PHYSICAL REVIEW D
PARTICLES AND FIELDS

THIRD SERIES, VOLUME 33, NUMBER 6 15 MARCH 1986

Generic instabilities in first-order dissipative relativistic fluid theories. II. Havas-Swenson-type theories

William A. Hiscock
Department of Physics, Montana State University, Bozeman, Montana 59717
(Received 29 July 1985)

The stability of a general class of first-order dissipative relativistic fluid theories developed by Havas and Swenson is examined. If either the shear viscosity coefficient or the coefficient of the acceleration term in the heat-flow vector is nonzero, then all equilibrium states in the theory are unstable.

I. INTRODUCTION

In this paper I investigate the stability of equilibrium states in a class of dissipative relativistic fluid theories developed by Havas and Swenson.[1] The analysis presented here is an extension of earlier work[2] (hereafter referred to as paper I) which dealt with the theories of Eckart[3] and Landau and Lifshitz.[4] The theories considered here are again "first order" in that the entropy current contains no terms of higher than first order in the deviations from equilibrium (heat flow, viscous stresses, etc.). The class of theories examined by Havas and Swenson is, however, more general than the class treated in paper I, and includes all paper-I theories as a subset.

The additional degrees of freedom present in the Havas-Swenson theory are known, in the Newtonian limit, to yield a finite speed for the propagation of heat pulses.[5] In view of the known connection between causality and stability[6] in the second-order Israel-type theories,[7,8] it seems not unreasonable to expect the Havas-Swenson theory to have improved stability behavior compared to the Eckart or Landau-Lifshitz theories treated in paper I. Analyzing the dynamics of plane-wave perturbations about a homogeneous equilibrium state, I find that the additional degrees of freedom present in these more general first-order theories do not solve the stability problems discovered in paper I. Even in the most general first-order theory considered here, small departures from equilibrium will grow exponentially in time, as long as either the shear viscosity coefficient or the coefficient of the acceleration term in the heat-flow vector is nonzero.

As the theories considered here are in many aspects quite similar to those treated in paper I, only the differences and new features will be emphasized. Readers wishing more details on other points are referred to paper I and the work of Havas and Swenson.[1]

In order to simplify the analysis, I will treat only a nonreacting single-component fluid. The notation has been chosen to agree with the usage of paper I.

II. HAVAS-SWENSON-TYPE FLUID THEORIES

The fundamental variables of a relativistic theory of fluids are the stress-energy tensor T^{ab} and the number current N^a. The fundamental equations of motion are that T^{ab} and N^a have zero divergence:

$$\nabla_a T^{ab}=0 , \tag{1}$$

$$\nabla_a N^a=0 . \tag{2}$$

All theories agree on the decomposition of these tensors in equilibrium:

$$T^{ab}=\rho u^a u^b+pq^{ab} , \tag{3}$$

$$N^a=nu^a , \tag{4}$$

where u^a is the four-velocity of the fluid, ρ is the energy density, p is the pressure, n is the number density, and q^{ab} is the projection tensor orthogonal to u^a.

In dealing with a nonequilibrium situation, the standard approach is to treat nonequilibrium states as an equilibrium state plus some "small" deviation. An arbitrary nonequilibrium set of tensors T^{ab}, N^a is thus associated with a distinct background equilibrium state about which the deviations from equilibrium are analyzed. In developing a theory of dissipative relativistic fluids, it is possible to associate a given T^{ab}, N^a with many different background equilibria. The choice of a specific set of rules for singling out the background state defines a theory.

The first area of freedom is in the choice of the four-velocity for the fluid away from equilibrium. For example, in the Eckart theory one chooses

$$q^{ab}N_b=0 \tag{5}$$

(the four-velocity parallel to the number current), while in the theory of Landau and Lifshitz one requires that

$$T^{ab}u_a q_{bc}=0 \tag{6}$$

(four-velocity parallel to the energy current). In the more general Havas-Swenson theory, and the theory of paper I, u^a is left arbitrary, required only to be a unit timelike vector field.

