New four-quark $\Delta S=2$ local operator

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We describe a new four-quark $\Delta S=2$ local operator, which we dub the "dipenguin" operator. Its form is derived and its $\bar{K}^0$-to-K^0 matrix element is estimated. Also, we comment upon bag-model evaluations of $\bar{K}^0$-to-K^0 matrix elements of $(V-A)\times(V-A)$ operators.

Throughout the development of gauge theories, the K^0-$\bar{K}^0$ system has played a fruitful role. The very tiny K_L-K_S mass difference Δm_{LS} requires that there be no $\Delta S=2$ effects until $O(G_F{}^2)$. The earliest "modern" treatment of the K^0-$\bar{K}^0$ system was that of Gaillard and Lee who calculated, in the four-quark model, short-distance "box" diagrams (Fig. 1) responsible for $\Delta S=2$ effects and estimated Δm_{LS} by use of the vacuum saturation method.[1]

Since then, there have been a number of papers extending the analysis of Δm_{LS} carried out in Ref. 1. Some of these stressed the point that long-distance effects could also in principle contribute to[2,3] Δm_{LS},

$$\Delta m_{LS}=\Delta m_{LS}^{\text{short}}+\Delta m_{LS}^{\text{long}}\ , \tag{1}$$

and there have been continuing efforts to estimate the magnitude of $\Delta m_{LS}^{\text{long}}$ (Refs. 4 and 5). Unfortunately the problem of determining the importance of long-distance effects is a severe one, and to this day there is no clear consensus regarding the relative size of $\Delta m_{LS}^{\text{short}}$ and $\Delta m_{LS}^{\text{long}}$.

In addition, K^0-$\bar{K}^0$ mixing plays a dominant role in the theory of CP violation. Here the long-distance effects appear to be unimportant and the short-distance box diagram is the main source of CP-odd mixing.

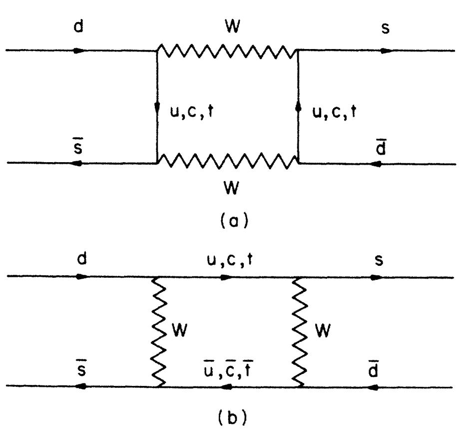

FIG. 1. Short-distance "box" diagrams.

There was also some effort devoted to incorporating the presence of the b quark (and by implication the t quark as well) to computations of K^0-$\bar{K}^0$ mixing. This work, largely concerned with quantifying the Kobayashi-Maskawa (KM) quark-mixing prescription and recomputing QCD renormalization-group correction factors, is lucidly reviewed in Ref. 6. Our purpose in this paper is to point out the existence of an *additional* contribution to $\Delta m_{LS}^{\text{short}}$ which, to our knowledge, has not previously been considered in the literature.[7] We call this contribution the "dipenguin" because it is a direct consequence of the "penguin" contribution in $\Delta S=1$ processes.

Recall that the $\Delta S=1$ penguin is associated with the presence of mass scales of heavy quarks.[8] In the usual penguin amplitude, a color gluon emitted from the $s\to d$ penguin vertex propagates and ultimately couples strongly to a quark line. Somewhat surprisingly the corresponding four-quark $\Delta S=1$ operator is local. This is because the q^{-2} behavior of the gluon propagator is compensated for by a q^2 factor from the loop integral.[8,9] Now suppose the gluon, rather than experiencing a subsequent strong interaction, instead hooks on to a $\bar{d}\to\bar{s}$ penguin. This process, depicted in Fig. 2(a), gives rise to a $\Delta S=2$ local four-quark operator, the dipenguin. Before displaying the form of this operator, we wish to stress that long-distance processes in which a $\Delta S=1$ penguin appears twice also generally occur. An example is shown in Fig. 2(b). The reader should not confuse the two processes.

