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Glueball theory of the $\theta(1700)$

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We consider, in the context of the glueball theory of the $\theta(1700)$, the recent measurements by the Mark III Collaboration at the SLAC e^+e^- storage ring SPEAR, of the θ helicity-amplitude ratios $\bar{x}$ and $\bar{y}$, $\bar{x}=A_1/A_0$, $\bar{y}=A_2/A_0$, for the decay sequence $\psi/J\to\theta\gamma$, $\theta\to K^-K^+$, where A_j is the Jacob-Wick amplitude for θ helicity j, $j=0,1,2$. We find that the values of $\bar{x}$ and $\bar{y}$ are not obviously inconsistent with the interpretation of the $\theta(1700)$ as a bound state of two transverse-electric constituent gluons.

I. INTRODUCTION

At present, the primary candidate for the theory of the strong interaction is QCD, quantum chromodynamics.[1] One of the most immediate predictions of QCD is that there should exist hadrons composed entirely of gluons—glueballs.[2] One of the most likely candidates for such a state[3] is the $\theta(1700)$, which, on this view, would most simply be viewed[2] as the lowest spin-2 bound state of two constituent TE (transverse-electric) gluons in the context of the MIT bag model[4] of hadrons, for example. Recently, the Mark III Collaboration,[5] operating at the SLAC e^+e^- storage ring SPEAR, has measured the Jacob-Wick helicity-amplitude ratios $\bar{x}$ and $\bar{y}$ for the θ in the decay sequence $\psi/J\to\theta\gamma$, $\theta\to K^+K^-$, where $\bar{x}=A_1/A_0$ and $\bar{y}=A_2/A_0$ if A_j is the decay amplitude for θ helicity j, $j=0,1,2$. It is therefore appropriate to check that the results in Ref. 5 for $\bar{x}$ and $\bar{y}$ are in fact consistent with the glueball theory of the θ. Such a check is the primary objective of this paper.

More specifically, the results[5] of the Mark III Collaboration are

$$\begin{aligned} x&=-1.07\pm0.20, \quad \varphi_x=0.6\pm0.8\ ,\\ y&=-1.09\pm0.25, \quad \varphi_y=-0.1\pm0.5\ , \end{aligned} \tag{1}$$

where $\bar{x}\equiv x\exp(i\varphi_x)$ and $\bar{y}\equiv y\exp(i\varphi_y)$ and where the errors are statistical errors. It is the set of data (1) which we wish to compare to the predictions of QCD on the glueball view of the θ.

Toward this end, we will rely on the effective-Lagrangian methods which we used in Ref. 6 to analyze, in some detail, the scenario that the[7] $\xi(2.22)$ is a bound state of two transverse-magnetic gluons, where the transverse magneticity is that of a massive constituent gluon in the context of the MIT bag model. Such methods are well known to yield reasonable results in other areas[8] of theoretical particle physics so that we feel it is indeed not obviously inappropriate to apply them to glueball physics.

Thus, our approach may be viewed as the completion of the analyses of Krammer[9] and Close[10] as they would relate to the decay $\psi/J\to\theta\gamma$, $\theta\to K^-K^+$. Both of these authors have emphasized the need for a complete calculation in any given model of a tensor meson $\mathscr{T}$ in order to verify the consistency of that meson model with such detailed data as the $\bar{x}$ and $\bar{y}$ values in $\psi/J\to\mathscr{T}\gamma$, $\mathscr{T}\to$ hadrons.

We should note that our methods would also apply to the decays $\psi/J\to\mathscr{T}\gamma$, $\mathscr{T}\to m\bar{m}$, for $\mathscr{T}=f,f'$, $m=\pi,K$, respectively, for example. Entirely for reasons of pedagogy, such quark-antiquark tensor-meson analyses will be taken up in a separate communication.

We should also note that we will not consider here the possibility that the θ is a $qq\bar{q}\,\bar{q}$ or $q\bar{q}G$ type state, where $q=u,d,s$, and G is a constituent gluon. Such possibilities could also be considered within our framework. We note that Liu[11] has already pointed out that the data in (1) are apparently inconsistent with the interpretation of $\theta(1700)$ as a $qq\bar{q}\,\bar{q}$ meson. There remain, then, the scenarios such as those of the hybrid ($q\bar{q}G$) type. These, too, will be taken up elsewhere.

Our work is organized as follows. In the next section, we review the relevant aspects of the effective-Lagrangian methods which we shall employ. In Sec. III we apply these methods to the process $\psi/J\to\theta\gamma$, $\theta\to K^-K^+$, in the context of the formalism of Jacob and Wick for the helicity amplitudes A_j, $j=0,1,2$; we compare our results with the Mark III measurements. Section IV contains our summary remarks.

II. EFFECTIVE-LAGRANGIAN TREATMENT OF GLUEBALL PRODUCTION AND DECAY

What we are interested in is a formalism which will allow us to compute $\psi/J\to\mathscr{T}\gamma$, $\mathscr{T}\to K^+K^-$, on the view that $\mathscr{T}$ is a spin-2 bound state of two transverse-electric gluons. In Ref. 6 we have illustrated such a formalism for the case $\mathscr{T}=\xi(2.22)$. Here, we will review that formalism as it relates to the case of $\mathscr{T}=\theta(1700)$.

Our starting points, then, are the effective Lagrangians for $\psi/J\to\mathscr{T}\gamma$ and $\mathscr{T}\to K^-K^+$. To lowest order in the QCD coupling constant, we have derived both of the respective effective-Lagrangian interaction operators in Ref. 6. On the view that $\mathscr{T}$ is composed of two gluons G_1 and G_2, we have, from Ref. 6, for $\psi/J\to\mathscr{T}\gamma$, by the diagrams in Fig. 1, the effective interaction

