$N=1$ supergravity grand unified theories with noncanonical kinetic energy terms

M. Drees
Institut für Physik, Universität Dortmund, 4600 Dortmund 50, West Germany
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$N=1$ supergravity theories with noncanonical kinetic energy terms for vector as well as chiral superfields are investigated. It is shown that, in general, *no* relation exists among the various soft-breaking parameters at the large scale. Nevertheless the gluino cannot be much heavier than scalar quarks of the first two generations due to renormalization effects. An explicit SU(5) model is constructed where both supergravity and SU(5) are broken by the help of a **24**. While $\sin^2\theta_W$ and m_b/m_τ can no longer be predicted, models of this type contain an additional "light" SU(3) octet, an SU(2) triplet, and a singlet, which might be, however, as heavy as 10^7 GeV. Some general constraints on masses of superpartners are derived from the requirement of a gauge hierarchy stable against radiative corrections. Finally the possible relevance of these types of models for cosmological considerations is pointed out.

I. INTRODUCTION

In the last couple of years a great amount of interest has been devoted to the construction of $N=1$ supergravity models.[1] Special attention has been paid to the mass spectrum of the light particles including the superpartners of "ordinary" fermions and gauge bosons. All these new particles (scalar quarks, scalar leptons, gauginos, and Higgs fermions) are usually assumed to be lighter than a few TeV in order not to destabilize the gauge hierarchy, the natural preservation of which has been one of the successes of supersymmetry. Most of the authors, however, restricted themselves to the case of so-called minimal or canonical supergravity, which means that kinetic energy terms for vector as well as chiral superfields are assumed to have their canonical form already at the Planck scale $M_P \simeq 2.4\times 10^{18}$ GeV. This is by far not obvious since the $N=1$ supergravity Lagrangian[2] contains in any case lots of terms which are *not* renormalizable at M_P.

It is thus tempting to investigate the phenomenological consequences of the introduction of noncanonical kinetic energy terms, still preserving renormalizability at scales well below M_P. Some research has been done for cases where kinetic energy terms of either chiral[3] *or* vector superfields[4] have a more general form. Effects due to a heavy grand-unified-theory (GUT) sector have, however, not been considered in Ref. 3. This was done in Ref. 5, but there the existence of an additional global $U(n)$ symmetry among the n chiral superfields was assumed which imposed strong constraints. In this paper I shall use the most general ansatz for all kinetic energy terms consistent with the requirement of renormalizability below M_P.

This paper is organized as follows. Section II will be devoted to the general formalism with special emphasis on the effects of general kinetic energy terms for chiral superfields. In order to keep the subject treatable, I shall assume that all kinetic energy terms can be parametrized as functions of one chiral supermultiplet Σ which is responsible for the spontaneous breakdown of supersymmetry. Nevertheless the results will be quite general.

In Sec. III this general formalism will be applied to a special GUT model based on the gauge group SU(5). In this case, Σ will be in the **24** representation, developing a nonvanishing vacuum expectation value (VEV) which breaks both supergravity and SU(5). The Σ-dependent part of the potential will be a generalization of a model proposed in Ref. 6; care has been taken to avoid the existence of absolute $SU(4)\times U(1)$-symmetric minima. It is shown that none of the strong predictions made by minimal $N=1$ supergravity theories survives. Especially neither the superpartners of the gauge bosons nor the scalar partners of quarks and leptons have to be degenerate in mass at the Planck scale.

Section IV is devoted to the analysis of constraints which can still be obtained for masses of the light particles within the theory. A rather weak upper bound can be derived from the requirement of a stable gauge hierarchy, while the gluino—to—scalar-quark mass ratio is bounded from above due to renormalization effects. Finally, in Sec. V, I shall present a short summary and draw some conclusions. Some details of the computations relevant for Secs. II and III are presented in the Appendixes.

II. GENERAL FORMALISM

Since the case of noncanonical kinetic terms for vector superfields has already been treated in some detail in Ref. 4, I shall only repeat some topics which are important for the further discussion.

The kinetic term for vector superfields is determined by the chiral function $f_{\alpha\beta}$ transforming like a symmetric product of two adjoints under the gauge group[2] G, $f_{\alpha\beta}=\delta_{\alpha\beta}$ being the canonical choice. Introducing a general ansatz for $f_{\alpha\beta}$ implies[4] that the gauge couplings of SU(3), SU(2), and U(1) do *not* have to be equal at the unification scale M_x which thus becomes a free parameter of the theory. This also holds for the gauge coupling of the GUT group. It is therefore natural to choose $M_x=O(M_P)$, where *both* the GUT group and supersymmetry are broken by the *same* VEV, thus getting rid of the somewhat arbitrary hidden sector.

