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Nucleon form factors in an independent-quark model based on Dirac equation with power-law potential

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The nucleon electromagnetic form factors $G_E^p(q^2)$ and $G_M^p(q^2)$ and the axial-vector form factor $G_A(q^2)$ are investigated in a simple model of relativistic quarks confined by a vector-scalar mixed potential $U_q(r)=(1+\gamma^0)(a^{\nu+1}r^\nu+V_0)$ without taking into account the center-of-mass correction and the pion-cloud effects. The respective rms radii associated with $G_E^p(q^2)$ and $G_A(q^2)$ come out as $\langle r_c{}^2\rangle^{1/2}=1.07$ fm and $\langle r_A{}^2\rangle^{1/2}=1.17$ fm. The possibility of restoring in this model the chiral symmetry in the usual way is discussed and the pion-nucleon form factor $G_{\pi NN}(q^2)$ is derived. The pion-nucleon coupling constant is obtained as $g_{\pi NN}=10.2$, as compared to $(g_{\pi NN})_{\rm expt}\simeq 13$.

I. INTRODUCTION

The usefulness of the study of electromagnetic form factors in connection with the description of the size and compositeness of hadrons led many authors to explain the available data within the framework of various potential models. Several authors[1] have recently attempted to study the nucleon form factors based on bag model and other confining quark potential models. Tegen and collaborators[1] have studied the same in a model with massless quarks in confining scalar potentials of the cr^2 and cr^3 type. They have reasonably accounted for the available experimental data by taking into account the center-of-mass corrections and the pion-cloud contributions. Here we present an attempt to study the quark-core contributions to the electromagnetic form factors in a model based on the Dirac equation with an independent-quark confining potential of the form

$$U_q(r)=(1+\gamma^0)(a^{\nu+1}r^\nu+V_0)\ , \tag{1.1}$$

which corresponds to an equal mixture of scalar and vector parts. Such a potential model met with reasonable success in explaining the quarkonium spectroscopy[2] as well as magnetic moments and other static properties[3] of baryons.

The organization of the paper is as follows. In Sec. II, we present a brief outline of the potential model as discussed in our earlier work.[3] Using expressions for the ground-state wave functions of the independently confined constituent quarks of the nucleon, the form factors are computed to order $\eta=|\mathbf{q}|^2/4M^2$ in the Breit frame. The $|\mathbf{q}|^2$ dependence of these form factors are studied in comparison with the experimental data in the range $0\le|\mathbf{q}|^2<1$ GeV^2. We have only considered the quark-core contributions to the form factors without taking into account the center-of-mass corrections and the pion-cloud contributions. Furthermore, we have computed the nucleon core radii associated with different form factors.

II. THE FORMALISM

A. Potential model

The potential model on which our present discussion is based upon, has been discussed in detail in our earlier work.[3] In order to make this paper self-contained, we briefly outline some important features of the model, which is based on the following assumptions.

The constituent quarks in a nucleon are assumed to move independently in an average flavor-independent potential defined in the nucleon center of mass by

$$U_q(r)=(1+\gamma^0)V(r)\ ,$$

where (2.1)

$$V(r)=(a^{\nu+1}r^\nu+V_0)\ ,$$

and a, ν, and V_0 are the potential parameters with a, $\nu>0$, and γ^0 is the usual Dirac matrix. The independent quarks of rest mass m_q ($m_u=m_d$) are believed to obey the Dirac equation

$$[\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m_q+U_q(r)]\psi_q(\mathbf{r})=E_q\psi_q(\mathbf{r})\ , \tag{2.2}$$

where $\psi_q(\mathbf{r})$ is the four-component quark wave function belonging to the energy eigenvalue E_q. It is further assumed that the quarks inside the nucleons in their ground state occupy $1S_{1/2}$ orbits and the nucleon mass (in the absence of c.m. corrections) is given by $M=3E_q$. Then in the two-component form $\psi_q(\mathbf{r})$ can be obtained as

$$\psi_q(r)=N_q\begin{pmatrix}\phi_q(\mathbf{r})\chi\uparrow\\ -i\dfrac{\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}}{\lambda_q}\phi_q'(\mathbf{r})\chi\uparrow\end{pmatrix}\ , \tag{2.3}$$

where $\lambda_q=(E_q+m_q)$ and $\phi_q(\mathbf{r})$ satisfies a Schrödinger-type equation,

$$\phi_q''+\frac{2}{r}\phi_q'+\lambda_q[E_q-m_q-2V(r)]\phi_q=0\ . \tag{2.4}$$

The overall normalization constant N_q is obtained as

$$N_q=\{\lambda_q/2[E_q-\langle V(r)\rangle]\}^{1/2}\ , \tag{2.5}$$

where $\langle V(r)\rangle$ is the weighted average of $V(r)$ with respect to $\phi_q(\mathbf{r})$.

