PHYSICAL REVIEW D VOLUME 33, NUMBER 1 1 JANUARY 1986

Calculation of the decay $\mu^- \to e^- e^+ e^- \nu_\mu \bar{\nu}_e$

Paul M. Fishbane
Physics Department, University of Virginia, Charlottesville, Virginia 22901

K. J. F. Gaemers
NIKHEF-H, P.O. Box 41882, 1009 DB Amsterdam, The Netherlands
(Received 22 July 1985)

We calculate rates for the allowed decay $\mu^- \to e^- e^+ e^- \nu_\mu \bar{\nu}_e$. We also study the dependence of these rates on non-standard-model couplings.

Many extensions of the standard model of electroweak interactions,[1] including those based on grand unification,[2] supersymmetry,[3] technicolor,[4] etc., as well as more conservative extensions,[5] imply family-mixing processes such as $\mu^+ \to e^+ e^- e^-$. It thus becomes important to understand as well as possible those processes allowed by the standard model which can provide backgrounds to new processes. One example is the $O(G^2\alpha^2)$ contribution to $\mu^- \to e^+ e^- e^- \nu_\mu \bar{\nu}_e$ [labeled as (A)], which is one region of phase space resembles $\mu^- \to e^+ e^- e^-$ [henceforth labeled as (B)].

One event of the allowed process (A) was observed[6] in 1959; subsequent experiments[7] revealed a total of some 20 events. The SINDRUM experiment,[8] as well as other work, revolutionizes the situation by detection of many thousands of events. Detailed comparison of distributions becomes possible.

Subsequent to a theoretical estimate[10] based on an equivalent-photon approximation, a calculation of the amplitude for (A) was carried out.[11] We report here a new evaluation of (A) in order to check Ref. 11, in order to improve the statistics, and in order to provide a source for distributions other than those in print.

We also found it convenient to evaluate this process with some currents other than those of the standard model (SM), in order to provide still another experimental check on the SM. We evaluated two cases.

Model (i). The electron has a charge current $V-A$ but the μ has a current $V+\alpha A$.

Model (ii). Both the muon and electron have charged currents of the form $V+\alpha A$.

In the limit $\alpha=-1$ we recover the SM.

Figure 1 shows two Feynman graphs which contribute to (A). There are two others corresponding to these with $q_1 \leftrightarrow q_2$. The graph in which the virtual intermediate photon is attached to the intermediate W^- will contribute only $O(m_\mu{}^2/M_W{}^2) \approx 1.6 \times 10^{-6}$ of the graphs calculated and is neglected. Hence our results are those of the four-fermion theory. The traces over the amplitudes (appropriately antisymmetrized over the electron momenta) were carried out with the aid of the SCHOONSCHIP symbolic-manipulation program.[12] Phase-space integration according to the distribution desired was performed using the Monte Carlo integration routine VEGAS (Ref. 13). Access to the spin-averaged amplitude squared as well as the distribution generator for any variable desired is available on request.

The numerical values of the static parameters of the problem are taken from the latest Particle Data Group tables:

$$\alpha^{-1} = 137.03604 ,$$

$$G_F = 1.166637 \times 10^{-5} \text{ GeV}^{-2} ,$$

$$m_e = 0.5110034 \text{ MeV} ,$$

$$m_\mu = 105.65932 \text{ MeV} .$$

The four-momentum labels of the detectable particles are as in Fig. 1. The four-vector q is defined as

$$q^\mu = (E_{\text{tot}} = q_{10} + q_{20} + q_{30}, \mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) .$$

The following variables of experimental interest were chosen to illustrate some results, always in the muon rest frame:

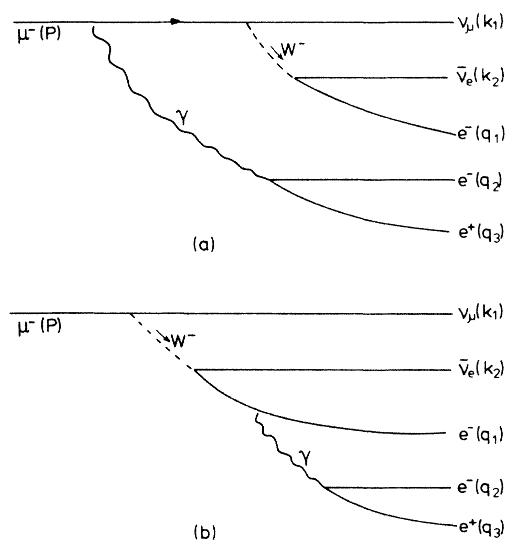

FIG. 1. Two out of four Feynman graphs in $O(G\alpha)$ for the process $\mu^- \to e^- e^- e^+ \bar{\nu}_e \nu_\mu$ in the standard model. (a) Bremsstrahlung from the initial μ^-, (b) bremsstrahlung from the final e^-.

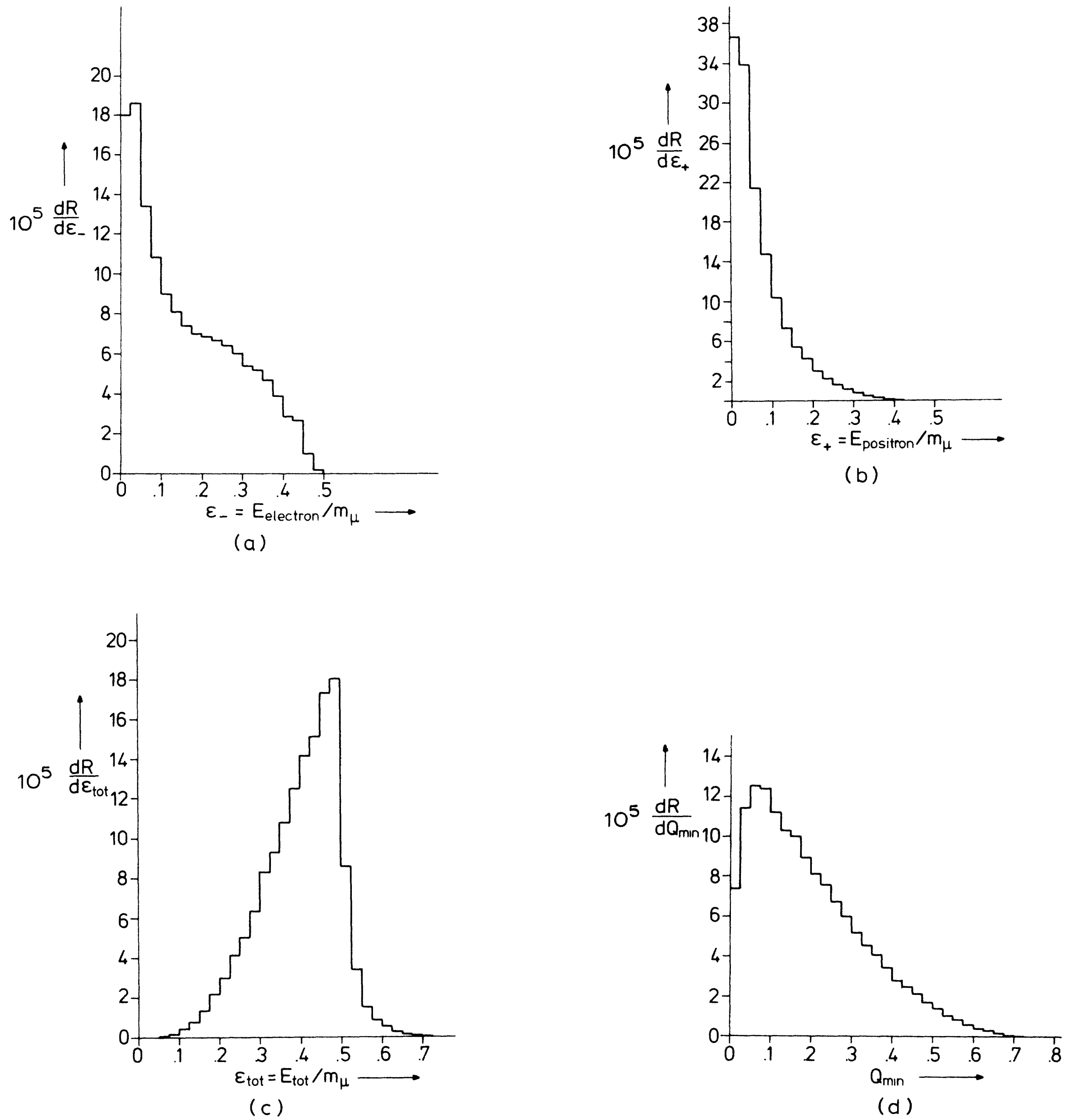

FIG. 2. Distributions in various variables for the process (A) as calculated in the standard model. The variables are described in the text. Note that to compare with experiment the distribution in an electron energy, case (a), should be doubled.

$$\epsilon_- = q_{10}/m_\mu , \tag{1}$$

where q_{10} is the energy of one electron;

$$\epsilon_+ = q_{30}/m_\mu , \tag{2}$$

where q_{30} is the energy of the positron;

$$\epsilon_{tot} = E_{tot}/m_\mu , \tag{3}$$

the total energy of the electrons and positrons in units of the muon mass;