$$\mathscr{L}_{\text{eff}}^{(1)}=\frac{-g^2e_c}{2N_c}\bar{\psi}_c\gamma_\nu\psi_c\left[\frac{2(-F^{a\nu}_{G\lambda_1'}A^a_{G\lambda_1}F_\gamma^{\lambda_1\lambda_1'}+F^\nu_{\gamma\lambda_1}F_G^{a\lambda_1\lambda_1'}A^a_{G\lambda_1'}-\frac{1}{2}A_G^{a\nu}F^a_{G\lambda_1'\lambda_1}F_\gamma^{\lambda_1\lambda_1'})}{m_{\psi/J}E_\gamma^{\text{lab}}(m_{\psi/J}E_G^{\text{lab}}-m_G{}^2)}\right.$$
$$\left.+\frac{F^\nu_{\gamma\lambda_1}A^a_{G\lambda_1'}F_G^{a\lambda_1\lambda_1'}+F_G^{a\nu\lambda_1'}A^a_{G\lambda_1}F^{\lambda_1}_{\gamma\lambda_1'}-\frac{1}{2}A_G^{a\nu}F_G^{a\lambda_1\lambda_1'}F_{\gamma\lambda_1'\lambda_1}}{(m_{\psi/J}E_G^{\text{lab}}-m_G{}^2)^2}+\frac{2A_G^{a\nu}A_\gamma\cdot A_G^a-A_\gamma^\nu A_G^a\cdot A_G^a}{m_{\psi/J}E_G^{\text{lab}}-m_G{}^2}\right]^2, \tag{2}$$

and for $\mathscr{T}\to K^-K^+$, by the diagrams in Fig. 2, the effective interaction (H.c. denotes Hermitian conjugation)

$$\mathscr{L}_{\text{eff}}^{(2)}=\frac{2g^2f_2\phi_{\mathscr{T}}^{\alpha_1\alpha_2}}{N_c m_{\mathscr{T}}{}^2}(i\bar{\psi}_q\gamma_{\alpha_2}D_{\alpha_1}\psi_q+\text{H.c.})\;, \tag{3}$$

where ψ_q is the field operator for quark q, $q=u,d,s,c$, A_γ^ν is the photon vector potential, A_G^ν is the gluon vector potential, $F_\gamma^{\mu\nu}$ is the photon field-strength tensor, $F_G^{a\mu\nu}$ is the gluon field-strength tensor, $\phi_{\mathscr{T}}^{\alpha_1\alpha_2}$ is the effective field operator for $\mathscr{T}$, e_c is the electric charge ($\frac{2}{3}e$) of the charm quark, g is the QCD coupling constant, N_c is the number of colors in QCD (we will always set $N_c=3$), and f_2 is the $\mathscr{T}$ decay constant which we define by

$$\langle 0|A^a_{G\lambda_1}(0)A^a_{G\lambda_2}(0)|\mathscr{T}\rangle=f_2\epsilon_{\lambda_1\lambda_2}/[2E_{\mathscr{T}}(2\pi)^3]^{1/2} \tag{4}$$

if $\epsilon_{\lambda_1\lambda_2}$ is the spin-2 polarization tensor and $E_{\mathscr{T}}$ is the energy of the $\mathscr{T}$. Further, $(D_\mu)_{\sigma\beta}=\partial_\mu\delta_{\sigma\beta}+ig\,\mathbf{A}_{G\mu}\cdot\boldsymbol{\tau}_{\sigma\beta}$ is the covariant derivative in the quark-color representation carried by τ. The kinematical parameters are $E_\gamma^{\text{lab}}\equiv(m_{\psi/J}{}^2-m_{\mathscr{T}}{}^2)/2m_{\psi/J}$, $E_G^{\text{lab}}=\frac{1}{2}E_{\mathscr{T}}^{\text{lab}}\equiv(m_{\psi/J}-E_\gamma^{\text{lab}})/2$, where m_B is the rest mass of B, $B=\psi/J,\mathscr{T},\ldots$.

The effective Lagrangians in (2) and (3) completely determine the dominant perturbative QCD contribution to $\psi/J\to\mathscr{T}\gamma$, $\mathscr{T}\to K^-K^+$. However, since we wish to make a reasonably precise comparison between the glueball hypothesis and the results (1), we must ask ourselves whether (2) and (3) are accurate to the level of $\sim 30\%$ as far as the relative values of A_0, A_1, and A_2 are concerned. We will turn to this issue in the general development of the next section.

III. EVALUATION OF A_0, A_1, AND A_2 FOR THE DECAY $\psi/J\to\theta\gamma$, $\theta\to K^+K^-$

In this section, we wish to apply the effective Lagrangians (2) and (3) to the decay sequence $\psi/J\to\theta\gamma$, $\theta\to K^+K^-$. We shall begin this application with a detailed discussion of the accuracy of (2) and (3) insofar as the ratios of A_0, A_1, and A_2 are concerned.

Thus, we are not concerned, at this time, with the uncertainty in f_2 because this parameter will cancel out of the values of $\bar{x}$ and $\bar{y}$. What we are concerned with is whether there is an additional nonperturbative contribution which is significant at the mass $m_{\mathscr{T}}=m_\theta$. From the standpoint of the Bethe-Salpeter equation, our perturbative calculation of $\mathscr{L}_{\text{eff}}^{(1)}$ in Ref. 6 does not describe the effects of resonances in the θ channel ($J^{PC}=2^{++}$). Thus, as has been emphasized by Lipkin,[12] the $\chi(3555)$ represents the source of a possible nonperturbative effect which is not included in (2) and which, on the glueball view of the θ, is not Zweig-rule suppressed relative to the interactions in (2) and (3). We wish to evaluate this possible nonperturbative effect in $\psi/J\to\theta\gamma$.

Toward this latter end, we wish to continue to rely on the method of the operator field[13] and to incorporate the $\chi(3555)$ as an effective point particle in the spirit of our effective-Lagrangian methods. Thus, the diagram which we have need of is shown in Fig. 3(a). The propagator is simply (we ignore the width of the χ and, for simplicity, we treat each of the five χ helicity states as propagating in a diagonal helicity basis so that our vertex functions will already contain the helicity spinor polarizations)

$$-i\delta_{\lambda\lambda'}/(m_\theta{}^2-m_\chi{}^2+i\epsilon)\;, \tag{5}$$

where λ,λ' denote the χ helicity states. The $-i$ in (5) is a standard result in the fermion-antifermion two-body Bethe-Salpeter bound-state problem. Thus, to compute the process in Fig. 3(a), we need the off-shell vertices for $\psi/J\to\chi\gamma$ and $\chi\to\theta$ shown in Figs. 3(b) and 3(c), respectively, where in Fig. 3(c), we have in mind that G_1 and G_2 are bound to form a θ. We consider first $\psi/J\to\chi\gamma$.

