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Thermodynamic instability of de Sitter space

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The generalized second law of thermodynamics is applied to fluctuations in the Hawking temperature of the cosmological event horizon in de Sitter spacetime. Fluctuations which decrease the temperature, thereby decreasing the scalar curvature, *increase* the generalized entropy S. Thus, the generalized second law implies that de Sitter space is *unstable* to fluctuations which decrease the effective cosmological constant.

I. INTRODUCTION

In de Sitter spacetime, as in curved space generally, the concept of "vacuum" for quantum field theory is not defined in a unique, unambiguous way. Because of the high degree of symmetry of de Sitter space, however, the wave equation for a noninteracting scalar field propagating in this curved background can be solved analytically in terms of standard functions. These wave functions may then be used to define the scattering states of the field theory in the remote past and the remote future. The two states of the field so defined, $|\text{in}\rangle$ and $|\text{out}\rangle$, are related by a Bogliubov transformation which mixes particle and antiparticle modes. According to the standard interpretation of this phenomenon in nongravitational background fields, mode mixing implies that the $|\text{in}\rangle$ state is unstable to the creation of particle-antiparticle pairs by the external field, in this case the de Sitter geometry.[1]

A standard example of such a pair-creation instability in a nongravitational background is that found by Schwinger for charged matter in a uniform constant electric field.[2] In that case few would suggest that the instability could or should be "fixed" by arranging the state of the field theory to be such that every pair created by the electric field is exactly compensated by another pair coming in from infinity and annihilating. Such a boundary condition involving electric currents flowing from infinity could be mathematically defined, of course, and the resultant state would necessarily be time-reversal invariant. Therefore the decay rate calculated in this state by Schwinger's method must vanish. To conclude from this that a uniform constant electric field is stable would be like arguing that nuclear α decay does not occur because Schrödinger's equation also possesses time-symmetric solutions for which the net probability flux at ∞ vanishes. The point is that although the fundamental equations are time-reversal invariant the physically correct boundary conditions generally are not.

Consider now a gravitational example of the same phenomenon—the Schwarzschild black hole. By canonically quantizing a scalar matter field in this background and identifying the appropriate scattering states of the field, Hawking showed[3] that the Bogliubov transformation between the corresponding $|\text{in}\rangle$ and $|\text{out}\rangle$ states (called Unruh vacua in the literature[4]) is a nontrivial mixing one. Like Schwinger, he concluded that this background field—the black hole—must be unstable to the creation of particle-antiparticle pairs. One member of the pair is drawn into the hole while the other emerges as a positive-energy flux at infinity with a Planck spectrum: the black hole appears to radiate like a blackbody at temperature $\beta^{-1}=(8\pi M)^{-1}$.

Because of the appearance of temperature in the Schwarzschild case it is natural to ask whether, in addition to the scattering states $|\text{in}\rangle$ and $|\text{out}\rangle$, there also exists a time-reversal-invariant equilibrium state. That is, can one arrange the boundary conditions far from the hole such that the outgoing Hawking flux is precisely compensated by an incoming flux? Physically the answer is clearly *yes* if one places the black hole in a very large container ($\gg 2GM/c^2$) which is also maintained at the Hawking temperature by contact with an external heat bath.

Mathematically the same condition can be realized by considering the Euclidean section $t \to it$ of the Schwarzschild line element. Since the points it and $it+\beta$ are identified, it follows that any function of the coordinates analytic under this continuation must have the *same* periodicity β. If this condition is demanded of the Feynman propagator of the scalar matter field, then the state of the field is fixed uniquely. It is the explicitly time-reversal-invariant Hartle-Hawking state[5] which describes a black hole in equilibrium with its own radiation. Does the mere existence of this equilibrium state imply that the black hole is stable after all?

Hawking subsequently showed that the answer to this question is *no*.[6] Although it clearly possesses no decay rate in the usual sense, Hawking found that the time-symmetric equilibrium state is *thermodynamically unstable* to macroscopic fluctuations in the temperature of the horizon. This thermodynamic instability can be traced to the fact that the black hole with temperature $T_H=1/8\pi M$ has a *negative* specific heat—a feature of gravitational systems familiar from other contexts, even in the nonrelativistic Newtonian limit. Thus, the rather artificial choice of time-symmetric boundary conditions is unstable to spontaneous fluctuations and time asymmetry reappears in a different guise.