$$Q_{min} = \frac{\ln(q_{min}^2/m_e^2)}{\ln(m_\mu^2/m_e^2)} , \tag{4}$$

$$Q_{max} = \frac{\ln(q_{max}^2/m_e^2)}{\ln(m_\mu^2/m_e^2)} , \tag{5}$$

where

$$q_{max}^2 = \max((q_1+q_3)^2, (q_2+q_3)^2) ,$$

$$q_{min}^2 = \min((q_1+q_3)^2, (q_2+q_3)^2)$$

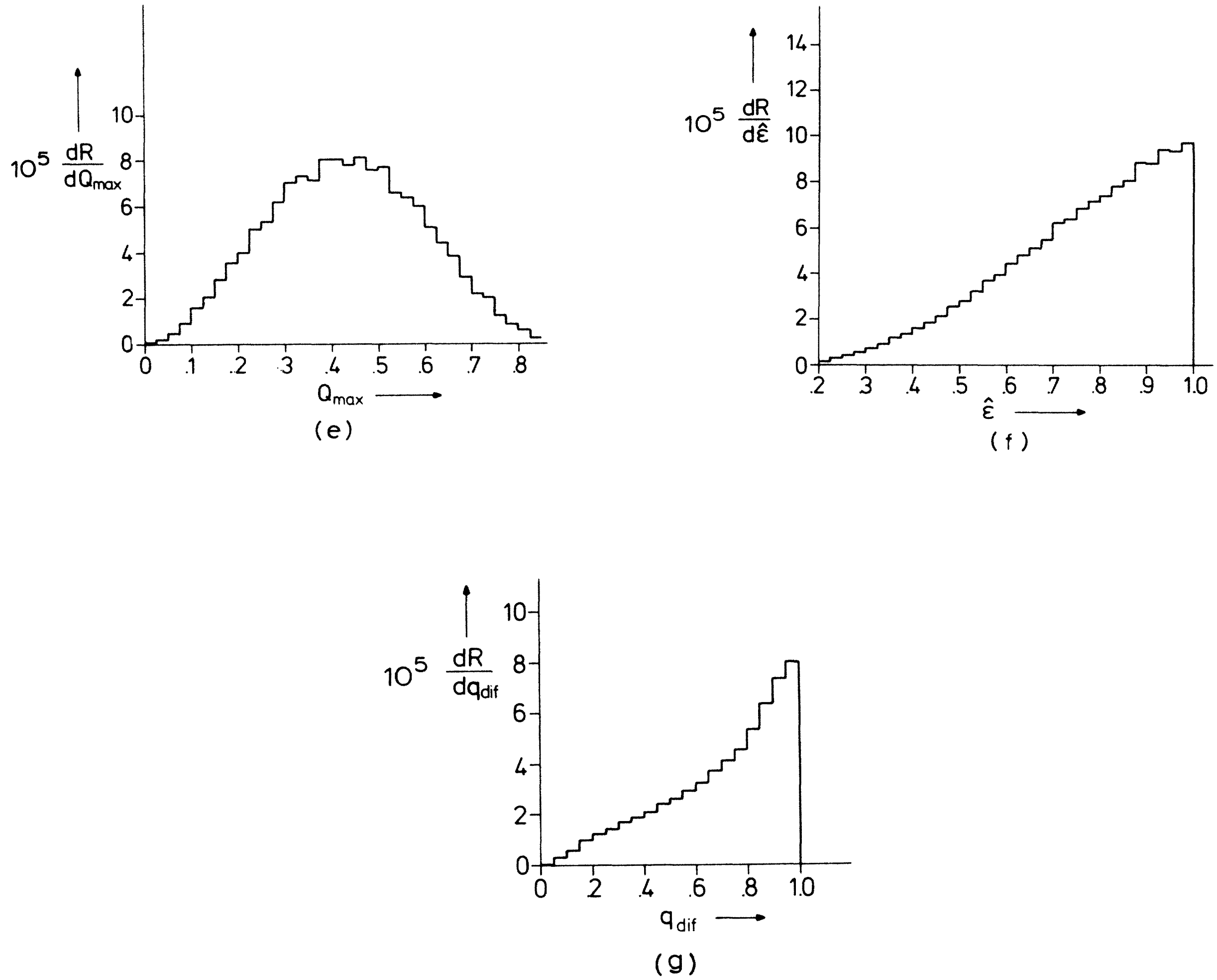

FIG. 2. (*Continued*).

[$q_{\min,\max}{}^2$ are the two possible choices for an invariant mass of an (e^+e^-) system];

$$\hat{\varepsilon} = (E_{tot} + |\mathbf{q}|)/m_\mu \ , \tag{6}$$

a variable used in the SINDRUM[8] experiment;

$$q_{dif} \equiv |\mathbf{q}_1 - \mathbf{q}_2| / |\mathbf{q}_1 + \mathbf{q}_2| \ . \tag{7}$$

This variable is sensitive to the antisymmetry of the electron momenta.

Figures 2(a)–2(g) show, respectively, distributions in these seven variables for the SM. In addition, we show in Table I the value of the branching ratio

$$R \equiv \frac{\Gamma(\mu^- \to e^- e^- e^+ \nu_\mu \bar{\nu}_e)}{\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e)}$$

as a function of a cut on $m_\mu - E_{tot}$. When this variable is taken large there is no constraint on the charged leptons and when it is small the charged leptons are constrained to the region corresponding to process (B). Our results are based on runs of 1.3×10^6 events. The results of Ref. 11 are confirmed; our statistics represents an improvement by roughly a factor of 40 in the errors. We find

