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Constituent-quark description of nonleptonic hyperon decays

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An elementary framework is presented in which both S-wave and P-wave nonleptonic hyperon decays can be described satisfactorily from a constituent-quark standpoint. The picture is likely to be of use in future discussions of kaon, hyperon radiative, or heavy-quark decays.

I. INTRODUCTION

Nonleptonic decays of hyperons provide a rich ground for the study of how weak- (or any pointlike) interaction processes occur in the presence of strong interactions. Even though many features of these decays have been understood for about 20 years, since the advent of current algebra,[1] many other questions remain timely. Among these are the following.

(a) How does one separate long-distance from short-distance effects? These questions return to haunt us in studies of the K^0-$\bar{K}^0$ system and of weak decays of heavy $(c,b,\ldots)$ quarks.

(b) Is the $\Delta I=\frac{1}{2}$ rule in hyperon (or kaon) decays fully understood?

(c) What can one predict for rates and asymmetries in the radiative decays of hyperons, such as $\Sigma^+\to p\gamma$?

(d) Can the technology of dealing with the weak nonleptonic interactions be applied to other pointlike processes, such as proton decay?

In the present article we describe nonleptonic hyperon decays primarily from a long-distance point of view, with as much appeal as possible to simple quark-model ideas. Our intention is to obtain a picture of quality and intuitive power similar to that provided by the constituent-quark description of magnetic moments and magnetic dipole transitions in baryons and mesons. Thus, we shall be satisfied with qualitative ($\approx 10\%$) results at the present stage. It is our hope that the techniques developed here may be helpful in shedding light on some of the questions mentioned above.

We set the stage for the discussion and mention some relevant previous work in Sec. II. Quark-model calculations are reviewed in Sec. III. The S-wave (parity-violating) decays of baryons are discussed in Sec. IV and the P-wave decays in Sec. V. Possible relations among amplitudes, motivated by nonrelativistic considerations, are mentioned in Sec. VI. A summary (and list of future problems) occupies Sec. VII.

II. PRELIMINARIES AND PREVIOUS WORK

We shall be concerned with the decays listed in Table I. Also shown are the experimental values for the corresponding amplitudes. For spin-$\frac{1}{2}$ baryon decays, we use the conventions of Ref. 2, taking the effective Lagrangian for the decay to be

$$\mathscr{L}_{\rm eff}=G_F m_\pi{}^2[\bar{\psi}_2(A+B\gamma_5)\psi_1]\phi_\pi\ , \tag{2.1}$$

where G_F is the Fermi coupling constant, so that the am-

TABLE I. Nonleptonic-hyperon-decay amplitudes.

	Decays $B_1(\frac{1}{2}^+)\to\pi+B_2(\frac{1}{2}^+)$ (from Ref. 2)	
Decay	S-wave amplitude	P-wave amplitude
$\Lambda\to\pi^-p$	$A(\Lambda^0_-)=1.47\pm0.01$	$B(\Lambda^0_-)=9.98\pm0.24$
$\Lambda\to\pi^0n$	$A(\Lambda^0_0)=-1.07\pm0.01$	$B(\Lambda^0_0)=-7.14\pm0.56$
$\Sigma^+\to\pi^+n$	$A(\Sigma^+_+)=0.06\pm0.01$	$B(\Sigma^+_+)=19.07\pm0.07$
$\Sigma^+\to\pi^0p$	$A(\Sigma^+_0)=1.48\pm0.05$	$B(\Sigma^+_0)=-12.04\pm0.58$
$\Sigma^-\to\pi^-n$	$A(\Sigma^-_-)=1.93\pm0.01$	$B(\Sigma^-_-)=-0.65\pm0.07$
$\Xi^0\to\pi^0\Lambda$	$A(\Xi^0_0)=1.55\pm0.03$	$B(\Xi^0_0)=-5.56\pm0.33$
$\Xi^-\to\pi^-\Lambda$	$A(\Xi^-_-)=2.04\pm0.01$	$B(\Xi^-_-)=-7.49\pm0.28$
	Decays $\Omega^-(\frac{3}{2}^+)\to M(0^-)+B_2(\frac{1}{2}^+)$	
Decay	P-wave amplitude (magnitude)	
$\Omega^-\to\pi^-\Xi^0$	$\lvert B(\Omega^-_-)\rvert=10.0\pm0.3$	
$\Omega^-\to\pi^0\Xi^-$	$\lvert B(\Omega^-_0)\rvert=6.0\pm0.3$	
$\Omega^-\to K^-\Lambda$	$\lvert B(\Omega^-_K)\rvert=30.6\pm0.6$	

plitudes A and B are dimensionless. A describes S-wave (parity-violating) decays; B describes P-wave (parity-conserving) decays. The corresponding partial width for $B_1 \to \pi B_2$ is

$$\Gamma(B_1 \to \pi B_2) = \frac{(G_F m_\pi{}^2)^2}{8\pi m_1{}^2} q\{[(m_1+m_2)^2 - m_\pi{}^2]\,|A|^2 + [(m_1-m_2)^2 - m_\pi{}^2]\,|B|^2\}\ , \tag{2.2}$$

where q is the magnitude of the final three-momentum of either particle in the rest frame of B_1.

For Ω^- decays to $\pi\Xi$ or $K\Lambda$, we take the effective Lagrangian[3] for the decay $\Omega^- \to MB_2$

$$\mathscr{L}_{\text{eff}} = G_F m_\pi{}^2 \bar{\psi}_2 (B + D\gamma_5)\psi_\mu (\partial^\mu \phi_M / m_\Omega)\ , \tag{2.3}$$

where ψ_μ is a Rarita-Schwinger spinor describing the Ω^-. The dimensionless amplitude B describes P-wave decays, while D describes D waves. The partial width is then

$$\Gamma(\Omega \to MB_2) = \frac{(G_F m_\pi{}^2)^2}{24\pi m_\Omega{}^4} q^3 [(m_\Omega + m_B)^2 - m_M{}^2]\,|B|^2\ , \tag{2.4}$$

plus a D-wave contribution which we shall henceforth neglect. We use partial decay widths for Ω^- based on the compilation in Ref. 4 to calculate $|B|$. We shall not be concerned here with the $\Xi^*\pi$ decays of Ω^-; discussions appear, for example, in Refs. 3 and 5.

The amplitudes in Table I satisfy a number of approximate regularities, some of which we will be able to describe quite simply.

The $\Delta I = \frac{1}{2}$ *rule* for the transformation property of the nonleptonic weak Hamiltonian implies

$$\Lambda^0_- = -\sqrt{2}\,\Lambda^0_0\ , \tag{2.5a}$$

$$\Sigma^+_+ + \sqrt{2}\,\Sigma^+_0 = \Sigma^-_-\ , \tag{2.5b}$$

$$\Xi^-_- = \sqrt{2}\,\Xi^0_0\ , \tag{2.5c}$$

$$\Omega^-_- = \sqrt{2}\,\Omega^-_0\ , \tag{2.5d}$$

for both A and B amplitudes. Deviations from these relations are small. We shall be treating only contributions respecting the $\Delta I = \frac{1}{2}$ rule.