To derive the $\Delta S=2$ dipenguin we consider first the $\Delta S=1$ penguin *vertex* in coordinate space:

$$\frac{G_F}{\sqrt{2}}\frac{g}{24\pi^2}\sin\theta_c\cos\theta_c\ln\frac{m^2}{\mu^2}\bar{d}\gamma_\mu(1+\gamma_5)\times\lambda^A s(g^{\mu\nu}\partial^2-\partial^\mu\partial^\nu)A_\nu^A\ , \tag{2}$$

where g is the QCD coupling constant, μ is a typical hadronic mass scale, and m is the fermion mass appearing in the loop integration. The dipenguin results upon "squaring" Eq. (2) and folding in the gluon propagator. In an effective four-quark description we obtain

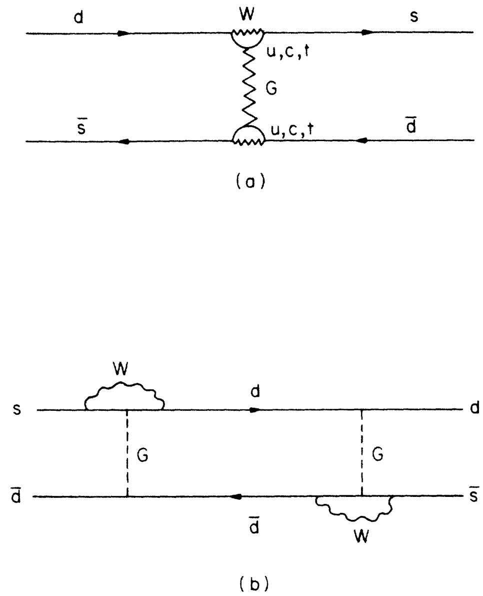

FIG. 2. "Penguin" diagrams as they contribute to (a) short-distance, (b) long-distance effects. (a) depicts the "dipenguin" process.

$$L^{\text{dip}}_{\Delta S=2}=\frac{G_F{}^2\alpha_s}{288\pi^3}\cos^2\theta_c\sin^2\theta_c \times\left[\ln\frac{m_c{}^2}{\mu^2}+CP\text{-odd piece}\right]^2\tilde{O}\ , \quad (3a)$$

where $\tilde{O}\equiv\tilde{O}_1-\tilde{O}_2$ is the four-quark operator:

$$\tilde{O}_1\equiv\bar{d}\gamma^\mu(1+\gamma_5)\lambda^A s\partial_\mu\partial_\nu\bar{d}\gamma^\nu(1+\gamma_5)\lambda^A s\ , \quad (3b)$$

$$\tilde{O}_2\equiv\bar{d}\gamma^\mu(1+\gamma_5)\lambda^A s\Box[\bar{d}\gamma_\mu(1+\gamma_5)\lambda^A s]\ . \quad (3c)$$

It is instructive to contrast the form of the dipenguin with that of the "box" operator:

$$L^{\text{box}}_{\Delta S=2}=\frac{G_F{}^2}{8\pi^2}\cos^2\theta_c\sin^2\theta_c m_c{}^2\bar{d}\gamma^\mu(1+\gamma_5)s \times\bar{d}\gamma_\mu(1+\gamma_5)s\ . \quad (4)$$

Observe that each operator has a common $(V-A)\times(V-A)$ chiral structure, and the same $G_F{}^2\cos^2\theta_c\sin^2\theta_c$ overall scale. The dipenguin betrays its QCD content with a factor of the strong fine-structure constant α_s as well as contracted color matrices in $\tilde{O}$. The presence of a mass parameter $m_c{}^2$ in the box is mimicked by the derivative terms in $\tilde{O}$.

To measure the importance of the dipenguin effect, we must compute its $\bar{K}^0$-to-K^0 matrix element. An exact calculation is beyond present-day capabilities. There do, of course, exist a variety of model-dependent approaches to employ. Unfortunately the $(V-A)\times(V-A)$ structure makes such estimates suspect because helicity suppression induces large cancellations. This is a point which has perhaps not been appreciated to the extent that it deserves, so we consider it in more detail in the Appendix. For completeness, we also compute there the dipenguin element in the bag model.