The situation in the chiral sector is a bit more complicated, since the form of the kinetic energy terms affects the scalar potential, too. The kinetic terms are determined[2] by the Kähler metric $\mathcal{G}_i^j \equiv \partial \mathcal{G} / \partial z^{*i} \partial z_j$, where $\mathcal{G}$ is a real function of the chiral multiplets $z_i = \{\Sigma, y_i\}$; here, y_i denote the light supermultiplets containing quarks, leptons, and the Higgs fermions responsible for the breakdown of $SU(2) \times U(1)_y$ to $U(1)_{em}$. The canonical choice for $\mathcal{G}$ is

$$\mathcal{G} = -\sum_i z_i z^{i*} - M_P^2 \ln \frac{|g|^2}{M_P^6} , \tag{1}$$

where $g \equiv g(z_i)$ is the superpotential, whereas I have considered the ansatz

$$\mathcal{G} = -\sum_i X_i(\Sigma) y_i y^{i*} - F(\Sigma) - M_P^2 \ln \frac{|g|^2}{M_P^6} , \tag{2}$$

where $X_i(\Sigma)$ are real dimensionless functions of the heavy supermultiplet Σ, while $F(\Sigma)$ has dimension 2. The combination $X_i(\Sigma) y_i y^{i*}$ has, of course, to be invariant under the GUT group G. Since Σ belongs to a real representation of G while all y_i belong to complex representations, no term linear in y_i is allowed. Since adding a constant to the right-hand side (RHS) of Eq. (2) would not have any effect and terms of third or higher order in y_i have to be suppressed by inverse powers of the Planck mass and are thus not relevant for the effective low-energy theory, Eq. (2) is the most general ansatz if one restricts oneself to the case of only one superheavy chiral superfield Σ.

In order to get some information about the soft-breaking parameters[7] of the scalar sector the potential has to be computed from Eq. (2). Its F-term contribution is generally given by[2]

$$V_F = -\exp(-\mathcal{G}/M_P^2)[3M_P^4 + M_P^2 \mathcal{G}^i (\mathcal{G}^{-1})_i^j \mathcal{G}_j] , \tag{3}$$

where $(\mathcal{G}^{-1})_i^j$ denotes the inverse matrix of $\mathcal{G}_j^i$. Some details of the computation of V_F will be presented in Appendix A. Here I just give the result in the phenomenologically relevant flat times where $M_P \to \infty$ while $m_{3/2} \equiv M_P \exp(-\mathcal{G}/2M_P^2)$ remains fixed:

$$V_F = \sum_i |\hat{g}_{,\hat{y}_i}|^2 + \sum_i m_i^2 |\hat{y}_i|^2 + m_{3/2}(A^{(1)}\hat{g} + A_i^{(2)} \hat{y}_i \hat{g}_{,\hat{y}_i} + \text{H.c.}) , \tag{4}$$

where $\hat{y}_i$ denotes the properly normalized scalar components of the corresponding superfield

$$\hat{y}_i \equiv \sqrt{X_i}\, y_i \,|_{\text{scalar components}} , \tag{5}$$

and $\hat{g} \equiv \exp(F/2M_P^2) g$ is the rescaled superpotential; finally, $\hat{g}_{,\hat{y}_i} \equiv \partial \hat{g} / \partial \hat{y}_i$ as usual.

Note that the fermionic components of y_i have to undergo the *same* rescaling as their scalar superpartners. Thus the rescaling (5) can be performed at the superfield level, manifesting the softness of supergravity breaking which is still preserved in the noncanonical case.[3] After the rescaling, which will in general not respect G invariance, parameters of the superpotential, which had to be equal because of symmetry under G transformations, do no longer need to be equal. This implies that, e.g., the value of the b-quark-to-τ mass can no longer be predicted if Σ is not a G singlet. In general, one has for trilinear superpotential couplings of the rescaled fields

$$h_{ijk} \to \hat{h}_{ijk} \equiv h_{ijk} / (X_i X_j X_k)^{1/2} . \tag{6}$$

The values of the soft-breaking parameters entering Eq. (4) are given by

$$m_i^2 = m_{3/2}^2 \left[1 - \left| F_{,\Sigma} + \frac{M_P^2 g_{,\Sigma}}{g} \right|^2 \frac{X_i X_{i,\Sigma\Sigma^*} - |X_{i,\Sigma}|^2}{X_i^2 F_{,\Sigma\Sigma^*}^2} \right] , \tag{7}$$

$$A^{(1)} = -3 + \left[F_{,\Sigma^*} + \frac{M_P^2 g^*_{,\Sigma^*}}{g^*} \right] \frac{F_{,\Sigma}}{M_P^2 F_{,\Sigma\Sigma^*}} , \tag{8}$$

$$A_i^{(2)} = 1 - \left[F_{,\Sigma^*} + \frac{M_P^2 g^*_{,\Sigma^*}}{g^*} \right] \frac{X_{i,\Sigma}}{X_i F_{,\Sigma\Sigma^*}} , \tag{9}$$

with $X_{i,\Sigma} \equiv \langle \partial X_i / \partial \Sigma \rangle$, etc. Setting $X_i \equiv 1$ one regains, of course, the well-known[8] results for the canonical case. The same holds if X_i is constant; thus a nontrivial dependence of X_i on Σ has to be assumed in order to produce any new effect. In general there are, however, *no* relations among the various soft-breaking parameters, i.e., there will be as many different masses m_i^2 and trilinear soft-breaking parameters $A_i^{(2)}$ as there are different X_i. Note that this even holds if one assumes the kinetic energy terms of Σ to have its canonical form, i.e., $F(\Sigma) = \Sigma\Sigma^*$. If one makes the natural assumptions $F_{,\Sigma\Sigma^*}, X_i \sim 1$, the order of magnitude of the soft-breaking parameters will be as in the canonical case. Note, however, that the m_i^2 might even become negative, thus allowing for a spontaneous breakdown of the $SU(2) \times U(1)$ symmetry without the need of any renormalization. Furthermore, the trilinear scalar couplings are determined by the superpotential alone while the *strength* of these couplings also depend on details of the heavy sector.

