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M^0-$\overline{M}^0$ matrix elements in a relativized quark model

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Matrix elements of the type $\langle M^0 | \mathcal{O}_i | \overline{M}^0 \rangle$ are evaluated using a relativized quark model for the K^0, D^0, B^0, and B_s^0 systems where $\mathcal{O}_1 = [\bar{q}\gamma^\mu(1-\gamma_5)q][\bar{q}\gamma_\mu(1-\gamma_5)q]$, $\mathcal{O}_2 = [\bar{q}(1-\gamma_5)q]\,[\bar{q}(1+\gamma_5)q]$, $\mathcal{O}_3 = [\bar{q}\gamma_\mu(1-\gamma_5)q][\bar{q}\gamma^\mu(1+\gamma_5)q]$, and $\mathcal{O}_4 = [\bar{q}(1-\gamma_5)q][\bar{q}(1-\gamma_5)q]$. The pseudoscalar-meson decay constants f_p are first calculated using various approximations to set the scale of the matrix elements' magnitude and to estimate their accuracy. The matrix elements are then calculated, again using various approximations, and are presented as ratios with respect to the vacuum saturation value of $\mathcal{O}_1$. When relativistic effects are included we find that significant enhancements are indicated for the $\mathcal{O}_2$, $\mathcal{O}_3$, and $\mathcal{O}_4$ matrix elements which will alter the resulting bounds on new interactions. As an illustration we find that the right-handed W boson of left-right-symmetric theories must have a mass of at least 3.8 TeV.

I. INTRODUCTION

At present, one of the preoccupations of particle physics is looking for effects in conflict with the standard model.[1] There are two approaches to this search: to go to higher and higher energies looking for new particles and interactions,[2] or alternatively, to look for violations of standard-model physics in low-energy phenomenology.[3] Ultimately, the high-energy route will be necessary to disentangle new effects, but, until the necessary energies are realized, low-energy phenomenology can give us a window to new physics or at least put bounds on what is possible. One of the most sensitive tests for new physics is in the K^0-$\overline{K}^0$ system where new interactions can contribute to the $\langle K^0 | H_{\rm eff} | \overline{K}^0 \rangle$ matrix element and hence to the K_L^0-$\overline{K}_S^0$ mass difference and to CP violation.[4] This has led to bounds on, for instance, left-right-symmetric theories of the electroweak interaction,[5] on supersymmetric theories,[6] and on composite theories.[7] More recently it has been pointed out that there exists the possibility of observing mixing effects and CP-violating effects in the B^0-$\overline{B}^0$ and B_s^0-$\overline{B}_s^0$ systems.[8] These bounds should be improved in the near future with large data samples of B^0 mesons. In addition, bounds on mixing effects in the D^0-$\overline{D}^0$ system can also put constraints on model building.[9]

The major uncertainty in the low-energy route is our inability to calculate hadronic matrix elements with reasonable reliability. In calculating the hadronic matrix elements of this type there are, typically, short-distance contributions from the box diagrams and long-distance effects from dispersive contributions of π, η, ρ, ω, $\pi\pi$, etc., intermediate states.[10-12] There are uncertainties in both contributions. For example, in the K^0-$\overline{K}^0$ system, estimates of the dispersive effects range from negligible[11] to the order of the K_L-K_S mass difference.[12] They are also large in the D^0-$\overline{D}^0$ system. However, in the B^0 and B_s^0 systems these long-distance effects are not nearly as important. Here we will restrict ourselves to the calculation of the hadronic matrix elements which correspond to the short-distance contributions. There have been many approaches to this problem: constituent-quark models,[13] bag models,[14] and chiral perturbation theory[15] to name a few. Unfortunately, the reliability of these results is questionable; bag-model calculations are extremely sensitive to the parameters of the model[13] and corrections to the chiral-perturbation-theory results are evidently large.[16]