At any rate, we can use vacuum saturation to provide an order-of-magnitude estimate for $\langle K^0|\tilde{O}|\bar{K}^0\rangle$. The matrix element of $\tilde{O}_1$ is straightforward to analyze. Upon integrating by parts and employing quark equations of motion we obtain

$$\begin{aligned}\tilde{O}_1=&m_s{}^2\bar{d}(1-\gamma_5)\lambda_A s\bar{d}(1-\gamma_5)\lambda_A s\\&-2m_s m_d\bar{d}(1+\gamma_5)\lambda_A s\bar{d}(1-\gamma_5)\lambda_A s\\&+m_d{}^2\bar{d}(1+\gamma_5)\lambda_A s\bar{d}(1+\gamma_5)\lambda_A s\ .\end{aligned} \quad (5)$$

Upon neglecting m_d relative to m_s and employing vacuum saturation we then find

$$\langle K^0|\tilde{O}_1|\bar{K}^0\rangle_{\text{vac}}\simeq\frac{32}{9}\frac{F_K{}^2m_K{}^4}{2m_K}\ , \quad (6)$$

where $F_K\simeq1.25F_\pi$ and $F_\pi\simeq0.0935$ GeV. The matrix element of $\tilde{O}_2$ is more troublesome to handle, and indeed we have no totally convincing method for computing it, even in vacuum saturation. Direct evaluation, neglecting m_d, yields

$$\langle K^0|\tilde{O}_2|\bar{K}^0\rangle_{\text{vac}}=-\frac{64}{9}\frac{F_K{}^2m_K{}^2m_s{}^2}{2m_K}+X\ , \quad (7)$$

where

$$\begin{aligned}X=&2\langle K^0|d_i^\dagger\gamma^\mu(1+\gamma_5)\lambda^A_{ij}\partial^\lambda\bar{s}_l^\dagger|0\rangle\langle0|(\partial_\lambda\bar{d}_k)\gamma_\mu(1+\gamma_5)\lambda^A_{kl}s_j|\bar{K}^0\rangle\\&+2\langle K^0|\partial_\lambda d_i^\dagger\gamma^\mu(1+\gamma_5)\lambda^A_{ij}\bar{s}_l^\dagger|0\rangle\langle0|\bar{d}_k\gamma_\mu(1+\gamma_5)\lambda^A_{kl}\partial^\lambda s_j|\bar{K}^0\rangle\ .\end{aligned} \quad (8)$$

To estimate X, let us assume that each derivative of a quark field in Eq. (8) can be interpreted as the average momentum of that quark, in which case

$$\begin{aligned}X&\simeq4p_s\cdot p_d\frac{32}{9}\frac{F_K{}^2m_K{}^2}{2m_K}\\&\simeq\tfrac{32}{9}F_K{}^2m_K(m_K{}^2-m_s{}^2)\end{aligned} \quad (9)$$

if the kaon energy-momentum is the sum of the s-quark and d-quark contributions. Altogether then we obtain

$$\langle K^0 | \tilde{O} | \bar{K}^0 \rangle_{\text{vac}} \simeq \tfrac{16}{9} F_K{}^2 m_K (4m_s{}^2 - m_K{}^2) . \tag{10}$$

The key quantity is of course the relative size of box and dipenguin matrix elements. In vacuum saturation, we obtain

$$\frac{\langle K^0 | L^{\text{dip}} | \bar{K}^0 \rangle_{\text{vac}}}{\langle K^0 | L^{\text{box}} | \bar{K}^0 \rangle_{\text{vac}}} \simeq \frac{\alpha_S}{54\pi} \ln^2 \frac{m_c{}^2}{\mu^2} \frac{4m_s{}^2 - m_K{}^2}{m_c{}^2} \ll 1 \tag{11}$$

for $m_c \simeq 1.5$ GeV, $m_s \simeq \mu \simeq 0.3$ GeV, and $\alpha_s \simeq 1$.