The Lee-Sugawara triangle relation,[6]

$$\Lambda^0_- + \sqrt{3}\,\Sigma^+_0 = 2\Xi^-_-\ , \tag{2.6}$$

holds well for S-wave (A) amplitudes but less well for P-wave (B) amplitudes. For S waves, it is a consequence of current algebra and partial conservation of axial-vector current (PCAC).

The S-wave amplitude $A(\Sigma^+_+)$ and the P-wave amplitude $B(\Sigma^-_-)$ nearly vanish. We shall be able to understand the first result, but the second will appear accidental.

Early approaches based on current algebra and PCAC were able to understand the S waves well, but the P waves less well.[6,7] This situation has persisted until fairly recently.[5,8] Explicit long-distance approaches to the problem have also appeared over the years.[3,8–15] It appears that satisfactory descriptions of both S- and P-wave decays can be obtained if one is prepared to abandon the current-algebra-inspired link between them. Various justifications for this procedure have been given.[3,10,13,14]

Our treatment is closest to the spirit of Ref. 3, but more directly tied to a quark picture. We find an adequate description of both S waves and P waves, leaving to further work the possible relation between the two sets of amplitudes as well as any connection with kaon decays.

The description of nonleptonic decays most directly tied to the present understanding of the strong and weak interactions is based on the short-distance behavior of operators.[16] A modest enhancement of the $\Delta I = \frac{1}{2}$ part of the nonleptonic weak Hamiltonian arises in QCD (Ref. 17). Internal quark loops play a role in generating an effective $s \to d$ transition[18,19] through the so-called "penguin" diagrams.[19] Short-distance analyses of radiative hyperon decays have been performed; the transition $s \to d + \gamma$ alone does not suffice to explain known data,[20] but $s \to d + \gamma + (\text{gluon})$ may do so.[21,22] It is necessary to evaluate matrix elements of short-distance operators via some theory of the hadrons, whether via lattice,[23] bag model,[12,15,22] or other methods. Our complementary long-distance approach should contain at least as many independent operators as those short-distance ones first classified in Ref. 18, and we have checked that it does. Relations among them may be possible; we leave these for future study.

One crucial aspect of the $\Delta I = \frac{1}{2}$ rule in baryon nonleptonic decays involves the subprocess shown in Fig. 1. For a pair of quarks in an S wave, the nonleptonic weak interaction leaves the final ud pair in $su \to ud$ in an $I=0$ state.[24] The process must then necessarily carry $\Delta I = \frac{1}{2}$ (the isospin of the initial u quark). This process will be one of several we consider.

Another subprocess in weak decay is loosely related to the penguin graph[18,19] shown in Fig. 2. A quark loop can lead to an effective $s \to d$ transition, as long as at least one gluon is radiated by the intermediate charge $\frac{2}{3}$ quark. Without this gluon, the $s \to d$ transition could be rotated away.[25]

A further operator suggested by the short-distance approach is illustrated in Fig. 3. A transition $s \to d$ can flip spin if another quark q flips its spin simultaneously. Thus, we anticipate an operator of the form $\sigma_{sd} \cdot \sigma_{qq}$ in

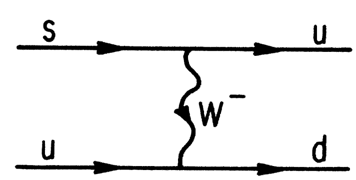

FIG. 1. $\Delta I = \frac{1}{2}$ four-quark contribution to hyperon decay from the transition $su \to ud$. Both the initial and final pairs are in a state with $I=0$, spin 0, and color 3^*, leading to a state with flavor antisymmetry.

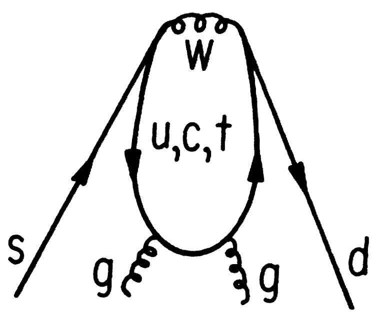

FIG. 2. "Penguin" graph depicting $s \to d$ weak transition.

baryon decays. The single gluon shown exchanged in Fig. 3 is only the lowest-order (in α_s) contribution to such an operator. For one-gluon exchange, the contribution is $O(v^2/c^2)$. For exchange of more than one gluon such suppression need not be present.

The penguin graph can manifest itself in another way, as shown in Fig. 4. There will be an effective $s \to \pi^- u$ or $s \to \pi^0 d$ decay amplitude, respecting the $\Delta I = \frac{1}{2}$ rule. We find the contribution of this amplitude to be of the same form as the $s \to d$ transition illustrated in Fig. 2.

One contribution which we explicitly neglect in the present treatment, but which is needed in a complete description including $\Delta I = \frac{3}{2}$ amplitudes, is the "spectator" diagram shown in Fig. 5. Because of color factors, it can be seen that this diagram predicts amplitudes for $s \to \pi^- u$ which are $3\sqrt{2}$ times those for $s \to \pi^0 d$, while this ratio would be $-\sqrt{2}$ for a pure $\Delta I = \frac{1}{2}$ transition (as in Fig. 4). The most prominent place to look for this effect is probably in $\Omega \to \Xi\pi$ decays, for which the $\Delta I = \frac{1}{2}$ rule predicts a $\Xi^0\pi^-/\Xi^-\pi^0$ ratio of 2. The experimental value[4] is closer to 3.

III. QUARK-MODEL CALCULATIONS

We shall consider only S-wave mesons and baryons. From this point of view the baryons may be considered as states symmetric in flavor×spin, since they are antisymmetric in space×color. It will then be convenient to treat quarks as *bosons* (as in Ref. 24) from the standpoint of creation and annihilation operators. This simplifies the construction of baryon wave functions; it is irrelevant for mesons, if we are careful about parity properties.

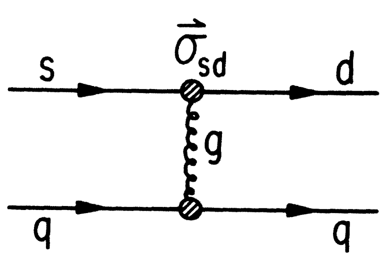

FIG. 3. Example of a spin-flipping $s \to d$ transition, giving rise to operator proportional to $\sigma_{sc}\sigma_{qq}$.

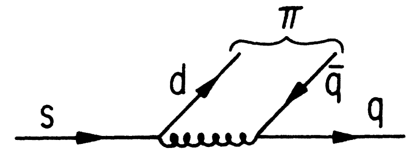

FIG. 4. Special contribution of "penguin" graph to pion emission in strange-quark decay.

A. Construction of baryon states

We will need quark-model wave functions for octet $(\frac{1}{2}^+)$ and decimet $(\frac{3}{2}^+)$ baryons. These are depicted in Tables II and III. The arrows denote components of spin along the z axis: ↑ for $J_z = \frac{1}{2}$, ↓ for $J_z = -\frac{1}{2}$. This notation also applies to the $\frac{3}{2}^+$ states, for which we will need only $J_z = \pm\frac{1}{2}$ spin projections. The operator $u^+\uparrow$, for example, creates a u quark with $J_z = +\frac{1}{2}$ from the vacuum $|0\rangle$. The states are correctly normalized, by virtue of the bosonic commutation relations of the creation and annihilation operators.

