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Dispersive effects in D^0-$\bar{D}^0$ mixing

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We estimate the size of dispersive effects in $\Delta C=2$ interactions and find that they represent in fact the dominant components of D^0-$\bar{D}^0$ mixing. The box diagram is suppressed by several orders of magnitude due to the small values of light-quark masses. Crucial to this argument is a discussion of how the Glashow-Iliopoulos-Maiani mechanism works in the dispersive sector.

I. INTRODUCTION

Some of the most sensitive probes of higher-order effects in weak-interaction gauge theories are mixing amplitudes which change a flavor quantum number by two units, e.g., K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, and B^0-$\bar{B}^0$ mixings. The standard way to generate such effects is via the box diagram[1] [Fig. 1(a)] which is second order in the weak interactions. However, it is also always possible to produce the flavor-violating transition by a product of weak-interaction processes to specific mesonic intermediate states, for example, as in the D^0-$\bar{D}^0$ system

$$\sum_I \frac{\langle D^0 | H_w | I\rangle\langle I | H_w | \bar{D}^0\rangle}{m_D{}^2-m_I{}^2+i\epsilon} \tag{1}$$

depicted in Fig. 1(b). Such dispersive effects were studied in the kaon system and were found to be comparable with the box-diagram contributions.[2] For B^0-$\bar{B}^0$ mixing we will argue below that dispersive effects are negligible, as expected. In the D^0-$\bar{D}^0$ sector, however, we will show that the natural scale of dispersive effects is between 1–2 orders of magnitude larger than that of the box diagram. While this seems at first sight to be surprising, it appears more obvious after the origin of the effect is understood.

Let us briefly summarize the status of the short-distance (box-diagram) component D^0-$\bar{D}^0$ mixing.[3] The intermediate quark lines are those of s,d quarks, and the Lagrangian has the structure

$$L^{\Delta C=2}=\frac{G_F}{\sqrt{2}}\frac{\alpha}{8\pi\sin^2\theta_W}\xi_s\xi_d\frac{(m_s{}^2-m_d{}^2)^2}{M_W{}^2m_c{}^2}(O+2O')\ , \tag{2}$$

where $\xi_i=V_{ic}^*V_{iu}$ [V is the Kobayashi-Maskawa (KM) matrix]. In addition to the expected four-fermion operator

$$O=\bar{u}\gamma^\mu(1+\gamma_5)c\,\bar{u}\gamma_\mu(1+\gamma_5)c\ , \tag{3a}$$

there is a new $\Delta C=2$ local operator

$$O'=\bar{u}(1-\gamma_5)c\,\bar{u}(1-\gamma_5)c\ , \tag{3b}$$

which is induced by the presence of the non-negligible charm-quark mass m_c carried by two of the four external legs of the box diagram. The most noteworthy aspect of Eq. (2) is the dimensionless factor $(m_s{}^2-m_d{}^2)^2/M_W{}^2m_c{}^2$. With the values $m_s\simeq0.3$ GeV and $m_c\simeq1.5$ GeV, this is roughly 600 times smaller than the corresponding term $m_c{}^2/M_W{}^2$ which appears for K^0-$\bar{K}^0$ mixing. This suppression has two sources. In K^0-$\bar{K}^0$ mixing, the dominant intermediate states are associated with the c quark whereas in the D^0-$\bar{D}^0$ case this role is played by the s quark. An additional suppression $(m_s{}^2-m_d{}^2)/m_c{}^2\sim0.04$ arises because the momentum of the heavy external-leg charm quark must be transferred through the light s,d fermion propagators. Incidentally, just as the t quark has essentially no effect in K^0-$\bar{K}^0$ mixing due to tiny KM angles, b-quark intermediate states are analogously negligible in the D^0-$\bar{D}^0$ system. They will henceforth be neglected in favor of the lighter quarks. However as we have seen, even these have highly suppressed short-distance contributions.