B. Nucleon electromagnetic form factors

The Dirac-Pauli nucleon form factors $F_1(q^2)$ and $F_2(q^2)$ in the spacelike region are defined as

$$\langle N(p_2)\,|\,J^\mu(0)\,|\,N(p_1)\rangle = \bar{u}(p_2,s')\left[\gamma^\mu F_1(q^2)+i\frac{\sigma^{\mu\nu}q_\nu}{2M}F_2(q^2)\right]u(p_1,s)\ , \tag{2.6}$$

where J^μ is the nucleon current and $u(p,s)$ is the positive energy Dirac spinor of the free nucleon of four-momentum p and spin s. p_1 and p_2 are the ingoing and outgoing nucleon four-momenta. $q=(p_2-p_1)$ is the four-momentum transferred from the photon to the nucleon and

$$\sigma^{\mu\nu}=\frac{i}{2}[\gamma^\mu,\gamma^\nu]\ .$$

In view of the fact that F_1 and F_2 mix effects due to the distribution of charge and magnetization, it is customary to adopt a convenient pair of form factors $G_{E,M}(q^2)$, called the Sachs[4] form factors. These are defined as

$$\begin{aligned} G_E(q^2)&=F_1(q^2)-\eta F_2(q^2)\ ,\\ G_M(q^2)&=F_1(q^2)+F_2(q^2)\ , \end{aligned} \tag{2.7}$$

where $\eta=-q^2/4M^2$ and $q^2=(q^{02}-\mathbf{q}^2)$. $G_E(q^2)$ and $G_M(q^2)$, respectively, represent the true charge and magnetic distribution and, hence, are called the electric and magnetic form factors. Using Eq. (2.7) and the usual Gordon decomposition technique, Eq. (2.6) can be reduced to

$$\langle N(p_2)\,|\,J^\mu(0)\,|\,N(p_1)\rangle = \frac{1}{(1+\eta)}\bar{u}(p_2,s')\left[(1+\eta)G_M\gamma^\mu-\frac{G_M-G_E}{2M}p^\mu\right]\times u(p_1,s) \tag{2.8}$$

when $p^\mu=(p_1+p_2)^\mu$. It is convenient to adopt the Breit frame in which $\mathbf{p}=(\mathbf{p}_1+\mathbf{p}_2)=0$. Then $q^\mu=(0,\mathbf{q})$, $p_1{}^2=p_2{}^2=M^2$, $p_0=E=M(1+\eta)^{1/2}$, and $\mathbf{p}_1=-\mathbf{p}_2=-\mathbf{q}/2$. Using these prescriptions in Eq. (2.8) and $u(p,s)$ with the normalization $\bar{u}u=1$, the time and space components of the nucleon current matrix elements are obtained as

$$\langle N(\mathbf{q}/2)\,|\,J^0(0)\,|\,N(-\mathbf{q}/2)\rangle=\chi^\dagger_{s'}(1+\eta)^{-1/2}G_E(q^2)\chi_s \tag{2.9}$$

and

$$\langle N(\mathbf{q}/2)\,|\,\mathbf{J}(0)\,|\,N(-\mathbf{q}/2)\rangle =\chi^\dagger_{s'}(1+\eta)^{-1/2}\frac{i(\boldsymbol{\sigma}\times\mathbf{q})}{2M}G_M(q^2)\chi_s\ . \tag{2.10}$$

We now proceed to determine the quark-core contributions to $G^p_{E,M}(q^2)$ without taking into account the recoil corrections and pion-cloud contributions. Assuming the quarks to be pointlike particles in $1S_{1/2}$ orbits, the nucleon current is taken as the sum of the quark currents $\bar{\psi}_q\gamma^\mu\psi_q$ where $\psi_q(\mathbf{r})$ is the single quark Dirac wave function of the form given in (2.3). Then Eq. (2.9) yields the electric form factor of the proton as

$$\begin{aligned}(1+\eta)^{-1/2}G^p_E(q^2)&=\int d^3\mathbf{r}\,\psi^\dagger_q(\mathbf{r})\psi_q(\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\\ &=4\pi N_q{}^2\int_0^\infty dr\, r^2 j_0(\,|\mathbf{q}|\,r)\times\left[\phi_q{}^2+\frac{1}{\lambda_q{}^2}\phi'^2\right],\end{aligned} \tag{2.11}$$