In de Sitter space, all of the same formal Euclidean periodicity structure of the classical metric is present. There is an event horizon for every freely falling inertial observer. The Feynman propagator can also be uniquely defined by demanding analyticity under the continuation to imaginary time. The corresponding state, sometimes called the Tagirov or Bunch-Davies[7] "vacuum," is invariant under the *full* de Sitter isometry group, including time reversal. Therefore, like the Hartle-Hawking state in the Schwarzschild case, it has no decay rate in the sense of Schwinger or Ref. 1. Is this the stable ground state of quantum field theory in de Sitter spacetime?

In this paper I show that the *same formal arguments* made by Hawking for the equilibrium state in the Schwarzschild case apply equally well to the de Sitter case. The logical basis of both and the assumption upon which both depend crucially is the existence of a *generalized second law of thermodynamics* as the condition for spontaneous change. In order that no extraneous considerations may enter, which might cause confusion and cast doubt on the result, I will adopt a purposely formal axiomatic presentation in the next section, in which this generalized second law is *assumed* as an axiom. Inequality (9) is then derived as a *sufficient* condition on the quantum matter energy density in order for the Bunch-Davies state to be unstable. Actually, this condition is not a necessary one. I postpone until Sec. III a fuller discussion of this and the axioms of Sec. II—in particular the physical basis for the generalized second law.

II. CONDITION FOR INSTABILITY

For the purposes of this section we will assume that the following axioms are valid generalizations of the laws of thermodynamics to arbitrary static backgrounds in general relativity.[8]

(i) The zeroth law of thermodynamics. Let quantum matter fields propagate in a globally static background solution of the classical Einstein equations (with or without a cosmological term):

$$ds^2 = -g_{tt}(x)dt^2 + h_{ij}(x)dx^i dx^j . \tag{1}$$

Denote the Killing field which generates the time translation $\partial/\partial t$ by K^a. Since g_{tt} and h_{ij} ($i,j=1,2,3$) are functions only of the spatial coordinates x^i, the line element becomes that of a real Euclidean signature metric under the replacement of t by it in (1). If the Feynman propagator for the matter field(s) is required to be analytic under the continuation $t \to it$ then a unique state of the quantum field is fixed by this requirement (in addition to requiring vanishing at spacelike infinity if the space is noncompact). Suppose that the orbits of the Killing field on the Euclidean section of the complexified manifold are periodic with periodicity β. Then the state of the matter field(s) is a thermal-equilibrium state with temperature

$$T_M = T_H \equiv \beta^{-1} . \tag{2}$$

The expectation value of any operator $\mathcal{O}$ in this state will be denoted by $\langle \mathcal{O} \rangle_\beta$.

Two clarifying points about this temperature may be helpful. First, T_M refers to the Planck distribution of quanta detected by any *freely falling* observer in the spacetime. For an observer with *fixed* x^i the local temperature θ will obey the usual Tolman red-shift formula

$$\theta = T_M(-g_{tt})^{-1/2} . \tag{3}$$

Second, the statement that the state is thermal is strictly true only for certain observables $\mathcal{O}$ (composed of operators with support localized within one horizon) for which the quantum expectation value can be replaced by a thermal density matrix.

(ii) The first law of thermodynamics. Let ρ be the mean energy density of the matter in the state defined in (i) and E_M the mean energy with respect to the Killing time:

$$\rho \equiv -\langle T^0_{\ 0} \rangle_\beta ,$$
$$E_M \equiv \int_\Sigma \langle T^a_{\ b} \rangle_\beta K^b d\Sigma_a . \tag{4}$$

The expectation values may be defined by regularizing and renormalizing the divergent matrix elements of $T^0_{\ 0}$ by any of the several covariant methods discussed in the literature.[9] Also define the isotropic pressure p and "projected 3-volume" V by

$$p \equiv \tfrac{1}{3}\langle T^i_{\ i} \rangle ,$$
$$V \equiv -\int_\Sigma K^a d\Sigma_a = \int_\Sigma \sqrt{-g_{tt}}\sqrt{h}\; d^3x . \tag{5}$$

Then under an arbitrary infinitesimal variation of the background geometry and/or perturbation of the matter,

$$\delta E_M = T_M \delta S_M - p\delta V , \tag{6}$$

where S_M is the total entropy of the matter in the same volume V.

