Baryonlike and mesonlike solitons in a one-dimensional Dirac model of extended particles

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A version in $1+1$ dimensions of a recently proposed model of an extended particle with confined constituents is presented. Several constitutive spinors, considered as classical c numbers, interact through a vector coupling. There are no solutions in which only one field is nonvanishing, because each of them acts as a source of the rest, so that they cannot be separated in any way. This gives a mechanism of confinement which operates at the classical level. Moreover, the bound states have the characteristic three-quark and quark-antiquark patterns and their analytic expression can be given in terms of the Thirring solitons. They keep their identity upon collisions, at least between particle-like solutions of the same kind. Accordingly, they are called baryonlike and mesonlike solitons.

I. INTRODUCTION

This work continues the study of a recently proposed unconventional approach to the problem of confinement and of extended particles with structure.[1-3] Its two basic elements are the use of c-number classical spinors and the representation of the interactions between them by nonlinear direct coupling, without any intermediate field.

The model, the details of which are explained in Refs. 1 and 2, makes use of six Dirac fields, ψ_k,ϕ_k, $k=1,2,3$, which interact through fourth-order four-fermion forces. The ψ_k can be interpreted as quarks and the ϕ_k as antiquarks, since they contribute with opposite signs to the conserved current. After defining an extended particle as a particlelike solution (PLS) of the field equations, an appealing representation of confinement arises, since there are no one-field solutions and the constitutive spinors cannot be separated in any way. Moreover, all the solitary waves are bound states of ψ_k and ϕ_k, either of three ψ's or of one ψ and one ϕ. Furthermore, its baryonlike solutions obey the Pauli exclusion principle in a certain sense,[5] while the mesonlike ones do not.

The model seems worthy of consideration, but is difficult to handle since it cannot be solved analytically. As a consequence, the behavior of the PLS upon collisions is not easily studied and one has to resort to numerical methods which are extremely complex in $1+3$ dimensions. For this reason it is convenient to develop a version which admits analytic solution, even if the price to be paid for it is reducing the dimensionality of the space-time to $(1+1)$ (Refs. 6 and 7). Fortunately enough, there is a very interesting nonlinear Dirac equation in one-space dimension, which has solitons and can be solved by the inverse-scattering transform (IST) method. It is the massive Thirring model which will be used here as the basis of an analytically solvable model.

The plan of the paper is the following. In Sec. II I will describe a model of extended particle with spinorial internal structure, in Sec. III the properties of its PLS will be discussed, and in Sec. IV several conclusions will be stated.

II. A DIRECT-COUPLING MODEL OF AN EXTENDED PARTICLE WITH CONFINED CONSTITUENTS

The Dirac-Thirring equation refers to a spinor field in one-space dimension and has the form

$$i\gamma^\mu\partial_\mu\psi-m\psi+2\lambda\bar{\psi}\gamma_\mu\psi\gamma^\mu\psi=0\ . \tag{1}$$

It can be obtained from the Lagrangian density

$$L=L_D(\psi)+\lambda\bar{\psi}\gamma_\mu\psi\bar{\psi}\gamma^\mu\psi\ , \tag{2}$$

where $L_D(\psi)$ is the Lagrangian of the linear Dirac equation. It was proposed by Thirring[8] in 1958, as a solvable model in quantum field theory, in the case $m=0$ and with a scalar nonlinearity $\lambda(\bar{\psi}\psi)^2$, instead of the vector nonlinearity, in (2). Its PLS's, which were first obtained by Chang, Ellis, and Lee,[9] have a curious and intriguing relationship with those of the sine-Gordon equation, as was shown by Coleman.[10-12] Soon after, it was discovered that (1) is solvable by the IST method, first by Mikhailov[13,14] and independently by Kaup and Newell,[15] the associated spectral problem being now well understood.[16-18]
spectral problem being now well understood.[14-16]