The case of noncanonical kinetic energy terms for chiral superfields has already been investigated in Ref. 3 in a somewhat less explicit manner. The more explicit equations (7)—(9) will be more suitable for the following discussions of effects of the GUT sector.

III. AN EXPLICIT SU(5) GUT MODEL

As stated in the last section the number of independent functions $X_i(\Sigma)$ determines the number of different soft-breaking parameters. It is evident that this number can be increased if Σ carries some indices, i.e., transforms nontrivially under G. I shall thus concentrate onto theories where the GUT group and supergravity are broken by the same VEV. To be definite I choose $G = SU(5)$ and $\Sigma = 24$ which has the virtue of being the simplest choice. Nevertheless all results will turn out to be quite general.

The light-particle spectrum contains N_f families, i.e., $N_f(\bar{5}+10)$, and N_H pairs of light Higgs doublets, i.e., $N_H(5+\bar{5})$. Different SU(5) multiplets may, of course, have different kinetic energy terms X_i. The question to be investigated is whether integrating out Σ means to create different X_i for members of the same SU(5) multiplet.

For a supermultiplet transforming as a 5 or $\overline{5}$ of SU(5) one has the most general ansatz for the kinetic term in $\mathcal{G}(a,b,=1,\ldots,5;\alpha=1,\ldots,24)$:

$$-y_a y^{*b}[f_1^{(5)}(\Sigma/M_P)\delta_b^a + f_2^{(5)}(\Sigma/M_P)(\lambda_\alpha)_b^a\Sigma^\alpha/M_P] . \tag{10}$$

Here, $f_1^{(5)}$ and $f_2^{(5)}$ are G-invariant functions of Σ. Using as in Ref. 4 the "physical" convention for the λ matrices one has to require

$$\langle \Sigma_\alpha \rangle = v\delta_{\alpha,24} \tag{11}$$

in order to have SU(3)$\times$SU(2)$\times$U(1) as the low-energy gauge group. Inserting Eq. (11) into Eq. (10) one sees that the kinetic term becomes diagonal in y_a with

$$X_a^{(5)} = f_1^{(5)} + c_a f_2^{(5)} v/M_P$$

$$\text{where } c_a = \begin{cases} 2/\sqrt{15}, & a=1,2,3 , \\ -3/\sqrt{15}, & a=4,5 . \end{cases} \tag{12}$$

From Eq. (12) one has the very important result that the two SU(3)$\times$SU(2) multiplets contained in **5** [$5=(3,1)+(1,2)$ (Ref. 9)] will have *different* kinetic energy terms if $f_2^{(5)} \neq 0$. That means from Eq. (7) that masses of the corresponding scalars might be different already at M_P. The same holds, e.g., for the Yukawa couplings of the b quark and the τ-lepton or of doublet and triplet components of the **5** of Higgs fermions due to different rescaling of these couplings according to Eq. (6).

In the case of a **10** of SU(5) the most general ansatz for the kinetic term is

$$y_{ab}y^{*cd}[f_1^{(10)}(\Sigma/M_P)\delta^a{}_c\delta^b{}_a + f_2^{(10)}(\Sigma/M_P)\delta^a{}_c\Sigma^\alpha/M_P(\lambda_\alpha)^b{}_d + f_3^{(10)}(\Sigma/M_P)\Sigma^\alpha/M_P(\lambda_\alpha)^a{}_c\Sigma^\beta/M_P(\lambda_\beta)^b{}_d] , \tag{13}$$

where y_{ab} is an antisymmetric 5×5 matrix: $y_{ab} = y_i(M^i)_{ab}$, $i=1,\ldots,10$ (Ref. 9). Replacing Σ by its VEV one has

$$X_i^{(10)} = 2f_1^{(10)} + C_i' v/M_P f_2^{(10)} + C_i'' v^2/M_P{}^2 f_3^{(10)} ,$$

with

$$C_i' = \begin{cases} 4/\sqrt{15}, & i=1,2,3 , \\ -1/\sqrt{15}, & i=4,\ldots,9 , \\ -6/\sqrt{15}, & i=10 , \end{cases}$$
$$C_i'' = \begin{cases} 8/15, & i=1,2,3 , \\ -4/5, & i=4,\ldots,9 , \\ 6/5, & i=10 . \end{cases} \tag{14}$$

One again finds that the different SU(3)$\times$SU(2) multiplets contained in the 10 of SU(5), i.e., $(\overline{3},1)$, (3,2), and (1,1) will in general have *different* kinetic energy terms. From Eqs. (12) and (14) one can in fact deduce that there might be as many different functions $X_i(\Sigma)$ as compatible with low-energy SU(3)$\times$SU(2)$\times$U(1) gauge invariance; as far as the X_i are concerned nothing remains of the grand unification.