(iii) The generalized second law of thermodynamics. If the metric (1) possesses a horizon, where $K^a K_a = g_{tt}$ vanishes, with finite area A_H, define the generalized entropy to be

$$S \equiv S_M + S_H \equiv S_M + \tfrac{1}{4}A_H . \tag{7}$$

Then it is S which must be maximized in stable equilibrium. Conversely, if there exist fluctuations of the matter field(s) and geometry for which

$$\delta S > 0 , \tag{8}$$

then the state defined in (i) is *unstable* to fluctuations of this kind. Based on these axioms it is possible to prove the following theorem about quantum matter in de Sitter spacetime.

Theorem.

Let $\rho(T_H, M, \ldots)$ be the energy density of some quantum matter field(s) in de Sitter spacetime, defined by Eq. (4) in the unique state with Feynman propagator regular on the Euclidean section of the de Sitter manifold S_4, i.e., in the Bunch-Davies vacuum. If

$$\frac{d\rho}{dT_H} < 0 , \tag{9}$$

then fluctuations in the Hawking—de Sitter temperature T_H with $\delta T_H < 0$ *increase* the generalized entropy S:

$$\delta S > 0 \quad \text{for} \quad \delta T_H < 0 . \tag{10}$$

Proof.

We first check the applicability of the axioms to de Sitter spacetime. The line element can be expressed in isotropic state coordinates

$$ds^2=-dt^2(1-H^2r^2)+\frac{dr^2}{1-H^2r^2}+r^2d\Omega^2 \tag{11}$$

with constant scalar curvature $R=12H^2$. This is of the same form as (1). The timelike Killing field K^a has components (1,0,0,0) in these coordinates. On the Euclidean section $t\to it$ the line element (11) becomes that of a four-sphere, so that the points

$$(it,r,\theta,\phi),\quad (it+\beta,r,\theta,\phi)$$

are identified with

$$T_H=\beta^{-1}=\frac{H}{2\pi}\equiv\frac{1}{2\pi r_H}\ , \tag{12}$$

the Hawking–de Sitter temperature. Thus the conditions of (i) are satisfied. The surface Σ of (ii) may be chosen to be any $t=$const slice extending from $r=0$ to $r=r_H$ where K^a becomes null:

$$K^aK_a\,|_{r=r_H}=0\ . \tag{13}$$

Thus

$$V=\frac{4\pi}{3}r_H{}^3 \tag{14}$$

and

$$A_H=4\pi r_H{}^2\ . \tag{15}$$

Therefore (ii) and (iii) are applicable as well.

Because de Sitter space is homogeneous and isotropic ρ and p must be independent of position, depending only on T_H, the mass(es) of the field(s), and any other fixed parameters of the field theory. Thus

$$E_M=\rho V\ . \tag{16}$$

In addition we must have

$$\rho=-p \tag{17}$$

by O(4,1) invariance of $\langle T^a{}_b\rangle_\beta$.

With these elementary observations the proof is now immediate, for

$$\begin{aligned}\delta E_M&=\delta\rho V+\rho\delta V \text{ by (16)}\\ &=\delta\rho V-p\delta V \text{ by (17)}\\ &=T_M\delta S_M-p\delta V \text{ by (ii)}\ .\end{aligned}$$

Hence

$$\delta S_M=\frac{\delta\rho V}{T_M} \tag{18}$$

and

$$\begin{aligned}\delta S&=\delta S_M+\delta S_H \text{ by (iii)}\\ &=\frac{\delta\rho V}{T_M}+\delta S_H \text{ by (18)}\ .\end{aligned}$$

But

$$\frac{\delta S_H}{\delta T_H}=-\frac{1}{2\pi T_H{}^3}<0 \text{ by (7) and (15)}\ ,$$

so that if $\delta\rho/\delta T_H<0$ then

$$\delta S=\left[\frac{\delta\rho}{\delta T_H}\frac{V}{T_H}+\frac{\delta S_H}{\delta T_H}\right]\delta T_H>0 \tag{19}$$

for $\delta T_H<0$. Q.E.D.