In this paper we will use the constituent-quark model in the mock-meson approach to calculate the matrix elements of various operators for the K^0-$\overline{K}^0$, D^0-$\overline{D}^0$, B^0-$\overline{B}^0$, and B_s^0-$\overline{B}_s^0$ systems. Our purpose is to present the results along with the uncertainties so that they are available to others. Similar calculations have been performed by Colic, Guberina, Tadić, and Trampetić and by Trampetić.[13] The present calculation differs from these in two respects. First, the integrals involved are evaluated numerically rather than using approximations. More importantly, here we use the wave functions of a relativized QCD-motivated quark model rather than the harmonic-oscillator wave functions of Ref. 13. This leads to significant effects which will be discussed. We begin in Sec. II with a brief description of our method. We then present our results for the pseudoscalar decay constants f_p and for the $\langle M^0 | \mathcal{O}_i | \overline{M}^0 \rangle$ matrix elements with a discussion of their respective sensitivity. As an example, these results are used to put a bound on the W_R mass in left-right-symmetric theories. In the final section we discuss the shortcomings of our approach and comment on the reliability of the results.

II. THE HADRONIC MATRIX ELEMENTS

To reduce the sensitivity of our results to the specifics of the calculation we will give the hadronic matrix elements as ratios with respect to the vacuum insertion value of $\langle M^0 | \mathcal{O}_1 | \overline{M}^0 \rangle$ matrix elements which are expressed in terms of the decay constants f_p. Since the f_p set the scale of the matrix elements it is important that we obtain reliable values for them and understand the uncertainties in their calculation. We begin this section by reviewing the mock-meson approach to hadronic matrix elements. Next we calculate the decay constants and discuss their reliability. Finally we calculate the various hadronic amplitudes.

A. The mock-meson method

The mock-meson approach is a prescription for relating quark-model matrix elements to the corresponding physical amplitudes.[17,18] In what follows we will use the prescription of Hayne and Isgur.[18] In this approach the mock meson, $\widetilde{M}$, is defined as a state of a free quark and antiquark with the wave function of the physical meson, M, and with the mock meson mass, $\widetilde{M}_M$, equal to the total mean energy of the quark and antiquark in $\widetilde{M}$. To calculate the hadronic matrix element, the physical matrix element $\mathscr{M}$ is expressed in terms of Lorentz covariants with Lorentz scalar coefficients A. In the simple cases when the mock-meson matrix element $\widetilde{\mathscr{M}}$ is the same form as $\mathscr{M}$ we simply take $A=\widetilde{A}$. The mock meson is defined by

$$|\widetilde{M}(\mathbf{K})\rangle=(2\widetilde{M}_M)^{1/2}\int d^3p\,\Phi_M(\mathbf{p})\chi_{s\bar{s}}\phi_{p\bar{p}}\,|q[m_q/\mu\mathbf{K}+\mathbf{p},s]\bar{q}[m_{\bar{q}}/\mu\mathbf{K}-\mathbf{p},\bar{s}]\rangle\ , \tag{1}$$

where $\Phi_M(\mathbf{p})$, $\chi_{s\bar{s}}$, and $\phi_{q\bar{q}}$ are momentum, spin, and flavor wave functions, respectively, $\mu=m_q+m_{\bar{q}}$, and $\mathbf{K}$ is the mock-meson momentum.

B. The pseudoscalar-meson decay constant: f_p

For the decay constants the hadronic matrix element can be expressed as

$$\langle 0|\bar{q}\gamma^\mu(1-\gamma_5)q\,|M(\mathbf{K})\rangle=\frac{i}{(2\pi)^{3/2}}f_pK^\mu\ . \tag{2}$$

We calculate $\langle 0|\bar{q}\gamma^\mu(1-\gamma_5)q\,|M(\mathbf{K})\rangle$ using a superposition of free-quark wave functions by evaluating the matrix element

$$\langle 0|\bar{q}\gamma^\mu(1-\gamma_5)q\,|q((m_q/\mu)\mathbf{K}+\mathbf{p},s)q((m_{\bar{q}}/\mu)\mathbf{K}-\mathbf{p},\bar{s})\rangle$$

using Dirac spinors. This leads to the expression

$$f_p=\frac{2\sqrt{3}}{\widetilde{M}_p^{\;1/2}}\int\frac{d^3p}{(2\pi)^{3/2}}\times\Phi_p(\mathbf{p})\left[\left(\frac{E_q+m_q}{2E_q}\right)\left(\frac{E_{\bar{q}}+m_{\bar{q}}}{2E_{\bar{q}}}\right)\right]^{1/2}\times\left[1-\frac{\mathbf{p}^2}{(E_q+m_q)(E_{\bar{q}}+m_{\bar{q}})}\right]. \tag{3}$$