The second area of freedom in selecting the background equilibrium reference state is in the measurement of the thermodynamic variables. It is in this area that the theory of Havas and Swenson is different than those of Eckart and Landau and Lifshitz. The background state of thermodynamic equilibrium can be defined by specifying the values of any two of the many possible thermodynamic variables (energy density, number density, pressure, temperature, chemical potential, entropy, etc.). The remaining thermodynamic variables are then obtained by using the equation of state and the first law of thermodynamics.

The theory examined in paper I, as well as the special cases of the theories of Eckart and Landau and Lifshitz, takes the stress-energy tensor and the number current vector to be the truly fundamental, measurable, quantities in the theory. The two needed thermodynamic variables are then defined as the appropriate contractions of T^{ab} and N^a with u^a and q^{ab}. In particular, all these theories define

$$\rho=T^{ab}u_a u_b\ , \tag{7}$$

$$n=N^a u_a\ , \tag{8}$$

for all states (not just equilibrium). The actual physical isotropic pressure $T^{ab}q_{ab}/3$ is then decomposed into an equilibrium pressure p determined from ρ, u, the equation of state, and first law of thermodynamics,

$$p=-\rho-n^2T\left[\frac{\partial s}{\partial n}\right]_\rho \tag{9}$$

and a nonequilibrium bulk viscous stress τ.

The theory of Havas and Swenson[1] results if one abandons the premise that only components of T^{ab} and N^a are measurable. If one can measure two thermodynamic potentials (say, μ, the chemical potential, and T, the temperature) other than ρ, p, or n, then it is possible for all three of the thermodynamic potentials which appear in T^{ab} and N^a, namely, ρ, p, and n, to differ from the physical local-energy density $T^{ab}u_a u_b$, the physical isotropic pressure $T^{ab}q_{ab}/3$, and the physical number density $N^a u_a$, i.e.,

$$T^{ab}u_a u_b=\rho+R\ , \tag{10}$$

$$T^{ab}q_{ab}/3=p+\tau\ , \tag{11}$$

$$N^a u_a=n+N\ , \tag{12}$$

where R, τ, and N describe deviations from equilibrium in the fluid.

In the general Havas-Swenson theory, the stress-energy tensor and number current vector then take the form

$$T^{ab}=(\rho+R)u^a u^b+(p+\tau)q^{ab}+q^a u^b+q^b u^a+\tau^{ab}\ , \tag{13}$$

$$N^a=(n+N)u^a+\nu^a\ , \tag{14}$$

where

$$u^a q_{ab}=u^a q_a=u^a\tau_{ab}=u^a\nu_a=\tau^a_a=\tau_{ab}-\tau_{ba}=0\ . \tag{15}$$

The nonequilibrium fields R, τ, N, q^a, ν^a, and τ^{ab} are then assumed to be of the form

$$R=R_1\nabla_a u^a+R_2u^a\nabla_a T+R_3u^a\nabla_a\mu\ , \tag{16}$$

$$\tau=-\zeta_1\nabla_a u^a-\zeta_2u^a\nabla_a T-\zeta_3u^a\nabla_a\mu\ , \tag{17}$$

$$N=N_1\nabla_a u^a+N_2u^a\nabla_a T+N_3u^a\nabla_a\mu\ , \tag{18}$$

$$q^a=-q^{ab}T\left[\kappa_1\frac{1}{T}\nabla_b T+\kappa_2u^c\nabla_c u_b+\kappa_3\nabla_b\mu\right]\ , \tag{19}$$

$$\nu^a=-q^{ab}T\left[\sigma_1\frac{1}{T}\nabla_b T+\sigma_2u^c\nabla_c u_b+\sigma_3\nabla_b\mu\right]\ , \tag{20}$$

$$\tau^{ab}=-\eta q^{ac}q^{bd}(\nabla_c u_d+\nabla_d u_c-\tfrac{2}{3}q_{cd}q^{ef}\nabla_e u_f)\ . \tag{21}$$