Our γ matrices will be $\gamma^0=\sigma_1$, $\gamma^1=i\sigma_2$, $\gamma^5=i\gamma^0\gamma^1$. In this representation, the Thirring solitons have the following expressions:

$$\begin{aligned}\psi_1&=\frac{\epsilon Z(1+v)^{1/2}}{\cosh[m\gamma\sin\delta(x-x_0-vt)+i\epsilon\delta/2]}\ ,\\ \psi_2&=\frac{Z(1-v)^{1/2}}{\cosh[m\gamma\sin\delta(x-x_0-vt)-i\epsilon\delta/2]}\ ,\end{aligned} \tag{3}$$

where $Z=(m\gamma/4\,|\,\lambda\,|\,)^{1/2}\sin\delta\exp[-i\epsilon m\gamma\cos\delta(t-vx)]$, $0<\delta<\pi$, $\epsilon=\text{sgn}(\lambda)$. Its energy and charge are

$$E=E(\delta)=\frac{m}{\lambda}\sin\delta,\quad B=B(\delta)=\frac{\delta}{\lambda}\ . \tag{4}$$

The model presented in this work uses six spinors ψ_k,ϕ_k, $k=1,2,3$, with a Lagrangian density with the same structure as in Refs. 1–3:

$$L = L_1 + L_2 + L_3 , \quad (5)$$

$$L_1 = \sum_k [L_D(\psi_k) + L_{\bar{D}}(\phi_k)] , \quad (5a)$$

$$L_2 = \frac{\lambda}{3}[V_\mu(\psi)V^\mu(\psi) + V_\mu(\phi)V^\mu(\phi) + 2\sigma V_\mu(\psi)V^\mu(\phi)] , \quad (5b)$$

$$L_3 = \frac{\lambda'}{2} \sum_{i,j} (\bar{\chi}_{ij}\chi_{ij})^2 , \quad (5c)$$

where $L_{\bar{D}}$ is obtained from the usual Lagrangian density of the linear Dirac field by changing the sign of the derivative terms in $\partial_\mu \phi_k$,

$$V_\mu(\theta) = \sum_i \bar{\theta}_i \gamma_\mu \theta_i ,$$

$$\chi_{ij} = \chi_i - \chi_j ,$$

$$\chi_i = \psi_i + \gamma^5 \phi_i ,$$

and σ is a parameter which characterizes the strength of the $\psi\phi$ interaction. In Refs. 1 and 2 $\sigma = 2$, because this implies that the mass of the baryon is $\frac{3}{2}$ times that of the spin-1 meson. On the other hand, if $\sigma = 1$, L_2 has the simpler form

$$L_2 = \frac{\lambda}{3} V_\mu V^\mu$$

with $V_\mu = V_\mu(\psi) + V_\mu(\phi)$. The field equations derived from (5) are

$$i\gamma^\mu \partial_\mu \psi_a - m\psi_a + \frac{2\lambda}{3}[V_\mu(\psi) + \sigma V_\mu(\phi)]\gamma^\mu \psi_a + 2\lambda' \sum_k (\bar{\chi}_{ak}\chi_{ak})\chi_{ak} = 0 ,$$

$$-i\gamma^\mu \partial_\mu \phi_a - m\phi_a + \frac{2\lambda}{3}[\sigma V_\mu(\psi) + V_\mu(\phi)]\gamma^\mu \phi_a + 2\lambda' \sum_k (\bar{\chi}_{ak}\chi_{ak})\gamma^5 \chi_{ak} = 0 , \quad (6)$$

where $a = 1,2,3$.

As in Refs. 1–3, the nonlinear terms coming from L_2 provide the necessary forces for localized solutions to exist, while the inseparability of the fields is due to L_3. The two basic aspects of confinement, localizability and inseparability, are thus incorporated by the two pieces of the Lagrangian density. It should be stressed that, although L_3 is very complicated (if fully developed it has 378 different terms), it vanishes in the case of the PLS. Moreover, because of it all the PLS's have zero triality.