These results are obtained for the simplest possible GUT model, i.e., $G=$SU(5) and $\Sigma=$**24**. More complicated models will certainly show the same amount of freedom if no further symmetry among the X_i is imposed. It remains to be shown, however, whether this simple model is already a realistic one. In other words, to complete the model one has to find a superpotential $g(\Sigma)$ resulting in a scalar potential $V(\Sigma)$ whose absolute minimum breaks both supergravity and SU(5) spontaneously, in agreement with the requirements of SU(3)$\times$SU(2)$\times$U(1) gauge symmetry at low energies and a vanishing cosmological constant. One existing model[6] has the disadvantage of an absolute SU(4)$\times$U(1) symmetric minimum. I have thus slightly generalized the ansatz of Ref. 6 to write

$$g(\Sigma) = \lambda[\mathrm{tr}\Sigma^4/M_P + a/M_P(\mathrm{tr}\Sigma^2)^2 + b\,\mathrm{tr}\Sigma^3 + cM_P\mathrm{tr}\Sigma^2] . \tag{15}$$

The constant λ has been factored out; it will turn out to be very small ($\sim 10^{-16}$–10^{-12}) in order to give a gravitino mass of order 100 GeV. Note, however, that the usually considered hidden sector superpotentials, e.g., the Polonyi ansatz,[10] contain a parameter which has the same order of magnitude. The superpotential (15) does not contain a constant term since this would imply $V(\Sigma=0)<0$; thus the SU(5)-symmetric state would be favored. A term linear in Σ is forbidden by SU(5) invariance. Thus Eq. (15) constitutes the most general ansatz which is at most quartic in Σ. In Ref. 6 only the case $a=0$ was considered. Just like the authors of Ref. 6 I assume the kinetic energy term of Σ to have its canonical form. As stated in Sec. II this does not impose any constraint on the low-energy phenomenology.

In order to minimize the corresponding potential one writes

$$\langle\Sigma\rangle = \frac{v}{\sqrt{30}} \begin{bmatrix} 2 & & & & 0 \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ 0 & & & & -3 \end{bmatrix} \tag{16}$$

and requires

$$V(v) = V'(v) = 0 . \tag{17}$$

Note that Eq. (16) automatically yields a vanishing D term. The ansatz (15) is general enough to treat $v \equiv M_X$ as a free parameter. This is possible since kinetic energy terms of vector superfields are assumed to be nonminimal, too, which in general destroys the relation $g_3{}^2(M_X) = g_2{}^2(M_X) = \frac{5}{3}g_1{}^2(M_X)$ (Ref. 4). Choosing a definite value for v thus means to fix, e.g., b and c via Eqs. (17), for a given value of a. One then has to check that this solution belongs to a minimum which also breaks supersymmetry, i.e., $g(v)\neq 0$. In the next step one minimizes V for this set of parameters a,b,c in $SU(4)\times U(1)$ direction. Combinations which give $V_{\min}(SU(4))<0$ have been discarded.

Some details of the computation can be found in Appendix B. It turns out that requiring an absolutely stable $SU(3)\times SU(2)\times U(1)$-invariant minimum imposes the bounds

$$-0.05 \gtrsim a \gtrsim -0.3 \; ; \tag{18}$$

i.e., only nonzero values of a are allowed. Note, however, that this "desired" minimum is degenerate with the SU(5)-symmetric minimum at $\Sigma=0$. This feature can be traced back to the absence of a linear term in the superpotential and is thus characteristic for all models where G and supersymmetry are broken by the same VEV.

Some typical values of v, a, b, and c are shown in Table I. The most natural choice is $M_X \simeq M_P$. This can be realized with $a,b,c\simeq 1$. On the other hand, λ has to be very small then, i.e., $\lambda \sim 10^{-16}$. Furthermore one expects those members of the **24** supermultiplets whose scalar components are not Goldstone bosons to be rather light.[6] This means that besides the usual light particles and their superpartners one has additional superfields in (8,1), (1,3), and (1,1) representations of $SU(3)\times SU(2)$ with masses of about $10m_{3/2}$ (Ref. 6). Formulas for the corresponding supersymmetric mass parameters are derived in Appendix B. In models which favor small values of $m_{3/2}$ (Ref. 11) these additional particles might be unwanted. It turns out, however, that their masses can be raised by a factor of about 10^4 if one chooses $M_X \simeq 0.01 M_P$, which, on the other hand, requires $c\simeq 10^{-4}$. λ can be raised to be $\simeq 10^{-12}$ now. It is interesting that there exists a region of parameter space (e.g., third line of Table I) where the SU(3) octet as well as the SU(2) triplet are heavy ($m_8, m_3 \simeq 10^4 m_{3/2}$) while the singlet is light ($m_1 \simeq m_{3/2}$). This provides a possibility to establish a light $SU(3)\times SU(2)\times U(1)$ singlet without destabilizing the gauge hierarchy, since it is embedded in a SU(5) nonsinglet.

IV. CONSTRAINTS ON THE LOW-ENERGY SPECTRUM

It has already been pointed out in the last of Ref. 4 that exploiting the possibility of noncanonical kinetic energy terms for vector superfields in general means to destroy all relations among masses of SU(3), SU(2), and U(1) gauginos. The analogous phenomenon was shown in the last section to appear also in the chiral sector: There are as many independent scalar masses as there are $SU(3)\times SU(2)$ multiplets. Nevertheless some statements can be made about the spectrum if one requires the gauge hierarchy to be stable against radiative corrections, and furthermore demands the $SU(3)\times U(1)_{\rm em}$ symmetry to be unbroken at all scales.

