RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 33, NUMBER 5 1 MARCH 1986

# Is $D^0 \to \phi \bar{K}^0$ really a clear signal for the annihilation diagram?

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(Received 1 November 1985)

It is shown that rescattering effects required by unitarity can produce the reaction $D^0 \to \phi \bar{K}^0$, even when the "$W$-exchange" or "annihilation" diagram is not present. This is addressed both in a general context and a specific model. In the latter, it is the mode $D^0 \to K^* \eta$ which plays the major role in generating the $\phi K$ final state, as the $K^* \eta$ state is produced using the $u\bar{u}$ component of the $\eta$, and scatters, $K^* \eta \to K\phi$, by quark exchange, utilizing the $s\bar{s}$ component of the $\eta$.

The quality of the data on exclusive $D$ decays has been steadily improving,[1,2] and this situation has attracted the attention of a number of theorists. The conventional way to analyze $D$ decays is in terms of a set of quark diagrams,[3] and the prime issues in the literature seem to be the size of "color suppression" and the existence of the "annihilation" and/or "$W$-exchange" diagrams. In the latter case, the decay $D \to \phi K$ has been claimed[4] to be an unambiguous signal for $W$ exchange (see Fig. 1), and this mode appears to have been seen experimentally.[2]

One problem with the quark-diagram approach is that it is only valid if the $S$ matrix for strong-interaction scattering is unity, i.e., no strong interactions. If the $S$ matrix is nontrivial, the relative sizes and strengths of the various amplitudes will be modified by the rescattering effects which are required by unitarity. The $S$-matrix elements involved are those of purely strong-interaction scattering in a $J=0$ state at $\sqrt{s}=m_D$. The mass of the $D$ meson lies in the heart of the resonance region, where all known scattering channels show a rich structure of both phase shifts and inelasticities. It is folly to proceed with the quark-diagram approach without considering these effects. The purpose of this Rapid Communication is to demonstrate that rescattering can also mix up the *classification* of diagrams, producing an apparent signal for the $W$-exchange diagram (i.e., $D \to \phi K$) even in the case when the diagram does not exist.

First the general mechanism for rescattering will be presented and a particular intermediate channel ($K^*\eta$) will be suggested. Later in the paper a simple model for the rescattering will be considered. In general, if we consider the scattering matrix $S$ and the representation in a particular $J^{PC}$,

$$(S_J)_{ij} = \delta_{ij} + 2i(T_J)_{ij}(q_i q_j)^{1/2} \, , \tag{1}$$

then the unitarity of the $S$ matrix requires

$$2\,\mathrm{Im}(T_J)_{ij} = \sum_K q_k (T_J)_{ik} (T_J^*)_{jk} \, , \tag{2}$$

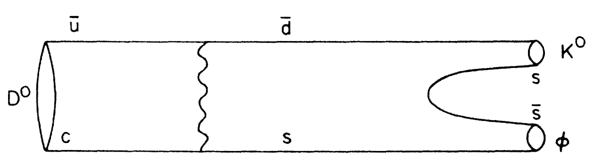


FIG. 1. The $W$-exchange or annihilation diagram applied to $D^0 \to \phi K^0$.

where $i,j,k$ are labels specifying the particle content of a state (e.g., $D^0$, $\phi K$, $K^*\eta$, etc.). In the case of purely elastic scattering, $S_L = \exp(2i\delta_L)$ and $T_L = \exp(i\delta_L)\sin\delta_L/q$. For a weak decay into a single final state (such as $K^+ \to \pi^+\pi^0$), application of the above formula yields Watson's theorem,[5] i.e., the phase of the weak amplitude is the same as the strong-interaction elastic-scattering phase $\delta_L$.