The magnetic moments of baryons in the quark model may be calculated by taking the expectation value of

$$\mu_3 = \sum_q (n_{q\uparrow} - n_{q\downarrow})\mu_q \ , \tag{3.1}$$

where, e.g.,

$$n_{u\uparrow} = u^+\uparrow u\uparrow \tag{3.2}$$

and so on. One then finds familiar relations such as

$$\mu_q = \tfrac{4}{3}\mu_u - \tfrac{1}{3}\mu_d \tag{3.3}$$

and

$$\mu_\Lambda = \mu_s \ . \tag{3.4}$$

B. Meson emission

We shall calculate meson-baryon couplings using the static quark model, which gives the SU(6) value of $F/D = \frac{2}{3}$. The nonrelativistic limit of pseudoscalar meson emission by a quark corresponds to an operator of the form $\boldsymbol{\sigma}\cdot\mathbf{k}$, where $\mathbf{k}$ is the meson momentum and $\boldsymbol{\sigma}$ is a Pauli matrix describing the quark spin. We shall always take $\mathbf{k}$ along the z axis, so the calculation of coupling-

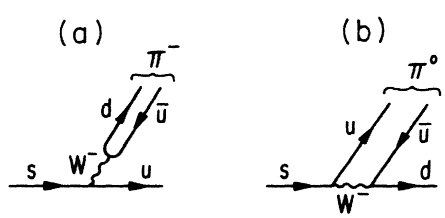

FIG. 5. "Spectator" diagrams contributing to pion emission by strange quarks. (a) π^- emission; (b) π^0 emission.

TABLE II. Quark-model wave functions for octet $(\frac{1}{2}^+)$ baryons with $J_z=+\frac{1}{2}$.

$p\uparrow$	$(\frac{1}{3})^{1/2}(u^+\uparrow u^+\uparrow d^+\downarrow - u^+\uparrow u^+\downarrow d^+\uparrow)\|0\rangle$
$n\uparrow$	$(\frac{1}{3})^{1/2}(d^+\uparrow d^+\downarrow u^+\uparrow - d^+\uparrow d^+\uparrow u^+\downarrow)\|0\rangle$
$\Lambda\uparrow$	$(\frac{1}{2})^{1/2}(u^+\uparrow d^+\downarrow s^+\uparrow - u^+\downarrow d^+\uparrow s^+\uparrow)\|0\rangle$
$\Sigma^+\uparrow$	$(\frac{1}{3})^{1/2}(u^+\uparrow u^+\downarrow s^+\uparrow - u^+\uparrow u^+\uparrow s^+\downarrow)\|0\rangle$
$\Sigma^-\uparrow$	$(\frac{1}{6})^{1/2}(u^+\uparrow d^+\downarrow s^+\uparrow + u^+\downarrow d^+\uparrow s^+\uparrow - 2u^+\uparrow d^+\uparrow s^+\downarrow)\|0\rangle$
$\Sigma^0\uparrow$	$(\frac{1}{3})^{1/2}(d^+\uparrow d^+\downarrow s^+\uparrow - d^+\uparrow d^+\uparrow s^+\downarrow)\|0\rangle$
$\Xi^0\uparrow$	$(\frac{1}{3})^{1/2}(s^+\uparrow s^+\uparrow u^+\downarrow - s^+\uparrow s^+\downarrow u^+\uparrow)\|0\rangle$
$\Xi^-\uparrow$	$(\frac{1}{3})^{1/2}(s^+\uparrow s^+\uparrow d^+\downarrow - s^+\uparrow s^+\downarrow d^+\uparrow)\|0\rangle$

constant ratios reduces to the evaluation of matrix elements of σ_3 between initial and final baryons. Relativistic effects renormalize the coupling strength (equivalently, the axial-vector coupling constant g_A) but not the pattern of couplings.[26]

The specific operators which we choose to describe meson emission are

$$O^{(\pi^-)}=u^+\uparrow d\uparrow - u^+\downarrow d\downarrow\ , \tag{3.5a}$$

$$O^{(\pi^0)}=\frac{1}{\sqrt{2}}(u^+\uparrow u\uparrow - u^+\downarrow u\downarrow - d^+\uparrow d\uparrow + d^+\downarrow d\downarrow)\ , \tag{3.5b}$$

$$O^{(\pi^+)}=-(d^+\uparrow u\uparrow - d^+\downarrow u\downarrow)\ , \tag{3.5c}$$

$$O^{(K^-)}=u^+\uparrow s\uparrow - u^+\downarrow s\downarrow\ , \tag{3.6a}$$

$$O^{(K^0)}=-(d^+\uparrow s\uparrow - d^+\downarrow s\downarrow)\ . \tag{3.6b}$$

The signs are chosen so that (π^-,π^0,π^+) and $(K^-,\bar{K}^0)$ correspond to isospin multiplets. SU(3) symmetry is assumed in passing from pions to kaons.

As an exercise, we may now calculate coupling constants for meson emission by $\frac{1}{2}^+$ or $\frac{3}{2}^+$ baryons. These coupling constants are needed in evaluating the contributions of pole terms to P-wave nonleptonic hyperon decays. Typical pole terms are shown in Fig. 6. To calculate the amplitude for $p\uparrow\to\pi^+n\uparrow$ we evaluate $\langle n\uparrow|O^{(\pi^+)}|p\uparrow\rangle$; the result is $-\frac{5}{3}$. Results of similar calculations are shown in Table IV. These isoscalar factors $(MB'|B)$ are related to the amplitude for $B\uparrow\to MB'\uparrow$ by an isospin Clebsch-Gordan coefficient:[4]

$$A(B\uparrow\to MB'\uparrow)=(MB'|B)(I_BI_{3B}|I_MI_{3M}I_{B'}I_{3B'})\ . \tag{3.7}$$

We have tabulated only those factors needed in subsequent calculations.

TABLE III. Quark-model wave functions for decimet $(\frac{3}{2}^+)$ baryons with $J_z=+\frac{1}{2}$.

$\Delta^{++}\uparrow$	$(\frac{1}{2})^{1/2}(u^+\uparrow u^+\uparrow u^+\downarrow)\|0\rangle$
$\Delta^+\uparrow$	$(\frac{1}{6})^{1/2}(u^+\uparrow u^+\uparrow d^+\downarrow + 2u^+\uparrow u^+\downarrow d^+\uparrow)\|0\rangle$
$\Delta^0\uparrow$	$(\frac{1}{6})^{1/2}(d^+\uparrow d^+\uparrow u^+\downarrow + 2d^+\uparrow d^+\downarrow u^+\uparrow)\|0\rangle$
$\Delta^-\uparrow$	$(\frac{1}{2})^{1/2}(d^+\uparrow d^+\uparrow d^+\downarrow)\|0\rangle$
$Y^{*+}\uparrow$	$(\frac{1}{6})^{1/2}(u^+\uparrow u^+\uparrow s^+\downarrow + 2u^+\uparrow u^+\downarrow s^+\uparrow)\|0\rangle$
$Y^{*0}\uparrow$	$(\frac{1}{3})^{1/2}(u^+\uparrow d^+\uparrow s^+\downarrow + u^+\uparrow d^+\downarrow s^+\uparrow + u^+\downarrow d^+\uparrow s^+\uparrow)\|0\rangle$
$Y^{*-}\uparrow$	$(\frac{1}{6})^{1/2}(d^+\uparrow d^+\uparrow s^+\downarrow + 2d^+\uparrow d^+\downarrow s^+\uparrow)\|0\rangle$
$\Xi^{*0}\uparrow$	$(\frac{1}{6})^{1/2}(s^+\uparrow s^+\uparrow u^+\downarrow + 2s^+\uparrow s^+\downarrow u^+\uparrow)\|0\rangle$
$\Xi^{*-}\uparrow$	$(\frac{1}{6})^{1/2}(s^+\uparrow s^+\uparrow d^+\downarrow + 2s^+\uparrow s^+\downarrow d^+\uparrow)\|0\rangle$
$\Omega^-\uparrow$	$(\frac{1}{2})^{1/2}(s^+\uparrow s^+\uparrow s^+\downarrow)\|0\rangle$