The first requirement is usually satisfied by choosing $m_{3/2}\lesssim 1$ TeV, which sets the mass scale for all light scalars in canonical theories where these masses are degenerate at M_P. This choice of $m_{3/2}$ is also suggested by naturalness,[12] since without fine-tuning one expects

$$m_{W^\pm} \sim m_{H,\bar H} \sim m_{3/2} \; , \tag{19}$$

where $m_{H,\bar H}$ are the masses of the light Higgs scalars. The second approximate equality in (19) holds, however, in canonical models only. In order to get some mass bounds from the requirement of a stable gauge hierarchy one now has to investigate the effect of each field separately. From naturalness one still has

$$m_{H,\bar H} \lesssim 1 \text{ TeV} \; . \tag{20}$$

A stable hierarchy means that these tree values are not changed too much by radiative corrections:

$$\delta m_{H,\bar H}{}^2 \lesssim m_{H,\bar H}{}^2 \; . \tag{21}$$

There are contributions to the left-hand side (LHS) of (21) from the gauge as well as the chiral sector. The gauge contributions are proportional to[13] $g_i{}^2 M_i{}^2$, $i=1,2$, where M_1, M_2 are masses of U(1) and SU(2) gauginos, respectively. One is thus led to the constraint

$$M_1, M_2 \lesssim 1 \text{ TeV} \; , \tag{22}$$

which is, of course, only an order-of-magnitude estimation. Contributions from the chiral sector are proportional to $h_i{}^2 m_i{}^2$ (Ref. 13), where $m_i{}^2$ is the mass squared of the corresponding scalar and h_i its Yukawa coupling to H or $\bar H$. From Eqs. (20) and (21) one thus finds

$$m_i \lesssim 1 \text{ TeV}/h_i \; , \tag{23}$$

which implies that scalars of the first generation might be as heavy as 10^7 GeV without destabilizing the hierarchy. This, of course, implies very large differences among the various X_i [see Eq. (7)], but the same holds for the standard Yukawa couplings anyway. If one introduces very heavy scalars of the first generation care has to be taken to cancel contributions from the U(1) D term to $m_{H,\bar H}{}^2$ (Ref. 13); this implies one relation among masses of scalars of the first generation:

TABLE I. Vacuum expectation values and parameters of the SU(5) model discussed in Sec. III. Values of the supersymmetric mass parameters m_8, m_3, and m_1 of the SU(3) octet, SU(2) triplet, and singlet, respectively, are given in columns 5—7 in units of $m_{3/2}$.

M_X/M_P	a	b	c	$m_8/m_{3/2}$	$m_3/m_{3/2}$	$m_1/m_{3/2}$
1	-0.2	0.543	0.0913	100.6	62.6	57
0.1	-0.2	0.049	6.86×10^{-4}	1.94×10^4	1.39×10^4	6.10^3
0.1	-0.13	0.152	2.13×10^{-3}	1.2×10^4	3.2×10^3	5.2
0.01	-0.1	0.0195	2.67×10^{-5}	1.4×10^6	5.4×10^{-5}	5.8×10^3

$$m_{\tilde{e}_R}{}^2 - m_{\tilde{e}_L}{}^2 + m_{\tilde{d}_R}{}^2 - 2m_{\tilde{u}_R}{}^2 + m_{\tilde{q}_L}{}^2 \lesssim (1\ \mathrm{TeV})^2 \ . \tag{24}$$

From (23) one sees that masses of scalars of the third generation are bounded from above by a few TeV. This in turn implies a similar bound for the gluino mass which contributes to scalar-quark masses.[13]

Another constraint can be deduced from the requirement of an exact $SU(3)\times U(1)_{em}$ symmetry at all energy scales, which implies

$$m_i{}^2(M_P) \ge 0 \tag{25}$$

for all scalar quarks and scalar leptons. These masses are renormalized between M_P and the weak-breaking scale. For particles with small Yukawa couplings this renormalization is dominated by gauge contributions[13] and is thus of major importance only for colored scalars, the scalar quarks. Starting from the extremal choice $m_{\tilde{q}}{}^2(M_P)=0$ one can thus deduce upper bounds for the ratio of the gluino to scalar quark mass. This ratio strongly depends on the evolution of the SU(3) gauge coupling and thus on the particle content of the theory at a given scale. Bounds on this ratio are given in Table II for some values of $\alpha_3(M_W)$ and the mass of the additional octet. Note that the one-loop β function for g_3 vanishes at scales larger than this mass. In any case one finds

$$m_{\tilde{g}}/m_{\tilde{q}} \lesssim 1 \ . \tag{26}$$

This bound has been obtained for $M_X = 2\times 10^{16}$ GeV and three generations of quarks. Modifications due to a change of M_X by 1 or 2 orders of magnitude are, however, minor. If one allows $SU(3)\times U(1)_{em}$ to be broken at some high energies the bound (26) is, of course, no longer valid.

As pointed out in Sec. III the theory contains additional (8,1), (1,3), and (1,1) "light" superfields. In principle, the scalar members of the octet superfield can be used to break $SU(3)_c$ at scale Λ_{QCD} (Ref. 14) which might be desired from the phenomenological point of view.[14] In this case these scalars would have to be very light which needs some fine-tuning [see Eq. (7)]. In Ref. 15 it was pointed out that this light octet, as well as the introduction of a fourth generation of quarks and leptons, results in an unacceptably large ratio of b-quark to τ mass. As stated in Sec. III this is no problem here since the corresponding Yukawa couplings need no longer be equal after Σ has been integrated out and the light fields have been rescaled.