Because it is important to understand the uncertainties in our calculation of f_p we will present nonrelativistic results, mock-meson results using the harmonic-oscillator wave functions of Colic *et al.*, of Haynes and Isgur, and the wave functions of the relativized model of Ref. 19, and compare them to previous calculations. These are presented in Table I. We find that the results are sensitive to the wave functions which we will discuss in detail below.

We first note that the nonrelativistic results for the decay constants are roughly proportional to $\widetilde{M}_p^{\;-1/2}$ with a small modification due to the mass dependence of β. That the nonrelativistic results are lacking is reflected in the predicted value of $f_K/f_\pi=0.94$ as compared to the measured value of ~ 1.25. Turning to the mock-meson results and comparing the results using the harmonic-oscillator wave functions of Refs. 13 and 18 to the results using the relativized wave function we find considerable disagreement. The disagreement stems from several sources. First, the harmonic-oscillator wave functions do not reflect the increase of the wave function at the origin for heavier mesons due to the Coulomb piece of the QCD-motivated potential. In addition, wave-function distortions of the relativistic model caused by the spin-dependent potentials lead to high-momentum components in the wave function which is illustrated in Fig. 1 where we have plotted the kaon momentum-space wave functions for the various models. For heavier mesons, the spin-dependent distortions are smaller and, in addition, the contributions of the high-momentum components to f_p are suppressed by a factor of roughly $1/(m_qm_{\bar{q}})$. We conclude that the naive harmonic-oscillator wave functions probably do not reflect the internal structure of the pseudoscalar mesons very well. This is supported by the rough agreement of our results with those of Krasemann[20] and of Claudson[21] who also use QCD-motivated potentials. The remaining disagreement with the latter results can be attributed to both the spin-dependent potentials included in the Hamiltonian of Ref. 19 and the relativized approach used in calculating f_p. We note that Krasemann has attempted to take into account SU(6)-breaking effects in the wave functions by using second-order perturbation theory. The magnitude of his corrections are consistent with the size of the effects calculated here (although the systematics differ). Thus, while our results disagree with those of Colić *et al.* we believe the results of model 5 to be reasonable estimates of the various f_p.

C. The hadronic matrix elements $\langle M^0|\mathscr{O}_i|\overline{M}{}^0\rangle$

With what we believe to be reasonably reliable values for the f_p we turn to the evaluation of the hadronic matrix elements of the operators:

$$\begin{aligned}\mathscr{O}_1&=[\bar{q}^i\gamma_\mu(1-\gamma_5)q^i][\bar{q}^j\gamma^\mu(1-\gamma_5)q^j]\ ,\\ \mathscr{O}_2&=[\bar{q}^i(1-\gamma_5)q^i][\bar{q}^j(1+\gamma_5)q^j]\ ,\\ \mathscr{O}_3&=[\bar{q}^i\gamma_\mu(1-\gamma_5)q^i][\bar{q}^j\gamma^\mu(1+\gamma_5)q^j]\ ,\\ \mathscr{O}_4&=[\bar{q}^i(1-\gamma_5)q^i][\bar{q}^j(1-\gamma_5)q^j]\ ,\end{aligned} \tag{4}$$

TABLE I. Results for the pseudoscalar decay constant f_p (all results are given in MeV).