$$R = (3.5916 \pm 0.0022) \times 10^{-5} \ ,$$

compared to $(3.54 \pm 0.09) \times 10^{-5}$ in Ref. 11.

We now make some comments.

(i) The value $R \approx 3.6 \times 10^{-5}$ should be compared with the measured μ lifetime,

TABLE I. Branching ratio as a function of cut on $m_\mu - E_{tot}$. Small values of the cut constrain the three electrons to mimic the family-symmetry-violating process $\mu^- \to e^- e^+ e^-$.

Cut on $m_\mu - E_{tot}$	$R = \frac{\Gamma(\mu^- \to e^- e^+ e^- \nu_\mu \bar{\nu}_e)}{\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e)}$
$5m_e$	$(4.608 \pm 0.008) \times 10^{-15}$
$10m_e$	$(3.059 \pm 0.005) \times 10^{-12}$
$50m_e$	$(7.149 \pm 0.014) \times 10^{-8}$
$100m_e$	$(2.116 \pm 0.004) \times 10^{-5}$
No cut	$(3.5916 \pm 0.0022) \times 10^{-5}$

$$\tau_\mu = (2.19709 \pm 0.00005) \times 10^{-6} \text{ sec} .$$

Since τ_μ is the standard method of measurement[14] of G_F, and since the branching ratio of (A) affects the last measured digit of τ_μ, the effect of (A) should henceforth be included in the determination of G_F; this correction has not to our knowledge been included.