where $j_0(\,|\mathbf{q}|\,r)$ is the zeroth-order spherical Bessel function. Now integrating the second term on the right-hand side of Eq. (2.11) and using Eqs. (2.4) and (2.1), we obtain

$$G^p_E(q^2)=\frac{N_q{}^2}{\lambda_q}\left[1+\frac{|\mathbf{q}|^2}{4M^2}\right]^{1/2}\left[\left[2E_q-2V_0-\frac{|\mathbf{q}|^2}{2\lambda_q}\right]\langle j_0(\,|\mathbf{q}|\,r)\rangle-2a^{\nu+1}\langle r^\nu j_0(\,|\mathbf{q}|\,r)\rangle\right]\ , \tag{2.12}$$

where $\langle j_0(\,|\mathbf{q}|\,r)\rangle$ and $\langle r^\nu j_0(\,|\mathbf{q}|\,r)\rangle$ are the expectation values with respect to $\phi_q(\mathbf{r})$. In the $q^2\to 0$ limit Eq. (2.12) yields $G^p_E(0)=1$ as expected.

We next obtain the expression for the magnetic form factor $G^p_M(q^2)$ of the proton. Using Eq. (2.10), we can obtain

$$(1+\eta)^{-1/2}\frac{i(\boldsymbol{\sigma}\times\mathbf{q})}{2M}G^p_M(q^2) =-\frac{2N_q{}^2}{\lambda_q}\int d^3\mathbf{r}(\boldsymbol{\sigma}\times\mathbf{r})e^{i\mathbf{q}\cdot\mathbf{r}}\phi_q(\mathbf{r})\phi'_q(\mathbf{r})\ . \tag{2.13}$$

Now choosing the z axis parallel to $\mathbf{q}$ inside the integration and equating the x component on both sides of Eq. (2.13), we obtain

$$(1+\eta)^{-1/2}\frac{|\mathbf{q}|}{2M}G^p_M(q^2)=-\frac{2N_q{}^2}{\lambda_q}4\pi\int_0^\infty dr\, r^2\phi_q(r)\phi'_q(r)\times j_1(\,|\mathbf{q}|\,r)\ . \tag{2.14}$$

Then integration by parts would yield

$$G_M^p(q^2)=\frac{2MN_q^2}{\lambda_q}\left[1+\frac{|\mathbf{q}|^2}{4M^2}\right]^{1/2}\langle j_0(|\mathbf{q}|r)\rangle\ , \qquad (2.15)$$

which in the $q^2\to 0$ limit reduces to

$$G_M^p(0)=\frac{2MN_q^2}{\lambda_q}\ . \qquad (2.16)$$

This is the expression for the magnetic moment μ_p of the proton as obtained in Ref. 3. If one does not take into account the coupling to the charged pion cloud it is trivial to obtain the electromagnetic form factors in this model for the neutron as

$$G_E^n(q^2)=0\ ,$$
$$G_M^n(q^2)=-\tfrac{2}{3}G_E^p(q^2)\ . \qquad (2.17)$$

Since the proton and neutron belong to an isotopic doublet, it is sometimes convenient to decompose the observed nucleon form factors into their isoscalar and isovector components. The isovector parts of the form factors $F_1(q^2)$ and $F_2(q^2)$ can be expressed here as

$$F_1^v(q^2)=\frac{(1+|\mathbf{q}|^2/4M^2)^{-1}}{2}\left[G_E^p(q^2)+\frac{5|\mathbf{q}|^2}{12M^2}G_M^p(q^2)\right]\ ,$$
$$F_2^v(q^2)=\tfrac{1}{2}(1+|\mathbf{q}|^2/4M^2)^{-1}[-G_E^p(q^2)+\tfrac{5}{3}G_M^p(q^2)]\ . \qquad (2.18)$$

C. Axial-vector form factor of the nucleon

For a nucleon consisting of three pointlike quarks $q=u,d$, in their lowest $1S_{1/2}$ orbits, the axial-vector current is the sum of the quark axial-vector currents and is given as

$$A_\lambda^\mu(\mathbf{r})=\sum_q\bar\psi_q(\mathbf{r})\gamma^\mu\gamma^5\frac{\tau_\lambda}{2}\psi_q(\mathbf{r})\ . \qquad (2.19)$$

