PHYSICAL REVIEW D VOLUME 33, NUMBER 5 1 MARCH 1986

# Induced second-class form factors in $\Sigma^-$ $\beta$ decay

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(Received 16 September 1985)

A study of induced second-class currents emerging as a consequence of broken SU(3) flavor symmetry in $\Sigma^-$ $\beta$ decay is presented. Working within the framework of the MIT bag model, all form factors describing the $\Sigma^- \rightarrow n$ weak transition are calculated, including QCD vertex corrections to order $\alpha_s$. It is found that the weak-electric dipole form factor $g_2$ is suppressed by confinement effects to be approximately $-0.1$. This is shown to have a negligible effect on the electron asymmetry $\alpha_e$, and hence strengthens the case that recent data for $\alpha_e$ in $\Sigma^-$ $\beta$ decay are consistent with Cabibbo-model predictions.

## I. INTRODUCTION

Recently the Fermilab E715 collaboration has reported results[1] of a high-statistics experiment measuring electron asymmetry $\alpha_e$ in the $\beta$ decay of polarized $\Sigma^-$ hyperons observed in coincidence with a neutron in the final state. Based on an initial sample of 25 000 events, they have determined

$$\alpha_e = -0.53 \pm 0.14 \ , \quad (1)$$

corresponding to a ratio of axial-vector to vector form factors of

$$(g_1/f_1)_{\text{expt}} = -0.29 \pm 0.07 \ . \quad (2)$$

The latter is in excellent agreement with phenomenological fits[2,3] to the semileptonic decays of the spin-$\frac{1}{2}$ baryon octet based on the Cabibbo model,[4] which predicts for the transition $\Sigma^- \rightarrow n$,

$$(g_1/f_1)_{\text{Cab}} = -0.28 \pm 0.02 \ . \quad (3)$$

To extract a value for $g_1/f_1$ from experimental quantities, however, data analyses must rely on untested assumptions regarding the value of other form factors which appear in baryon weak-current matrix elements. In particular, the second-class form factors,[5] $f_3$ and $g_2$, whose definitions we give below, are usually assumed to vanish by virtue of the generalized $G$-parity invariance of SU(3) flavor symmetry. However, unequal quark masses break this invariance and strong-interaction effects are expected to generate nonzero values for $f_3$ and $g_2$ to the extent SU(3) is broken.

Theoretical estimates of induced second-class currents in flavor-changing transitions were orignally provided for $\Delta S = 0$ transitions by Halprin, Lee, and Sorba[6] in a study of neutron $\beta$ decay. Employing the quark model with unequal masses for $u$ and $d$ quarks, these authors showed that gluon vertex corrections induce large second-class currents, of order $\Delta m_q/m_q$, when quarks are treated as free particles. (Here $m_q$ is the current-quark mass.) However, they also argued that when confinement effects are taken into account, this effect is suppressed to order $\Delta m_q/\omega_q$, where $\omega_q$ is the constituent mass of a quark confined inside a baryon. This ratio is very small for $\Delta S = 0$ decays ($\sim 0.01-0.02$) but much larger for $\Delta S = 1$ transitions since $m_s \sim \omega_s$.

It was this latter possibility that prompted us to evaluate second-class currents for the strangeness-changing decay $\Sigma^- \rightarrow ne\bar{\nu}_e$, to determine whether they appreciably contribute to the electron asymmetry $\alpha_e$ and thus affect the experimental determination of $g_1/f_1$. In this work, we calculate all form factors of the weak decay $\Sigma^- \rightarrow n$ in the framework of the MIT bag model,[7] including gluon vertex corrections to lowest order in $\alpha_s$. Our results are close to phenomenological fits based on the Cabibbo model. In particular, we find the second-class form factor $g_2$ to be relatively small, even for this strangeness-changing transition, and conclude that its effect in the phenomenological analysis of $\Sigma^-$ $\beta$ decay is negligible compared to other theoretical and experimental uncertainties.