This is the second area in which the theory of Havas and Swenson is more general than that of paper I: nonequilibrium fields are allowed to depend on all first derivatives of the background fields u^a, T, and μ having the appropriate tensor rank. The restriction to terms linear in the first derivatives, and the lack of derivatives of the nonequilibrium fields in Eqs. (16)—(21), guarantees that the theory is first order in the sense of paper I. While the theories of Eckart or of Landau and Lifshitz are the simplest first-order theories, the theory of Havas and Swenson is perhaps the most general. There are 12 new phenomenological coefficients in the theory which are not present in the theory discussed in paper I. The theory of paper I may be recovered by setting

$$R_1=R_2=R_3=N_1=N_2=N_3=\zeta_2=\zeta_3=\sigma_2=\kappa_3=0\ , \tag{22}$$

$$\kappa_1=\kappa_2\ , \tag{23}$$

and

$$\sigma_1=-\mu\sigma_3\ . \tag{24}$$

The phenomenological coefficients of the derivatives (R_i, N_i, ζ_i, κ_i, σ_i, η) are functions of the background-state variables. They are not completely arbitrary: they can be constrained by the second law of thermodynamics and by a study of the equilibrium states. The constraints which follow from the second law are analyzed in detail in the work of Havas and Swenson.[1] For the purposes of this paper, it is sufficient to note that the second law requires that η, κ_2, ζ_1, $R_2+\mu N_2$, and $-N_3$ must be greater than or equal to zero. These conditions are necessary, but not in themselves sufficient, for the second law to hold true.

A second set of constraints on the coefficients is obtained by demanding that the usual equilibrium states, with

$$\nabla_a u^a=\tau_{ab}=u^a\nabla_a T=u^a\nabla_a\mu=\nabla_a(\mu/T)=0\ , \tag{25}$$

$$\nabla_b T=-Tu^c\nabla_c u_b\ , \tag{26}$$

have vanishing nonequilibrium fields. Applying Eqs. (25) and (26) to Eqs. (19) and (20) and demanding that the nonequilibrium vector fields q^a and ν^a vanish in equilibrium yields the following constraints:

$$\kappa_1-\kappa_2+\mu\kappa_3=0\ , \tag{27}$$

$$\sigma_1-\sigma_2+\mu\sigma_3=0\ . \tag{28}$$

Further constraints may be obtained if one insists that the only states with unchanging entropy are the equilibrium states. The theory of Havas and Swenson, as described in Ref. 1, does not have this property; there exist nonequilibrium states which have unchanging entropy. If one wishes to allow such behavior in the theory, then additional constraining equations [such as Eqs. (25) and (26)] are needed to define equilibrium states.

Finally, the physical meaning of the various phenomenological coefficients is best understood by examining the Newtonian limit of the theory. One thus finds[1] that η is the shear viscosity coefficient, that ζ is the bulk viscosity coefficient, and that the Newtonian thermal conductivity (κ_N) is given by

$$\kappa_N=\kappa_1+\left[\frac{\rho+p}{n}\right]^2\sigma_3\ , \tag{29}$$

as in paper I. The remaining coefficients describe less familiar dissipative phenomena;[1] their values and relative signs will not be relevant to the stability calculations presented here.

III. STABILITY OF EQUILIBRIUM STATES

In this section the dynamics of small perturbations about a homogeneous equilibrium state is studied. First the equations governing perturbations about a homogeneous equilibrium state are derived, and then exponential plane-wave solutions to these equations are found. The resulting dispersion relations show that there exist unstable, growing, transverse modes whenever $\kappa_2\neq 0$. In addition, there are unstable long-wavelength longitudinal modes if $\kappa_2\neq 0$. The stability of the longitudinal modes for arbitrary wavelengths has not been determined; the long-wavelength modes are expected to be the most important physically, as one can argue that the theory breaks down at short wavelengths. The exponential plane-wave solutions are then examined on a homogeneous but moving background. In this case a growing transverse mode exists even if $\kappa_2=0$, so long as $\eta\neq 0$.

The perturbations about equilibrium will be analyzed in the Eulerian framework; i.e., δQ is the difference between the actual nonequilibrium value of a field Q at a given spacetime point and the value of Q in the background equilibrium state at that point. The perturbations are assumed to be small enough so that their evolution is adequately described by the linearized equations of motion. Variables which do not possess the prefix δ refer to the background equilibrium state. The background equilibrium state is assumed to be homogeneous, so that all background-field variables have vanishing gradients and, by virtue of being an equilibrium state, to have $R=\tau=N=q^a=\nu^a=\tau^{ab}=0$.