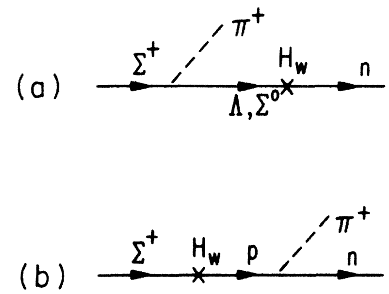

FIG. 6. Typical pole terms contributing to nonleptonic hyperon decay. (a) H_W acts "after" meson emission; (b) H_W acts "before" meson emission.

C. Parity-violating and parity-conserving weak Hamiltonian

The nonleptonic strangeness-changing Hamiltonian is

$$H_W=\frac{G_F}{\sqrt{2}}\sin\theta_C\cos\theta_C[\bar{u}\gamma^\mu(1-\gamma_5)s] \times[\bar{d}\gamma_\mu(1-\gamma_5)u]+\text{H.c.} \tag{3.8}$$

To set the stage for the nonrelativistic discussions which follow in Secs. IV and V, we note that the parity-violating part of (3.8) in this limit always carries $\Delta I=1$ and is of order v/c in comparison with the dominant part of the parity-conserving part (with $\Delta I=0$). A similar distinction applies to the penguin diagrams giving rise to $s\to d$ transitions.

TABLE IV. Quark-model predictions for isoscalar factors in $(J_z=\frac{1}{2})\to(J_z=\frac{1}{2})$ transitions.

Isoscalar factor	Value
$(\pi N\|N)$	$-5/\sqrt{6}$
$(\pi\Sigma\|\Lambda)$	$-\sqrt{2}$
$(\pi\Lambda\|\Sigma)$	$\sqrt{2/3}$
$(\pi\Sigma\|\Sigma)$	$-\frac{4}{3}$
$(\pi\Xi\|\Xi)$	$1/\sqrt{6}$
$(\bar{K}N\|\Lambda)$	$-\sqrt{3}$
$(\bar{K}N\|\Sigma)$	$-\frac{1}{3}$
$(\bar{K}\Lambda\|\Xi)$	$1/\sqrt{6}$
$(\pi\Xi\|\Xi^*)$	$2/\sqrt{3}$
$(\bar{K}\Lambda\|\Xi^*)$	$2/\sqrt{3}$
$(\bar{K}\Xi\|\Omega)$	$4/\sqrt{3}$

IV. S-WAVE DECAYS

A satisfactory description of S-wave nonleptonic hyperon decays is possible from many points of view. In particular, current algebra and PCAC can be used to relate the amplitudes for $B \to B'\pi$ (parity violating, S wave) to the matrix elements $\langle B' | H_W^{(\mathrm{PC})} | B \rangle$, where PC stands for parity conserving. These matrix elements then may be calculated via SU(3). The results involve an f/d ratio which is approximately the same $(-3 \le f/d \le -2)$ as that characterizing octet or electromagnetic splittings $\langle B | \Delta M | B \rangle$.[5,7,10]

We would like to see from a quark standpoint how some of these results arise. The parity-violating weak interaction leads to an effective $su \to ud$ term in the effective Hamiltonian of the following form when both initial and final pairs are in a color $\mathbf{3}^*$:

$$H^{(\mathrm{PV})}_{(su\to ud)} \sim (u^+{\uparrow} d^+{\downarrow} + u^+{\downarrow} d^+{\uparrow})(s{\uparrow} u{\downarrow} - s{\downarrow} u{\uparrow}) + (u^+{\uparrow} d^+{\downarrow} - u^+{\downarrow} d^+{\uparrow})(s{\uparrow} u{\downarrow} + s{\downarrow} u{\uparrow}) \ . \tag{4.1}$$

The form of (4.1) expresses the fact that the PV transition acts between quark pairs of $S=1$ (and, incidentally, $I=1$) and those of $S=0$. We consider only those $S=I=1$ states with $S_z=I_z=0$, as will be clear presently.

The quarks in the initial or final state of the $su \to ud$ transition must be involved in pion emission in order for a nonzero contribution to arise in the processes of interest. Allowed diagrams are shown in Fig. 7. If the pion is not "connected" to the quarks in $H^{(\mathrm{PV})}_{(su\to ud)}$, as in Fig. 8(a), the initial or final states of the term (4.1) will not match those of the quarks in the initial or final baryon, because such quarks are in a relative S wave with respect to one another. The effective operators leading to pion emission via the parity-violating contribution (4.1) then have the form

$$O_1(\pi^-,\mathrm{PV}) \equiv x(u^+{\uparrow} d^+{\downarrow} - u^+{\downarrow} d^+{\uparrow})(s{\uparrow} d{\downarrow} - s{\downarrow} d{\uparrow}) \ , \tag{4.2a}$$

$$O_1(\pi^0,\mathrm{PV}) \equiv x(u^+{\uparrow} d^+{\downarrow} - u^+{\downarrow} d^+{\uparrow})(s{\uparrow} u{\downarrow} - s{\downarrow} u{\uparrow})/\sqrt{2} \ , \tag{4.2b}$$

where x is an arbitrary strength. The result in (4.2a) comes from Fig. 7(a) alone [Fig. 7(c) will not contribute], while that in (4.2b) is the sum of equal positive contributions from Figs. 7(a) and 7(b) and an equal negative contribution from Fig. 7(c). As we shall see, there is no operator corresponding to π^+ emission from these graphs.

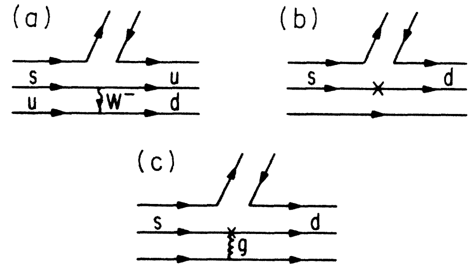

FIG. 8. Diagrams contributing only to parity-conserving nonleptonic decays. (a) $su \to ud$ transition; (b) $s \to d$ transition; (c) $s \to d$ transition with a spin flip.

A similar distinction among quarks involved in pion emission and those which are not applies to PV $s \to d$ transitions generated by penguin graphs. Since such transitions must involve $\Delta I=1$, graphs such as Figs. 8(b)and 8(c) cannot contribute to PV decays, while those in which the final d quark ends up in the meson can contribute.