Then the axial-vector form factor of the nucleon which measures the spin distribution of the quarks inside the nucleon is given by

$$G_A(q^2)\langle\sigma_N\tau_N^\lambda/2\rangle=\left\langle N\left|\int d^3\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}\mathbf{A}^\lambda(\mathbf{r})\right|N\right\rangle\ . \qquad (2.20)$$

Now substituting (2.19) and (2.3) in (2.20) we get,

$$G_A(q^2)\langle\sigma_N\tau_N^\lambda/2\rangle=N_q^2\left\langle N\left|\int d^3\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}\sum_q\left[\phi_q^2\sigma+\frac{(\sigma\cdot\hat{\mathbf{r}})\sigma(\sigma\cdot\hat{\mathbf{r}})}{\lambda_q^2}\phi_q'^2\right]\tau^\lambda/2\right|N\right\rangle\ . \qquad (2.21)$$

After a little spin algebra and the angular integration, one obtains with the substitution

$$\left\langle N\left|\sum_q\sigma\tau^\lambda/2\right|N\right\rangle=\tfrac{5}{3}\langle\sigma_N\tau_N^\lambda/2\rangle\ ,$$

$$G_A(q^2)=\frac{5N_q^2}{3}\left[\langle j_0(|\mathbf{q}|r)\rangle-\frac{4\pi}{\lambda_q}\int_0^\infty dr\,r^2\phi_q'^2[j_0(|\mathbf{q}|r)-2j_1'(|\mathbf{q}|r)]\right]\ . \qquad (2.22)$$

The integral in (2.22) can be simplified further by using Eq. (2.4) which yields an expression for $\phi_q'^2$ as

$$\phi_q'^2=-\frac{\lambda_q}{4}[E_q-m_q-2V(r)]r\frac{d}{dr}(\phi_q^2)-\frac{r}{4}\frac{d}{dr}(\phi_q'^2) \qquad (2.23)$$

and the Bessel-function identity

$$[j_0(|\mathbf{q}|r)-j_q'(|\mathbf{q}|r)]=\frac{2j_1(|\mathbf{q}|r)}{|\mathbf{q}|r} \qquad (2.24)$$

to give

$$G_A(q^2)=\frac{5}{3}\frac{2N_q^2}{\lambda_q}\left[\langle V_1(r)j_0(|\mathbf{q}|r)\rangle+\left\langle r\frac{dV(r)}{dr}\frac{j_1(|\mathbf{q}|r)}{|\mathbf{q}|r}\right\rangle\right]\ . \qquad (2.25)$$

Here

$$V_1(r)=m_q+V(r)$$

and

$$V(r)=(a^{\nu+1}r^\nu+V_0)\ .$$

Hence, written explicitly, one gets

$$G_A(q^2)=\frac{5}{3}\frac{2N_q^2}{\lambda_q}\left[(m_q+V_0)\langle j_0(|\mathbf{q}|r)\rangle+a^{\nu+1}\langle r^\nu j_0(|\mathbf{q}|r)\rangle+\nu a^{\nu+1}\left\langle r^\nu\frac{j_1(|\mathbf{q}|r)}{|\mathbf{q}|r}\right\rangle\right]\ . \qquad (2.26)$$

Since the $q^2\to 0$ limit of (2.25) defines the axial-vector constant g_A, we find that

$$G_A(0)=\tfrac{5}{9}(4N_q^2-1) \qquad (2.27)$$

which is the expression for g_A derived in our earlier work.[3]

D. Chiral-symmetry breaking and pion-nucleon form factor

The quark-core contribution to the axial-vector current of the nucleon $A_\lambda^\mu(\mathbf{r})$ given in (2.19) breaks chiral symmetry as a necessary consequence of confinement. With the quark fields $\psi_q(x)$ satisfying the Dirac equation (2.2) it can be shown that

$$\partial_\mu A^\mu_\lambda(x)=iV_1(r)\sum_{j=1}^{3}\bar{\psi}_j(x)\gamma^\mu\gamma^5\tau_\lambda\psi_j(x) . \quad (2.28)$$

