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Neutrino mass limits from the Fritzsch mass matrix

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Assuming that neutrinos are Dirac particles and have a mass matrix of the Fritzsch form, we constrain the masses of the second and third neutrinos using the following experimental data: the recently improved upper bounds on ν_μ and ν_τ masses, the results of Lubimov *et al.*, and neutrino-oscillation data. The masses of the second and third neutrinos are found to lie in a small region of the m_{ν_2}-m_{ν_3} plane.

Are neutrinos Dirac or Majorana particles? What are the masses of the neutrinos? These questions have been widely discussed in the literature[1] and are still unanswered experimentally. The lack of observation of neutrinoless double-β decay has put an upper bound on the electron-neutrino Majorana mass of $M_{\nu_e} < 5$ eV.[2] However, the experiment of Lubimov *et al.*[3] has reported an electron-neutrino mass of $M_{\nu_e} \sim 33$ eV. Together these experiments lead to the idea that neutrinos are Dirac particles. Grand unified theories which lead to light Dirac neutrinos have recently been discussed in the literature.[4] These models are generally more complicated and involve larger unification groups than SU(5). If neutrinos are Dirac particles, then horizontal symmetries in the electroweak Lagrangian may act on quarks and leptons in an identical fashion. If this is so, then the unitary matrices which rotate the mass eigenstates into the weak eigenstates will be of the same form for both the quark and leptonic sectors. There have been extensive analyses[5] of the relationships between the Kobayashi-Maskawa[6] mixing angles and mass ratios in the quark sector. The scenario which has received the most attention in the literature[7] is the Fritzsch[8] model. In the quark sector the predictions of the Fritzsch model are in agreement with present experiment.[7] The purpose of this Rapid Communication is to examine the implications of the Fritzsch model in the leptonic sector, i.e., to calculate the neutrino mixing angles in terms of lepton masses. We will find that the Fritzsch model predicts a small mass region for ν_μ and ν_τ which is consistent with neutrino-oscillation experiments.

The Fritzsch model results from imposing certain discrete horizontal symmetries on the electroweak Lagrangian.[7] A mass matrix of the Fritzsch type takes the form

$$M_F=\begin{pmatrix} 0 & |A|e^{i\phi_A} & 0 \\ |A|e^{i\phi_A'} & 0 & |B|e^{i\phi_B} \\ 0 & |B|e^{i\phi_B'} & |C|e^{i\phi_C}\end{pmatrix} .$$

The phases in this matrix allow for the possibility of *CP* violation in the leptonic sector. The weak-charged-current mixing matrix then takes the form

$$U=V_\nu^T\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\sigma} & 0\\ 0 & 0 & e^{i\tau}\end{pmatrix}V_l .$$

The matrices V_i ($i=\nu,l$) are obtained in terms of the fermion masses in the work of Ref. 9. σ and τ are two free phase parameters which are related to the phases in the mass matrix M_F.[7] Note that the separate rotations of the charged leptons and the neutrinos are included in the mixing matrix U.

Combining the results of Lubimov *et al.*[3] with the improved upper mass limits on ν_μ and ν_τ from π and τ decays,[10] gives the following constraints: $M_{\nu_1}=33\pm1.1$ eV, 33 eV $< M_{\nu_2} <$ 250 keV, $M_{\nu_2} < M_{\nu_3} <$ 70 MeV. The lower limits on the masses of ν_2 and ν_3 come from the assumption that massive neutrinos obey the usual hierarchal generation structure. Exploring the above mass region, and allowing for arbitrary phases σ and τ, we have calculated the probability of observing neutrino oscillations. The standard result[11] for the probability of $\nu_\alpha\to\nu_\beta$ for large $\Delta M^2_{\alpha\beta}\equiv M_{\nu_\alpha}{}^2-M_{\nu_\beta}{}^2$ ($\Delta M^2_{\alpha\beta}\gtrsim 10$ eV^2) is given by