For $\psi/J\to\chi\gamma$, then, we have to compute the off-shell diagram shown in Fig. 3(b). We use the Bethe-Salpeter method and we specialize to the photon helicity $\lambda_\gamma=+1$ without loss of content. Now, we must be careful to note that the χ is off-shell. Thus, we use the explicit form of Bjorken and Drell[13] for the Dirac spinors so that we can continue the χ to the appropriate off-shell point. With this in mind, we follow Ref. 14 and write the ψ/J and χ states as

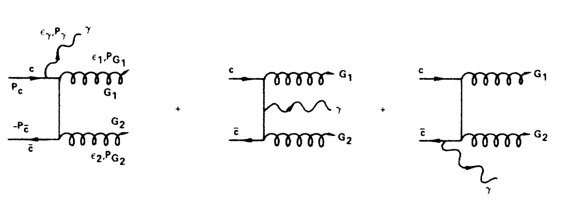

FIG. 1. The process $c+\bar{c}\to G_1+G_2+\gamma$ to lowest order in g and e.

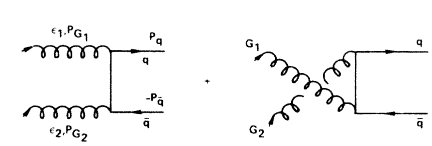

FIG. 2. The process $G_1+G_2\to q\bar{q}$ to lowest order in g.

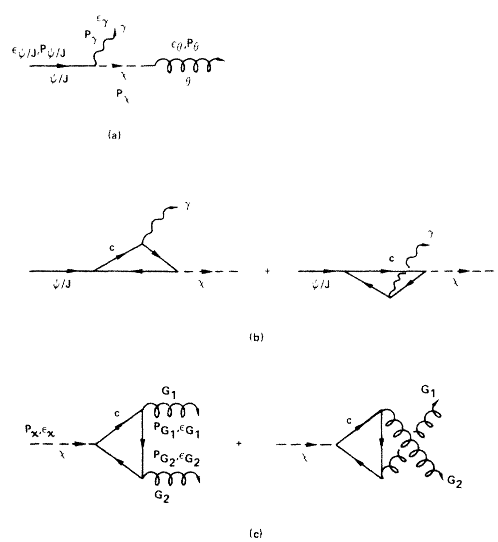

FIG. 3. The process $\psi/J \to \chi\gamma$, $\chi \to \theta$. In (a), we show the process at the level of the respective effective Lagrangian in an obvious Feynman-diagrammatic notation. In (b), we show $\psi/J \to \chi\gamma$ at the level of QCD. In (c), we show $\chi \to \theta$ at the level of QCD. The kinematical symbols are such that P_A is the four-momentum of A and ϵ_A is the polarization of A, $A = \psi/J$, θ, γ, χ, G_1, and G_2, where G_1 and G_2 are gluons, so that ϵ_θ and ϵ_χ are actually symmetric traceless two-tensors.

$$| Y\rangle = \sum_{\substack{s,s',\\ L_z}} \mathscr{S}_{YL_z}(s,s') \int d^4p' F_{YL_z}(p') \frac{b_c^{\dagger\beta}(P_Y - P',s)}{\sqrt{N_c}} \times d_c^{\dagger\beta}(p',s') | 0\rangle, \quad Y = \psi/J, \chi, \tag{6}$$

where P_Y is the four-momentum of Y, $\mathscr{S}_{YL_z}$ is the appropriate spin configuration for a Y with z component of orbital angular momentum L_z, F_{YL_z} are the respective Bethe-Salpeter wave functions, and β is a color label for the fundamental representation of color SU(N_c). Here $b_c^\dagger$ ($d_c^\dagger$) is a creation operator for c ($\bar{c}$). The spinor wave functions associated with the quarks and antiquarks will be identified with[13] (henceforth, $p^0 \equiv E$)

$$u(s) = \frac{(\not{p} + m_c)}{(E^2 + \mathbf{p}^2 + m_c{}^2 + 2Em_c)^{1/2}} u_0(s) ,$$
$$v(s) = \frac{(m_c - \not{p})}{(E^2 + \mathbf{p}^2 + m_c{}^2 + 2Em_c)^{1/2}} v_0(s) , \tag{7}$$

where p and s are the respective four-momentum and four-spin so that we have ($\hat{s}^3$ is the 3 component of s in a frame with $s^0 = 0$)

$$u_0(s)|_{\hat{s}^3 = 1/2} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad u_0(s)|_{\hat{s}^3 = -1/2} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix},$$
$$v_0(s)|_{\hat{s}^3 = 1/2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad v_0(s)|_{\hat{s}^3 = -1/2} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix};$$

here the 3 direction is identified with the z direction. We note that, for the χ, we may have $p^2 \neq m_c{}^2$ in (7) but that, for the ψ/J, in the spirit of Van Royen and Weisskopf,[14] we will take $p^2 = m_{\psi/J}{}^2/4 \equiv m_c{}^2$. For definiteness, let us note further that our conventions are such that, if, for the electromagnetic current J_μ^{EM}, we write

$$\langle 0 | J_\mu^{\mathrm{EM}}(0) | \psi/J\rangle = f_{\psi/J} m_{\psi/J} \epsilon_\mu / [(2\pi)^3 2E_{\psi/J}]^{1/2} , \tag{8}$$

then [henceforth, we suppress the $L_z \equiv 0$ label on $\bar{F}_{\psi/J}(\mathbf{p}) \equiv \int_{-\infty}^{\infty} dp^0 F_{\psi/J_0}(p)$ and on F_{ψ/J_0}]

$$f_{\psi/J} = -\tfrac{2}{3}(2\sqrt{N_c}) \int d^3p \frac{\bar{F}_{\psi/J}(\mathbf{p})}{(2\pi)^{3/2}\sqrt{E_{\psi/J}}} , \tag{9}$$

where we note that, in the Van Royen—Weisskopf limit in the ψ/J rest frame, $F_{\psi/J}(p) = \delta(p_0 - m_{\psi/J}/2)\bar{F}_{\psi/J}(\mathbf{p})$ for $\bar{F}_{\psi/J}(\mathbf{p}) = a_{\psi/J}\delta^{(3)}(\mathbf{p})$. Here, ϵ_μ is the ψ/J polarization four-vector and $a_{\psi/J}$ is given by

$$-3(2\pi)^{3/2} f_{\psi/J}\sqrt{m_{\psi/J}}/4\sqrt{N_c} .$$

Experimentally,[15] we know that $f_{\psi/J} \cong 0.254$ GeV. With these kinematical preliminaries, we may now proceed with the computation of Fig. 3(b).