After linearization in the perturbation variables and restricting the equilibrium state as described above, the equations of motion [Eqs. (1), (2), and (16)–(21)] take the form

$$-u_b\nabla_a\delta T^{ab}=u^a\nabla_a\delta\rho+u^a\nabla_a\delta R+(\rho+p)\nabla_a\delta u^a+\nabla_a\delta q^a=0\ , \tag{30}$$

$$q^c_b\nabla_a\delta T^{ab}=(\rho+p)u^a\nabla_a\delta u^c+q^{ac}\nabla_a(\delta p+\delta\tau)+u^a\nabla_a\delta q^c+\nabla_a\delta\tau^{ac}=0\ , \tag{31}$$

$$\nabla_a\delta N^a=n\nabla_a\delta u^a+\nabla_a\delta\nu^a+u^a\nabla_a\delta n+u^a\nabla_a\delta N=0\ , \tag{32}$$

$$\delta R=R_1\nabla_a\delta u^a+R_2u^a\nabla_a\delta T+R_3u^a\nabla_a\delta\mu\ , \tag{33}$$

$$\delta N=N_1\nabla_a\delta u^a+N_2u^a\nabla_a\delta T+N_3u^a\nabla_a\delta\mu\ , \tag{34}$$

$$\delta\tau=-\zeta_i\nabla_a\delta u^a-\zeta_2u^a\nabla_a\delta T-\zeta_3u^a\nabla_a\delta\mu\ , \tag{35}$$

$$\delta q^a=-q^{ab}T\left[\kappa_1\frac{1}{T}\nabla_b\delta T+\kappa_2u^c\nabla_c\delta u_b+\kappa_3\nabla_b\delta\mu\right]\ , \tag{36}$$

$$\delta\nu^a=-q^{ab}T(\sigma_1\nabla_b\delta T+\sigma_2u^c\nabla_c\delta u_b+\sigma_3\nabla_b\delta_\mu)\ , \tag{37}$$

$$\delta\tau^{ab}=-\eta q^{ab}q^{cd}(\nabla_c\delta u_d+\nabla_d\delta u_c-\tfrac{2}{3}q_{cd}q^{ef}\nabla_e\delta u_f)\ . \tag{38}$$

Exponential plane-wave solutions of the above equations will have the form

$$\delta Q=\delta Q_0\exp(\Gamma t+ikx)\ . \tag{39}$$

First consider an equilibrium state in which the fluid is at rest, so that

$$u^a\partial_a=\partial_t\ . \tag{40}$$

The set of perturbation equations can then be written in matrix form:

$$M^A{}_B\delta Y^B=0\ , \tag{41}$$

where, as in paper I, δY^B represents the 19 perturbation fields and $M^A{}_B$ is the 19-by-19 complex-valued matrix which describes the linearized equations of motion. The matrix $M^A{}_B$ block diagonalizes as follows:

$$\mathbf{M}=\begin{bmatrix}\mathbf{Q}&0&0&0\\0&\mathbf{R}&0&0\\0&0&\mathbf{R}&0\\0&0&0&\mathbf{I}\end{bmatrix}\ , \tag{42}$$

when one chooses the following set of perturbation variables

$$\delta Y^A = \{\delta T, \delta\mu, \delta R, \delta N, \delta\tau, \delta u^x, \delta q^x, \delta v^x, \delta\tau^{xx}, \delta u^y, \delta q^y, \delta v^y, \delta\tau^{xy}, \delta u^z, \delta q^z, \delta v^z, \delta\tau^{xz}, \delta\tau^{yz}, \delta\tau^{yy} - \delta\tau^{zz}\} . \tag{43}$$