Our discussion begins in Sec. II with a review of the spin-$\frac{1}{2}$ baryon weak-current matrix element, thereby defining six form factors, two of which are second class. Taking the nonrelativistic limit, linear functions of the form factors are produced which have a direct correspondence to moments of vector- and axial-vector-current matrix elements in coordinate space. Our approach illustrates the necessity of evaluating all form factors simultaneously when working in a specific Lorentz frame and, in particular, reveals commonly neglected contributions to second-class form factors. Up to this point the formalism we develop is quite general and independent of the specific form of quark- and gluon-field operators. In Sec. III we specialize to the MIT bag model and present numerical results for the form factors of $\Sigma^-$ $\beta$ decay. Included in this section are examples of the explicit construction of matrix elements, both for "bare" vertex diagrams and for lowest-order QCD vertex corrections. We also present an approach to renormalization in the bag model, which treats the net effects of quark confinement on form factors perturbatively and by which we shall be able to derive physically meaningful renormalized form factors. In Sec. IV the implications of our results are discussed, with particular attention paid to the extraction of $g_1/f_1$ from experiment. We relegate to the Appendix expressions for bag quark and gluon wave functions and propagators needed for the evaluation of matrix elements.

## II. FORM FACTORS—PRELIMINARIES

The matrix element of the charged weak current between spin-$\frac{1}{2}$ baryon states may be expressed in its most general form consistent with Lorentz covariance and time-reversal ($T$) invariance as

$$\langle B_f | J_\lambda(q) | B_i \rangle = \bar{u}_f \left[ \gamma_\lambda f_1(q^2) - \frac{i\sigma_{\lambda\alpha} q^\alpha f_2(q^2)}{m_{fi}} + \frac{q_\lambda f_3(q^2)}{m_{fi}} - \left[ \gamma_\lambda g_1(q^2) - \frac{i\sigma_{\lambda\alpha} q^\alpha g_2(q^2)}{m_{fi}} + \frac{q_\lambda g_3(q^2)}{m_{fi}} \right] \gamma_5 \right] u_i \,, \tag{4}$$

where $|B_f\rangle$, $|B_i\rangle$, $m_f$, and $m_i$ are the final and initial baryon states and masses, respectively. In Eq. (4), $m_{fi} \equiv m_f + m_i$ while $q = p_i - p_f$ is the four-momentum transfer, where $p_i$ and $p_f$ are the initial- and final-state momenta. $T$ invariance requires that all six form factors, $f_i$ and $g_i$, $i=1,2,3$, are real. Our metric is $g_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ and we shall follow the conventions of Bjorken and Drell[8] for Dirac matrices and spinors.

The form factors[9] $f_1$ and $g_1$ are the familiar vector and axial-vector form factors, $f_2$ and $g_2$ are the weak-magnetic and electric dipole form factors, while $f_3$ and $g_3$ are the induced-scalar and pseudoscalar form factors. If SU(3) flavor symmetry were unbroken, $G$ parity, suitably generalized to SU(2) $V$-spin multiplets, would require that both $f_3$ and $g_2$ be zero;[10] these are the so-called second-class form factors[5] which are usually neglected in analyses of baryon decay. However, since SU(3) is broken, $f_3$ and $g_2$ are expected to be nonzero and, particularly for $\Delta S = 1$ transitions, could be important.

In evaluating the form factors, we first note that the timelike momentum transfer in semileptonic decays is typically small compared to the mass of the participating baryons ($\Delta m / m_{fi} \sim 12\%$ for $\Sigma^- \to n$). Thus it is reasonable to take the nonrelativistic limit of Eq. (4) and to neglect terms of order $(\mathbf{q}/m_i)^2$ or $(\mathbf{q}/m_f)^2$, where $\mathbf{q}$ is the three-momentum transfer. Implementing this limit in the Lorentz frame where $\mathbf{p}_i = -\mathbf{p}_f = \mathbf{q}/2$, we make the replacements,

$$q_0 \to \Delta m \equiv m_i - m_f \,, \quad \bar{u}_f \to \chi_f^\dagger \left[ 1, \frac{\boldsymbol{\sigma}\cdot\mathbf{q}}{4m_f} \right] , \quad u_i \to \begin{bmatrix} 1 \\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{q}}{4m_i} \end{bmatrix} \chi_i \tag{5}$$