Toward this end, we note that the standard methods allow us to write, in the θ rest frame, the amplitude in Fig. 3(b) as

$$\mathscr{A}(\psi/J \to \chi\gamma) = \frac{\int d^4p \int d^4p'}{N_c} \sum_{\substack{s_i,s_i'\\ L_z}} \mathscr{S}^*_{\chi L_z}(s_2,s_2')\mathscr{S}_{\psi/J}(s_1,s_1')F^*_{\chi L_z}(p')F_{\psi/J}(p)$$
$$\times[\bar{u}_c(P_\chi - p',s_2)(-ie_c\not{\epsilon}^*_\gamma)u_c(P_{\psi/J} - p,s_1)\delta^{(3)}(\mathbf{p}' - \mathbf{p})$$
$$- \bar{v}_{\bar{c}}(p,s_1')(-ie_c\not{\epsilon}^*_\gamma)v_{\bar{c}}(p',s_2')\delta^{(3)}(\mathbf{P}_\chi - \mathbf{p}' - \mathbf{P}_{\psi/J} + \mathbf{p})] . \tag{10}$$

Since[16] $\epsilon^*_\gamma = (\hat{\mathbf{x}} + i\hat{\mathbf{y}})/\sqrt{2}$ in our conventions (here, we take the z axis along the direction of the θ in the ψ/J rest system—we shall always do this—thus, $\hat{\mathbf{x}}, \hat{\mathbf{y}}$ are unit vectors along the x and y axes in this ψ/J rest system) and since $F_{\psi/J}(p) \propto \delta^{(3)}(\mathbf{p} - \mathbf{P}_{\psi/J}/2)\delta(p^0 - E_{\psi/J}/2)$, $F^*_{\chi L_z}(p')$ will only be evaluated at $\mathbf{p}' = \mathbf{P}_{\psi/J}/2$ or $\mathbf{p}' = -\mathbf{P}_{\psi/J}/2$ in (10). Since these momenta are parallel to the z axis, only $L_z = 0$ components of the χ wave function can participate in (10). This means that no helicity $\lambda = 2$ χ states or θ states are involved in Fig. 3(a).

Further, if the helicity of the χ is $\lambda_\chi=1$, we note, trivially, that this would imply helicity $\lambda_\theta=1$ for the θ. We assess this scenario as follows. Since our formalism is gauge invariant, we may pass to the gauge $A^a_{G0}=0$ without loss of content. In this gauge, to leading order in g one can show that the TE gluon wave function is proportional to $\mathbf{L}Y_{1m}$ where Y_{1m} is the usual spherical harmonic and $\mathbf{L}$ is the orbital angular momentum operator. Thus, if $\hat{\mathbf{P}}_{G_1}$ is the direction of the 3 momentum $\mathbf{P}_{G_1}$ of the gluon G_1 in Fig. 3(c) inside of the θ, then we have the bound-state polarizations[17] $\hat{P}_{G_1z}(\hat{\mathbf{x}}\pm i\hat{\mathbf{y}})/\sqrt{2}+(-\hat{P}_{G_1x}\mp i\hat{P}_{G_1y})\hat{\mathbf{z}}/\sqrt{2}$ and $-i(\hat{P}_{G_1y}\hat{\mathbf{x}}-\hat{P}_{G_1x}\hat{\mathbf{y}})$, respectively , for $m=\pm 1$ and 0 for G_1. Here, $\hat{P}_{G_1a}$ is the a component of $\hat{\mathbf{P}}_{G_1}$, $a=x,y,z$. Using these polarizations, we will be able to show that, to the order of our approximations, $\lambda_\chi=\lambda_\theta=1$ transitions are suppressed in Fig. 3.

The net result will be that only $\lambda_\chi=0$, $L_z=0$ states participate in (10). On computing (10) in this latter scenario we arrive at the vertex (by definition the vertex has factors such as $1/[(2\pi)^3 2E_{\psi/J}]^{1/2}$ removed from it)

$$i\Lambda_{\psi\to\chi_{\lambda_\chi=0}\gamma}=\frac{4ie}{\sqrt{6}}\frac{f_{\psi/J}}{\sqrt{N_c}}\frac{[(E_{\psi/J}/2+m_c)p_{c_\chi z}-(E_{c_\chi}+m_c)P_{\psi/Jz}/2]E_{\psi/J}\sqrt{m_\theta}}{[\mathbf{P}_{\psi/J}{}^2/4+(E_{\psi/J}/2+m_c)^2]^{1/2}(E_{c_\chi}{}^2+\mathbf{p}_{c_\chi}{}^2+m_c{}^2+2m_cE_{c_\chi})^{1/2}}[(2\pi)^{3/2}\bar{F}^*_{\chi 0}(-\mathbf{P}_{\psi/J}/2)] , \quad (11)$$

where $\bar{F}_{\chi 0}(\mathbf{p}')\equiv\int_{-\infty}^{\infty}dp'_0F_{\chi 0}(p')$, $E_{c_\chi}\equiv m_\theta-E^{\text{lab}}_\theta m_{\psi/J}/2m_\theta$, and $p_{c_\chi z}=E^{\text{lab}}_\gamma m_{\psi/J}/2m_\theta=-P_{\psi/Jz}/2$. Here, E^{lab}_θ and E^{lab}_γ are the energies of the θ and the γ in the ψ/J rest system whereas, to repeat, all other kinematic variables refer to their values in the θ rest system. This completes the specification of the process in Fig. 3(b).

The vertex in Fig. 3(c) may be addressed in a manner which is entirely similar to the analysis of (11). We have the amplitude, from Fig. 3(c) (here, the kinematics is summarized in the figure),

$$\begin{aligned}\mathscr{A}(\chi\to G_1G_2)&\\=\frac{1}{(2\pi)^3}\int d^3p'\frac{-ig^2}{\sqrt{N_c}}\sum_{s,s',L_z}\mathscr{S}_{\chi L_z}(s,s')\bar{F}_{\chi L_z}(\mathbf{p}')\bar{v}^{\sigma}_{\bar{c}}(p',s')&\left[\frac{\not\epsilon_1^{*a}\tau^a_{\sigma\sigma'}(\not P_\chi-\not p'-\not P_{G_2}+\bar{m}_c)\not\epsilon_2^{*a'}\tau^{a'}_{\sigma'\sigma}}{(P_\chi-p'-P_{G_2})^2-\bar{m}_c{}^2+i\epsilon}\right.\\&\left.+\frac{\not\epsilon_2^{*a'}\tau^{a'}_{\sigma\sigma'}(\not P_\chi-\not p'-\not P_{G_1}+\bar{m}_c)\not\epsilon_1^{*a}\tau^{a}_{\sigma'\sigma}}{(P_\chi-p'-P_{G_1})^2-\bar{m}_c{}^2+i\epsilon}\right]u^{\sigma}_c(P_\chi-p',s)\end{aligned}\quad (12)$$