(ii) The branching ratio for process (A) as a function of a cut on $m_\mu - E_{tot}$ drops very rapidly as the $(e^+e^-e^-)$ system is constrained on the kinematic region of process (B). The value 4.6×10^{-15} for the cut as $m_\mu - E_{tot} = 5m_e{}^2$ should be compared to the recent SINDRUM limit[8] 1.6×10^{-10}. Process (A) is still far from producing a measurable background for process (B).

(iii) Figure 2(c) is very similar in shape to the spectrum of electron energy in the normal decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$. In contrast the spectra of individual lepton energies are quite different from the normal spectrum. This result can be attributed to the fact that photon emission from the electron line, Fig. 1(b), dominates emission from the muon line, Fig. 1(a). The three charged leptons together then behave roughly as the single e^- emitted in the first part of the graph, which corresponds to ordinary μ decay.

(iv) The zero in Fig. 2(g) at the zero of the variable is due to the Fermi-Dirac statistics obeyed by the electrons. This spectrum thus provides an opportunity for a microscopic test of the statistics of free electrons.

(v) If we order the electrons by their energy, so that q_1 labels the most energetic electron, we can study the quantity $\mathbf{q}_3 \cdot (\mathbf{q}_1 \times \mathbf{q}_2)$, which is odd under time reversal. If the decay is T invariant then the average value of this quantity will be zero.

Let us turn now to a discussion of the nonstandard models tested. We tried a series of values of α for the two models ranging from $+1.0$ to -1.0. In model (ii), the universality model, differential decays depend only on $|\alpha|$, so that process (A) does not provide a distinction between $V-A$ and $V+A$ universal currents. The curves plotted in Fig. 2 are almost completely independent of α for both models with one notable exception: Fig. 2(c) in model (i) for the differential decay width as a function of E_{tot}/m_μ. In Fig. 3 we show this distribution for $\alpha = 1.0$ and $\alpha = -1.0$. The $\alpha = -1$ curve is indistinguishable from cases with $\alpha = -0.5$ to $\alpha = -1.0$ and the $\alpha = +1$ curve is indistinguishable from cases with $\alpha = +0.5$ to $\alpha = +1.0$. The two curves are quite different from each other, however, and so provide a good test of $V-A$ versus $V+A$ muon couplings.

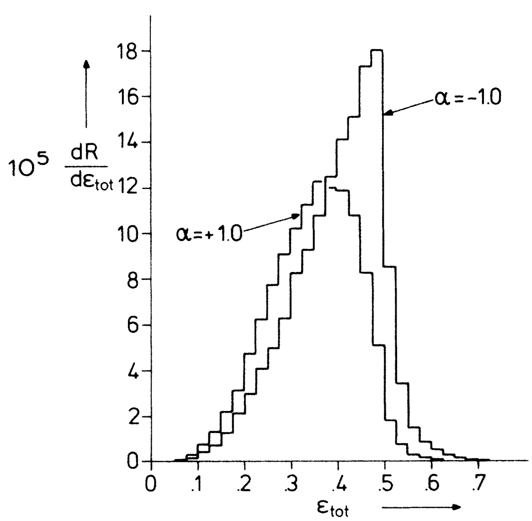

FIG. 3. Distributions in the variable E_{tot}/m_μ for the muon charged current $V-A$ ($\alpha = -1.0$) and $V+A$ ($\alpha = 1.0$). The electron charged current is $V-A$.