The chiral-symmetry breaking here is in fact due to the scalar part of the confining potential $V(r)$ and the quark mass m_q. Then, to restore the chiral symmetry, one usually introduces an elementary pion field $\Phi_\lambda(x)$ such that the generalized axial-vector current

$$A^\mu_\lambda(x)=\sum_{j=1}^{3}\bar{\psi}_j(x)\gamma^\mu\gamma^5\frac{\tau_\lambda}{2}\psi_j(x)+f_\pi\partial^\mu\Phi_\lambda(x) \quad (2.29)$$

is a quantity satisfying the PCAC (partially conserved axial-vector current) condition

$$\partial_\mu A^\mu_\lambda(x)=-m_\pi^{\,2}f_\pi\Phi_\lambda(x) , \quad (2.30)$$

where $f_\pi=93$ MeV is the pion decay constant. The pion field $\Phi^\lambda(x)$ satisfies the resulting field equation

$$(\Box+m_\pi^{\,2})\Phi^\lambda(x)=J_5^\lambda(x) . \quad (2.31)$$

The source function $J_5^\lambda(x)$ provides the coupling of a pion to the quarks in the nucleon and is given by

$$J_5^\lambda(x)=-i\frac{V_1(r)}{f_\pi}\sum_{j=1}^{3}\psi_j(x)\gamma_5\tau^\lambda\psi_j(x) . \quad (2.32)$$

Then one can define the pion-nucleon form factor $G_{\pi NN}(q^2)$ for the static source $J_5^\lambda(\mathbf{r})$ as

$$iG_{\pi NN}(q^2)\langle\sigma_N\cdot\mathbf{q}\tau_N^\lambda\rangle=2M\left\langle N\left|\int d^3\mathbf{r}\,e^{i\mathbf{q}\cdot\mathbf{r}}J_5^\lambda(\mathbf{r})\right|N\right\rangle . \quad (2.33)$$

For the quarks in $1S_{1/2}$ orbits, using (2.3) and (2.32) one obtains

$$G_{\pi NN}(q^2)\langle\sigma_N\cdot\mathbf{q}\tau_N^\lambda\rangle=\frac{iM}{f_\pi}\frac{4N_q^{\,2}}{\lambda_q}\int_0^\infty dr\,r^2V_1(r)\phi_q(r)\phi_q'(r)\left\langle N\left|\sum_{j=1}^{3}\int d\Omega(\sigma_j\cdot\hat{\mathbf{r}})e^{i\mathbf{q}\cdot\mathbf{r}}\tau_j^\lambda\right|N\right\rangle . \quad (2.34)$$

Using the integral result

$$\int d\Omega(\sigma_j\cdot\hat{\mathbf{r}})e^{i\mathbf{q}\cdot\mathbf{r}}=4\pi i j_1(|\mathbf{q}|r)$$

and the identity

$$\left\langle N\left|\sum_j(\sigma_j\cdot\mathbf{q})\tau_j^\lambda\right|N\right\rangle=\tfrac{5}{3}\langle\sigma_N\cdot\mathbf{q}\tau_N^\lambda\rangle ,$$

we find

$$G_{\pi NN}(q^2)=-\frac{8\pi M}{f_\pi|\mathbf{q}|}\frac{5N_q^{\,2}}{3\lambda_q}\int_0^\infty dr[r^2V_1(r)j_1(|\mathbf{q}|r)]\times\frac{d}{dr}(\phi_q^{\,2}) . \quad (2.35)$$

Integrating by parts (2.35) yields

$$G_{\pi NN}(q^2)=\frac{M}{f_\pi}\frac{10}{3}\frac{N_q^{\,2}}{\lambda_q}\left[\langle V_1(r)j_0(|\mathbf{q}|r)\rangle+\left\langle r\frac{dV(r)}{dr}\frac{j_1(|\mathbf{q}|r)}{|\mathbf{q}|r}\right\rangle\right] . \quad (2.36)$$

Now comparing (2.36) with (2.26) we find that

$$G_{\pi NN}(q^2)=\frac{M}{f_\pi}G_A(q^2) , \quad (2.37)$$

which relates the pion-nucleon form factor to the axial-vector form factor of the quark core of the nucleon. Although strictly at the $q^2=-m_\pi^{\,2}$ limit $G_{\pi NN}(q^2)$ defines the pion-nucleon coupling constant $g_{\pi NN}$, we ignore the minor deviation in writing the $q^2\to 0$ limit of Eq. (2.37) as

$$g_{\pi NN}\simeq\frac{M}{f_\pi}g_A , \quad (2.38)$$

which is the Goldberger-Treiman relation

III. RESULTS AND CONCLUSION

In the preceding section we have obtained expressions for nucleon electromagnetic form factors, axial-vector form factor, and pion-nucleon form factor in an independent quark-model based on the Dirac equation with equally mixed scalar and vector potentials taken in a non-Coulombic power-law form as given in (1.1). This potential model has been employed before with reasonable success in explaining the quarkonium spectroscopy[2] as well as the magnetic moment and other static properties of baryons.[3] Our purpose here is basically to see to what extent the same model can explain the nucleon form factors. Therefore, we take the potential and quark-mass parameters obtained in Ref. 3,

$$(\nu,a,V_0)\equiv(0.1,1.5562\text{ GeV},-1.89\text{ GeV}) ,\qquad (m_u=m_d)=146.95\text{ MeV} . \quad (3.1)$$