The distinction between allowed and forbidden contributions to the PV decays immediately tells why $A(\Sigma_+^+)$ must vanish. The only potential contribution to this amplitude would come from the graph of Fig. 7(b), in which the meson consists of $u\bar{d}$ and the quark q is a d. Then, by statistics, the dd pair in the final state must have $S=1$, $S_z=0$. But the emitted pion and the ud pair just after W exchange also have $S=1$, $S_z=0$. This amplitude then vanishes, since the Clebsch-Gordan coefficient $(10|10\,10)$ vanishes.

To summarize, the only contribution in Fig. 7 comes from Fig. 7(a); Figs. 7(b) and 7(c) always sum to zero, for any pion charge.

We shall distinguish between $s{\uparrow} \to d{\uparrow}$ and $s{\downarrow} \to d{\downarrow}$ PV transitions (recalling that baryons are always being taken to have $J_z=+\frac{1}{2}$ here). (See Fig. 9.) We then parametrize the effective operators leading to pion emission as

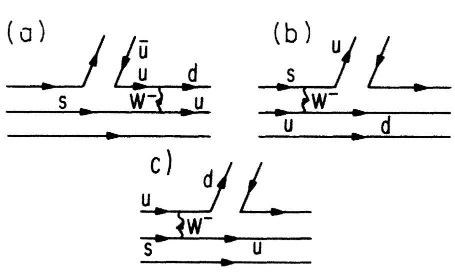

FIG. 7. Examples of allowed diagrams in parity-violating nonleptonic decays involving the $su \to ud$ transition.

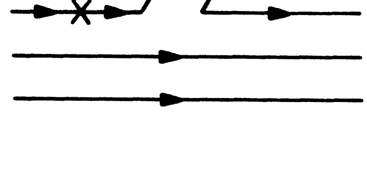

FIG. 9. Diagram involving $s \to d$ transition in which final d appears in a pion. The K^* pole (PV decays) or K pole (PC decays) gives a contribution which may be estimated via this diagram.

$$O_2(\pi^-,\mathrm{PV}) \equiv y(u^+\uparrow s\uparrow)+y'(u^+\downarrow s\downarrow)\,, \tag{4.3a}$$

$$O_2(\pi^0,\mathrm{PV}) \equiv -[y(d^+\uparrow s\uparrow)+y'(d^+\downarrow s\downarrow)]/\sqrt{2}\,. \tag{4.3b}$$

One source of this term can be thought of as the K^* pole discussed (for example) in Ref. 10. The ratio y'/y is characteristic of the f/d ratio whereby the K^* couples: $y'=y$ for pure f-type coupling, for example.

A further spin-dependent contribution can be anticipated on the basis of graphs such as those in Fig. 10. We expect only the graph in Fig. 10(a) to contribute to PV processes, of interest here, since the $s\to d$ transition involves $\Delta I=1$ and hence is forbidden for final ground-state baryons. The graphs of Figs. 10(b), 10(c), and 8(c) can contribute only to P-wave (PC) transitions. The effective operators for pion emission then take the form

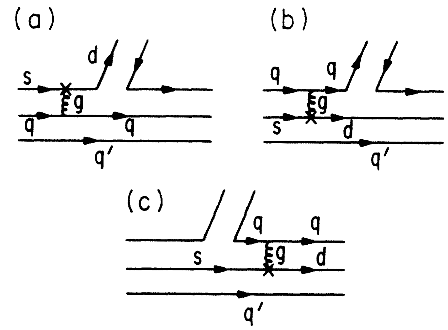

FIG. 10. Sources of spin-dependent $s\to d$ transitions. (a) Allowed for PV and PC decays; (b), (c) allowed only for PC decays. The $s\to d$ transition is marked with a cross.

$$\begin{aligned} O_3(\pi^-,\mathrm{PV}) &= z(\boldsymbol{\sigma}_{su}\cdot\boldsymbol{\sigma}_{qq}) \\ &= z:[(u^+\uparrow s\uparrow - u^+\downarrow s\downarrow)(q^+\uparrow q\uparrow - q^+\downarrow q\downarrow)]:\,, \\ O_3(\pi^0,\mathrm{PV}) &= -\frac{z}{\sqrt{2}}(\boldsymbol{\sigma}_{sd}\cdot\boldsymbol{\sigma}_{qq}) \\ &= -\frac{z}{\sqrt{2}}:[(d^+\uparrow s\uparrow - d^+\downarrow s\downarrow)(q^+\uparrow q\uparrow - q^+\downarrow q\downarrow)+2(d^+\uparrow s\downarrow q^+\downarrow q\uparrow + d^+\downarrow s\uparrow q^+\uparrow q\downarrow)]:\,, \end{aligned} \tag{4.4}$$

where the symbol : denotes normal ordering. This ensures that the quarks q are distinct from those involved in $s\to u$ or $s\to d$.

Here we have used the identity

$$\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b = \sigma_{3a}\sigma_{3b}+2(\sigma_{+a}\sigma_{-b}+\sigma_{-a}\sigma_{+b})\,. \tag{4.5}$$

Again, no π^+ emission occurs.

The calculation of decay amplitudes is thus a simple matter of evaluating

$$O(\pi,\mathrm{PV}) \equiv \sum_{i=1}^{3} O_i(\pi,\mathrm{PV}) \tag{4.6}$$

between initial and final baryon states of Table II. The results are shown in Table V. The seven amplitudes satisfy the three $\Delta I=\frac{1}{2}$ relations (2.2), the Lee-Sugawara relation (2.3), and the relation $A(\Sigma^+_+)=0$. There are thus only two linearly independent combinations of the four parameters in Table V. Thus, for example, we may define

$$u=(\tfrac{3}{2})^{1/2}(x+y)\,, \tag{4.7a}$$

$$v=\frac{\sqrt{2}}{3}(y'-4y)-2\sqrt{2}z\,, \tag{4.7b}$$

so that

$$A(\Lambda^0_-)=u=-\sqrt{2}A(\Lambda^0_0)\,, \tag{4.8a}$$

$$A(\Sigma^+_0)=\sqrt{3}\,u+v=A(\Sigma^-_-)/\sqrt{2}\,, \tag{4.8b}$$

$$A(\Xi^-_-)=2u+\frac{\sqrt{3}}{2}v=\sqrt{2}\,A(\Xi^0_0)\,. \tag{4.8c}$$

A simultaneous least-squares fit to the amplitudes in Table I predicts the values in the last column of Table V with

$$u=1.525\,, \tag{4.9a}$$

$$v=-1.203\,. \tag{4.9b}$$

The agreement is no worse than violations of the $\Delta I=\frac{1}{2}$ rule, which we have ignored: for example, experimentally,

$$\sqrt{2}A(\Xi^0_0)/A(\Xi^-_-)=0.93\pm0.02\,.$$

Notice that although we cannot separate all the contributions from one another, certain ones cannot be ignored. For example, we cannot assume that a spin-independent $s\to d$ transition alone is responsible for the S-wave decays; the ansatz $x=z=0$, $y=y'$ would predict $u/v=-\sqrt{3}/2=-0.87$, whereas the fit implies $u/v=-1.27$. If we were to insist on a pure $s\to d$ description (without an exchange term, i.e., with $x=0$),

TABLE V. Parity-violating nonleptonic-decay amplitudes A in terms of contributions of Figs. 7 (x), 9 (y,y'), and 10 (z).