We conclude that the presence of a $\Delta S=1$ penguin necessarily implies the existence of a distinct short-distance $\Delta S=2$ effect. However this dipenguin contribution is much smaller than the standard short-distance box contribution. There are basically two reasons for this: a smaller coefficient and a smaller mass scale ($m_K{}^2$ vs $m_c{}^2$).

We have ignored QCD radiative correction renormalization-group-summed factors in our discussion. The dipenguin effect is so small as to make any impact on the phenomenology of $\Delta S=2$ processes highly unlikely. A renormalization-group analysis of such $\Delta S=2$ operators would thus be mainly of academic interest.

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APPENDIX

Several papers have addressed the issue of estimating in the bag model the $\bar{K}^0$-to-K^0 matrix element of the chiral operator

$$O \equiv :\bar{d}_i \gamma^\mu (1+\gamma_5) s_i \bar{d}_j \gamma_\mu (1+\gamma_5) s_j: . \tag{A1}$$

Our evaluation yields

$$\langle K^0 | O | \bar{K}^0 \rangle = \frac{4N_d{}^2 N_s{}^2}{\pi} \int_0^R r^2 dr\, I(r) , \tag{A2}$$

where

$$I(r) = f_d{}^2 f_s{}^2 + g_d{}^2 g_s{}^2 - 4 f_d f_s g_d g_s - f_d{}^2 g_s{}^2 - g_d{}^2 f_s{}^2 . \tag{A3}$$

In Eqs. (A2) and (A3), the subscripts d and s refer to the d-quark and s-quark kinematics, respectively, the N factors are bag normalization factors, and f,g are the usual spherical Bessel functions:

$$\begin{aligned} f &= j_0(pr/R) , \\ g &= [(\omega - mR)/(\omega + mR)]^{1/2} j_1(pr/R) \end{aligned} \tag{A4}$$

for the ground-mode wave number $p \cong 2.0428$, and $\omega^2 = p^2 + m^2 R^2$. We agree with the corresponding result in Ref. 10 but differ by a minus sign with Ref. 11.

Historically the quantity in Eq. (A2) has been the focus of much attention because it provides an estimate of the "B parameter,"

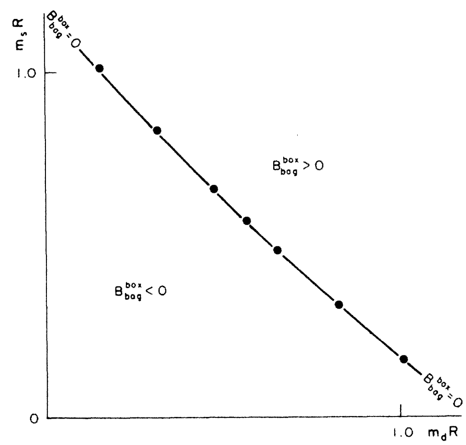

FIG. 3. Bag-model evaluation of $\bar{K}^0$-to-K^0 matrix element of local four-quark box operator.

$$B = \langle K^0 | O | \bar{K}^0 \rangle / \langle K^0 | O | \bar{K}^0 \rangle_{\text{vac}} \tag{A5}$$

and in particular predicts $B \cong -0.4$. The point we wish to emphasize in this appendix is that bag-model evaluations of $(V-A)\times(V-A)$ operators such as the one in Eq. (A3) are likely to be rather sensitive functions of the input parameters due to helicity suppression. As such, the resulting value must be viewed as suspect. To demonstrate this point we have plotted in Fig. 3 the bag-model value for B as a function of $m_d R$ and $m_s R$. Observe the line of zeros which separates the parameter space into regions of $B>0$ and $B<0$. For the usual choice $m_d R \cong 0$, $m_s R \cong 0.9$, one obtains $B<0$, yet for a reasonable SU(3)-invariant choice $m_d R = m_s R \cong 0.7$, one finds $B>0$.