where we will use the off-shell point $P_\chi=(m_\theta,\mathbf{0})$ as the reference point for p': $p'=(m_\theta/2,\mathbf{p}')$. Further, we will work to lowest order in $|\mathbf{p}'|/\bar{m}_c$, where $\bar{m}_c$ is determined to be ~ 1.16 GeV following Ref. 18. From (12) we can now see that the $\lambda_\chi=\lambda_\theta=1$ transition is of order $\hat{P}_{G_1j}{}^2/\hat{P}_{G_1z}{}^2$, $j=x,y$, relative to the $\lambda_\chi=\lambda_\theta=0$ transition if the ϵ_α in (12) are associated with our TE G. However, in the spirit of our effective-Lagrangian methods, the momenta of the two gluons in Fig. 3 are supposed to be evaluated in the Van Royen—Weisskopf limit $P_{G_2j}=P_{G_1j}\equiv 0$, $j=x,y$, $P_{G_iz}\equiv q\to 0$, $i=1,2$. In this limit, then, we see that, within our approximations the $\lambda_\chi=\lambda_\theta=1$ $\chi\to\theta$ vertex vanishes, as we have already anticipated. As far as the $\lambda_\chi=\lambda_\theta=0$ transition is concerned, we may continue with our effective-Lagrangian methods and specialize (12) to the case in which $q\to 0$ and in which $2\epsilon_1^{*a\alpha_1}\epsilon_2^{*a\alpha_2}$ is identified with the color-singlet gluon-field product $A_G^{a\alpha_1}A_G^{a\alpha_2}$; for, this is what has been done in arriving at (2). The definition (4) of the θ decay constant f_2 then allows us to identify, in the Van Royen—Weisskopf limit, the vertex for $\chi_{\lambda_\chi=0}\to\theta$ as

$$\begin{aligned}i\Lambda(\chi_{\lambda_\chi=0}\to\theta)&\\=\frac{2g^2}{3\bar{m}_c{}^2}\left[\frac{m_\theta}{N_c}\right]^{1/2}&f_2(-1/\sqrt{2})(\partial/\partial x-i\,\partial/\partial y)\Psi_{\chi 1}(\mathbf{0}),\end{aligned}\quad (13)$$

where $\Psi_{\chi 1}(\mathbf{r})$ is the $L_z=1$ spatial wave function for the χ and where, to repeat, we have specialized (13) to the helicity-0 scenario since this is the only θ helicity which can participate in Fig. 3(a) in the context of our approximations. This completes the specification of the vertex in Fig. 3(c).

Our amplitude for $\psi/J\to\chi\gamma$, $\chi\to\theta$ is now easily obtained as

$$\begin{aligned}\mathscr{A}(\psi/J\to\chi\gamma,\chi\to\theta)&=i\Lambda(\psi/J\to\chi\gamma)\frac{(-i)}{m_\theta{}^2-m_\chi{}^2}i\Lambda(\chi\to\theta)\\&=\frac{8eg^2}{3\sqrt{6}}\frac{f_{\psi/J}f_2}{\bar{m}_c{}^2N_c}\frac{p_{c_\chi z}m_\theta(m_\theta+2m_c)E_{\psi/J}[(2\pi)^{3/2}\bar{F}^*_{\chi 0}(-\mathbf{P}_{\psi/J}/2)]}{(m_\theta{}^2-m_\chi{}^2)\{[\mathbf{P}_{\psi/J}{}^2/4+(E_{\psi/J}/2+m_c)^2][E_{c_\chi}{}^2+\mathbf{p}_{c_\chi}{}^2+m_c{}^2+2m_cE_{c_\chi}]\}^{1/2}}\\&\quad\times(-1/\sqrt{2})(\partial/\partial x-i\partial/\partial y)\Psi_{\chi 1}(\mathbf{0})\ .\end{aligned}\quad (14)$$

To evaluate (14) we need to compute $(\partial/\partial x - i\partial/\partial y)\Psi_{\chi 1}(0)$ and $(2\pi)^{3/2}\tilde{F}^*_{\chi 0}(-\mathbf{P}_{\psi/J}/2)$. To these computations we now turn.

For definiteness we will use the Cornell model[18] for the ψ/J system with potential

$$V(r) = -\kappa/r + r/a^2 \tag{15}$$

with $a = 2.34\ \text{GeV}^{-1}$ and $\kappa \cong 0.52$. Further, we will treat the off-shell displacements of the quark and the antiquark in Fig. 3(b) as additive shifts in their respective momenta so that $V(r)$ may still be viewed as a nonrelativistic interaction between a quark and an antiquark of effective mass[18] $m_{\text{eff}} \cong 1.84$ GeV; the total four-momentum of the quark (or the antiquark) is then just the sum of its off-shell displacement and the four-momentum associated with the Schrödinger problem for (15). Physically, we are simply saying that, in the zero-momentum frame, the momentum distribution in the off-shell χ in Fig. 3 is that in an on-shell χ shifted by the off-shell displacement four vectors of the χ constituents; this is consistent with time-ordered old-fashioned perturbative practice. On this view, one can then use finite-difference methods to compute

$$\begin{aligned}(2\pi)^{3/2}\tilde{F}^*_{\chi 0}(-\mathbf{P}_{\psi/J}/2) &= i\sqrt{12\pi}\frac{a}{(m_{\text{eff}})^{1/2}}\int_0^\infty d\rho\, j_1(\eta\rho)u(\rho)\rho \\ &\cong i\sqrt{12\pi}\frac{a}{(m_{\text{eff}})^{1/2}}(0.74)\end{aligned} \tag{16}$$

and

$$\begin{aligned}(-1/\sqrt{2})(\partial/\partial x - i\partial/\partial y)\Psi_{\chi 1}(0) \\ \cong 0.90\left[\frac{3}{4\pi}\right]^{1/2}(m_{\text{eff}})^{5/6}/a^{5/3}\,,\end{aligned} \tag{17}$$

where $\eta = (a^{2/3}/m_{\text{eff}}^{1/3})E_\gamma^{\text{lab}} m_{\psi/J}/2m_\theta \cong 1.38$, $j_1(x)$ is the spherical Bessel function of order 1 and $u(\rho)$ is the $1P$ solution of the reduced Schrödinger problem for (15) and satisfies

$$(-d^2/d\rho^2 + 2/\rho^2 - \nu/\rho + \rho)u = E_{1P}u \tag{18}$$

with $\nu = (am_{\text{eff}})^{2/3}\kappa$ and,[18] by our difference methods, $E_{1P} \cong 2.61$. On introducing (16) and (17) into (14), we arrive at

$$\begin{aligned}&\mathcal{A}(\psi/J \to \chi\gamma, \chi \to \theta) \\ &\quad\cong -i(0.179/\text{GeV}^2)eg^2 f_{\psi/J} f_2 m_{\psi/J}/N_c\,.\end{aligned} \tag{19}$$

This is the desired contribution of the virtual χ state in Fig. 3 to $\psi/J \to \theta\gamma$.