The submatrix **Q**, which describes the evolution of The longitudinal modes, is given by

$$\mathbf{Q} = \begin{bmatrix} \left[\frac{\partial\rho}{\partial T}\right]_\mu \Gamma & \left[\frac{\partial\rho}{\partial\mu}\right]_T \Gamma & \Gamma & 0 & 0 & i(\rho+p)k & ik & 0 & 0 \\ i\left[\frac{\partial p}{\partial T}\right]_\mu k & i\left[\frac{\partial p}{\partial\mu}\right]_T k & 0 & 0 & ik & (\rho+p)\Gamma & \Gamma & 0 & ik \\ \left[\frac{\partial n}{\partial T}\right]_\mu \Gamma & \left[\frac{\partial n}{\partial\mu}\right]_T \Gamma & 0 & \Gamma & 0 & ink & 0 & ik & 0 \\ -R_2\Gamma & -R_3\Gamma & 1 & 0 & 0 & -iR_1k & 0 & 0 & 0 \\ -N_2\Gamma & -N_3\Gamma & 0 & 1 & 0 & -iN_1k & 0 & 0 & 0 \\ \zeta_2\Gamma & \zeta_3\Gamma & 0 & 0 & 1 & i\zeta_1 k & 0 & 0 & 0 \\ i\kappa_1 k & i\kappa_3 Tk & 0 & 0 & 0 & \kappa_2 T\Gamma & 1 & 0 & 0 \\ i\sigma_1 Tk & i\sigma_3 Tk & 0 & 0 & 0 & \sigma_2 T\Gamma & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{4}{3}i\eta k & 0 & 0 & 1 \end{bmatrix} . \tag{44}$$

The submatrix **R**, which describes the transverse modes, is given by

$$\mathbf{R} = \begin{bmatrix} (\rho+p)\Gamma & \Gamma & 0 & ik \\ \kappa_2 T\Gamma & 1 & 0 & 0 \\ \sigma_2 T\Gamma & 0 & 1 & 0 \\ i\eta k & 0 & 0 & 1 \end{bmatrix} , \tag{45}$$

and **I** is the two-by-two unit matrix. There will exist exponential plane-wave solutions of Eq. (41) whenever Γ and k satisfy the dispersion relation,

$$\det\mathbf{M} = (\det\mathbf{Q})(\det\mathbf{R})^2 = 0 . \tag{46}$$

Examining first the transverse modes, one finds

$$-\det\mathbf{R} = \kappa_2 T\Gamma^2 - (\rho+p)\Gamma - \eta k^2 = 0 . \tag{47}$$

The frequencies of the transverse modes are then

$$\Gamma_\pm = \frac{1}{2\kappa_2 T}\{(\rho+p) \pm [(\rho+p)^2 + 4\eta\kappa_2 Tk^2]^{1/2}\} . \tag{48}$$

The transverse modes are essentially identical to those of paper I, except that δv^y and/or δv^z need not be zero if $\sigma_2 \neq 0$. If $\kappa_2 > 0$, then one root of Eq. (47), Γ_+, will be positive for all real wave numbers k. The existence of a positive real root implies a growing mode; hence the fluid is unstable to a growing transverse mode for perturbations of all wavelengths. In the special case where $\kappa_2 = 0$, δq^y and δq^z are identically zero, and, there are fewer transverse modes. There is only a single root for Γ in this case,

$$\Gamma = -\eta k^2/(\rho+p) , \tag{49}$$

a decaying nonpropagating mode.

The dispersion relation for the longitudinal modes is a complicated sixth-order polynomial in Γ; in the limit $k \to 0$ (long-wavelength perturbations), however, it simplifies to

$$\lim_{k\to 0}(\det\mathbf{Q}) = \Gamma^3[\kappa_2 T\Gamma - (\rho+p)]\left\{(R_2N_3 - R_3N_2)\Gamma^2 + \left[N_3\left[\frac{\partial\rho}{\partial T}\right]_\mu - N_2\left[\frac{\partial\rho}{\partial\mu}\right]_T + R_2\left[\frac{\partial n}{\partial\mu}\right]_T - R_3\left[\frac{\partial n}{\partial T}\right]_\mu\right]\Gamma + \left[\frac{\partial\rho}{\partial T}\right]_n\left[\frac{\partial n}{\partial\mu}\right]_T\right\} + O(k^2) = 0 , \tag{50}$$

where $O(k^2)$ represents terms of order k^2 and higher. At least one root of Eq. (50) is positive:

$$\Gamma_1 = (\rho+p)/\kappa_2 T . \tag{51}$$

Thus, for at least some open neighborhood in k about $k=0$, there is a longitudinal growing mode.