This phenomenon is not limited to the operator of Eq. (A1). Consider the local, chiral quantity

$$\tilde{O}_2 = :\bar{d}_i \gamma^\mu (1+\gamma_5) \lambda_A s_i \Box [\bar{d}_j \gamma_\mu (1+\gamma_5) \lambda_A s_j]: \tag{A6}$$

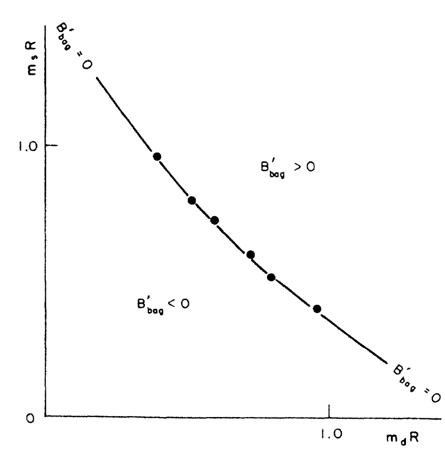

FIG. 4. Bag-model evaluation of $\bar{K}^0$-to-K^0 matrix element of local four-quark dipenguin operator $\tilde{O}_2$.

which appears as part of the dipenguin operator. In the bag model, we obtain

$$\langle K^0 | \tilde{O}_2 | \bar{K}^0 \rangle = \frac{16 N_d{}^2 N_s{}^2}{3\pi R^2} \int_0^R r^2 dr\, J(r) \,, \tag{A7}$$

where $J(r)$ is a complicated and lengthy function which we do not reproduce here. However defining a quantity B' as

$$B' = \langle K^0 | \tilde{O}_2 | \bar{K}^0 \rangle / \langle K^0 | 0 | \bar{K}^0 \rangle_{\rm vac} \,, \tag{A8}$$

we plot B' as a function of its input parameters in Fig. 4. Observe a pattern of behavior similar to that in Fig. 3, a line of zeros separating regions where $B' > 0$ and $B' < 0$.

Although this reinforces our point regarding the difficulty of estimating such quantities, for the sake of completeness we have computed the ratio of dipenguin to box contributions in the bag model. We find

$$\frac{\langle K^0 | L^{\rm sp} | \bar{K}^0 \rangle}{\langle K^0 | L^{\rm box} | \bar{K}^0 \rangle} = \frac{\alpha_s \ln^2(m_c/\mu)}{9\pi} \frac{\langle K^0 | \tilde{O} | \bar{K}^0 \rangle}{m_c{}^2 \langle K^0 | O | \bar{K}^0 \rangle}$$

$$\cong 0.06 \,. \tag{A9}$$

This small ratio is consistent with our vacuum saturation estimate. The two factors in Eq. (A9) equal 0.09 and 0.67, respectively.

[1]M. K. Gaillard and B. W. Lee, Phys. Rev. D **10**, 897 (1974).
[2]L. Wolfenstein, Nucl. Phys. **B160**, 501 (1979).
[3]C. Hill, Phys. Lett. **97B**, 275 (1980).
[4]J. F. Donoghue, E. Golowich, and B. R. Holstein, Phys. Lett. **135B**, 481 (1984).
[5]I. I. Bigi and A. I. Sanda, Phys. Lett. **148B**, 205 (1984); M. R. Pennington, *ibid.* **153B**, 439 (1985); P. Cea and C. Nardulli, *ibid.* **152B**, 251 (1985).
[6]L. L. Chau, Phys. Rep. **95**, 1 (1983).
[7]A related diagram occurring in the Higgs model of *CP* violation has been considered by Y. DuPont and T. N. Pham, Phys. Rev. D **28**, 2169 (1983).
[8]M. A. Shifman, A. I. Vainshtein, and V. J. Zakharov, Nucl. Phys. **B120**, 315 (1977); Pis'ma Zh. Eksp. Teor. Fiz. **22**, 123 (1975) [JETP Lett. **22**, 55 (1975)].
[9]M. Wise and E. Witten, Phys. Rev. D **20**, 1216 (1979).
[10]P. Colic *et al.*, Report No. MP1-PAE/P, 1982 (unpublished); I. I. Bigi and A. Sanda, Phys. Rev. D **29**, 1393 (1984).
[11]R. Schrock and S. Treiman, Phys. Rev. D **19**, 2148 (1979); B. McWilliams and O. Shankar, *ibid.* **22**, 2853 (1980).