The result (19) may be compared with the perturbative prediction of (2) for the amplitude for $\psi/J \to \theta\gamma$: we have, using the standard methods,

$$\begin{aligned}&\mathcal{A}(\psi/J \to \theta\gamma)_{\text{perturbative}} \\ &\quad = -\frac{ieg^2}{2N_c} f_{\psi/J} f_2 m_{\psi/J}\frac{m_{\psi/J}(E_\theta^{\text{lab}} - E_\gamma^{\text{lab}}) - 2m_G^2}{(m_{\psi/J}E_\theta^{\text{lab}}/2 - m_G^2)^2} \\ &\quad\quad \times \epsilon^\nu_{\psi/J}(s_z)\epsilon^{*\mu}_\gamma(\lambda_\gamma)\epsilon^*_{\nu\mu}(\lambda_\theta)\,,\end{aligned} \tag{20}$$

where $\epsilon^\nu_{\psi/J}(s_z)$ is the ψ/J four-polarization for spin projection s_z along the z direction, $\epsilon^\mu_\gamma(\lambda_\gamma)$ is the photon four-polarization with helicity λ_γ, and $\epsilon^*_{\nu\mu}(\lambda_\theta)$ is the θ spin-2 polarization for helicity λ_θ. In the case analyzed for (20), we have $s_z = -1$ for the ψ/J, $\lambda_\gamma = 1$, and $\lambda_\theta = 0$. Thus, we see that, in this case (we take[6,14] $m_G = m_\theta/2$),

$$\begin{aligned}&\mathcal{A}(\psi/J \to \theta\gamma)_{\text{perturbative}}\big|_{\lambda_\theta=0,\,\lambda_\gamma=1} \\ &\quad \cong -\frac{1}{3.4}\mathcal{A}(\psi/J \to \chi\gamma, \chi \to \theta)\big|_{\lambda_\theta=0,\,\lambda_\gamma=1}\,.\end{aligned} \tag{21}$$

We would like to emphasize that the relation (21) does not depend very much on the specific values of f_2 and $f_{\psi/J}$. We conclude that the χ state does indeed make a significant contribution to the $\lambda_\theta = 0$ production of the θ in the ψ/J decay systematics.

Our objective was to compute the values of $\bar{x}$ and $\bar{y}$ in this QCD scenario as we have manifested it via the method of the effective Lagrangian. Turning to this computation now, we note that the amplitudes A_j, $j = 0,1,2$, used to define $\bar{x}$ and $\bar{y}$ may be identified, for $\lambda_\gamma = 1$, as the invariant coefficients in the respective amplitude T_{M1} of Jacob and Wick[19] for the two-step process $\psi/J \to \theta\gamma$, $\theta \to K^+K^-$, where M is the projection of the ψ/J spin in the laboratory frame (the ψ/J rest frame) along the e^-

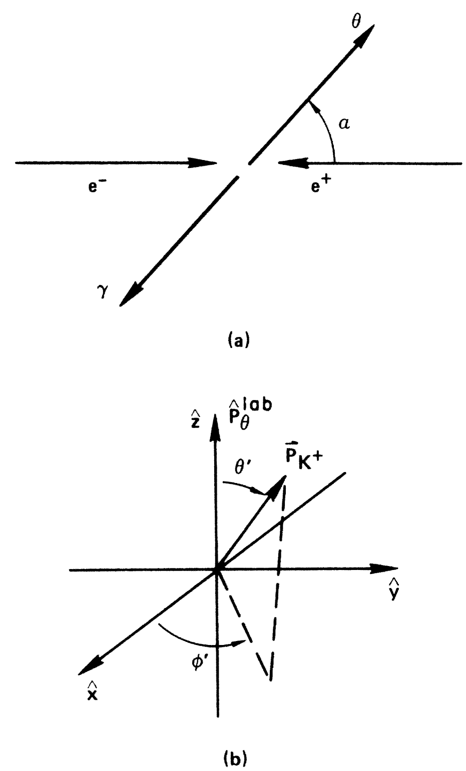

FIG. 4. Kinematics for $e^+e^- \to \psi/J$, $\psi/J \to \theta\gamma$, $\theta \to K^+K^-$. The laboratory frame is the ψ/J rest frame so that α, as shown in (a), is the θ production angle in this frame. In (b), the spherical angles of the K^+ momentum $\mathbf{P}_{K^+}$ in the θ rest frame are shown. Thus, $\hat{\mathbf{P}}_\theta^{\text{lab}} = \hat{\mathbf{z}}$ is the direction of the θ three-momentum in the laboratory frame.

beam direction [we denote this direction by $\hat{\mathbf{z}}_{\rm lab}$; it is related to the z direction (the direction of the θ in the laboratory) by $\hat{\mathbf{z}}_{\rm lab}=\cos\alpha\,\hat{\mathbf{z}}-\sin\alpha\,\hat{\mathbf{x}}$ so that α is the production angle of the θ in the laboratory]. All of these kinematics are summarized in Fig. 4. As we indicate in this figure, we take the K^+ to have the spherical angles (θ',ϕ') about the z direction in the θ rest frame. Specializing to $M=1$, we may write

$$T_{11}=D^{2*}_{20}(\phi',\theta',-\phi')D^1_{11}(0,\alpha,0)A_2+D^{2*}_{00}(\phi',\theta',-\phi')D^1_{1-1}(0,\alpha,0)A_0+D^{2*}_{10}(\phi',\theta',-\phi')D^1_{10}(0,\alpha,0)A_1\,, \quad (22)$$

where $D^J_{mm'}$ are the usual D functions of the rotation group. Thus, (22) is the same as

$$T_{11}=e^{2i\phi'}\sin^2\theta'\left[\frac{\sqrt{6}}{4}\right]\frac{(1+\cos\alpha)}{2}A_2+\left(\tfrac{3}{2}\cos^2\theta'-\tfrac{1}{2}\right)\frac{(1-\cos\alpha)}{2}A_0+e^{i\phi'}\left[\frac{3}{2}\right]^{1/2}\sin\theta'\cos\theta'\frac{\sin\alpha}{\sqrt{2}}A_1\,. \quad (23)$$