III. DISCUSSION

The fluctuations that we have been considering may be thought of as fluctuations in the Hawking–de Sitter temperature as determined by the geometry through Eq. (12) or (15); alternately because of the zeroth law, we may regard the fluctuations as the fluctuations of quantum matter in the de Sitter background, which then drives the geometry through Einstein's equations. The two points of view are equivalent because the concept of temperature for a gravitational field horizon makes sense only when the quantum effects of the matter sources are taken into account. The ρ and $d\rho/dT$ appearing in our previous analysis are not arbitrary, but instead must be determined by the one-loop (semiclassical) properties of quantum fields in the de Sitter background. The $T^a{}_b$ of Eqs. (4) and (5) will then be identified with the (properly regularized and renormalized) equilibrium expectation value of the quantum stress tensor of the matter field(s) in the de Sitter-invariant thermal state. The variations and instability condition of Sec. II can then be understood in terms of familiar ideas of linear-response analysis. In particular, the condition (9) for a spontaneous small variation *away* from equilibrium can be calculated completely in terms of quantities in the equilibrium state.

Actually this condition, though certainly sufficient, is not necessary for the theorem's validity. The reason is that $(V/T_H)\delta\rho$ is always much less than δS_H in absolute magnitude, so that the second term alone in (19) always dominates. To see this recognize that ρ is of order $R^2\sim T_H{}^4$ or higher, the lower powers of R having been absorbed into renormalization counterterms for Λ and G. Thus $\delta\rho V/T_H\sim 1$ while $\delta S_H\sim 1/GR\gg 1$ provided the curvature scale is far below the Planck scale. Thus for curvatures small compared to the Planck scale the second term in (19)—the gravitational horizon contribution to the entropy—always dominates the matter term. Since this term gives the horizon a negative specific heat, the cosmological horizon is always unstable to fluctuations in the Hawking–de Sitter temperature in the limit $GR\ll 1$, provided the axioms of Sec. II are valid. We now proceed to discuss those axioms.

Classical thermodynamics can be formulated in a self-contained logically deductive way. Concepts such as temperature and entropy are defined in an implicit manner through the axioms of the system—the laws of classical thermodynamics. This approach has been followed in Sec. II. Of course, classical thermodynamics is supported both by an enormous body of empirical evidence and by the deeper theoretical understanding of its laws provided by statistical mechanics. Since it is unlikely that the laws of gravitational event horizons used in Sec. II will receive experimental support of refutation in the near future, we are forced to rely totally upon their logical cogency as evidence of their truth. Any circumstantial evidence we can obtain from considerations of *Gedankenexperimente* in curved spacetime will be extremely useful to this evaluation.

The starting point for a deeper understanding of the thermodynamic laws must be the semiclassical Einstein equations

$$R^a_b - \tfrac{1}{2} R \delta^a_b + \Lambda \delta^a_b = 8\pi \langle T^a{}_b \rangle \ . \quad (20)$$

Whatever the ultimate form of a quantum theory of gravitation it is reasonable to suppose that there is a semiclassical limit to the theory such that spacetime curvature can be treated classically even in the presence of quantum matter. This is, after all, an excellent approximation to the world we live in. The formal divergences of the matrix element in (20) can be handled by absorbing them into redefinitions of Λ, in Newtonian constant, and coefficients of possible terms of order R^2 in the gravitational action. Provided we are dealing with curvatures much less than the Planck scale, it is possible to neglect these explicit R^2 contributions to Eq. (20).

The Feynman propagator for a (noninteracting) field in curved space is the inverse of some Hermitian differential operator, such as $-\Box + M^2$ for a scalar field. If the Euclidean section of the spacetime manifold is compact (S_4 for example) this operator has a *unique* inverse in the space of regular functions. If the Euclidean section is noncompact ($\mathbf{R}_2 \times S_2$ in the Schwarzschild case) then "the space of regular functions" must be defined by their behavior at infinity as well. The quantum state of the field theory in the given background is then fixed by the analyticity requirement on the propagator under $t \to it$. Thus the matrix element in (20) is completely well defined (and finite, once regularized and renormalized).