Finally I want to comment on the light Higgs sector. As stated in the last section it will in general contain a light neutral SU(2) singlet in addition to the two usual doublets. The couplings of this singlet depend on the mechanism chosen to achieve the double-triplet mass splitting. If one considers, as in Ref. 6, the missing partner mechanism there will be no coupling between the singlet and the doublets at small scales. In this case the Higgs-boson phenomenology would be similar to those in minimal models, including the prediction of a light neutral Higgs scalar with mass not larger than that of the Z^0 boson.[16] Improvements of this bound due to the radiative breaking of $SU(2)\times U(1)_y$ symmetry[17] would, of course, not survive, since they rely on the relation $m_H{}^2 = m_{\bar{H}}{}^2$ at M_P. Furthermore, the couplings of this Higgs boson to quarks and leptons can deviate from their standard values since the ratio of the VEV's of H and H' is completely unrestricted now.

The missing partner mechanism requires, however, the introduction of an additional $50+\overline{50}$ superfield as well as nonrenormalizable couplings between light and superheavy Higgs fermions. Using the fine-tuning mechanism means, on the other hand, to introduce a term $\sim NH\bar{H}$ in the superpotential, where N denotes the singlet. This would certainly affect the Higgs-boson phenomenology. In any case the N^3 term in the superpotential can be neglected because the coupling is tiny, while the supersymmetric mass term will in general survive. It might even become so large (see Table I) that the singlet decouples from physics at the weak scale.

TABLE II. Upper bounds for the ratio of gluino to scalar-quark mass for various combinations of $\alpha_s(M_W)$ and m_8. M_X was chosen to be 2×10^{16} GeV.

$\alpha_s(M_W)$	m_8 (GeV)	$(m_{\tilde{g}}/m_{\tilde{q}})_{\max}$
0.10	100	0.6
0.10	10^7	1.0
0.12	100	0.55
0.12	10^7	0.98

V. SUMMARY AND CONCLUSIONS

$N=1$ supergravity GUT models with general kinetic energy terms have been considered. It was shown that the usual relations[5,8] among soft-breaking operators do no longer hold if the corresponding kinetic terms are not equal. Even the GUT symmetry at high-energy scales does not restrict the number of free parameters of the effective low-energy theory, if one uses[6] the same field to break supersymmetry and the GUT symmetry. This means that one loses all predictions following from gauge unification or canonical $N=1$ supergravity in this general case: M_X as well as the SU(5) gauge coupling and the couplings of the heavy Higgs-boson triplets to matter become arbitrary so that nothing can be said about nucleon lifetime or decay modes. Since neither the various gauge couplings nor the Yukawa couplings of τ lepton and b-quark need to be degenerate at M_X the successful predictions of $\sin^2\theta_W$ and m_b/m_τ appear accidental here. Furthermore the mass of each superpartner will have to be determined separately; upper bounds which are, however, rather weak can only be imposed from naturalness[12] and stability of the gauge hierarchy under radiative corrections.[13] The only rather restrictive constraint can be imposed for the gluino to scalar quark mass ratio due to strong renormalization effects.

Nevertheless the combination of noncanonical kinetic energy terms and the idea[6] of supergravity breaking in the GUT sector has its virtues. The latter idea is interesting since one gets rid of the "hidden sector" which was introduced in a rather *ad hoc* manner. Introduction of noncanonical kinetic terms for the light superfields in some sense *simplifies* these theories since the introduction of additional fields and intermediate scales[6] to yield $M_X \simeq M_P$, which is the most natural choice, is no longer necessary. These models generally predict the existence of further "light" supermultiplets. In the simplest version [$G=\mathrm{SU}(5)$, broken by $\Sigma=24$] these transform as $(8,1)+(1,3)+(1,1)$ under $\mathrm{SU}(3)\times\mathrm{SU}(2)$. If chiral kinetic energy terms are assumed to have their canonical form problems with m_b/m_τ show up[15] which might again necessitate the introduction of further fields.

Canonical $N=1$ supergravity theories usually contain three distinct superheavy sectors, one being responsible for the breakdown of supersymmetry, the second for the breakdown of GUT symmetry, while the third produces the inflation[18] at a very early stage of the Universe. The first two have been unified in this paper. Perhaps the introduction of noncanonical kinetic terms allows one to come back to the first inflationary Universe scenarios where inflation was correlated to the breakdown of GUT symmetry.[19] In canonical models this leads to unacceptably large density fluctuations.[20] This problem can probably be solved here since the relevant GUT couplings can be made very small without changing the couplings of the low-energy theory. It might thus be that noncanonical $N=1$ supergravity GUT models can be constructed which reduce the number of independent heavy sectors from three to one.