($\chi_{i,f}$ are two-component Pauli spinors) in Eq. (4), and reduce the matrix element to two-component form:[11]

$$\langle B_f | J_\lambda(q) | B_i \rangle \Big|_{q_0=\Delta m}^{\mathbf{p}_i=-\mathbf{p}_f=\mathbf{q}/2} \equiv \langle B_f | \mathcal{J}_\lambda(\mathbf{q}) | B_i \rangle \equiv V_\lambda(\mathbf{q}) - A_\lambda(\mathbf{q}) \,, \tag{6}$$

where

$$V^0(\mathbf{q}) \equiv \chi_f^\dagger [\mathscr{V}_0(q^2)] \chi_i \,, \tag{7a}$$

$$V^i(\mathbf{q}) \equiv \chi_f^\dagger [q^i \mathscr{V}_V(q^2) + i\epsilon^{ijk} q^j \sigma^k \mathscr{V}_M(q^2)] \chi_i \,, \tag{7b}$$

$$A^0(\mathbf{q}) \equiv \chi_f^\dagger [\mathbf{q}\cdot\boldsymbol{\sigma} \mathscr{A}_0(q^2)] \chi_i \,, \tag{7c}$$

$$A^i(\mathbf{q}) \equiv \chi_f^\dagger [\sigma^i \mathscr{A}_S(q^2) + (q^i q^j - \tfrac{1}{3} q^2 \delta^{ij}) \sigma^j \mathscr{A}_T(q^2)] \chi_i \,. \tag{7d}$$

[In Eqs. (7) and henceforth, $q$ denotes $|\mathbf{q}|$.] The coefficients, $\mathscr{V}_0$, $\mathscr{V}_V$, $\mathscr{V}_M$, $\mathscr{A}_0$, $\mathscr{A}_S$, and $\mathscr{A}_T$ are even-parity scalar functions of $q^2$ given by the following linear combinations of form factors:[12]

$$\mathscr{V}_0(q^2) = f_1(q^2) + \frac{\Delta m}{m_{fi}} f_3(q^2) + O(q^2) \,, \tag{8a}$$

$$\mathscr{V}_V(q^2) = -\frac{\Delta m}{4m_i m_f}[f_1(q^2) + f_2(q^2)] + \frac{f_3(q^2)}{m_{fi}} + O(q^2) \,, \tag{8b}$$

$$\mathscr{V}_M(q^2) = \frac{m_{fi}}{4m_i m_f}[f_1(q^2) + f_2(q^2)] + O(q^2) \,, \tag{8c}$$

$$\mathscr{A}_0(q^2) = \frac{\Delta m}{4m_i m_f}[g_3(q^2) - g_1(q^2)] - \frac{g_2(q^2)}{m_{fi}} + O(q^2) \,, \tag{8d}$$

$$\mathscr{A}_S(q^2) = g_1(q^2) - \frac{\Delta m}{m_{fi}} g_2(q^2) + O(q^2) \,, \tag{8e}$$

$$\mathscr{A}_T(q^2) = \frac{1}{4m_i m_f}\left[ g_3(q^2) - \frac{1}{2}\left[ g_1(q^2) - \frac{\Delta m}{m_{fi}} g_2(q^2) \right] \right] + O(q^2) \,, \tag{8f}$$

where all dependence of the form factors $f_i$ and $g_i$ and coefficients $\mathscr{V}_i$ and $\mathscr{A}_i$ on $m_i$ and $m_f$ is suppressed. Given a model in which the matrix elements $V_\mu(\mathbf{q})$ and $A_\mu(\mathbf{q})$ may be evaluated, Eqs. (7) determine the coefficients $\mathscr{V}_i$ and $\mathscr{A}_i$ by suitable projection.

Alternatively, the two-component form (7) of vector and axial-vector currents may be derived by noting that in the Lorentz frame we have selected, $q^i$ and $\sigma^i$ are the only vectors at our disposal; Eqs. (7) thus exhaust all operators constructed from $q^i$ and $\sigma^i$ that can specify $V_\lambda$ and $A_\lambda$ consistent with the transformation properties of these currents under spatial rotations and parity. The coefficients $\mathscr{V}_i$ and $\mathscr{A}_i$ may also be classified according to $G$ parity, which in the two-component form transforms an arbitrary matrix element $\chi_f^\dagger M(\mathbf{q}) \chi_i$ according to

$$\chi_f^\dagger M(\mathbf{q}) \chi_i \underset{G}{\to} \chi_f^\dagger \sigma_2 M^T(-\mathbf{q}) \sigma_2 \chi_i \,. \tag{9}$$

Second-class (axial-) vector currents transform odd (even) under $G$ parity. It is easy to verify that $\mathscr{V}_V(q^2)$ and

$\mathscr{A}_0(q^2)$ are second-class coefficients, in agreement with the identifications (8b) and (8d), respectively, and like $f_3$ and $g_2$ must vanish in the SU(3)-symmetry limit, $\Delta m \to 0$.