It has long been appreciated that phases required by Watson's theorem can dramatically modify quark-model predictions.[6] For example, $D^0 \to K^0\pi^0$, $K^-\pi^+$ produces $I=\frac{1}{2}$ and $\frac{3}{2}$ final states with

$$\begin{aligned} A(D^0 \to K^0\pi^0) &= A_{1/2} - \sqrt{2} A_{3/2} \, , \\ A(D^0 \to K^-\pi^+) &= \sqrt{2} A_{1/2} + A_{3/2} \, , \end{aligned} \tag{3}$$

and simple valence-quark models predict $A_{1/2}$ and $A_{3/2}$ real and in a combination that cancels strongly in the $K^0\pi^0$ mode (i.e., "color suppression"),

$$\begin{aligned} \frac{\sqrt{2} A_{3/2}}{A_{1/2}} &= \frac{4}{3} \frac{C_+}{(C_- + \frac{2}{3} C_+)} \simeq 0.40 \, , \\ \frac{\Gamma(D^0 \to K^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} &\simeq 0.1 \, , \end{aligned} \tag{4}$$

where $C_\pm$ are QCD coefficients, $C_- \sim 1.9$, $C_+ \sim 0.7$. Final-state rescattering will produce different phases for the $I=\frac{1}{2},\frac{3}{2}$ final states, and will destroy the cancellation

$$\frac{\Gamma(D^0 \to K^0\pi^0)}{\Gamma(D^0 \to K^-\pi^+)} = \frac{1}{2} \left| \frac{(C_- + \frac{2}{3} C_+) - \frac{8}{3} C_+ e^{i(\delta_3 - \delta_1)}}{(C_- + \frac{2}{3} C_+) - \frac{4}{3} C_+ e^{i(\delta_3 - \delta_1)}} \right|^2 \, . \tag{5}$$

The above ratio reproduces Eq. (4) if $\delta_3 - \delta_1 = 0$, but can be as large as 4 for other values of the phase difference. Such phases can completely mask this color-suppression effect. In addition, unitarity of the $S$ matrix can require specific relations between various final states regardless of the initial weak-interaction production mechanism. Sorenson[6] has demonstrated this for a two-channel problem and applied it to $D$ decays.

However, phases are not the only manifestations of rescattering. The final-state interactions can also mix reactions which come from different *types* of quark diagrams. Consider the two processes $D^0 \to \phi \bar{K}^0$ and $D^0 \to \eta \bar{K}^{*0}$. At the level of valence-quark diagrams, the former proceeds through the annihilation diagram of Fig. 1, while the latter can be produced by the more obvious "$W$-decay" diagram of Fig. 2. However, the two channels are connected by the rescattering required by unitarity. For example, the $s\bar{s}$ component of the $\eta$ can be used in a simple quark-exchange picture, Fig. 2(b), to produce $K^0\phi$. Hence, even if the annihilation diagram is completely absent we expect the reaction $D^0 \rightarrow K^0\phi$ to be observed. Note that the intermediate state has been produced by the $u\bar{u}$ component of the $\eta$ and, in this picture, rescatters by the $s\bar{s}$ component. Since we know that the $\eta$ has large amounts of both components in its wave function, such mixing effects will occur. If the rescattering reactions proceed by quark exchange only, the $K^*\eta$ intermediate state should be the dominant one. The only other similar mode, $K^*\eta'$, is strongly suppressed by the lack of phase space. In a more general model of rescattering, in which quark pairs can be created and destroyed, other modes, such as $\rho K$, could also feed the $\phi K$ channel.

How large do we expect rescattering effects to be? The naive valence-quark model predictions will emerge unmodified by final-state interactions only if the strong-interaction $S$ matrix is unity. In the meson channels accessible to experiment, such as $\pi\pi$ and $KK$, the $S$ matrix already has large phases at about 1 GeV of center-of-mass energy. Above 1.5 GeV the inelasticities grow, indicating scattering out of the elastic channel and a mixing of modes. The $D$ mass, 1.87 GeV, is in a region where inelastic effects are very large, and the $S$ matrix is very far from unity in both magnitude and phase. It would be remarkable if any remnant of the quark-diagram pattern were to survive.