Decay	Amplitude A	Fitted value
Λ^0_-	$\left[\frac{3}{2}\right]^{1/2}x+\left[\frac{3}{2}\right]^{1/2}y$	1.52
Λ^0_-	$-\Lambda^0_-/\sqrt{2}$	-1.08
Σ^+_+	0	0
Σ^+_0	$\frac{3}{\sqrt{2}}x+\frac{y+2y'}{3\sqrt{2}}-2\sqrt{2}z$	1.44
Σ^-_0	$\sqrt{2}\,\Sigma^+_0$	2.03
Ξ^-_-	$\sqrt{6}x+\frac{2y+y'}{\sqrt{6}}-\sqrt{6}z$	2.01
Ξ^0_0	$\Xi^-_-/\sqrt{2}$	1.42

we would either need $y \neq y'$ or $z \neq 0$ (or both). If only the exchange term (x) were nonzero, we would have $|u/v| = \infty$, again in contradiction with experiment. A more microscopic examination in principle could give relations among x, y, y', and z.

V. *P*-WAVE DECAYS

Many descriptions of nonleptonic hyperon decays have encountered difficulty with *P*-wave (parity-conserving or PC) amplitudes. A recent summary of such problems is given in Ref. 5. Part of the problem seems to be the rapid variation of amplitudes with respect to off-shell pion momenta, as a result of pole terms (e.g., Fig. 6).

We shall adopt a highly simplified description of pole-term contributions to *P*-wave hyperon decays. We find that higher-mass pole terms, which we parametrize in a simple fashion, appear necessary for a satisfactory description.

We proceed as in Sec. IV to identify various types of operators responsible for weak transitions. These are then incorporated into baryon-pole terms and couplings to mesons calculated with the help of the quark model (Table IV). There will also be a contribution from kaon pole terms (Fig. 11), in which $\langle \pi | H_W^{(\mathrm{PC})} | K \rangle$ is taken to have arbitrary strength for present purposes.

The $su \to ud$ parity-conserving transition is now described by an operator

$$H^{(\mathrm{PC})}_{(su \to ud)} \sim (u^+\uparrow d^+\downarrow - u^+\downarrow d^+\uparrow)(s\uparrow u\downarrow - s\downarrow u\uparrow) \ . \tag{5.1}$$

The PC $s \to d$ transition will be described by

$$H^{(\mathrm{PC})}_{(s \to d)} \sim d^+\uparrow s\uparrow + d^+\downarrow s\downarrow \ . \tag{5.2}$$

Here we do not distinguish between $s\uparrow \to d\uparrow$ and $s\downarrow \to d\downarrow$. If we were to do so, we would find that $H^{(\mathrm{PC})}_{(s \to d)}$ would not conserve angular momentum. This is in contrast with the distinction drawn in Sec. IV, where the final d quark must end up in a pion.

The spin-dependent $s \to d$ transition is described by an operator

$$H^{\mathrm{PC}}_{(\sigma_{sd}\cdot\sigma_{qq})} \sim :(d^+\uparrow s\uparrow - d^+\downarrow s\downarrow)(q^+\uparrow q\uparrow - q^+\downarrow q\downarrow) + 2(d^+\uparrow s\downarrow q^+\downarrow q\uparrow + d^+\downarrow s\uparrow q\uparrow^+ q\downarrow): \ , \tag{5.3}$$

where, as in Eq. (4.4), the normal ordering : ensures that q are quarks distinct from those involved in the $s \to d$ transition.

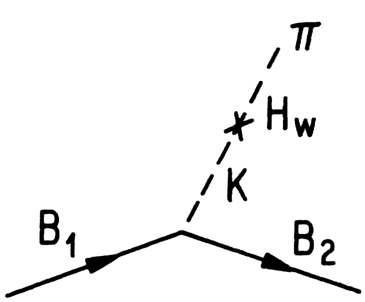

FIG. 11. Kaon-pole term in PC nonleptonic decays.

Each of the operators (5.1)–(5.3) may be used to calculate transitions between initial or final and intermediate baryon states. The meson couplings are calculated via the quark model. The resulting contributions to PC amplitudes will be proportional to arbitrary parameters: $\tilde{x}$ for (5.1), $\tilde{y}$ for (5.2), and $\tilde{z}$ for (5.3). The resulting PC amplitudes are summarized in Table VI. Here we have presented relations for the quantities

$$\tilde{B} \equiv \left[\frac{2}{3}\right]^{1/2} \frac{E_Y + m_Y}{m_\Omega} B \tag{5.4}$$

for Ω decays, since it is these that are most directly related to the B amplitudes for octet baryon decays via quark-model calculations. The energy E_Y is that of the final baryon Y in the Ω rest frame. Note that all amplitudes obey the $\Delta I = \frac{1}{2}$ relations (2.5) (and $\Omega_0^- = -\Omega_-^-/\sqrt{2}$).

We now discuss the contributions of Table VI in the context of pole terms. The most crude description, which would be an unsatisfactory approximation to the data,[5] is based on the assumption that only octet baryon poles (or Ξ^* poles, for Ω^- decays) contribute to the *P*-wave amplitudes. In pole terms there appear energy denominators which are always proportional to a mass splitting between states differing by one unit of strangeness. The assumption just mentioned is approximately equivalent to taking all such energy denominators equal to a common value $\pm\Delta M$: $+$ for graphs such as Fig. 6(b), where the weak Hamiltonian acts "before" the meson emission, and $-$ for graphs such as Fig. 6(a), where the weak Hamiltonian acts "after" the meson emission. We also would neglect differences in the momenta of emitted pions in various processes when using this lowest-pole approximation. The arbitrary amplitudes in the second row of Table VI would then be related by

$$\tilde{x}_{1b} = -\tilde{x}_{1a} \equiv \tilde{x}, \quad \tilde{x}_2 = 0 \ , \tag{5.5}$$

$$\tilde{y}_1 \equiv \tilde{y}, \quad \tilde{y}_2 = 0 \ , \tag{5.6}$$

$$\tilde{z}_{1b} = -\tilde{z}_{1a} \equiv \tilde{z}, \quad \tilde{z}_2 = 0 \ . \tag{5.7}$$

Note, in particular, the vanishing of terms corresponding to graphs 8(a) ($\tilde{x}_2$), 8(b) ($\tilde{y}_2$), and 8(c) ($\tilde{z}_2$). This comes about as a result of cancellation of pole terms of the form [6(a)] with those of the form [6(b)]. One would then find, e.g.,

$$\Lambda_-^0 = \frac{\tilde{x}}{\sqrt{6}} + \tilde{y}(\tfrac{3}{2})^{1/2} + 4\tilde{z}(\tfrac{2}{3})^{1/2} \ , \tag{5.8}$$

$$\Sigma_+^+ = -2\tilde{x} - 4\tilde{z} \ , \tag{5.9}$$

$$\Xi_-^- = (\tfrac{2}{3})^{1/2}\tilde{x} + \tilde{y}/\sqrt{6} + \tilde{z}\sqrt{6} \ , \tag{5.10}$$

with remaining amplitudes related to these by the Lee-Sugawara relation (2.6) and the $\Delta I = \frac{1}{2}$ relations (2.5).