The ground-state energy eigenvalue solutions for the confined u (or d) quark, assumed to be in their $1S_{1/2}$ orbit in the nucleon gives

$$(E_u=E_d)=312.77\text{ MeV} ,\qquad (N_u^{\,2}=N_d^{\,2})=0.703\,88 . \quad (3.2)$$

Now taking this typical set of parameters and the corresponding solutions, we compute the various form factors like $G_E^p(q^2)$, $G_M^p(q^2)$, $F_{1,2}^V(q^2)$, and $G_A(q^2)$ as provided by

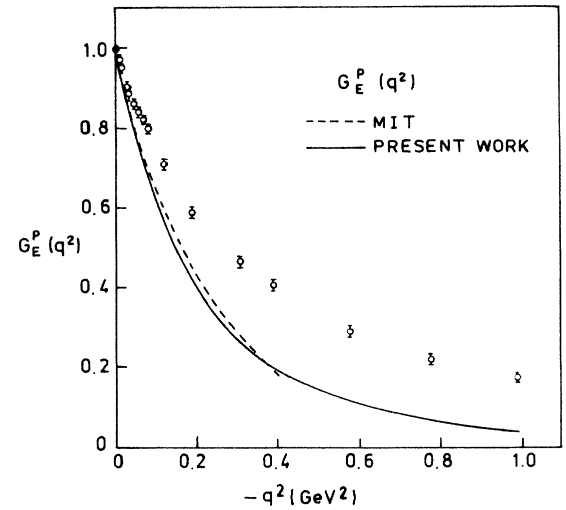

FIG. 1. Charge form factor $G_E^p(q^2)$, calculated in the present model, in comparison with the results of MIT bag-model calculations with bag radius $R=1$ fm. Experimental data are taken from Ref. 5.

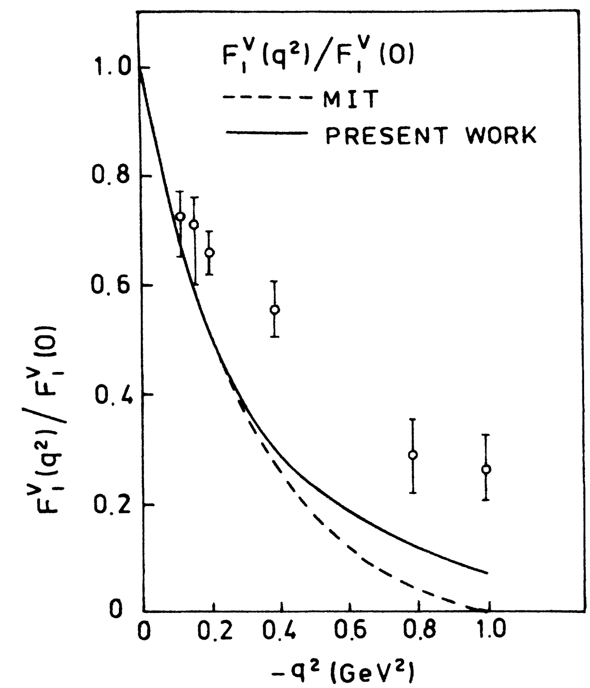

FIG. 3. Isovector form factor $F_1^v(q^2)/F_1^v(0)$, as obtained in the present model, in comparison with the MIT bag-model calculations with bag radius $R=1$ fm, as well as with the experimental data.

the expressions (2.12), (2.13), (2.18), and (2.26), respectively. The expectation values $\langle r^\alpha \rangle$, $\langle j_0(|\mathbf{q}|r)\rangle$, $\langle r^\nu j_0(|\mathbf{q}|r)\rangle$, and $\langle r^\nu j_1(|\mathbf{q}|r)/|\mathbf{q}|r\rangle$ appearing in these expressions are evaluated numerically for a range of $|\mathbf{q}|^2$ values, $0 \le |\mathbf{q}|^2 < 1$ GeV, which enables the computation of the q^2 dependence of the form factors.