To illustrate the effects under discussion, we will consider a simple model. We seek a parametrization of the $S$ matrix, where the unmodified valence-quark weak amplitudes can be introduced, and the effect of final-state strong rescattering studied. For this purpose the $K$-matrix parametrization is useful.[7,8] A time-reversal-invariant $S$ matrix can always be written in the form

$$S = \frac{1+iK}{1-iK} , \tag{6}$$

where the $K$ matrix is real and symmetric. Because $D$ decay may violate parity, the $S$ matrix splits into two sectors, given by angular momentum and parity $J^P = 0^+$ and $J^P = 0^-$, which do not mix with each other. Our discussion of the $K\phi$ signal will concern only those channels in the $J^P = 0^-$ sector. Let us consider a situation with $n$ two-body channels (this can easily be generalized to multibody modes) plus the $D^0$ which decays weakly into those $n$ modes. If we label the two-body modes by $i$ (or $j,k$) with $i = 1,n$, the naive quark-model amplitudes can be given by

$$\mathrm{Amp}^{(\mathrm{QM})}(D^0 \rightarrow i) = 2\epsilon_i , \tag{7}$$

where $\epsilon_i$ is of order $G_F$. The $K$ matrix will be $(n+1)\times(n+1)$ in size and will contain an $n\times n$ submatrix $K_0$ describing the strong-interaction scattering of the two-body modes:

$$K = \begin{pmatrix} K_0 & \epsilon \\ \epsilon & O(\epsilon^2) \end{pmatrix} . \tag{8}$$

The $D^0 \rightarrow \bar{D}^0$ element is second order in $G_F$ and we will neglect it henceforth. In the presence of rescattering, the quark-model amplitudes are mixed to form the observed $S$-matrix elements. From Eq. (6), one obtains

$$S(D^0 \rightarrow i) = 2\left(\frac{1}{1-iK_0}\right)_{ij} \epsilon_j$$

$$= (1+S_0)_{ij}\epsilon_j , \tag{9}$$

where in the second line $S_0$ is the $n\times n$ $S$ matrix

$$S_0 = \frac{1+iK_0}{1-iK_0} . \tag{10}$$

In the $n=1$ case, this reproduces Watson's theorem.

To study the specific case $D^0 \rightarrow K^0\phi$, let us consider the set of final states $i = [K^*\eta, \phi K, (K^*\pi)_{I=1/2}, (\rho K)_{I=1/2}, \omega K]$, i.e., $n=5$. If we make the assumption that all strong scattering is by quark exchange, the matrix $K_0$ is determined by various counting factors for quark rearrangement, plus an overall parameter $a$ for the strength of the interaction. We find

$$K_0 = a\sqrt{\rho}L_0\sqrt{\rho} , \tag{11}$$

where

$$L_0 = \begin{pmatrix} \frac{1}{36} & \frac{2}{3} & \frac{1}{4} & \frac{1}{4} & \frac{1}{12} \\ \frac{2}{3} & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{4} & 0 & \frac{1}{4} & \frac{1}{4} & \frac{3}{4} \\ \frac{1}{12} & 0 & \frac{3}{4} & \frac{3}{4} & \frac{1}{4} \end{pmatrix} , \tag{12}$$

and $\sqrt{\rho}$ is a phase-space factor

$$\rho = \frac{2p}{M_D} = \left[\left(1 - \frac{m_1{}^2 + m_2{}^2}{M_D{}^2}\right)^2 - \frac{4m_1m_2{}^2}{M_D{}^4}\right]^{1/2} . \tag{13}$$

This can be studied as a function of the strength $a$. Because the parameter $a$ is not of much physical interest, I have chosen to quote results in terms of the inelasticity in the $K^*\eta$ channel, $\eta_{K^*\eta}$. The quantity $(1-\eta_{K^*\eta})$ gives the fraction of events where an incoming $K^*\eta$ state scatters out of the elastic channel and into other modes. Of course, this particular number is unknown, but typical inelasticities at the energy produce $(1-\eta) \sim 0.5$ with significant variation. To be more realistic, this model could be supplemented by