Indeed, both (2.6) and (2.5) are marginally adequate for *P*-wave amplitudes. However, Eqs. (5.8)–(5.10) themselves are not linearly independent, and one finds them related to one another by

$$B(\Lambda_-^0) = \frac{5}{2\sqrt{6}} B(\Sigma_+^+) + 3B(\Xi_-^-) \ , \tag{5.11}$$

TABLE VI. Contributions to parity-conserving nonleptonic-decay amplitudes B. For Ω^- decays the quantities B, defined in the text, are listed.

Decay \ Graph of figure: / Amplitude:	7(a) $\tilde{x}_{1a}$	7(b) + 7(c) $\tilde{x}_{1b}$	8(a) $\tilde{x}_2$	9 $\tilde{y}_1$	8(b) $\tilde{y}_2$	10(c) $\tilde{z}_{1a}$	10(a) + 10(b) $\tilde{z}_{1b}$	8(c) $\tilde{z}_2$
Λ^0_-	$-\left(\frac{3}{2}\right)^{1/2}$	$-\left(\frac{2}{3}\right)^{1/2}$	$-\left(\frac{3}{2}\right)^{1/2}$	$-\left(\frac{3}{2}\right)^{1/2}$	$-\frac{2}{3}$	$-2\left(\frac{2}{3}\right)^{1/2}$	$2\left(\frac{2}{3}\right)^{1/2}$	$-2\left(\frac{2}{3}\right)^{1/2}$
Λ^0_0	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$	$\frac{2}{\sqrt{3}}$
Σ^+_+	0	-2	-3	0	$\frac{5}{3}$	$\frac{5}{3}$	$-\frac{7}{3}$	$-\frac{13}{3}$
Σ^+_0	$\frac{1}{\sqrt{2}}$	$\sqrt{2}$	$\frac{3}{\sqrt{2}}$	$-\frac{1}{3\sqrt{2}}$	$-\frac{2\sqrt{2}}{3}$	$\frac{\sqrt{2}}{3}$	$\sqrt{2}$	$\frac{7\sqrt{2}}{3}$
Σ^-_-	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{7}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$
Ξ^-_-	$-\left(\frac{2}{3}\right)^{1/2}$	0	0	$\frac{1}{\sqrt{6}}$	$\frac{1}{\sqrt{6}}$	$-\left(\frac{3}{2}\right)^{1/2}$	$\left(\frac{3}{2}\right)^{1/2}$	$\frac{1}{\sqrt{6}}$
Ξ^0_0	$-\left(\frac{1}{3}\right)^{1/2}$	0	0	$\frac{1}{2\sqrt{3}}$	$\frac{1}{2\sqrt{3}}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2\sqrt{3}}$
Ω^-_-	0	0	0	$-2\left(\frac{2}{3}\right)^{1/2}$	0	0	$-4\left(\frac{2}{3}\right)^{1/2}$	0
Ω^-_0	0	0	0	$\frac{2}{\sqrt{3}}$	0	0	$\frac{4}{\sqrt{3}}$	0
Ω^-_K	-2	0	0	0	2	-6	2	2

which is not well satisfied. [It gives ≈ -3.2 for $B(\Xi^-_-)$ instead of the experimental -7.5.]

If one relates P-wave amplitudes to S-wave ones by current algebra and retains only the lowest baryon poles, an even worse prediction is obtained: $B(\Sigma^+_+)=0$. Discussions of this behavior are given in the second of Ref. 1 and in Ref. 5. Clearly the assumption that P-wave amplitudes are dominated by the lowest baryon poles does not agree with experiment.

A somewhat less drastic approximation to pole terms is to ascribe different weights to graphs in which the weak Hamiltonian acts before or after meson emission. Specifically, we take

$$\tilde{x}_{1a}\equiv -x_A,\quad \tilde{x}_{1b}\equiv x_B,\quad \tilde{x}_2\equiv x_B-x_A\ , \tag{5.12}$$

$$\tilde{y}_1\equiv y_B,\quad \tilde{y}_2\equiv y_B-y_A\ , \tag{5.13}$$

$$\tilde{z}_{1a}\equiv -z_A,\quad \tilde{z}_{1b}\equiv z_B,\quad \tilde{z}_2\equiv z_B-z_A\ . \tag{5.14}$$

The previous limit corresponded to $x_A=x_B=\tilde{x}$, etc. The models (5.12)–(5.14) allow for some contribution of higher-mass states, which will not follow the simple pattern leading to (5.5)–(5.7). The limit in which a high-mass continuum dominates corresponds to $x_A=-x_B$, etc. The A amplitudes correspond to Fig. 6(a), in which H_W acts "after" meson emission, while the B amplitudes correspond to Fig. 6(b), where H_W acts "before" meson emission. Note that the present decomposition is equivalent to that in terms of $\tilde{y}_1$ and $\tilde{y}_2$, but is more restrictive than that of Table VI for the $\tilde{x}$ and $\tilde{z}$ amplitudes. The amplitude $\tilde{y}_1$ in fact should contain a kaon pole contribution in addition to a baryon pole one, but the two will not be separable from one another on the basis of phenomenology alone.

Applying the assumptions (5.12)–(5.14) to Table VI, we find

$$B(\Lambda^0_-)=\sqrt{6}(x_A-y_A)+2(\tfrac{2}{3})^{1/2}(y_A+2z_A) -\frac{5}{\sqrt{6}}(x_B-y_B)\ , \tag{5.15}$$

$$B(\Sigma^+_+)=3(x_A-y_A)-5(x_B-y_B) +\tfrac{4}{3}(y_A+2z_A)-\tfrac{10}{3}(y_B+2z_B)\ , \tag{5.16}$$

$$B(\Sigma^-_-)=-(x_A-y_A)-\tfrac{4}{3}(y_A+2z_A)\ , \tag{5.17}$$

$$B(\Xi^-_-)=(\tfrac{2}{3})^{1/2}(x_A-y_A)+\frac{1}{\sqrt{6}}(y_A+2z_A) +(\tfrac{2}{3})^{1/2}(y_B+2z_B)\ , \tag{5.18}$$

with remaining amplitudes given by the $\Delta I=\frac{1}{2}$ rule. These four independent amplitudes (the ones with the smallest experimental errors) determine the independent combinations

$$x_A-y_A=-32.7\ , \tag{5.19}$$

$$x_B-y_B=-24.1\ , \tag{5.20}$$

$$y_A+2z_A=25.0\ , \tag{5.21}$$

$$y_B+2z_B=11.0\ . \tag{5.22}$$

Comparing Eq. (5.19) with Eqs. (5.20) and (5.21) with (5.22), we see that A amplitudes are consistently larger than B amplitudes. The energy denominator for an A amplitude [described by Fig. 6(a)] is $E_{\text{final}}-E_{\text{intermediate}}$, which is always negative. The energy denominator for a B amplitude [Fig. 6(b)] is $E_{\text{initial}}-E_{\text{intermediate}}$, which is positive when one takes the lowest intermediate state (belonging to the same octet, or decimet for the Ω) but changes sign for higher intermediate states. (Note that our method does not require detailed specification of the quark configuration of these intermediate states.) Thus, the continuum can be expected to *add* to the contribution of the lowest poles of the type in Fig. 6(a), but *cancel* that of the lowest poles in Fig. 6(b). Although our description of the P waves contains no predictive power (recall we have restricted our attention to amplitudes satisfying the $\Delta I=\frac{1}{2}$ rule), the inadequacy of the lowest-pole approximation can be understood and its magnitude gauged. In fact, x_A-y_A and x_B-y_B differ by only $\pm15\%$ from their average, while y_A+2z_A and y_B+2z_B differ by $\pm40\%$ from their average. Continuum effects thus appear more important for these latter combinations.