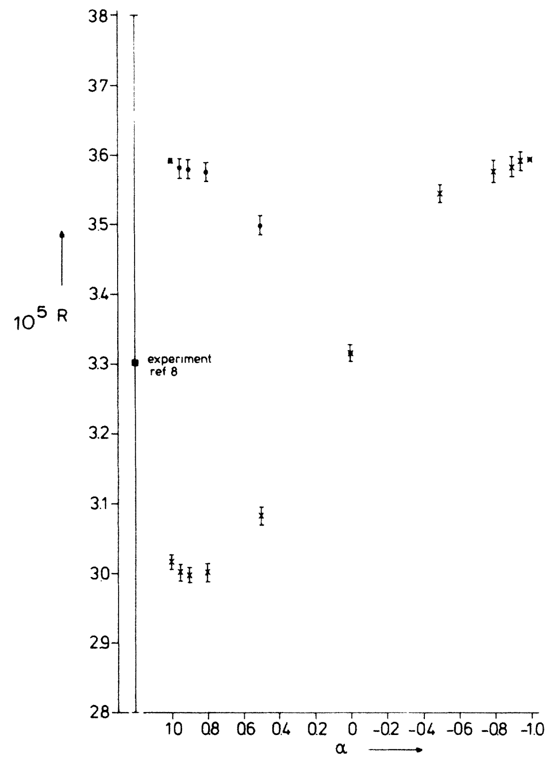

FIG. 4. Values of R as a function of α for model (i) (crosses) and model (ii) (dots). R is a function only of α^2 in model (ii). We also show the result of the recent measurement of Ref. 8.

The best test for α in $V+\alpha A$ using this particular decay mode comes from the total branching ratio R. We present our results for R in Fig. 4. The distinction between $\alpha = \pm 1$ for model (i) is a 20% effect and should be measurable. The origin of this effect is that the different e^- spectra (e.g., as in Fig. 4) for different values of α make the probability of bremsstrahlung different. Unfortunately this ratio distinguishes, say, $\alpha = -1$ from $\alpha = -0.95$ or even $\alpha = -0.8$ at only the 1% level, so that experiment cannot easily distinguish these cases.

We thank J. Vermaseren and several members of the SINDRUM experiment for useful discussions. P. M. F. wishes to thank W. Hoogland and NIKHEF-H for generous support and hospitality. We also thank the CERN Theory Division for their generous support.

[1]See, e.g., M. Bég and A. Sirlin, Phys. Rep. **88C**, 3 (1982).

[2]See, e.g., P. Langacker, Phys. Rep. **72C**, 186 (1981).

[3]See, e.g., *Supersymmetry Confronting Experiment*, proceedings of the CERN Workshop, 1982, edited by D. V. Nanopoulos and A. Savoy-Navarro [Phys. Rep. **105**, 1 (1984)].

[4]See, e.g., R. K. Kaul, Rev. Mod. Phys. **55**, 449 (1983).

[5]P. Fishbane, S. Meshkov, and P. Ramond, Phys. Lett. **134B**, 81 (1984); P. Fishbane, K. Gaemers, S. Meshkov, and R. Norton, Phys. Rev. D **32**, 1186 (1985).

[6]I. Gurevich, B. Nikolskii and L. Sarkova, Zh. Eksp. Teor. Fiz. **37**, 318 (1960) [Sov. Phys. JETP **10**, 225 (1960)].

[7]J. Lee and N. Samios, Phys. Rev. Lett. **3**, 55 (1959); R. Crittenden, W. Walker, and J. Ballam, Phys. Rev. **121**, 1823 (1961).

[8]W. Bertl *et al.*, Phys. Lett. **140B**, 299 (1984).

[9]R. E. Mischke *et al.*, contributed paper No. 618 to *Proceedings of the XXII International Conference on High Energy Physics, Leipzig, 1984,* edited by A. Meyer and E. Wieczorek (Academie der Wissenschaften der DDR, Zeuthen, DDR, 1984).

[10]S. Eckstein and R. Pratt, Ann. Phys. (N.Y.) **8**, 297 (1959).

[11]D. Bardin, T. Astatkov, and G. Mitsel'makher, Yad. Fiz. **15**, 284 (1972) [Sov. J. Nucl. Phys. **15**, 161 (1972)].

[12]H. Strubbe, Comp. Phys. Commun. **8**, 1 (1974).

[13]P. Lepage, J. Comput. Phys. **27**, 192 (1978); J. Vermaseren (private communication).

[14]O. Dumbrajs *et al.*, Nucl. Phys. **B216**, 277 (1983).