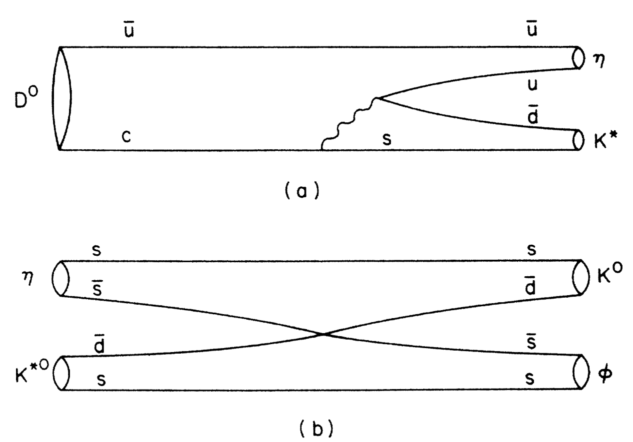


FIG. 2. (a) The $W$-decay diagram producing $D^0 \rightarrow K^*\eta$. (b) A quark-exchange contribution to the rescattering $K^*\eta \rightarrow K\phi$.

resonances which decay into the various final states (e.g., see Donoghue and Holstein, Ref. 6). Such an addition could modify the details of rescattering, but should leave the basic message unchanged.

The prime question is whether $D^0 \to K\phi$ can be generated significantly if the annihilation diagram is absent. To answer this, I will always keep $\epsilon_2 = 0$, i.e., the quark-model transition for $D^0 \to \phi K$ vanishes. Thus, the full $D^0 \to K\phi$ signal is generated by rescattering. As expected, in this model the $K^*\eta$ mode is most important in producing $\phi K$. Other modes require two rescatterings, e.g., $\pi K^* \to K^*\eta \to \phi K$, and are not very efficient in yielding $\phi K$. The basic result of this calculation is given in Fig. 3, which shows the decay rate of $\Gamma(D^0 \to \phi K^0)/\Gamma(D^0 \to K^*\eta)$ as a function of inelasticity. Any values in the range shown are physically reasonable, and we find a significant amount of $\phi K^0$ generated.

This particular model yields the prediction that the mode $D^0 \to K^*\eta$ should be sizable. Since the ARGUS Collaboration finds $B(D^0 \to \phi K^0) \simeq 1\%$, we would expect $B(D^0 \to K^*\eta) > 2\%$. In a completely general model this prediction could, in principle, be violated, but it is probably reasonable in a wider class of models than that considered here.

We have seen that the mode $D^0 \to \phi K$ is not necessarily a clear signal for the presence of the annihilation or $W$-exchange diagram. Indeed, the effects of final-state interactions appear significant enough to make the whole program of sorting out the quark diagrams in $D$ decays impossible to carry out. Given full knowledge of the strong-interaction $S$ matrix one could, in principle, invert the data to obtain the quark amplitudes. However, in practice this is impossible as we know very little about the relevant $S$-matrix elements. Unfortunately, the pattern which we see in $D$-meson decay is probably determined as much by final-state effects as by the underlying quark dynamics.

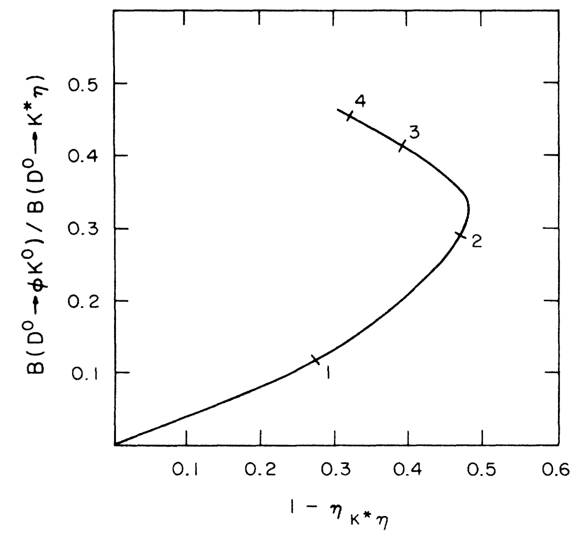


FIG. 3. The relation between $\Gamma(D^0 \to K\phi)/\Gamma(D^0 \to K^*\eta)$ and the inelasticity in $K^*\eta$ scattering, predicted in the model given in the text.

I would like to thank D. Hitlin for stimulating conversations, and the theory group at CERN for their hospitality in the initial phase of this project. This work was supported in part by the National Science Foundation.

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