}+\bar{H}^v Z^0(h_v+h_A\gamma_5)H^v] , \tag{4.1}$$

where

$$D^u = UD = U\begin{bmatrix} d \\ s \\ b \end{bmatrix} ,$$

$$H^v = VH = V\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} , \tag{4.2}$$

$$U^{\tilde{u}} = \tilde{U}\begin{bmatrix} u \\ c \\ t \end{bmatrix} ,$$

and $\tilde{U}^\dagger U = K$ is the usual Kobayashi-Maskawa (KM) matrix. The matrix V describes the corresponding mixings among the generations (here assumed to be three) of h-type quarks and it need not be the same as U. The $\bar{D}Z^0D$ and $\bar{H}ZH$ terms are, of course, generation diagonal since U and V are unitary but the off-diagonal $\bar{D}Z^0H$ terms can lead to flavor-changing neutral currents (unless $U = V$). The form of these interactions is basically identical to the usual charged-current form,

$$\frac{ig}{\sqrt{2}}\bar{D}^u W U_L^{\tilde{u}} + \text{H.c.} , \tag{4.3}$$

with the substitutions

$$g/\sqrt{2} \to -e\sin\alpha\cos\alpha/\sin(2\theta_W) ,$$

$K \to V^\dagger U$ and $M_W \to M_Z$ in any propagator. This means that the usual analyses of such processes as K^0-$\bar{K}^0$ mixing and $K_L \to \mu^+\mu^-$ can be taken over directly by using the substitutions above. (There will be small differences as, for example, in the calculation of the effective $d\bar{s}Z^0$ vertex due to the lack of $Z^0Z^0Z^0$ couplings.)

Because of the unknown masses and mixings of the h quarks we will only illustrate schematically the constraints that are possible from considering such processes. For example, in the K^0-$\bar{K}^0$ system we would require that the contributions of the box diagram now possible with intermediate h quarks and Z bosons be small compared to the usual box with u-type quarks and W's as intermediate states.[14] In a simplified two-generation model this would require that

$$\frac{\cos^2(\theta-\phi)\sin^2(\theta-\phi)}{\cos^2\theta_C\sin^2\theta_C}\sin^4\alpha\frac{M_h{}^2}{m_c{}^2} \ll 1 , \tag{4.4}$$

where θ (ϕ) is the mixing in the d- (h-) quark sector. Since $\sin\alpha \sim m/M$, where m (M) is a $\Delta I = \frac{1}{2}$ (0) mass term, we could argue that such a new box diagram is naturally a factor of $(m/M)^2$ smaller than the usual contribution and so would already be suppressed by present limits on $\sin\alpha$. The process $K_L \to \mu^+\mu^-$ requires fewer powers of the mixing angle to yield a flavor-changing result and would presumably give more stringent bounds. As an example in this sector we might assume a specific mixing in the h sector ($V = \tilde{U}$) and roughly equal masses for the three h quarks. Then, following Inami and Lim[15] we would be led to the (approximate) bound

$$\sin^4\alpha(M_h{}^2/4M_Z{}^2)^2 \lesssim 10^{-4} \tag{4.5}$$

or

$$\sin\alpha\, M_h \lesssim 20 \text{ GeV} \tag{4.6}$$

if we use the presently accepted values (or limits) for the KM angles. One might argue (as above) that the ratio of this new contribution to $K_L \to \mu^+\mu^-$ to the usual one might be expected to be approximately

$$\sin^4\alpha\frac{M^4}{m^4} \sim \left[\frac{m}{M}\right]^4\frac{M^4}{m^4} \sim \text{order unity} . \tag{4.7}$$

Somewhat weaker bounds on masses and mixings can also be derived from constraints on the ρ parameter due to the renormalization of the W and Z masses due to loops with heavy fermions.[16] With one light partner (the d-type quarks) and one heavy (the h-type quarks) we can estimate that the new diagrams due to fermion mixing would contribute

$$\Delta\rho \simeq 3\frac{G_F}{8\sqrt{2}\pi^2}\sin^4\alpha \sum_h M_h{}^2 . \quad (4.8)$$

If we require that $\Delta\rho \lesssim 0.01$ this would yield the bound

$$\sin^2\alpha \left[\sum_h M_h{}^2 \right]^{1/2} \lesssim 180 \text{ GeV} . \quad (4.9)$$

In conclusion, we have considered the possibility of mixings of the 16-plet and 10-plet fermions in superstring motivated E_6 electroweak theories, derived constraints on exotic fermion masses and mixings, and provided examples of new mechanisms for producing these new fermions singly. Because the usual $N=1$ supersymmetry is assumed to operate in such theories to solve the hierarchy problem, these theories will also allow for new, exotic scalar fermions with mixings between the **16** and **10**. Such mixings are not directly constrained by experiments testing weak universality or neutral currents (because, of course, no scalar fermions have yet been observed). Any bounds on scalar-fermion masses or processes involving their weak interactions may then need to be rethought. (We have in mind, for example, the K^0-$\bar{K}^0$-system box diagram with intermediate scalar quarks and gauge fermions, etc.[17])

Noted added in proof. After submitting this manuscript, I learned of work by Barr,[18] who also considered the possibility of flavor-changing processes due to exotic E_6 fermions.

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