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APPENDIX A: COMPUTATION OF THE GENERAL EFFECTIVE LOW-ENERGY POTENTIAL

In general the F-term contribution to the scalar potential is given by[2]

$$V_F = -\exp(-\mathcal{G}/M_P{}^2)[3M_P{}^4 + M_P{}^2 \mathcal{G}^i (\mathcal{G}^{-1})_i^j \mathcal{G}_j] , \tag{A1}$$

where $\mathcal{G}$ is now given by Eq. (2). One finds for n light fields $y_i, i=1,\ldots,n$,

$$\mathcal{G}_i = \left(-X_1 y_1 - M_P{}^2 \frac{g^*_{,y_1^*}}{g^*}, \ldots, -X_n y_n - M_P{}^2 \frac{g^*_{,y_n^*}}{g^*}, -\sum_j X_{j,\Sigma^*} y_j y^{j*} - F_{,\Sigma^*} - M_P{}^2 \frac{g^*_{,\Sigma^*}}{g^*} \right) , \tag{A2}$$

$$(\mathcal{G}^{-1})_i^j = \begin{pmatrix} -\dfrac{1}{X_1} + \dfrac{|X_{1,\Sigma} y_1|^2}{X_1{}^2 D} & \dfrac{X_{1,\Sigma} y_1 X_{2,\Sigma^*} y_2^*}{X_1 X_2 D} & \cdots & \dfrac{X_{1,\Sigma} y_1 X_{n,\Sigma^*} y_n^*}{X_1 X_n D} & \dfrac{-X_{1,\Sigma} y_1}{X_1 D} \\ & \dfrac{-1}{X_2} + \dfrac{|X_{2,\Sigma} y_2|^2}{X_2{}^2 D} & \cdots & \dfrac{X_{2,\Sigma} y_2 X_{n,\Sigma^*} y_n^*}{X_2 X_n D} & \dfrac{-X_{2,\Sigma} y_2}{X_2 D} \\ & & \ddots & \vdots & \vdots \\ & & & \dfrac{-1}{X_n} + \dfrac{|X_{n,\Sigma} y_n|^2}{X_n{}^2 D} & \dfrac{-X_{n,\Sigma} y_n}{X_n D} \\ & & & & \dfrac{1}{D} \end{pmatrix} , \tag{A3}$$

where the lower left of this matrix is determined by Hermiticity. D is given by

$$D = -\sum_i X_{i,\Sigma\Sigma^*} y_i y^{i*} - F_{,\Sigma\Sigma^*} + \sum_i \frac{|X_{i,\Sigma} y_i|^2}{X_i} , \tag{A4}$$

with $X_{i,\Sigma\Sigma^*} \equiv \partial^2 X_i / \partial\Sigma\partial\Sigma^*$, etc. Only terms which do not vanish as $M_P \to \infty$, $m_{3/2} \equiv M_P e^{-\mathcal{G}/2M_P{}^2}$ fixed, are important for the low-energy theory. Note that the last term in $\mathcal{G}_i$, Eq. (A2), rises linearly with M_P. This implies that the $1/D$ term in Eq. (A3) has to be expanded up to $O(M_P{}^{-2})$.

Inserting Eqs. (A2)–(A4) into Eq. (A1) and keeping only terms of $O(M_P{}^0)$ one has

$$\begin{aligned} V_F = {} & -3m_{3/2} e^{F/2M_P{}^2} (g^{(L)} + \text{H.c.}) + \sum_i m_{3/2}{}^2 X_i |y_i|^2 + e^{F/M_P{}^2} \sum_i |g^{(L)}_{,y_i}|^2 \frac{1}{X_i} + m_{3/2} e^{F/2M_P{}^2} \sum_i (y_i g^{(L)}_{,y_i} + \text{H.c.}) \\ & - m_{3/2} e^{F/2M_P{}^2} \left[\left(F_{,\Sigma^*} + M_P{}^2 \frac{g^*_{,\Sigma^*}}{g^*} \right) \sum_i \frac{X_{i,\Sigma}}{X_i F_{,\Sigma\Sigma^*}} y_i g^{(L)}_{,y_i} + \text{H.c.} \right] \\ & - m_{3/2}{}^2 \left| F_{,\Sigma} + \frac{M_P{}^2 g_{,\Sigma}}{g} \right|^2 \sum_i \frac{|y_i|^2}{F_{,\Sigma\Sigma^*}{}^2} \left(X_{i,\Sigma\Sigma^*} - \frac{|X_{i,\Sigma}|^2}{X_i} \right) + \frac{m_{3/2}}{F_{,\Sigma\Sigma^*}} \left[\left(F_{,\Sigma} + \frac{g_{,\Sigma} M_P{}^2}{g} \right) e^{F/2M_P{}^2} F_{,\Sigma^*} \frac{g^{*(L)}}{M_P{}^2} + \text{H.c.} \right] , \end{aligned} \tag{A5}$$

where $g^{(L)}$ denotes that part of g which depends on light fields only. From Eq. (A5) one obtains Eqs. (4) and (7)–(9) after the rescalings

$$g^{(L)} \rightarrow \hat{g} \equiv e^{F/2M_P^2} g^{(L)} , \tag{A6}$$

$$y_i \rightarrow \hat{y}_i \equiv \sqrt{X_i}\; y_i , \tag{A7}$$

where (A7) is necessary in order to give the kinetic energy terms of the low-energy theory the correct normalization.