As discussed in paper I, in a general equilibrium state there will not exist a spacelike surface which is everywhere orthogonal to the fluid four-velocity u^a. In order to determine the stability of the fluid in a general equilibrium state, it is then necessary to examine perturbations in a frame which is not comoving with the fluid. This is accomplished by Lorentz transforming to a frame moving in the direction of wave propagation with velocity v. The frequency and wave number transform in the following fashion:

$$k=\gamma\tilde{k}+iv\gamma\tilde{\Gamma} , \quad (52)$$

$$\Gamma=\gamma\tilde{\Gamma}-iv\gamma\tilde{k} , \quad (53)$$

where $\tilde{\Gamma}$ and $\tilde{k}$ are the frequency and wave number in the boosted, noncomoving frame, and

$$\gamma=(1-v^2)^{-1/2} . \quad (54)$$

The dispersion relations for exponential plane-wave solutions in the boosted frame may be easily found by substituting Eqs. (52) and (53) into the rest-frame dispersion relations. Since a nonzero value of κ_2 is already known to lead to instabilities, the value of κ_2 will be taken to be zero in the following analysis. The boosted transverse-mode dispersion relation becomes

$$\gamma v^2\eta\tilde{\Gamma}^2-[(\rho+p)+2iv\gamma\eta\tilde{k}]\tilde{\Gamma}+i(\rho+p)v\tilde{k}-\eta\gamma\tilde{k}^2=0 , \quad (55)$$

exactly again the dispersion relation found for the boosted transverse modes in paper I. The roots of Eq. (55) are always complex except when $\tilde{k}=0$ and

$$\tilde{\Gamma}=(\rho+p)/\gamma v^2\eta . \quad (56)$$

The real parts of the complex roots satisfy

$$\tilde{\Gamma}_{R1}+\tilde{\Gamma}_{R2}=\frac{\rho+p}{\gamma v^2\eta}>0 , \quad (57)$$

$$\tilde{\Gamma}_{R1}\tilde{\Gamma}_{R2}=-\left[\tilde{\Gamma}_{I1}-\frac{\tilde{k}}{v}\right]^2\leq 0 . \quad (58)$$

Equations (57) and (58) imply that exactly one of the two transverse modes has a positive real part, and hence represents a growing, unstable mode.

The dispersion relation for the longitudinal modes in the boosted frame is again an excessively complicated sixth-order polynomial in $\tilde{\Gamma}$: without further constraints on the signs and values of the N_i, R_i, ζ_i, σ_i, and κ_i coefficients, it appears extremely difficult to determine the stability of those modes.

IV. DISCUSSION

In the previous section it has been shown that if $\kappa_2\neq 0$, then there exist growing exponential plane-wave solutions to the linearized perturbation equations for both transverse and long-wavelength longitudinal modes. In addition, even if $\kappa_2=0$, so long as η, the shear viscosity coefficient, is nonzero, there exists a growing transverse mode in the boosted, noncomoving frame. The arguments given in Sec. IV of paper I then lead to the conclusion that real, physically acceptable perturbations which are spatially bounded (unlike plane waves) will also grow exponentially in time. The equilibrium states of the Havas-Swenson theory are thus all unstable, with the possible exception of the special case where $\kappa_2=\eta=0$. The stability or instability of the equilibrium states in this special case is unknown at present. For a generic, realistic, fluid, however, the shear viscosity coefficient will be nonzero, and the most general first-order theory, that of Havas and Swenson, will be unstable.

In both paper I and this study, it is clear that the evolution of perturbations depends on the choice of fluid frame (choice of u^a). The evolution of perturbations in, say, the Eckart frame, satisfying Eq. (5), is thus different than the evolution in, say, the Landau-Lifshitz frame, satisfying Eq. (6). Israel, however, has shown that the predictions of the fluid theory should be invariant under first-order (in q^a, etc.) changes in the choice of four-velocity.[7] The resolution of this seeming conflict lies in the assumptions underlying Israel's proof. Israel's demonstration of the equivalence of fluid frames assumed that the gradients of first-order deviations from equilibrium are themselves first order. This is not, however, true for certain of the solutions to the perturbation equations found in this paper and in paper I. In these solutions, the derivatives with respect to time of the first-order quantities are zero order (since the frequencies are proportional to κ^{-1}, η^{-1}, etc.), and hence the assumptions of Israel's proof are violated.