In Figs. 1 and 2, we show the results for the proton electromagnetic form factors $G_E^p(q^2)$ and $G_M^p(q^2)$. We observe only an overall qualitative agreement with experimental data[5] with discrepancies more prominent for the higher-$|\mathbf{q}|^2$ region only. We obtain $G_E^p(0)=1$ as expected and $G_M^p(0)=\mu_p=2.874\mu_N$ as compared to $(\mu_p)_{\text{expt}}=2.793\mu_N$. It is also instructive to obtain the charge rms radius of the proton which is related to $G_E^p(q^2)$ as

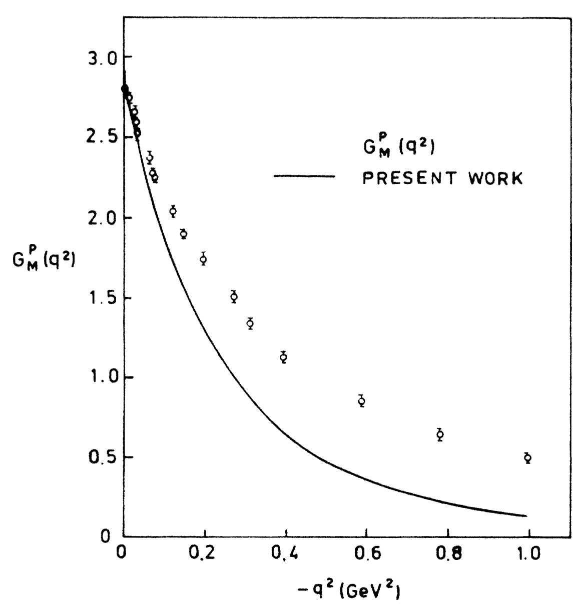

FIG. 2. Magnetic form factor $G_M^p(q^2)$, calculated in the present model, in comparison with the experimental data taken from Ref. 5.

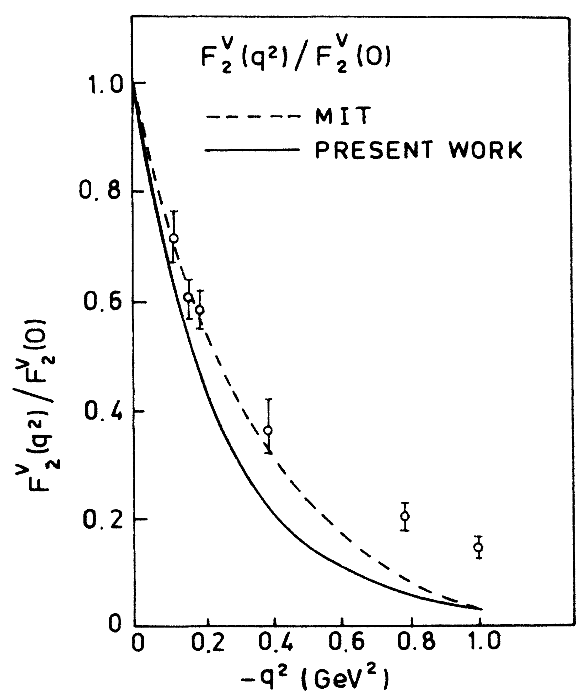

FIG. 4. Isovector form factors $F_2^v(q^2)/F_2^v(0)$, as calculated in the present model, in comparison with the MIT bag-model calculations (with $R=1$ fm) and with the experimental data taken from Ref. 7.

$$\langle r_c^2\rangle = -6\frac{dG_E^p(q^2)}{d\,|\mathbf{q}|^2}\bigg|_{|q|^2=0}$$
$$= \frac{N_q^2}{\lambda_q}\left[2E_q\langle r^2\rangle - 2\langle r^2V(r)\rangle + \frac{3}{\lambda_q}\right] - \frac{3}{4M^2}\ . \tag{3.3}$$

Calculating the above expression we obtain $\langle r_c^2\rangle^{1/2}=1.072$ fm, in somewhat poor agreement with $\langle r_c^2\rangle_{\rm expt}^{1/2}=0.88$ fm. However, our result compares well with the predictions of the MIT bag model[6] with $\langle r_c^2\rangle^{1/2}\simeq 1.12$ fm.