The main point of our paper is that the Glashow-Iliopoulos-Maiani (GIM) suppression does not appear to be nearly as strong in the dispersive sector. First, we must explain how the GIM mechanism works for such contributions. In view of the unimportance of b-quark contributions we shall find it convenient to use the four-quark KM matrix in our discussion. The $\Delta C=1$ weak Hamiltonian thus has the form

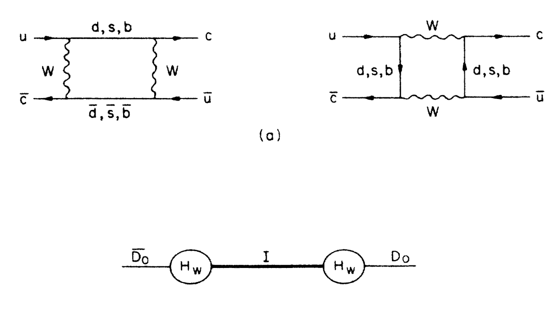

FIG. 1. Mechanisms for generating $\bar{D}^0$-D^0 mixing. The short-distance (box) contribution and the long-distance contribution from intermediate-state I are depicted in (a) and (b), respectively.

$$H_w^{\Delta C=1}=\cos^2\theta_C \bar{u}\Gamma_L^\mu d\,\bar{s}\Gamma_{L\mu}c+\sin\theta_C\cos\theta_C(\bar{u}\Gamma_L^\mu d\,\bar{d}\Gamma_{L\mu}c-\bar{u}\Gamma_L^\mu s\,\bar{s}\Gamma_{L\mu}c)-\sin^2\theta_C\bar{u}\Gamma_L^\mu s\,\bar{d}\Gamma_{L\mu}c\ , \tag{4}$$

where $\Gamma_L^\mu \equiv \gamma^\mu(1+\gamma_5)$. Now consider that piece of the dispersive component arising from the two-charged-pseudoscalar-meson states, viz., $K^-\pi^+, K^-K^+, \pi^+\pi^-, K^+\pi^-$. SU(3) symmetry predicts these transitions to occur in D^0 decay with relative strengths

$$K^-\pi^+:K^-K^+:\pi^-\pi^+:\pi^-K^+=\cos^4\theta_C:\cos^2\theta_C\sin^2\theta_C:\cos^2\theta_C\sin^2\theta_C:\sin^4\theta_C\ . \tag{5}$$

Likewise they occur as intermediate states in D^0-$\bar{D}^0$ mixing, all with [in the SU(3) limit] identical strength, $\cos^2\theta_C\sin^2\theta_C$, but with relative signs

$$K^-\pi^+:K^-K^+:\pi^-\pi^+:\pi^-K^+=-1:1:1:-1\ . \tag{6}$$

The GIM mechanism is thus realized in the SU(3) limit by the vanishing sum of these dispersive contributions. However, for D^0 decays SU(3) symmetry is known to be badly broken. From available experimental data on D^0 decay widths we find[5]

$$K^-\pi^+:K^-K^+:\pi^-\pi^+:\pi^-K^+=\cos^4\theta_C:\cos^2\theta_C\sin^2\theta_C(2.42\pm0.36\pm0.28):\cos^2\theta_C\sin^2\theta_C(0.65\pm0.20\pm0.12):f_{K^+\pi^-}\sin^4\theta_C\ , \tag{7}$$

to be compared with Eq. (5). The parameter called $f_{K^+\pi^-}$ has not yet been measured. The SU(3) violation of Eq. (7) can occur for a number of reasons since these decay amplitudes are sensitive to all aspects of hadronic physics. Thus, for example, the D^0 lies in the middle of the resonance region. The presence of resonances or final-state interactions can strongly shift the decay strengths. We note that in the $J^P=0^+$ channel relevant here, there is a nonstrange resonance at 1300 MeV, a strange $K\pi$ resonance at 1450 MeV, and therefore an expected $s\bar{s}$ resonance near 1600—1700 MeV. The closeness of these resonances to the decay region might well play a role in explaining the disparate relative strengths of the $K^-\pi^+$, K^-K^+, and $\pi^+\pi^-$ signals. In any case whatever the origin of the SU(3) breaking, we do *not* obtain a substantial GIM cancellation in the two charged-pseudoscalar dispersive components. Instead we expect an effect of order

$$|A_0|^2\cos^2\theta_C\sin^2\theta_C[2.42+0.65-2(f_{K^+\pi^-})^{1/2}]\ . \tag{8}$$

While unfortunately we do not have an experimental measure of $f_{K^+\pi^-}$, we will argue below that $f_{K^+\pi^-}\sim 1$ is reasonable, yielding a sizable remainder. Our point is then that the GIM cancellation in the dispersive component, while present, is appreciably modified by long-distance dynamics, and is *not* expected to be as severe as the result, Eq. (3) obtained for the box diagrams.