The first-order fluid theories can either be derived axiomatically as in this paper, or they can be constructed (in the dilute-gas limit) from kinetic theory using the relativistic version of the Chapman-Enskog approximation. The Chapman-Enskog approximation systematically eliminates time derivatives, i.e., it assumes that all quantities vary only on time scales much longer than the collisional time scale. Since the unstable growing modes in the first-order fluid theories all have very large time derivatives (order zero in κ, η, etc.) of quantities which are assumed to be small (first-order q^a, τ^{ab}, etc.), it is clear that, at least in the dilute-gas limit, the fluid theory has been pushed beyond the domain rigorously justified by a kinetic theory derivation. In this sense, one might not be surprised by the existence of instabilities or some sort of odd behavior in this regime. It should be noted, however, that in the equivalent Newtonian theory, the kinetic theory derivation breaks down in a similar fashion for rapid variations, yet the Navier-Stokes-Fourier theory of fluids does not contain any unstable growing modes; equilibrium states are stable in that theory. The origin of the instabilities is thus not simply explained by the breakdown in the kinetic theory derivation of the fluid theory; rather, the additional relativistic terms in the equations of motion [such as the acceleration term in the heat flow Eq. (19)] are responsible.

Faced with unstable equilibria in even the most general first-order theories of dissipative relativistic fluids, what is one to do? It is clear that these instabilities are unphysical; since they are ubiquitous, however, (*all* equilibria have unstable modes), they will likely appear in any evolution calculation using the first-order theories. They are perhaps especially likely to cause problems in numerical evolution calculations. Even with purely decaying-mode initial data, numerical round-off error will falsely excite the growing modes, which will then subsequently dominate the evolution.

One approach to remedying these problems would be to develop a set of supplemental rules to append to the first-order theories which would distinguish between "real" physical solutions to the equations of motion and "false," spurious, growing solutions.

A second approach to eliminating the spurious instabilities is to abandon the first-order theories in favor of the second-order theories developed by Israel and Stewart.[7,8] It is known that in the second-order theories, if the phenomenological coefficients are chosen so that the perturbations propagate causally, then the equilibrium states are stable.[6] Furthermore, from the viewpoint of kinetic theory, the second-order theories can be derived from a better approximation than that used to derive the first-order theories[8–10] (i.e., terms which are neglected in deriving the first-order theories are retained in the derivation of the second-order theories). While the second-order theories are more complicated than the first-order theories, it is not at all clear that they are more complicated than the first-order theories with supplemental rules appended to eliminate unphysical solutions.

ACKNOWLEDGMENTS

I wish to thank L. Lindblom and R. Swenson for helpful discussions. This research was supported in part by the National Science Foundation, Grants Nos. ISP80-11449 and PHY8505484.

[1]P. Havas and R. J. Swenson, Ann. Phys. (N.Y.) **118**, 259 (1979).

[2]W. A. Hiscock and L. Lindblom, Phys. Rev. D **31**, 725 (1985).

[3]C. Eckart, Phys. Rev. **58**, 919 (1940).

[4]L. Landau and E. M. Lifshitz, *Fluid Mechanics* (Addison-Wesley, Reading, Mass., 1958), Sec. 127.

[5]R. J. Swenson, J. Non-Equilib. Thermodyn. **3**, 39 (1978).

[6]W. A. Hiscock and L. Lindblom, Ann. Phys. (N.Y.) **151**, 466 (1983).

[7]W. Israel, Ann. Phys. (N.Y.) **100**, 310 (1976).

[8]W. Israel and J. M. Stewart, Ann. Phys. (N.Y.) **118**, 341 (1979).

[9]J. M. Stewart, Proc. R. Soc. London **A357**, 59 (1977).

[10]W. Israel and J. M. Stewart, Proc. R. Soc. London **A365**, 43 (1979).