$$P(\nu_\alpha\to\nu_\beta)=\sum_{i=1}^{3}|U_{\alpha i}U_{\beta i}|^2 .$$

We compared these calculations to the results of neutrino-oscillation experiments[12] and found that only a small set of values for the masses of ν_2 and ν_3 agrees with experiment. This small region is shown in Fig. 1. The predicted phases σ and τ are relatively small with $|\sigma|$, $|\tau|\le 0.6$. It should be noted that the numerical values of the masses, mixing angles, and oscillation probabilities are not significantly altered as the phases vary within their allowed ranges. The mixing matrix for a sample value of M_{ν_2}, M_{ν_3}, σ, and τ in the middle of the allowed region ($M_{\nu_2}=25$ keV, $M_{\nu_3}=500$ keV, $\sigma=0°=\tau$) is

$$U=\begin{pmatrix} 9.99\times10^{-1} & -3.32\times10^{-2} & 1.26\times10^{-3}\\ 3.32\times10^{-2} & 9.99\times10^{-1} & 1.83\times10^{-2}\\ -1.87\times10^{-3} & -1.83\times10^{-2} & 9.99\times10^{-1}\end{pmatrix} .$$

For an arbitrary 3×3 unitary mixing matrix U, Jarlskog has shown[13] that the nine quantities

$$\mathrm{Im}(U_{ij}U_{kl}U^*_{kj}U^*_{il})$$

are all equal up to a sign and may be used as a measure of *CP* violation resulting from the matrix U. This result is independent of the parametrization of the mixing matrix U. In order to estimate the amount of *CP* violation in the leptonic sector for the present model, we have calculated the quantity

$$J=\mathrm{Im}(U_{11}U_{22}U^*_{21}U^*_{12})$$

for neutrino masses in the allowed region. In all cases we find that $|J|\lesssim 10^{-5}$. Thus, we expect any *CP* violation in

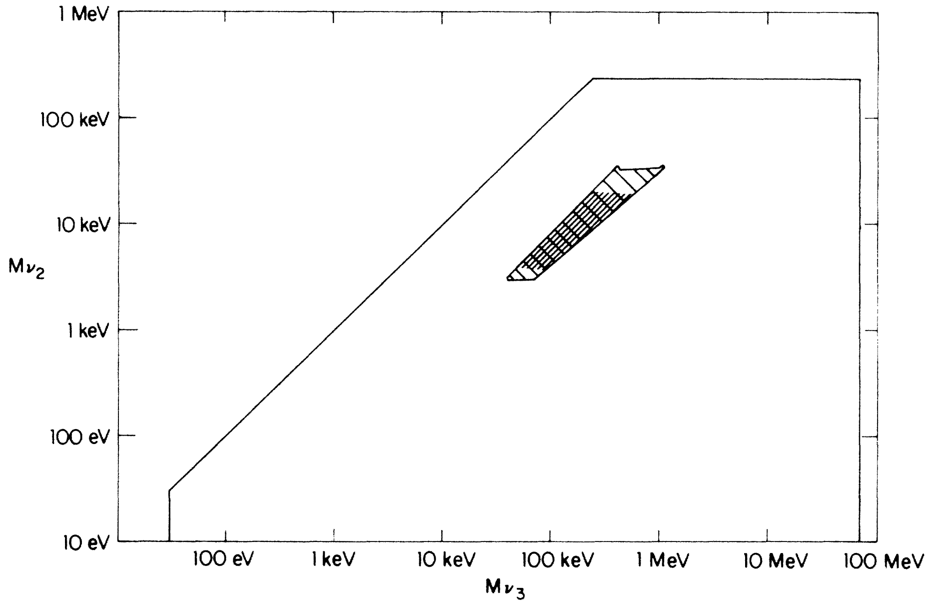

FIG. 1. A plot of the ν_2-ν_3 mass plane showing the mass region set by the experimental upper mass bounds and the hierarchical generation constraint. The crosshatched area is the region where the Fritzsch model is consistent with present neutrino-oscillation experiments. The shaded area corresponds to an improvement by a factor of 2 in $\nu_\mu \to \nu_e$ oscillation experimental limits.

the leptonic sector to be very small. The oscillation probabilities, along with the present limits (for large $\Delta M^2_{\alpha\beta}$) from neutrino-oscillation experiments are shown in Table I. We note that Simpson's 17.1-keV neutrino[14] lies within the allowed mass range for ν_2, but in order to have agreement with oscillation experiments the mixing-matrix elements U_{13} must be far less than that found by Simpson. New experiments[15] have failed to verify the existence of a 17.1-keV neutrino with $|U_{13}|^2 \simeq 3\%$, but have not ruled out a 17.1-keV neutrino with a much smaller value of $|U_{13}|^2$.