On the other hand, from (3), we have the θ-decay amplitude

$$\mathcal{A}(\theta\to K^+K^-)=\frac{4ig^2(m_\theta{}^2)}{N_c m_\theta{}^2}f_2\langle K^+K^-\,|\,O_{\alpha_1\alpha_2}(0)\,|\,0\rangle\epsilon^{\alpha_1\alpha_2}(\lambda_\theta)\,, \quad (24)$$

where $O_{\alpha_1\alpha_2}$ is the quark contribution to the QCD energy-momentum tensor:

$$O_{\alpha_1\alpha_2}=(i\bar\psi_q\gamma_{\alpha_2}D_{\alpha_1}\psi_q+\text{H.c.})/2\,. \quad (25)$$

On very general grounds (Lorentz covariance, C, P, and T) we may write

$$\langle K^+K^-\,|\,O_{\alpha_1\alpha_2}(0)\,|\,0\rangle=[F_1(P_{K^+}+P_{K^-})_{\alpha_1}(P_{K^+}+P_{K^-})_{\alpha_2}+F_2(P_{K^+}-P_{K^-})_{\alpha_1}(P_{K^+}-P_{K^-})_{\alpha_2}/4+F_4 m_\theta{}^2 g_{\alpha_1\alpha_2}]/[(2\pi)^6 4P^0_{K^+}P^0_{K^-}]^{1/2} \quad (26)$$

for some invariant functions $F_i(m_\theta{}^2,m_K{}^2)$, where P_B is the four-momentum of $B, B=K^+,K^-$. Thus, we have

$$\epsilon^{\alpha_1\alpha_2}(\lambda_\theta)\langle K^+K^-\,|\,O_{\alpha_1\alpha_2}(0)\,|\,0\rangle=F_2\epsilon^{\alpha_1\alpha_2}(\lambda_\theta)P_{K^+\alpha_1}P_{K^+\alpha_2}/[(2\pi)^6 4P^0_{K^+}P^0_{K^-}]^{1/2}\,. \quad (27)$$

In Ref. 6, we have argued, using the methods of Lepage and Brodsky,[20] that for glueballs of sufficient mass $F_2=-\frac{14}{3}F_K$, where F_K is the kaon form factor. Since $m_\theta\cong 1.722$ GeV, the appropriateness of this result for F_2 is somewhat unclear here and, hence, to this extent, the absolute normalization of F_2 is uncertain. For our present purposes, this uncertainty is irrelevant since we only want to compute A_2/A_0 and A_1/A_0. Indeed, toward this end we note that, for $M=1$, in the θ rest system

$$\epsilon_{\psi/J}=\frac{-\sin\alpha}{\sqrt{2}}\epsilon'_{\psi/J}(0)+\frac{(1+\cos\alpha)}{2}\epsilon'_{\psi/J}(1)+\frac{(1-\cos\alpha)}{2}\epsilon'_{\psi/J}(-1)\,,$$

where $\epsilon'_{\psi/J}(s_z)$ is the four-polarization of the ψ/J particle in the θ rest system when the ψ/J has spin projection s_z. Thus, if we combine (20), (21), (24), and (27) we have, suppressing irrelevant kinematic factors,

$$T_{11}=\frac{4g^2(m_\theta{}^2)f_2F_2}{N_c m_\theta{}^2}\frac{eg^2(m_{\psi/J}{}^2)}{2N_c}f_2 f_{\psi/J}m_{\psi/J}\left[\frac{m_{\psi/J}(E^{\rm lab}_\theta-E^{\rm lab}_\gamma)-2m_G{}^2}{(m_{\psi/J}E^{\rm lab}_\theta/2-m_G{}^2)^2}\right]\times\left[e^{2i\phi'}\sin^2\theta'\frac{(1+\cos\alpha)}{2}\left(-\tfrac{1}{2}\right)|\mathbf{P}_K|^2+\frac{(1-\cos\alpha)}{2}\left(\tfrac{3}{2}\cos^2\theta'-\tfrac{1}{2}\right)\frac{|\mathbf{P}_K|^2}{3}(-1+3.4)+\frac{\sin\alpha}{2}e^{i\phi'}\sin\theta'\cos\theta'(-|\mathbf{P}_K|^2)E^{\rm lab}_\theta/m_\theta\right]\,, \quad (28)$$

where $|\mathbf{P}_K|$ is the K^+ three-momentum magnitude in the θ rest frame. On comparing (23) and (28), we conclude that A_2, A_1, and A_0 lie in the ratio

$$A_2{:}A_1{:}A_0=-2/\sqrt{6}{:}-E^{\rm lab}_\theta/m_\theta\sqrt{3}{:}2.4/3=-0.816{:}-0.68{:}0.8\,. \quad (29)$$

In this way, we find

$$\bar{x}\cong-0.85,\quad \bar{y}\cong-1.0\,, \quad (30)$$

in reasonable agreement with the data in (1) when one allows for the uncertainty in our methods and in the data.

A natural question to ask is whether the branching-ratio product $B(\psi/J\to\theta\gamma)B(\theta\to K^+K^-)$ is in reasonable agreement with the prediction which would follow from (30). We re-emphasize that, while the decay $\psi/J\to\theta\gamma$ is expected to be given accurately by our methods, the decay $\theta\to K^+K^-$ involves the computation of the exclusive function F_2 in a regime in which the corresponding fragmenting (anti)quarks have energy $<1_+$ GeV. It is expected that the methods used in Ref. 6 to compute F_2 are incomplete here. Rather, a large-distance method should be used to augment the perturbative methods in Ref. 6. Such a large-distance method exists,[21] but its use here would take us beyond the scope of the current discussion. For this reason, the value of $B(\psi/J\to\theta\gamma)B(\theta\to K^+K^-)$ will be taken up elsewhere. Our prediction for the width $\Gamma(\psi/J\to\theta\gamma)$ is in fact expected to be reliable and we record it here for completeness (here, $\alpha=e^2/4\pi$):