Next we obtain the isovector form factors $F_1^v(q^2)$ and $F_2^v(q^2)$ as prescribed in Eq. (2.8). Figures 3 and 4 provide a comparison of this result with the MIT bag-model results as well as with the experimental data.[7] We observe again that our results are not very different from the MIT bag-model calculations.[7]

Finally, the results for the axial-vector form factor $G_A(q^2)$, evaluated according to Eq. (2.26), are shown in Fig. 5. Here we have plotted the normalized ratio $G_A(q^2)/G_A(0)$ in comparison with the experimental data.[8] We find here that the agreement is rather poor. Nevertheless, we can derive the rms radius associated with $G_A(q^2)$, which is given by

$$\langle r_A^2\rangle = -\frac{6}{g_A}\frac{dG_A(q^2)}{d\,|\mathbf{q}|^2}\bigg|_{|\mathbf{q}|^2=0}$$
$$= \frac{10}{3g_A}\frac{N_q^2}{\lambda_q}\left[(V_0+m_q)\langle r^2\rangle + \left[1+\frac{\nu}{5}\right]a^{\nu+1}\langle r^{\nu+2}\rangle\right]\ . \tag{3.4}$$

Calculating this expression with $g_A=G_A(0)=1.01$ we find $\langle r_A^2\rangle^{1/2}=1.172$ fm which is roughly of the same order as $\langle r_c^2\rangle^{1/2}=1.072$ fm, found in this model without considering the pion-cloud effects. Then calculating $g_{\pi NN}$ from (2.38) we find $g_{\pi NN}\simeq 10.2$ which should be compared with $(g_{\pi NN})_{\rm expt}\simeq 13$.

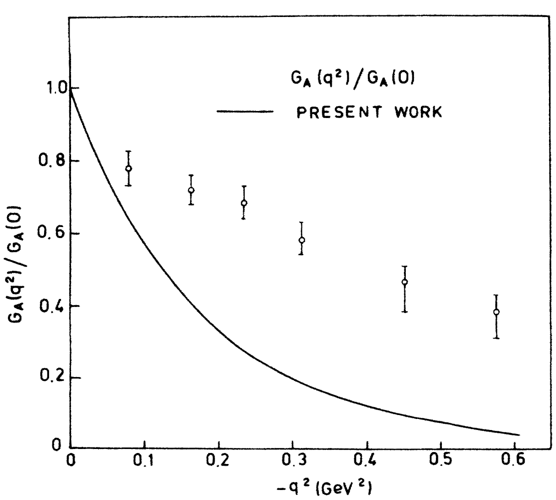

FIG. 5. Axial-vector form factor $G_A(q^2)/G_A(0)$, calculated in the present model, in comparison with the experimental data taken from Ref. 8.

It is a well-known fact that in models as this where the pion is introduced by a $f_\pi\partial_\mu\Phi$ term in the axial-vector current, the pionic contribution to the axial form factor $G_A(q^2)$ vanishes[9] in the Breit frame. Hence, in such a description $G_A(q^2)$ should be determined entirely by the quark-core alone, while $G_E^p(q^2)$ and $G_M^p(q^2)$ should as well receive contributions from the pion-cloud effects. Therefore, we believe that the overall discrepancies observed in the present calculations may be because of the absence of possible corrections due to the pion-cloud effects and the center-of-mass motion.

One can also further note that in a composite system like the nucleons, the high-q^2 form factors measure the high-momentum tail of the relative-momentum wave functions of the constituents. This tail is generated by the short-distance dynamics of the system which in this case is believed to be governed by the one-gluon-exchange part of the interaction. Therefore, the high-q^2 discrepancy seen in Figs. 1 and 2 may partly be an *a priori* indicator that the present model, like the MIT bag model, fails to take short-range correlations into account properly. This was because of the fact that there is no straightforward and clear cut mechanism to generate the three-body central potential out of the given two-body potential including the short-distance one-gluon-exchange part. Therefore, the best one can do is to assume some phenomenological average potential for independent quarks. Keeping in view the fact that the spin-orbit splitting is not very significant in ordinary baryons and also the short-distance Coulombic part of the interaction is supposedly less dominant in baryonic dimensions; a suitable average potential for independent quarks was modeled in the form of

$$(1+\gamma^0)(a^{\nu+1}r^\nu+V_0)\ ,$$

which was found to be quite successful in predicting the static properties of baryons[3] as well as the weak electric and magnetic form factors[3] in semileptonic baryon transitions. In view of this success it was natural to extend the applicability of the model in the present work to the study of the nucleon form factors arising out of the core contributions mainly in the low-q^2 region, which has been found to yield results comparable with those obtained without c.m. and pionic corrections in the MIT bag model. However, these results might be improved by incorporating chiral symmetry so as to bring in the pionic effects and including the c.m. corrections in the present model, which is being taken up in a more detailed study in our subsequent work. Nevertheless, in view of testing the applicability of the model in this area of study, it has been found to be meaningful with its simple and straightforward approach, yielding qualitatively encouraging and reasonable results.

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