	Experiment	Potential model 1[a]	Potential model 2[b]	Bag model[b]	Mock-meson results Nonrelativistic[c,d]	Model 3[e,d]	Model 4[f,d]	Model 5[g]
f_π	133	139	280	161	130	126	123	130
f_K	166	176	198	198	122	135	149	169
f_D		150	259	171	91	122	151	234
f_F		210	327	195	101	142	190	391
f_B		125	153	148	58	84	106	191
f_{B_s}		175	198	170	68	102	140	236
f_{B_c}		425	447	255	87	139	216	421

[a]H. Krasemann, Phys. Lett. **96B**, 397 (1980).
[b]M. Claudson, Harvard University Report No. 81-546, 1981 (unpublished).
[c]Harmonic-oscillator wave functions with $\kappa=0.0027$ GeV^3, $m_{\bar{u}}=m_d=0.33$ GeV, $m_s=0.55$ GeV, $m_c=1.628$ GeV, $m_b=4.977$ GeV.
[d]The $L=O$ ground-state harmonic-oscillator wave function is

$$\Phi(\mathbf{p})=\frac{1}{(\sqrt{\pi}\beta)^{3/2}}e^{-\mathbf{p}^2/2\beta^2} \text{ with } \beta=\left[\frac{2m_q m_{\bar{q}}}{m_q+m_{\bar{q}}}\kappa\right]^{1/4} .$$

[e]Harmonic-oscillator wave functions of Ref. 13 with $\kappa=0.0106$ GeV^3, $m_u=m_d=0.33$ GeV, $m_s=0.55$ GeV, m_c, m_b as above.
[f]Extended model parameters of Ref. 18 with $\kappa=0.037$ GeV^3, $m_u=m_d=0.22$; $m_s=0.43$, m_c, m_b as above.
[g]Using wave functions of Ref. 19 with $m_u=m_d=0.22$ GeV, $m_s=0.419$ GeV, $m_c m_b$ as above. The values for f_p in this column were normalized to give the correct f_π by multiplying all results by an overall constant.

where i and j are color indices which are summed over. We denote the matrix elements of these operators by

$$\mathscr{M}_i=-\langle M^0 | \mathscr{O}_i | \overline{M}^0\rangle . \tag{5}$$

In the mock-meson approach the amplitudes are given by

$$(2\pi)^3\mathscr{M}_i=-(2\widetilde{M})\int \frac{d^3p}{(2\pi)^{3/2}}\frac{d^3p'}{(2\pi)^{3/2}}\Phi_p(\mathbf{p})\Phi_p(\mathbf{p}')A_i , \tag{6}$$

where

$$A_1=4I_- , \tag{7a}$$

$$A_2=3I_+ + \tfrac{1}{2}I_- , \tag{7b}$$

$$A_3=-3I_- -2I_+ , \tag{7c}$$

$$A_4=-\tfrac{5}{2}I_+ , \tag{7d}$$

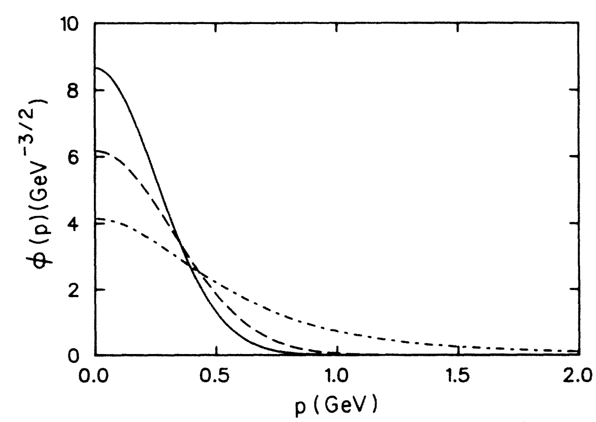

FIG. 1. The momentum-space wave function for the K meson. The wave function of Ref. 13 is given by the solid line, of Ref. 18 by the dashed line, and of Ref. 19 by the dash-dot line.

$$I_\pm=4\left[1\pm\frac{\mathbf{p}^2}{(E_q+m_q)(E_{\bar{q}}+m_{\bar{q}})}\right] \times\left[1\pm\frac{\mathbf{p}'^2}{(E_q'+m_q)(E_{\bar{q}}'+m_{\bar{q}})}\right] \times\left[\left[\frac{E_q+m_q}{2E_q}\right]\left[\frac{E_{\bar{q}}+m_q}{2E_{\bar{q}}}\right] \times\left[\frac{E_q'+m_q}{2E_q'}\right]\left[\frac{E_{\bar{q}}'+m_{\bar{q}}}{2E_{\bar{q}}'}\right]\right]^{1/2} , \tag{8}$$

and $E=(p^2+m^2)^{1/2}$, $E'=(p'^2+m^2)^{1/2}$.