How difficult would it be to observe neutrinos with masses in the crosshatched region of Fig. 1? τ-decay experiments are insensitive to ν_3 masses below 1 MeV. In principle, measurements of neutrino masses from π decay are sensitive to the range $m_\nu \gtrsim 100$ keV if the mixing angles are not too small. In the model presented here we find that $|U_{23}|^2 \lesssim \text{few} \times 10^{-4}$, which is far too small to allow for the observation of neutrinos in this mass range. On the other hand, β-decay experiments are sensitive to small masses (< 20 keV), but again require large mixing angles ($\gtrsim 10^{-2}$). In this model, however, the mixing angles are at least a factor of 10 too small.

If there is an improvement in neutrino-oscillation experiments, the area of the crosshatched region of Fig. 1 can be reduced. Note that the experimental limits for $\nu_e \leftrightarrow \nu_\mu$ oscillations from Table I are closest to the predicted values. A factor-of-2 improvement in both these experiments shrinks the allowed region to the shaded area in Fig. 1. Thus, neutrino-oscillation experiments are the only way to reduce the allowed mass region in this model.

The cosmological requirement that neutrinos do not dominate the energy density of the Universe leads to the neutrino mass limit of $\sum m_\nu < 100$ eV.[16] This constraint can be avoided if neutrinos with masses above this range are unstable, i.e., if their lifetime is much shorter than the lifetime of the Universe. The decay rates of heavy unstable neutrinos have recently been discussed in the literature. Many authors have already excluded an unstable neutrino heavier than 1 MeV.[17] For masses $\lesssim 1$ MeV, neutrinos may decay radiatively ($\nu \to \nu'\gamma$, $\nu \to \nu'\gamma\gamma$), into lighter neutrinos ($\nu \to \nu'\nu'\bar{\nu}'$), or into Goldstone bosons ($\nu \to \nu' G$). These Goldstone-boson decay modes occur in theories with spontaneously broken global horizontal family symmetry and seem to be cosmologically acceptable depending on the symmetry-breaking scale.[18] The neutrino mass range in the model presented here is not excluded by cosmology if such decay modes are allowed.

Although we have allowed the phases σ and τ to be arbitrary in our calculation, an extension of the Fritzsch scheme allows these quantities to become calculable. In a recent paper, Gronau, Johnson, and Schechter[19] have combined the Fritzsch matrix with the ansatz of Stech,[20] leading to calculable phase parameters for the quark-mixing matrix. If the Stech hypothesis is also made in the leptonic sector, we can

TABLE I. A comparison of the predicted values of neutrino-oscillation probabilities with the experimental limits, for $M_{\nu_2} = 25$ keV, $M_{\nu_3} = 500$ keV, and $\sigma = 0° = \tau$.

	Prediction	Experiment
$P(\nu_e \to \nu_e)$	9.98×10^{-1}	>0.85
$P(\nu_e \to \nu_\mu)$	2.20×10^{-3}	<0.004
$P(\nu_e \to \nu_\tau)$	5.43×10^{-6}	<0.30
$P(\nu_\mu \to \nu_e)$	2.20×10^{-3}	<0.003
$P(\nu_\mu \to \nu_\mu)$	9.97×10^{-1}	>0.70
$P(\nu_\mu \to \nu_\tau)$	6.65×10^{-4}	<0.007
$P(\nu_\tau \to \nu_e)$	5.43×10^{-6}	$\cdots$
$P(\nu_\tau \to \nu_\mu)$	6.65×10^{-4}	<0.022
$P(\nu_\tau \to \nu_\tau)$	9.99×10^{-1}	$\cdots$

substantially reduce the size of the allowed region in Fig. 1.[21]

We thus conclude that given the hypotheses (i) neutrinos are Dirac particles, (ii) the lepton mass matrix is of the Fritzsch type, and (iii) the mass of $\nu_1 \sim 33$ eV, we can constrain the masses of ν_2 and ν_3 to a relatively small region. Neutrino-oscillation experiments are apparently the only way to reduce the allowed region in this model. We need more experimental data to resolve the question on the nature of the neutrinos and to pinpoint the neutrino mass.

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