Like many other authors (see, e.g., Ref. 3), we have no natural explanation of why $B(\Sigma_-^-)\approx 0$. The Lee-Sugawara relation does not follow automatically in this approach.

The prediction for $\Omega^-\to\Xi\pi$ decays involves just the combination y_B+2z_B:

$$\tilde{B}(\Omega^-\to\Xi^-\pi^0)\equiv\tilde{B}(\Omega_-^-)$$
$$=-2(\tfrac{2}{3})^{1/2}(y_B+2z_B)\ , \tag{5.23}$$

where $\tilde{B}$ is related to B, the amplitude in Eqs. (2.3) and (2.4), by

$$\tilde{B}=\left[\frac{E_\Xi+m_\Xi}{m_\Omega}\right]\left[\frac{2}{3}\right]^{1/2}B\ . \tag{5.24}$$

The amplitude $\tilde{B}$ is what emerges most directly from a quark-model calculation; it is most closely related to the corresponding octet-baryon decay helicity amplitude.

We find the predictions

$$B(\Omega_-^-)=-13.9\ \ (\text{expt. } 10.0\pm0.3)\ , \tag{5.25}$$

$$B(\Omega_0^0)=-9.8\ \ (\text{expt. } 6.0\pm0.3)\ . \tag{5.26}$$

These are too large, as noted previously.[3] However, in contrast with the results of Ref. 3, we do not find any terms in the $\Omega\to\Xi\pi$ decay amplitudes that have not been encountered previously, and the deviations from experiment are not as large as those which would have been found in Ref. 3 in the absence of such terms.

The amplitudes for $\Omega^-\to\Lambda K^-$ involve new combinations:

$$\tilde{B}(\Omega^-\to\Lambda K^-)=2(x_A-y_A)+2(y_B+2z_B)+4z_A\ . \tag{5.27}$$

Given the value of B in Table I, we find

$$\tilde{B}(\Omega^-\to\Lambda K^-)=\frac{E_\Lambda+m_\Lambda}{m_\Omega}\left[\frac{2}{3}\right]^{1/2}B(\Omega^-\to\Lambda K^-)$$
$$=\pm3.6\pm0.6\ . \tag{5.28}$$

For the $-$ sign in (5.28), we find

$$z_A=2.45\to y_A=20.1,\ \ x_A=-12.6\ , \tag{5.29}$$

while for the $+$ sign, we find

$$z_A=19.3\to y_A=-13.5,\ \ x_A=46.2\ . \tag{5.30}$$

It will be interesting to compare these numbers with the predictions of a more microscopic theory (combining the present results with S-wave nonleptonic hyperon decays and kaon decays), but that is beyond the scope of the present admittedly descriptive work. One could also hope to apply these methods to two-body decays of charmed baryons, such as $\Lambda_c^+\to p\bar{K}^0$, $\Sigma^+\pi^0$, $\Sigma^+\eta$, $\Sigma^+\eta'$, $\Sigma^0\pi^+$, $\Lambda\pi^+$, Ξ^0K^+, and final states involving decimet baryons.

VI. RELATIONS AMONG AMPLITUDES

In most treatments, the P-wave amplitudes have been expressed in terms of S waves, with unsatisfactory results.[5,7,8] Our P-wave contributions are expressed as free parameters. At least the lowest-pole terms should be related to the S-wave decays. However, we have not made a clean separation between those pole terms and the higher ones. To some extent this problem has been addressed in the second of Ref. 13.

A further way in which the connection between S waves and P waves can be broken has been discussed by Gronau.[10] A large K^* pole contribution to the A amplitudes will not be related to corresponding contributions to B amplitude poles. [Gronau's specific model, utilizing *only* pole contributions to P waves, appears to us incapable of fitting the data because we disagree with the sign of his Ξ^0 pole contribution to $B(\Xi_-^-)$.]

Note added. One point still not clear to us is the different result we obtain, in comparison with Ref. 3, for the P-wave Lee-Sugawara relation (2.6). The $SU(6)_W$ approach of Ref. 3 leads to a modified relation of the form

$$\frac{B(\Lambda_-^0)}{m_\Lambda+m_n}+\frac{\sqrt{3}B(\Sigma_0^+)}{m_\Sigma+m_p}=\frac{2B(\Xi_-^-)}{m_{\Xi^0}+m_\Lambda}\ , \tag{6.1}$$

which holds to an accuracy of 17% with the values quoted in Table I. We do not obtain the relation (2.6) or any modifications of it in the most general case [Eqs. (5.15)–(5.18)]. If (2.6) were to hold, the four amplitudes on the left-hand side of Eqs. (5.19)–(5.22) would be related by

$$5[(x_A-y_A)-(x_B-y_B)]=6[(y_A+2z_A)-(y_B+2z_B)]\ . \tag{6.2}$$

In fact, using the values in (5.19)–(5.22), we find the right-hand side of (6.2) is nearly twice as large as the left-hand side, though this discrepancy does arise from a cancellation of large contributions. Since our approach has much in common with one based on $SU(6)_W$, there may be a simple assumption that would lead to the P-wave Lee-Sugawara relation, but we do not see it at present.

VII. SUMMARY

We have searched for and found a set of long-distance operators that describes nonleptonic hyperon decays satisfactorily (except for $\Omega \rightarrow \Xi\pi$) from a constituent-quark standpoint. This forms a first step toward a more general program of understanding nonleptonic weak decays. We outline several questions remaining for future work, some of which in fact have already been addressed in other contexts.

(1) Are there relations among the free parameters needed for our descriptions of *S*- and *P*-wave decays? Can these parameters be related to other quantities measurable, for example, in nonleptonic kaon decays or even to the fundamental structure of the weak interaction itself? The approach to this problem closest in spirit to the present one was taken in Ref. 10, and a reexamination of those results in the present context would probably be enlightening.

(2) What are the contributions from "spectator" diagrams and other processes violating the $\Delta I = \frac{1}{2}$ rule?

(3) What does the present scheme predict for processes of current experimental interest, such as K^0-$\bar{K}^0$ mixing, matrix elements of the *CP*-violating weak Hamiltonian, hyperon radiative decays,[27,28] heavy-quark decays, and proton decay?

We have found that a mixture of contributions is necessary; present data require only certain linear combinations of these contributions to have nonvanishing values. It is likely that much more progress in sorting out these contributions is possible if one addresses question (1)—(3) above. Our purpose in the present article was to present the descriptive results as a stepping stone to these problems.

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