The current

$$J^\mu = \sum_k (\bar{\psi}_k \gamma^\mu \psi_k - \bar{\phi}_k \gamma^\mu \phi_k) \quad (7)$$

is conserved, although its separate terms are not. This justifies our interpretation of ϕ_k as charge conjugate to ψ_k. The corresponding conserved quantity

$$B = \int_R \sum_k (\psi_k^\dagger \psi_k - \phi_k^\dagger \phi_k) dx \quad (8)$$

will be called the baryonic charge or simply the charge.

The field equations (6) may be interpreted in an alternative way as those generated by the Lagrangian density $L_1 + L_2$, but with the fields submitted to the condition

$$\sum_{i,j} (\bar{\chi}_{ij}\chi_{ij})^2 = 0 . \quad (9)$$

In this case, however, λ' would be a Lagrangian multiplier depending in general on x and t. Nonetheless, all the results in this paper are equally valid for both interpretations.

A solution of (6), in which only the spinors θ_i, $i = 1, \ldots, n$ are different from zero, will be called an n-field solution or, more precisely, a $(\theta_1, \ldots, \theta_n)$ solution or of type $(\theta_1, \ldots, \theta_n)$. They will play an important role in the following.

III. PROPERTIES OF THE FIELD EQUATIONS: BARYONLIKE AND MESONLIKE SOLUTIONS

The field equations (6) have several interesting properties which parallel those of the standard quark model, just as in the three-dimensional case of Refs. 1 and 2. The details of the proofs are given in Ref. 6.

(1) There are no one-field solutions. In other words, none of the spinors may appear without being associated with some of the rest, clearly because of the last term in (6). This means that it is impossible that in a region of space only one spinor is nonvanishing, so that they cannot be separated in any way. As this applies, in particular, to the special case of the solitary waves, the fields cannot manifest themselves as particles, although they do appear as constituents of composite systems. Following the terminology of the quark model, it can be said, as in Refs. 1 and 2, that the fundamental fields ψ_k, ϕ_k are confined.

The proof is simple. If ψ is the only spinor different from zero, (6) takes the form

$$i\gamma^\mu \partial_\mu \psi - m\psi + \tfrac{2}{3}\lambda \bar{\psi}\gamma_\mu \psi \gamma^\mu \psi = 0 , \quad (10a)$$

$$\bar{\psi}\psi = 0 , \quad (10b)$$

and it happens that its only solution is $\psi = 0$. For, because of (10b), ψ can be written as

$$\psi = \begin{bmatrix} A \\ iB \end{bmatrix} \exp(i\tfrac{2}{3}\lambda F), \quad A, B, F \in R \quad (11)$$

and, after inserting (11) into (10a), it is easy to see that either $\psi = 0$ or

$$(\partial_\mu \partial^\mu + m^2)A = (\partial_\mu \partial^\mu + m^2)B = 0 ,$$

$$\partial_\mu F = \bar{\psi}\gamma_\mu \psi, \quad \partial_\mu \partial^\mu F = 0 . \quad (12)$$

It is clear, then, that $F = f(t+x) + g(t-x)$ and that (the prime means derivative with respect to the argument)

$$\bar{\psi}\gamma_0 \psi = A^2 + B^2 = f' + g' ,$$

$$\psi \gamma_1 \psi = A^2 - B^2 = f' - g' .$$

As $A^2 = f'(t+x)$, $B^2 = g'(t-x)$, A and B obey the classical wave equation and, since they also obey the Klein-

Gordon one with nonzero mass (12), they must vanish so that $\psi=0$. In a completely symmetric way, it can be proved that there are no solutions in which only one ϕ is different from zero.