We do *not* have sufficient data or calculational power to evaluate with any reliability the full D^0-$\bar{D}^0$ mixing parameters through dispersive effects, and hence will have to settle for an order-of-magnitude estimate. One worry is that, although one particular channel might be sizable, through some sort of "closure" or "duality" effect the *sum* of all dispersive contributions could yield a suppression like that of the box diagram. While we cannot rule this scenario out, it would require very artificial cancellations between processes which have quite different long-distance dynamics, and hence would be very unnatural. We argue that D^0-$\bar{D}^0$ mixing is *not* short-distance dominated. We now turn to more quantitative estimates to support this assertion.

II. NUMERICAL ESTIMATES

In order to have a standard against which to compare our dispersive calculation, we recall that the box contribution to Δm_D is given by (omitting b-quark contributions and taking $m_d \simeq 0$)

$$\Delta m_D^{\text{box}}=\frac{G_F}{\sqrt{2}}\frac{\alpha}{4\pi\sin^2\theta_W}\xi_s\xi_d\frac{m_s{}^4}{M_W{}^2m_c{}^2}\frac{8}{3}\times m_D F_D{}^2(B_D-2B_D')\ , \tag{9}$$

where the quantities B_D, B_D' are defined by

$$\langle D^0|O|\bar{D}^0\rangle=\frac{16}{3}\frac{m_D{}^2F_D{}^2}{2m_D}B_D\ , \tag{10a}$$

$$\langle D^0|O'|\bar{D}^0\rangle=-\frac{10}{3}\left[\frac{m_D}{m_c}\right]^2\frac{m_D{}^2F_D{}^2}{2m_D}B_D'\ , \tag{10b}$$

and we have taken $(m_D/m_c)^2\simeq 1.6$. Numerically we find

$$\Delta m_D^{\text{box}}\simeq 2.5\times10^{-17}\ \text{GeV}\left[\frac{m_s}{0.3}\right]^4\left[\frac{F_D}{F_\pi}\right]^2 \tag{11}$$

which is considerably smaller than the quoted upper bound[6]

$$|\Delta m_D^{\text{expt}}|\lesssim 6\times10^{-13}\ \text{GeV}\ . \tag{12}$$

In Eq. (11) we have for simplicity taken[7] $B_D=B_D'=1$, scaled F_D with F_π (Ref. 8), and scaled m_s with 0.3 GeV (Ref. 9). Consider next an estimate of the dispersive contribution.

There is, of course, no strictly reliable procedure for calculating the dispersive component to the weak mixing. We will use a method which is certainly adequate for obtaining order-of-magnitude estimates. If one calculates the loop diagram of Fig. 2 or equivalently if one uses dispersive methods one obtains a self-energy of the form

$$\Sigma(p^2)=A(g)[\ln(-p^2)+\cdots]\ , \tag{13}$$

where A depends on the interaction employed but is quad-

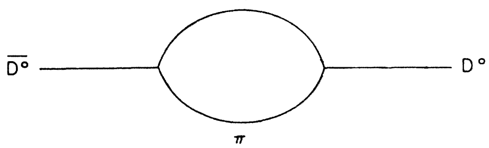

FIG. 2. Self-energy loop diagram. For definiteness, the intermediate state is taken to be $K\pi$.

ratic in the coupling constant g. The ellipses denote various constant factors whose numerical value depends upon the form of the vertex. The logarithm however is universal, in that the imaginary part, which arises when one evaluates

$$\ln(-p^2)=\ln p^2+i\pi \tag{14}$$

must yield exactly the decay rate into that channel. In other words the coefficient of the logarithm is determined by the decay rate

$$\frac{\Delta m}{\Gamma}=\frac{1}{2\pi}\ln p^2/\mu^2+\cdots . \tag{15}$$

Here μ is a parameter with dimensions of mass which makes the argument of the logarithm dimensionless. It arises naturally in dispersive or loop calculations as the effective cutoff of the vertex function, and we will use $\mu \approx 1$ GeV. Our procedure to obtain the order of magnitude of the dispersive component is then to retain the logarithmic term only. As to the adequacy of this estimate, we note that a dispersive approach to calculation of the 2π contribution to Δm_K yields in a chiral model with a cutoff Λ (Ref. 2)

$$\cdots=\frac{1}{2\pi}\frac{\Lambda^2}{p^2} \tag{16}$$

while a perturbative method yields[2]

$$\cdots=\frac{1}{2\pi}\left[\frac{19}{12}\frac{\Lambda^2}{p^2}+\frac{121}{120}-\frac{93}{140}\frac{p^2}{\Lambda^2}\right], \tag{17}$$

where 700 MeV $\lesssim \Lambda \lesssim 1$ GeV is a high-energy cutoff. We see that in either case the omitted terms are of order

$$\frac{\Lambda^2/p^2}{\ln\Lambda^2/p^2}\lesssim 3 \tag{18}$$

so that the ln indeed provides a reasonable *order-of-magnitude* estimate of the dispersive component.