One first notes that in the constituent-quark model the $\mathscr{M}_i$ will always factorize so that $\mathscr{M}_1$ is proportional to the vacuum saturation value

$$\mathscr{M}_1=\tfrac{8}{3}\langle M^0 | \bar{q}\gamma_\mu(1-\gamma_5)q\,|\,0\rangle\langle 0\,|\,\bar{q}\gamma^\mu(1-\gamma_5)q\,|\,\overline{M}^0\rangle \tag{9}$$

giving a value of unity for the B parameter. This is a consequence of restricting ourselves to the valence-quark sector and the fact that the crossed terms can be Fierz transformed into the direct term.[22]

The values of the K^0, D^0, B^0, and B_s^0 amplitudes for the nonrelativistic results and for the mock-meson results using the parameters of Refs. 13 and 18 and the relativized wave functions of Ref. 19 are given in Table II. To obtain numerical values for these matrix elements one should multiply the entries by the vacuum saturation value $\langle M^0 | \bar{q}\gamma^\mu(1-\gamma_5) | 0\rangle^2=-f_p{}^2m_p{}^2$. One can see that there are enhancements for the $\mathscr{O}_2$, $\mathscr{O}_3$, and $\mathscr{O}_4$ matrix elements in all models with the relativized model giving the most extreme results. That the $\mathscr{M}_1$ values are always the smallest is a consequence of its integrand being entirely I_- while the other amplitudes contain at least

TABLE II. The hadronic matrix elements of the $\mathcal{O}_i$ defined in the text. They are given as ratios of M_i/M_1^{VI} where $M_1^{VI}=-f_p{}^2M_p{}^2$.

	Nonrelativistic value	Model 3[a]	Mock-meson results Model 4[b]	Model 5[c]
		K^0-$\bar{K}^0$		
M_1/M_1^{VI}	8/3	8/3	8/3	8/3
M_2/M_1^{VI}	7/3	4.0	6.7	30
M_3/M_1^{VI}	−10/3	−4.5	−6.2	−22
M_4/M_1^{VI}	−5/3	−3.1	−5.3	−24
		D^0-$\bar{D}^0$		
M_1/M_1^{VI}	8/3	8/3	8/3	8/3
M_2/M_1^{VI}	7/3	2.9	3.4	6.9
M_3/M_1^{VI}	−10/3	−3.7	−4.1	−6.4
M_4/M_1^{VI}	−5/3	−2.2	−2.6	−5.5
		B^0-$\bar{B}^0$		
M_1/M_1^{VI}	8/3	8/3	8/3	8/3
M_2/M_1^{VI}	7/3	2.5	2.7	3.5
M_3/M_1^{VI}	−10/3	−3.5	−3.6	−4.1
M_4/M_1^{VI}	−5/3	−1.8	−1.9	−2.6
		B_s^0-$\bar{B}_s^0$		
M_1/M_1^{VI}	8/3	8/3	8/3	8/3
M_2/M_1^{VI}	7/3	2.5	2.6	3.4
M_3/M_1^{VI}	−10/3	−3.4	−3.5	−4.0
M_4/M_1^{VI}	−5/3	−1.8	−1.9	−2.5

[a]Harmonic-oscillator wave functions of Ref. 13. See footnotes e and d to Table I.
[b]Extended-model parameters of Ref. 18. See footnotes f and d to Table I.
[c]Wave functions of Ref. 19. See footnote g to Table I.