If the Euclidean section of the classical manifold is periodic in the imaginary-time coordinate *it*, then this definition of the Feynman propagator ensures that it has the same periodicity. Thus axiom (i) of Sec. II really amounts to a *definition* of what temperature for the matter field is in this particular curved space, and so is trivial, in one sense. In a different sense, axiom (i) is quite nontrivial, however, because it implies that this is the *unique* value of temperature that can be assigned to the field in this background, in contrast with the ordinary situation in flat-space field theory where *any* temperature at all is allowed. The curved spacetime has a natural temperature scale associated with the surface gravity at the horizon and consistency of the semiclassical equations (20) forces the matter to be at the same temperature. This is the only *strictly static* equilibrium configuration of matter consistent with the given background. Any other configuration and, in particular, a different temperature for the matter would lead to singularities of $\langle T_a{}^b \rangle$ on the horizon or a time-dependent metric which no longer possesses a Killing field.

Axiom (ii) expresses the equivalent of energy conservation for the quantum matter, which results from the Killing symmetry of de Sitter spacetime. Actually it is possible to *derive* this from the density matrix form that the matrix elements $\langle \mathcal{O} \rangle_\beta$ take in the Tagirov-Bunch-Davies state:

$$\langle \mathcal{O} \rangle_\beta = \mathrm{Tr}[\hat{\rho} \mathcal{O}] \ , \quad (21)$$

where

$$\hat{\rho} = \frac{e^{-\beta \hat{H}}}{Z}, \quad \hat{H} = \int_\Sigma T^a{}_b K^b d\Sigma_a \ , \quad (22)$$

and

$$Z = \mathrm{Tr} e^{-\beta \hat{H}} \ . \quad (23)$$

The fact that matrix elements of operators $\mathcal{O}$ with support within one horizon volume actually takes this form follows immediately from the Euclidean periodicity of the Feynman function in this state.[10] The appearance of a density matrix formula in what was originally a *pure quantum state* should not cause surprise. It arises for the simple reason that half of the fully Cauchy data of the global de Sitter metric are *outside* the horizon volume V. Thus, with respect to observables localized within V the information represented by this data is totally inaccessible and must be averaged over. It is this summing over inaccessible degrees of freedom, a kind of course graining enforced by the causality structure of the spacetime, that results in the density matrix (22). Thus, for this restricted class of physical observables $\mathcal{O}$ it is possible to define an entropy in the standard manner:

$$\begin{aligned} S_M &\equiv -\mathrm{Tr}[\hat{\rho} \ln \hat{\rho}] \\ &= \beta E_M + \ln Z \ . \end{aligned} \quad (24)$$

In order to relate the variation of S_M to the energy-momentum tensor we also require

$$\begin{aligned} \beta \frac{d}{d\beta} \ln Z &= -T_H \frac{d}{dT_H} \ln Z \\ &= \int_{S_4} d^4x (|g|)^{1/2} g_{ab}(x) \frac{\delta \ln Z}{\delta g_{ab}(x)} \\ &= \langle T^a{}_a \rangle_\beta \beta V = -4\rho\beta V \ , \end{aligned} \quad (25)$$

which follows from the definition of the one-loop trace of the energy-momentum tensor as a conformal variation of the Euclidean effective action, $-\ln Z$.

Under such a variation of β,

$$\delta V = 3V \frac{\delta\beta}{\beta} = -3V \frac{\delta T_H}{T_H} \quad (26)$$

from (12) and (14). Hence

$$\delta S_M = \delta\beta\rho V + \beta\delta\rho V + \beta\rho\delta V - 4\rho V \delta\beta = \beta V \delta\rho \ . \quad (27)$$

But

$$\delta E_M = \delta(\rho V) = \delta\rho V - p\delta V \tag{28}$$

so

$$\delta E_M = T_M \delta S_M - p\delta V \tag{29}$$

by (27). So, given the density matrix expression (21) for expectation values in the Tagirov-Bunch-Davies state, the first law (ii) follows from the usual definition of matter entropy associated with this density matrix and the loss of information it represents.