With the use of this simple prescription, one can readily calculate the size of the dispersive effects. However, care must be taken to include all the modes needed to produce the GIM cancellations, as described above. Let us give the estimate for the case of pairs of charged pseudoscalars. We find

$$\Delta m_D^{\text{disp}} \simeq \frac{1}{2\pi}\ln\frac{m_D{}^2}{\mu^2}\{\Gamma(D^0\to K^+K^-)+\Gamma(D^0\to\pi^+\pi^-)-2[\Gamma(D^0\to K^-\pi^+)\Gamma(D^0\to K^+\pi^-)]^{1/2}\}$$
$$\simeq 2\times 10^{-15}\ \text{GeV}[1-0.65(f_{K^+\pi^-})^{1/2}] . \tag{19}$$

The unknown parameter $f_{K^+\pi^-}$ is expected in quark models to be quite close to unity. In models where final-state effects are the prime determination of SU(3) breaking, $K^-\pi^+$ and $K^+\pi^-$ have the same final-state interactions, a situation which would lead to $f_{K^+\pi^-}$ equaling unity. In any case the GIM cancellation is to be expected to reduce the basic amplitude to some fraction of the original amplitude. Typically for SU(3) breaking that fraction should be 20—30 %, and there does not appear to be a reason to suspect any smaller effects here, as there is known to be large SU(3) breaking in the K^+K^- and $\pi^+\pi^-$ decay rates. We thus find that from the two charged pseudoscalar intermediate states we expect

$$\Delta m_D \simeq 0.7\times 10^{-15}\ \text{GeV} \tag{20}$$

to be compared to the box-diagram contribution

$$\Delta m_D^{\text{box}} \approx 2.5\times 10^{-17}\ \text{GeV} . \tag{21}$$

Thus this particular dispersive channel is a factor of 30 larger than the box contribution, for reasons which were outlined above.

Unfortunately it is impossible to examine other intermediate states with any degree of reliability, since complete data on other sets of Cabibbo suppressed modes ($\Delta C=1$, $\Delta S=0$) does not exist. However, there is no reason to suspect that SU(3) breaking should be any smaller in these cases. Thus, for example, if final-state interactions are the source of the large SU(3) violation, the same 0^+ resonance should play a role in the four-pseudoscalar channels

$$K^-\pi^+\pi^+\pi^-,\ \bar{K}^-\pi^+\pi^0\pi^0,\ \pi^+\pi^+\pi^-\pi^-,\ \text{etc.} ,$$

although the three-pseudoscalar channels

$$K^-\pi^+\pi^0,\ \bar{K}^0\pi^+\pi^-,\ \text{etc.} ,$$

would have different dynamical breaking. Taking 20% SU(3) breaking for these channels, we anticipate additional dispersive effects of order

$$\begin{aligned}
\Delta m_D(\text{4-pseudoscalar}) &\sim \frac{1}{2\pi}\ln\frac{{m_D}^2}{\mu^2}2\sin^2\theta_C B(K^-\pi^+\pi^+\pi^-)\Gamma_{\text{tot}}\times 20\% \\
&\sim 0.6\times 10^{-15}\,, \\
\Delta m_D(\text{3-pseudoscalar}) &\sim \frac{1}{2\pi}\ln\frac{{m_D}^2}{\mu^2}2\sin^2\theta_C B(K^-\pi^+\pi^0)\Gamma_{\text{tot}}\times 20\% \\
&\sim 1\times 10^{-15}\,.
\end{aligned} \tag{22}$$

Now the sign of these effects is *a priori* unknown so that there *could* exist cancellation among these (and other) components which might affect the order of magnitude. However, this would require a rather miraculous set of circumstances, and we think this to be unlikely (nor do we feel that the other possibility—a constructive interference—is a reasonable likelihood). Thus we believe that the dispersive contribution to the D-$\bar{D}$ mass difference is

$$\Delta m_D^{\text{disp}} \sim 10^{-15}\ \text{GeV} \tag{23}$$

which is much smaller than the experimental number, Eq. (12), but is also more than an order of magnitude larger than the box diagram.