some I_+ contribution. The relativized model gives the most extreme result which, again, reflects the high-momentum components of those wave functions compared to the harmonic-oscillator wave functions. The relativistic corrections decrease as the meson masses increase, as in the case of the f_p, because of the smaller spin-dependent effects in the heavy mesons and because the relativistic corrections go roughly as $1/(m_q m_{\bar{q}})$. To demonstrate these effects we plot in Fig. 2 the matrix elements (as ratios with respect to M_1^{VI}) versus β the oscillator parameter. We see that matrix elements for the B^0 and B_s^0 systems are relatively insensitive to β while the K^0 matrix elements are very sensitive to the relativistic corrections. Thus, we find that there are large differences between the results using the naive and the relativized-model wave functions.

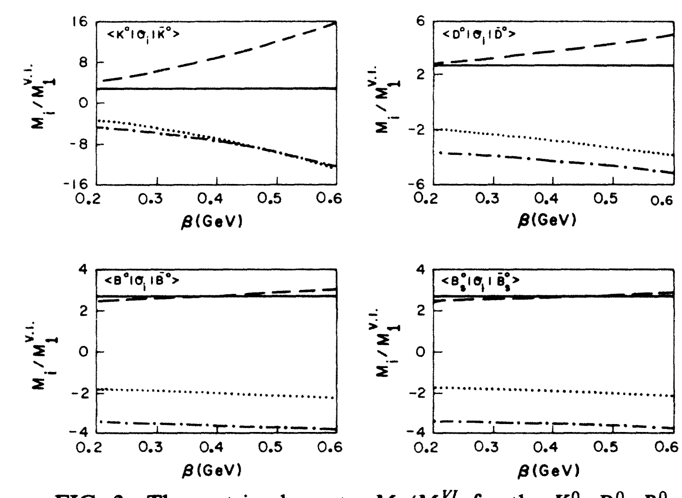

FIG. 2. The matrix elements, M_i/M_1^{VI}, for the K^0, D^0, B^0, and B_s^0 systems as a function of β, the oscillator parameter, with $m=0.22$ GeV, $m_s=0.43$ GeV, $m_c=1.628$ GeV, and $m_b=4.977$ GeV. M_1/M_1^{VI} is given by the solid line, M_2/M_1^{VI} by the dashed line, M_3/M_1^{VI} by the dot-dash line, and M_4/M_1^{VI} by the dotted line.

III. AN APPLICATION: CONSTRAINTS ON THE W_R MASS

As an application of these matrix elements we consider the constraint that the K_L-K_S mass difference puts on the mass of the charged right-handed gauge field occurring in a left-right-symmetric (LRS) extension of the standard model. These models have the attraction that above a sufficiently high-energy scale, parity is restored and the electroweak interaction would be based on the gauge group $SU(2)_L\times SU(2)_R\times U(1)_{B-L}$ (Ref. 23). Although these theories are not necessary for phenomenological reasons and are therefore purely speculative, it is interesting to consider what constraints we can put on the associated mass scales. This question has been addressed previously;[24–26] in what follows we reconsider the analysis of the K_L-K_S mass difference of Beall, Bander, and Soni[25] using more recent information for the Kobayashi-Maskawa (KM) matrix elements[27] and the top-quark mass.[28]

The effective $|\Delta S|=2$ Hamiltonian for K^0-$\bar{K}^0$ mixing in the LRS theory, to leading order in the W-boson, t-quark, and c-quark masses, and neglecting terms proportional to β^2, with $\beta=(M_{W_L}/M_{W_R})^2$, is given by[25]

$$H_W^{\rm eff}=-\frac{G_F^2}{4\pi^2}\{[\eta_1 m_c^2\lambda_c^2+\eta_c m_t^2\lambda_t^2+2\eta_3 m_c^2\lambda_c\lambda_t\ln(m_t^2/m_c^2)]\mathcal{O}_{LL}$$
$$+8\beta[\eta_1 m_c^2\lambda_c^2(1+\ln\eta_c)+\eta_2 m_t^2\lambda_t^2(1+\ln\eta_t)+2\eta_3 m_c m_t\lambda_c\lambda_t\ln\eta_t]\mathcal{O}_{LR}\}, \tag{10}$$