(2) The only two-field solutions with finite energy are of type (ψ_k,ϕ_k). Or, otherwise stated, there are no solutions of the types (ψ_i,ψ_j), (ϕ_i,ϕ_j) or (ψ_i,ϕ_j), $i\neq j$ with finite energy. The reason is that, in these cases, the sources of four fields must be zero and this gives four algebraic conditions which happen to be incompatible with the two remaining differential equations, save for some exceptional infinite-energy solutions. For instance, the only (ψ_1,ψ_2) solutions are of the form

$$\psi_i=\begin{bmatrix} A_i \\ iB_i \end{bmatrix}\exp(i\tfrac{2}{3}\lambda F)\ ,$$

$$A_1=A\cos\alpha,\quad B_1=B\sin\alpha\ ,$$

$$A_2=A\sin\alpha,\quad B_2=-B\cos\alpha\ ,$$

$$F=A^2(t+x)+B^2(t-x)+C\ ,$$

$$\alpha=-\frac{m}{2AB}F+\alpha_0\ ,$$

where A, B, C, and α_0 are real constants. Their energy density is constant $T^{00}=-\frac{2}{3}\lambda(A^4+B^4)$.

The details of the proof can be found in Ref. 6 and are omitted here for brevity.

(3) Mesonic and mesonlike solutions. As a consequence of (2), the finite-energy two-field solutions can only be of type (ψ_k,ϕ_k), $k=1,2,3$. As in Refs. 1 and 2, they will be called mesonic solutions, since they can be considered as analogous to the $q\bar{q}$ states in the quark model. If $\psi_k=\psi$, $\phi_k=\phi$, the algebraic conditions expressing the annihilation of the sources of the other four fields reduce to

$$\bar{\psi}\psi-\bar{\phi}\phi+\bar{\psi}\gamma^5\phi+\bar{\phi}\gamma^5\psi=0\ . \tag{13}$$

There is a very interesting family of solutions of (6) which verify (13) and have the form

$$\psi=\left[\frac{3}{1+\sigma}\right]^{1/2}\psi_T,\quad \phi=\psi^* e^{i\epsilon}\ , \tag{14}$$

where ψ_T is any solution of the Dirac-Thirring equation (1) and ϵ is an arbitrary constant phase. They have zero baryonic charge, as is clear from (8). If ψ_T is a Thirring soliton (3), (14) is a mesonlike solution, its rest energy and baryonic charge being equal to

$$E=\frac{6}{1+\sigma}E(\delta)=\frac{6}{1+\sigma}\frac{m}{\lambda}\sin\delta,\quad B=0\ . \tag{15}$$

If ψ_T is an n-soliton solution, (14) has the form indicated in Fig. 1 and can certainly be interpreted as a system of n mesons in elastic scattering. It is convenient to stress that these mesons are solitons in the strong sense, as they recover their identity after colliding among themselves.

Are there other $(\psi\phi)$ mesonic solutions? As (13) is very restrictive, the conjecture that the answer is negative in the sense that the only finite-energy two-field solutions are of the form (14) seems plausible.

As in Refs. 1 and 2, there are also mesonic solutions with two and three pairs $(\psi\phi)$ (Ref. 6). They have the same energy and charge as the two-field mesons.

(4) Baryonic and baryonlike solutions. The (ψ_1,ψ_2,ψ_3) solutions will be called baryonic solutions, as they are analogous to the $3q$ states in the quark model. It is not difficult to prove[6] that all of them, having finite energy, are of the form

$$\psi_1=\psi_2=\psi_3=\psi_T,\quad \phi_k=0,\quad k=1,2,3\ , \tag{16}$$

where ψ_T is again an arbitrary solution of the Dirac-Thirring equation. Their baryonic charge is positive as

$$B=3\int_R \psi_T^\dagger\psi_T dx>0\ . \tag{17}$$

If ψ_T is a Thirring soliton, we obtain a baryonlike solution with rest energy and baryonic charge given by

$$E=3E(\delta)=3\frac{m}{\lambda}\sin\delta,\quad B=3B(\delta)=\frac{3}{\lambda}\delta\ . \tag{18}$$

If ψ_T is an n-soliton, (16) has the form indicated in Fig. 2 and can be interpreted as n baryons in elastic scattering. Obviously there are also antibaryonic solutions which, if they have finite energy, are of the form

$$\phi_1=\phi_2=\phi_3=\psi_T^*,\quad \psi_k=0,\quad k=1,2,3\ . \tag{19}$$

They have the same energy, but opposite charge as the baryonic ones built up with the same ψ_T. An n-antibaryon solution is represented in Fig. 3.