In the B meson sector, the dispersive effects are not likely to be important. There the dominant intermediate states are top and charm quarks, so that the short-distance analysis of the box diagram should be valid. As long as the KM matrix element connecting top and down quarks is not too small, the top-quark contribution will be largest. A reasonable approximation to the box diagram is then found to be[10]

$$\Delta m_B^{\text{box}} \simeq \frac{G_F}{\sqrt{2}}\frac{\alpha}{8\pi\sin^2\theta_W}\frac{8M_B{F_B}^2B_B}{3}{\xi_t}^2\left[\frac{m_t}{M_W}\right]^2, \tag{24}$$

where $\xi_t = V_{tb}^{*}V_{td}$, and B_B is defined as in Eq. (10a) except for the $B^0\bar{B}^0$ system. Numerically we obtain

$$\Delta m_B^{\text{box}} \approx 1.6\times 10^{-13}\ \text{GeV}\left[\frac{\xi_t}{0.024}\right]^2\left[\frac{F_B}{F_\pi}\right]^2\left[\frac{m_t}{40\ \text{GeV}}\right]^2 \tag{25}$$

for $B_B = 1$. The dispersive component could only arise at the level of intermediate states with no charm or strangeness. However, the dominant decays of B^0 all contain either net charm or net strangeness, and hence any possible dispersive effect is also strongly suppressed by small KM angles. To give an estimate, let us consider the $\pi\pi$ intermediate state. The dispersive Δm would involve the standard sizes of two body branching ratios (we will use 0.01 for this) and also the $b\to u$ probability $[(b\to u)/(b\to c) < 0.04]$ to produce an estimate

$$\begin{aligned}
\Delta m_B^{\text{disp}} &\lesssim \frac{1}{2\pi}\ln {m_B}^2/\mu^2(0.01)(0.04)\Gamma_B \\
&\approx 10^{-16}\ \text{GeV}\,.
\end{aligned} \tag{26}$$

Thus we conclude that the box diagram is most important for $B^0\bar{B}^0$ mixing. It is interesting to note that all three examples of flavor mixing lead to different results

$$\begin{aligned}
&\Delta m_K^{\text{disp}} \approx \Delta m_K^{\text{box}}\,, \\
&\Delta m_D^{\text{disp}} \gg \Delta m_D^{\text{box}}\,, \\
&\Delta m_B^{\text{disp}} \ll \Delta m_B^{\text{box}}\,.
\end{aligned} \tag{27}$$

III. CONCLUSIONS

Our estimates support the idea that dispersive effects are much larger than the box diagram in D^0-$\bar{D}^0$ mixing. The reason is not that the dispersive contributions are any larger than expected, but rather the box diagram is suppressed strongly due to the small quark masses which appear. The main concern regarding this conclusion is the difficulty of making reliable statements about the long-distance components. In particular, at very high mass one could argue that the sum over all intermediate states of physical hadrons should reproduce the box diagram by a form of completeness. For sufficiently large mass this probably is correct, but the charm-quark mass is in an intermediate region where the argument is of dubious validity. There are many dynamical effects in the energy range 1—2 GeV which can influence the final states in D decay, each mode in a different way. As we have seen, reasonably small SU(3) breaking produces a dispersive mixing with a natural scale much larger than that of the box diagram.

Even with the larger estimate of $D^0\bar{D}^0$ mixing provided here, it appears difficult to *observe* a signal of this size. The present experimental bound[6]

$$|\Delta m| < 6\times 10^{-13}\ \text{GeV} \tag{28}$$

is still much larger than our estimate. However should experimental techniques improve sufficiently to make such measurements possible, it has been claimed that a mixing larger than that of the box diagram would be a signal of new physics beyond the standard model.[11] For mixing in the range discussed here, this conclusion does not appear to be justified. However mixing observed at a much larger rate than discussed here would imply new physics such as a fourth fermion generation,[12] etc.

Note added in proof. After submission of this paper, we received a paper by L. Wolfenstein in which this problem is studied and similar conclusions are reached.

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