For the generalized second law (iii) it is possible to give both a formal argument based on the Euclidean approach to quantum gravity and a more intuitive one. After the matter degrees of freedom have been integrated out the effective action for the gravitational field in Euclidean signature is given by

$$e^{-I_{\rm eff}} = \int \mathscr{D}\phi\, e^{-I_M[\phi,g]} e^{-I_{\rm grav}[g]} \tag{30}$$

or

$$\begin{aligned} I_{\rm eff} &= I_{\rm grav} - \ln Z \\ &= -\frac{1}{16\pi}\int_{S_4} \sqrt{g}\, d^4x (R - 2\Lambda) - \ln Z \\ &= -\frac{1}{2\pi}\left[\beta^2 - \frac{\Lambda\beta^4}{24\pi^2}\right] - \ln Z(\beta)\ . \end{aligned} \tag{31}$$

The first derivative of $I_{\rm eff}$ just gives the Euclidean form of the trace of semiclassical Einstein equations (20):

$$\beta\frac{dI_{\rm eff}}{d\beta} = 0 = \frac{1}{48\pi^3}\beta^4[-12(2\pi/\beta)^2 + 4\Lambda - 8\pi\langle T_a{}^a\rangle]\ . \tag{32}$$

The *second* derivative of $I_{\rm eff}$ describes conformal fluctuations *away* from the semiclassical solution of (20):

$$\beta\frac{d}{d\beta}\left[\beta\frac{d}{d\beta}I_{\rm eff}\right]\Bigg|_{(32)} = 4\beta\left[+\frac{\beta}{2\pi} + \frac{1}{6\pi^2}\beta^4\frac{d\rho}{d\beta}\right]\ . \tag{33}$$

The term in large parentheses on the right-hand side in (33) is equal to

$$\frac{dS_H}{d\beta} + V\beta\frac{d\rho}{d\beta}\ ,$$

which shows that

$$\beta\frac{d}{d\beta}\left[\beta\frac{d}{d\beta}I_{\rm eff}\right]\Bigg|_{(32)} = 4\beta\frac{dS}{d\beta} > 0\ , \tag{34}$$

where S is the generalized entropy of (iii).

If gravity were like any other field theory we would expect that positivity of the second variation of the Euclidean action with respect to a real parameter to be a necessary condition for the *stability* of the solution to (32). However, the variation with respect to β is a conformal variation and the Euclidean Einstein action (31) is *unbounded from below* with respect to local conformal variations: the kinetic term has the wrong sign. Hawking has suggested that this may be remedied by integrating over *imaginary* conformal variations instead. This prescription would change the condition (34) to an *instability* condition on the Euclidean de Sitter solution. Recently Hartle and Schleich[11] have reexamined this issue and argue that Hawking's suggestion is actually *required* by a canonical approach to gravity in which only physical degrees of freedom appear.

A very different sort of argument for the validity of the generalized second law comes from considerations of *Gedankenexperimente* of the kind familiar in black-hole spacetimes. Davies has shown that these can equally well be imagined in the de Sitter case.[12] For example, a box filled with radiation in thermal equilibrium can be brought to the cosmological horizon and the lid opened. The radiation flows out through the horizon. To the experimenter remaining inside the horizon the entropy would then seem to decrease spontaneously. One can easily construct a closed cycle representing a perpetual motion machine which makes use of this fact—unless the term $\frac{1}{4}A_H$ is included in the entropy. Also from the work of Gibbons and Hawking,[13] extending the laws of event horizons to the cosmological case one can see that the horizon area there plays a role analogous to the Schwarzschild case, increasing whenever the matter density within decreases.

Thus the generalized second law receives support both at the formal level and the intuitive level. *If* the imaginary conformal prescription of the Euclidean path integral for quantum gravity is in fact correct, the discussion following Eq. (34) would actually constitute a proof of the generalized second law (iii) as an (in)stability criterion following directly from the path-integral approach to canonical quantization. As in (34) the concept of entropy would then be a derivative—nearly superfluous—notion replaced by a more fundamental analysis of the conformal degree(s) of freedom of the gravitational field.

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