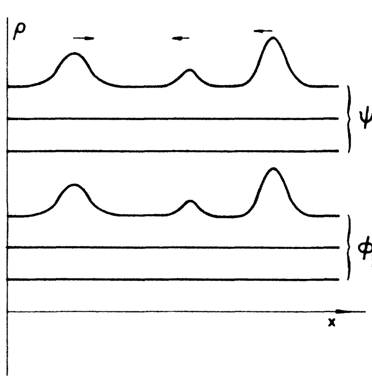

FIG. 1. Shapes of the charge densities of the six spinors in the case of an n-meson-like solution. Each solitary wave has two constituents.

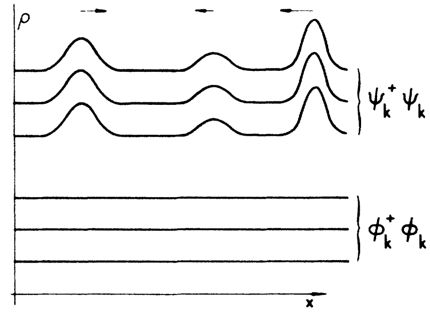

FIG. 2. Same as in Fig. 1, but in the case of an n-baryon-like solution. The three constituents of the solitary waves are clearly seen.

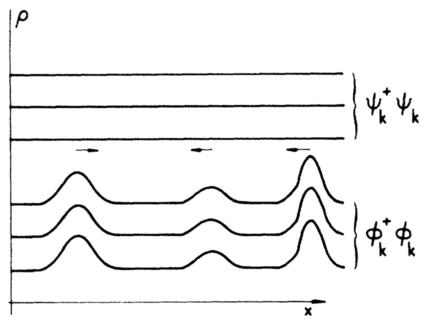

FIG. 3. Same as in Fig. 2, but in the case of n-antibaryonlike solitary waves. The three constituents are clearly seen.

IV. CONCLUSIONS

The particlelike solutions of the model presented in this paper have a striking similarity with the real existing hadrons. First of all, they can be classified in baryonlike and mesonlike solutions, all of which are bound states of several constituents with the characteristic pattern of three-quark baryons and quark-antiquark mesons. Second, the constitutive spinors are confined, since there are no solutions with only one nonvanishing field, this mechanism operating at the prequantum classical level.

These properties are shared with the three-dimensional model presented in Refs. 1 and 2, in which the PLS's are of three kinds: baryons, spin-1, and spin-0 mesons, their masses and mean-square radii being of the right order of magnitude. In the particular case of the vector coupling in $(1+3)$ dimensions,[2] if $m=286$ MeV, $\lambda m^2=6.5$, the masses of the three kinds of PLS are 1200, 800, and 582 MeV, respectively, which are close to those of Δ, ρ, and η. The mean-square radii are 1.2, 1.2 and 2.3 fm, respectively.

The case of one-space dimension is less realistic, but, on the other hand, has the appealing feature that not only the PLS, but the states with n PLS of the same kind as well, have analytic expressions in terms of the well-known solution of the massive Thirring model.[13–18] As a consequence, the baryonlike and mesonlike solutions are solitons in the sense that they are stable upon collisions between members of the same class.

Two problems arise naturally. The first one is to determine the behavior of the PLS in collisions between members of different kind, between a baryon and a meson, for instance. Second, to characterize the evolution from initial Cauchy data in which only one field is different from zero, since, if the final state contains PLS, this could model the self-dressing of an isolated quark. These two questions will be discussed in a forthcoming work.

To sum up, this paper proposes a model of a one-dimensional hadron, based on a mechanism of confinement which operates at the prequantum classical level and in which the particles are represented by solitons with internal structure. The main conclusion is that the use of nonlinear direct coupling to represent strong interactions should be seriously considered and that the confinement might be described as a nonquantum effect.

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