APPENDIX B: MINIMIZATION OF THE SU(5) POTENTIAL OF SEC. III

Inserting the superpotential g, Eq. (15), into Eq. (A1) one has for a canonical kinetic energy term for Σ

$$\begin{aligned} V_F = M_P{}^4\lambda^2 e^{\mathrm{tr}\hat{\Sigma}^2}\{ &\tilde{g}^2(\mathrm{tr}\hat{\Sigma}^2-3)+2\tilde{g}[4\,\mathrm{tr}\hat{\Sigma}^4+4a(\mathrm{tr}\hat{\Sigma}^2)^2+3b\,\mathrm{tr}\hat{\Sigma}^3+2c\,\mathrm{tr}\hat{\Sigma}^2]+16\,\mathrm{tr}\hat{\Sigma}^6 \\ &-\tfrac{16}{5}(\mathrm{tr}\hat{\Sigma}^3)^2+32a\,\mathrm{tr}\hat{\Sigma}^4\mathrm{tr}\hat{\Sigma}^2+16a^2(\mathrm{tr}\hat{\Sigma}^2)^3+24b\,\mathrm{tr}\hat{\Sigma}^5 \\ &+\mathrm{tr}\hat{\Sigma}^3\mathrm{tr}\hat{\Sigma}^2(24ab-\tfrac{24}{5}b)+\mathrm{tr}\hat{\Sigma}^4(9b^2+16c)+(\mathrm{tr}\Sigma^2)^2(16ac-\tfrac{9}{5}b^2)+12bc\,\mathrm{tr}\hat{\Sigma}^3+4c^2\mathrm{tr}\hat{\Sigma}^2\} , \end{aligned} \tag{B1}$$

with $\hat{\Sigma} \equiv \Sigma/M_P$ and $\tilde{g} \equiv g/\lambda$.

A $SU(3)\times SU(2)\times U(1)$ symmetric minimum can be parametrized by

$$\hat{\Sigma} = \frac{v}{\sqrt{30}} \begin{bmatrix} 2 & & & & \\ & 2 & & 0 & \\ & & 2 & & \\ 0 & & & -3 & \\ & & & & -3 \end{bmatrix} \tag{B2}$$

and has to satisfy simultaneously

$$\tilde{g} = v^4\left[\frac{7}{30}+a\right] - \frac{bv^3}{\sqrt{30}} + cv^2 \neq 0 , \tag{B3}$$

in order to break supersymmetry,

$$\begin{aligned} &\tilde{g}^2(v^2-3)+2\tilde{g}\left[v^4(\tfrac{14}{15}+4a)-\frac{3bv^3}{\sqrt{30}}+2cv^2\right]+v^6(\tfrac{14}{15}+4a)^2-\frac{v^5}{\sqrt{30}}(24ab+\tfrac{28}{5}b) \\ &\qquad +v^4(\tfrac{56}{15}c+\tfrac{3}{10}b^2+16ac)-\frac{12v^3bc}{\sqrt{30}}+4c^2v^2=0 , \end{aligned} \tag{B4}$$

in order to have a vanishing cosmological constant and

$$\begin{aligned} &\tilde{g}\left[5v^4(\tfrac{7}{30}+a)-\frac{4bv^3}{\sqrt{30}}+v^2(\tfrac{14}{15}+4a+3c)-2c\right]+\left[4v^3(\tfrac{7}{30}+a)-\frac{3bv^2}{\sqrt{30}}+2cv\right]^2-3v^4(\tfrac{14}{15}+4a)^2-\frac{5v^3}{\sqrt{30}}(12ab-\tfrac{14}{5}b) \\ &\qquad +2v^2(\tfrac{56}{15}c+\tfrac{3}{10}b^2+16ac)-\frac{18bcv}{\sqrt{30}}+4c^2=0 \end{aligned} \tag{B5}$$

in order to have an extremum, which has to be checked to be a minimum. For given values of a,v Eqs. (B4) and (B5) fix b,c; in general some solutions are discarded by (B3). For these values of a,b,c one finally requires

$$\begin{aligned} &\left[v^4(\tfrac{13}{20}+a)-\frac{3bv^3}{\sqrt{20}}+cv^2\right]^2(v^2-3)+2\left[v^4(\tfrac{13}{20}+a)-\frac{3bv^3}{\sqrt{20}}+cv^2\right]\left[4v^4(\tfrac{13}{20}+a)-\frac{9bv^3}{\sqrt{20}}+2cv^2\right] \\ &\qquad +16v^6(\tfrac{13}{20}+a)^2-\frac{72bv^5}{\sqrt{20}}(\tfrac{13}{20}+a)+v^4[\tfrac{81}{20}b^2+16c(\tfrac{13}{20}+a)]-\frac{36bcv^3}{\sqrt{20}}+4c^2v^2\geq 0 \end{aligned} \tag{B6}$$

for all v, in order to forbid lower-lying $SU(4)\times U(1)$-symmetric solutions. The supersymmetric mass parameters m_8, m_3, and m_1 of the additional (8,1), (1,3), and (1,1) superfields (see Sec. III) are obtained by shifting Σ into the $SU(3)\times SU(2)\times U(1)$-invariant minimum and gathering the relevant quadratic terms in g:

$$m_8 = 2\lambda M_P\left[v^2(\tfrac{4}{5}+a)+\frac{6bv}{\sqrt{30}}+c\right] , \tag{B7}$$

$$m_3 = 2\lambda M_P\left[v^2(\tfrac{9}{5}+a)-\frac{9bv}{\sqrt{30}}+c\right] , \tag{B8}$$

$$m_1 = 2\lambda M_P\left\{v^2\left[\tfrac{7}{5}+a\left[1+\frac{\sqrt{2}}{2}\right]\right]-\frac{3bv}{\sqrt{15}}+c\right\} . \tag{B9}$$

By comparison of Eqs. (B7)–(B9) with Eq. (B3) one sees that $m_i/m_{3/2}$ increases with decreasing v, since $m_{3/2}\sim g$.

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