with $\mathcal{O}_{LL}=\frac{1}{4}\mathcal{O}_1$, $\mathcal{O}_{LR}=\frac{1}{4}\mathcal{O}_2$, $\eta_{c,t}=m_{c,t}^2/M_{W_L}^2$, $m_c=1.628$ GeV, $m_t=40$ GeV, $m_{W_L}\approx 81$ GeV, η_1, η_2, and η_3 are the QCD coefficients which we take from Gilman and Wise[29] and $\lambda_c=V_{dc}V_{cs}^*$ and $\lambda_t=V_{dt}V_{ts}^*$ are the Kobayashi-Maskawa matrix elements the values of which we take from Chau and Keung.[27] In the above, the assumption has been made that the mixing angles in the right-handed sector are equal to those in the left-handed sector. We have also made the assumption that the flavor-changing Higgs boson is very massive and does not contribute.[26]

With $H_W^{\rm eff}$ thus defined the K_L-K_S mass difference is given by the real part of the K^0-$\overline{K}^0$ matrix element:

$$m_L-m_S=2\ \mathrm{Re}\langle\overline{K}^0|H_W^{\rm eff}|K^0\rangle\ . \tag{11}$$

Substituting in the values for the various parameters we obtain

$$m_L-m_S=2.07\times10^{-13}M_{LL}(1-54\beta M_{LR}/M_{LL}) \tag{12}$$

with $M_{LL}=\mathcal{M}_1/2m_K$ and $M_{LR}=\mathcal{M}_2/2m_K$. Relating this to the experimental value of $\Delta m_K=3.52\times10^{-15}$ GeV we obtain the limit that $M_{W_R}\gtrsim 3.8$ TeV although in the most extreme case of model 5 this limit becomes 7.9 TeV. Clearly, one should be extremely cautious when using hadronic matrix elements to obtain bounds on new physics and should take such bounds as only an indication of the order of magnitude.

IV. DISCUSSION AND CONCLUSIONS

In calculating hadronic amplitudes using the mock-meson approach there are several sources of error; the first is that we have used the amplitudes of free quarks weighted by the momentum distributions of the meson wave functions neglecting off-mass-shell effects. In principle these effects could be included by using a mean-field approach to calculate both the hadron wave functions and the amplitudes.[30] Unfortunately the second source of error would remain—that we have restricted ourselves to the valence-quark sector of the QCD Fock space. The constituent-quark model is an effective model with degrees of freedom above some momentum scale integrated out leaving the constituent quarks. How important multiquark components of the Fock space are to the amplitudes is not clear. That they will alter the results is indicated by the effects of gluon corrections to the short-distance behavior of the free-quark diagrams.

In addition to the short-distance box-diagram contributions there are also the dispersive long-distance contributions mentioned previously, arising from intermediate states of π, $\pi\pi$, η, etc., which can propagate over larger distances. These dispersive contributions are related to the multiquark components of the Fock space. In light of the above uncertainties one should view results involving the K^0-$\overline{K}^0$ and D^0-$\overline{D}^0$ systems with skepticism. Fortunately, for the B^0-$\overline{B}^0$ and B_s^0-$\overline{B}_s^0$ systems, one can use the box-diagram contributions with a greater degree of confidence.

To conclude, given the importance of having an accurate estimate of the M^0-$\overline{M}^0$ amplitudes we have used the relativized quark model to calculate hadronic matrix elements for various operators which arise in effective Hamiltonians of new interactions. We find indications that in the relativized approach there are significant enhancements of the results compared to naive calculations. If one takes this mock-meson approach seriously then these enhancements exist. Although the quantitative results may be altered by a more rigorous treatment the qualitative results of the model will remain. These would lead to significant enhancements of new effects which would put tighter constraints on new physics than is currently expected and could have important consequences in evaluating the prospects for new interactions. As indicated by the example of the W_R mass, because of the uncertainties in the matrix elements, one should take results which use hadronic matrix elements as order-of-magnitude estimates at best.

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