$$\Gamma(\psi/J\to\theta\gamma)=16\alpha g^4(1-E_\gamma^{\rm lab}/E_\theta^{\rm lab}-2m_G{}^2/E_\theta^{\rm lab}m_{\psi/J})^2E_\gamma^{\rm lab}f_{\psi/J}{}^2f_2{}^2\frac{\frac{2}{3}+\frac{1}{2}[(E_\theta^{\rm lab})^2+m_\theta{}^2]/m_\theta{}^2+\frac{1}{6}[(\mathcal{A}_0^\chi/\mathcal{A}_0^p+1)^2-1]}{3(2N_c)^2(E_\theta^{\rm lab})^2m_{\psi/J}{}^2(1-2m_G{}^2/E_\theta^{\rm lab}m_{\psi/J})^4}, \tag{31}$$

where $\mathcal{A}_0^\chi$ is the contribution to the $\lambda_\theta=0$ amplitude for $\psi/J\to\theta\gamma$ from the θ-χ mixing process and $\mathcal{A}_0^p$ is the perturbative contribution to the $\lambda_\theta=0$ amplitude for $\psi/J\to\theta\gamma$. In our lattice-bag treatment of f_2, as it is described in Ref. 6, we would estimate $f_2\cong0.387$ GeV. Hence, the value $\mathcal{A}_0^\chi/\mathcal{A}_0^p=-3.4$ derived herein implies

$$B(\psi/J\to\theta\gamma)\cong0.99\% \ , \tag{32}$$

where we have taken[6] $g^2(m_{\psi/J}{}^2)/4\pi\cong0.179$ and[15] $\Gamma(\psi/J\to\text{all})=63$ KeV. We know of no obvious problem of (32) in relation to observation. A more direct check of our methods will ensue with the detailed treatment of $B(\theta\to K^+K^-)$; to repeat, such a treatment will be taken up elsewhere.

Thus, on the basis of our results in this section, we may say that there is no obvious disagreement between the data in (1) and the TE-TE glueball view of the $\theta(1700)$. Such a conclusion is significant enough that we feel a few remarks are appropriate concerning the possible sources of errors in our various approximations. The key approximation is the off-shell theory of the χ interactions. Both in Fig. 3(b) and in Fig. 3(c), we have used a nonrelativistic theory of the χ in the context of the Van Royen—Weisskopf approximation. We can only cite the success of this approximation in providing a reasonable phenomenology of the decay of mesons such as the ρ meson to $l\bar{l}$, $l=e,\mu$. Correspondingly, we do feel that our χ analysis is not without some justification. The specific potential model (15) may also be questioned. We feel, however, that the various alternatives all are known[22] to be very similar in the regime wherein most of the support of $u(\rho)$ lies for the χ system. Thus, again, we do not expect our results to be very sensitive to the specific choice of (15) for $V(r)$. These remarks, then, appear to substantiate the position that there is no obvious source of a large error in our work on the process $\psi/J\to\theta\gamma$.

IV. CONCLUSION

We feel it is very encouraging that the data in (1) are actually consistent with the TE-TE glueball view of the $\theta(1700)$ as interpreted via our methods. Such consistency supports the candidacy of QCD as the theory of the strong interaction.

It should be re-emphasized, however, that, for example, the $q\bar{q}G$ interpretation of $\theta(1700)$ is not excluded by our analysis since we have not considered such a scenario.

Further, we cannot exclude the interesting possibility[23] that the θ is a mixture of a glueball and a $q\bar{q}$ state at the $\sim$20—30% level since this is the level of the uncertainty in our methods. Thus, this scenario, too, should be pursued.

We end by noting that, from the perspective of QCD, the interpretation of the $\theta(1700)$ as an alternative scenario ($q\bar{q}G$, etc.) would not be a disaster — rather, it would be something in nature that, from a theoretical standpoint, could very well have been otherwise. There are many things like this.

Notes added

(1) Our use of the TE, TM classification of Ref. 3 of the gluon states for a massive gluon in a spherical cavity is, strictly speaking, only claimed to be correct to leading order in g in the MIT bag theory. Thus, our effective-Lagrangian methods, as we have implemented them, are only correct to leading order in g. Indeed, recall from Ref. 6 and from the work of J. M. Cornwall [Phys. Rev. D **10**, 500(1974); Nucl. Phys. **B157**, 392 (1979)] that the auxiliary fields ϕ^a associated with $A_{G\mu}^a$ in a gauge-invariant formulation of massive QCD satisfy $-\Box\phi^a=g\partial\cdot A_G^a$, to the order of our approximations, inside our spherical MIT bag. The $A_{G\mu}^a$ field equations give

$$\Box A_{Gi}^a-\partial_i\partial^jA_{Gj}^a=-m_G{}^2\left|A_{Gi}^a+\frac{1}{g}\partial_i\phi^a\right| \ ,$$
$$\partial^0\partial^iA_{Gi}^a=(m_G{}^2/g)\partial^0\phi^a$$

in our $A_{G0}^a=0$ gauge, to the order of our approximations. Thus, $m_G{}^2\phi^a/g=\partial^iA_{Gi}^a+C^a(\mathbf{x})$ for some time-independent $C^a(\mathbf{x})$. This means that $(\Box+m_G{}^2)\partial^0\phi^a=0$ inside the bag so that, since either $\phi^a|_S=0$ or $n\cdot\partial\phi^a|_S=0$ (n^μ is the unit-normal four-vector to S of Ref. 4), the Green's theorem implies [see J.D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975)], for an eigenmode ϕ_l^a, either $\partial^0\phi_l^a=0$ or $\partial^0\phi_l^a=C_l^a$ inside the bag, for some constant C_l^a, where S is the bag's boundary. For ϕ_l^a we may write $\phi_l^a=e^{-iw_lt}\bar{\phi}_l^a(\mathbf{x})$ so that $\partial^0\phi_l^a=0$ or $\partial^0\phi_l^a=C_l^a$ implies either $w_l=0$ or $\phi_l^a=0$. If $w_l=0$, the electric field vanishes identically, whereas the magnetic field B_{Glk}^a satisfies $(-\nabla^2+m_G{}^2)B_{Glk}^a=0$; the non-negativity of $-\nabla^2$ then forces $B_{Glk}^a=0$. If $w_l\neq0$, $\phi_l^a=0$ so that the $A_{Gl\mu}^a$ field satisfies $(\Box+m_G{}^2)A_{Gli}^a=0$, the MIT quadratic boundary condition picks up a mass term $m_G{}^2\mathbf{A}_G{}^2$, and the confinement condition on $\mathbf{F}_{G\mu\nu}$ still reads $n^\mu\mathbf{F}_{G\mu\nu}|_S=\mathbf{0}$. Thus, the classification of states used in Ref. 3 still applies.

(2) The spinors in the manuscript have an extra factor of $\sqrt{E/m_c}$ relative to those in Ref. 13.

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