By working in a specific Lorentz frame, we see that it is not the Lorentz-invariant form factors, $f_i$ and $g_i$, $i=1,2,3$, but the rotational invariants $\mathscr{V}_i$, $i=0,V,M$, and $\mathscr{A}_i$, $i=0,S,T$, which naturally determine the structure of the weak current. Thus in models such as the MIT bag in which a particular frame is implicitly selected, it is simpler to first compute $\mathscr{V}_i$ and $\mathscr{A}_i$. Equations (8), evaluated at $q^2=0$, then determine six linear relations which may be inverted to extract the desired form factors.

Besides calculational simplicity, Eqs. (8) also have important consequences for the evaluation of second-class form factors. Since these are already of order $\Delta m/m_{fi}$, it is necessary to include all contributions of this order from the first-class form factors. To illustrate this point we consider $g_2$ in detail. From Eq. (8d), we have

$$\frac{g_2}{m_{fi}} = \frac{\Delta m}{4m_i m_f}(g_3 - g_1) - \mathscr{A}_0 , \tag{10}$$

showing that $g_2$ depends not only on $\mathscr{A}_0$, but also receives contributions of order $\Delta m/m_{fi}$ from $g_1$ and $g_3$ (or, equivalently, $\mathscr{A}_S$ and $\mathscr{A}_T$). Since $\mathscr{A}_0$ is second-class and proportional to $\Delta m$, all three terms of the right-hand side (RHS) of Eq. (10) are of comparable magnitude, and consequently all these terms must be retained in determining $g_2$. This corrects the practice often found in the literature of prematurely neglecting $f_3$ and $g_3$ in the *evaluation of form factors.* Of course, these induced form factors may be discarded afterwards in the calculation of $\beta$-decay *transition amplitudes* as they appear there suppressed by the factor, $(m_e/m_{fi})^2 \ll 1$.

The currents $V_\mu(\mathbf{q})$ and $A_\mu(\mathbf{q})$ are obtained by transforming the relevant weak-current matrix element to momentum space. For example, in the absence of gluon vertex corrections (Fig. 1), we simply have

$$\begin{aligned} 2\pi\delta(E_i - E_f - \Delta m)\langle B_f \,|\, \mathscr{J}_\lambda(\mathbf{q}) \,|\, B_i\rangle &= \sum_c \int dy\, e^{iq_0 y_0 - i\mathbf{q}\cdot\mathbf{y}} \\ &\quad \times \langle B_f \,|\, :\bar{\psi}^c(y)\gamma_\lambda(1-\gamma_5)\psi^c(y): \,|\, B_i\rangle , \end{aligned} \tag{11}$$

where $E_i$ and $E_f$ are the initial- and final-state baryon energies, respectively, and $\psi^c(y)$ is the canonical quark field operator,

$$\psi^c(y_0,\mathbf{y}) = \sum_n [a_n^c q_n(\mathbf{y})e^{-i\omega_n y_0} + b_n^{c\dagger}\tilde{q}_n(\mathbf{y})e^{i\omega_n y_0}] . \tag{12}$$

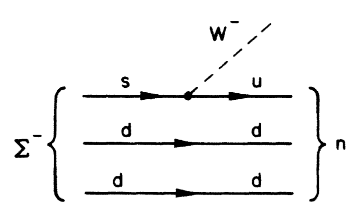


FIG. 1. $\Sigma^-$ $\beta$ decay.

Here $q_n(\mathbf{y})$ $[\tilde{q}_n(\mathbf{y})]$ are static quark [antiquark] wave functions of energy $\omega_n$, $c$ is a color index and $n$ denotes all other quantum numbers (spin, flavor, etc.). The operators $a_n^c$ and $a_n^{c\dagger}$ obey the usual relations,

$$\{a_n^c, a_{n'}^{c'\dagger}\} = \delta^{cc'}\delta_{nn'} , \tag{13}$$

all other anticommutators being zero, with similar relations for the antiquark operators $b_n^c$ and $b_n^{c\dagger}$.

For the baryon states we shall use the standard SU(6) wave functions.[13] In evaluating the weak current describing the $\Sigma^- \to n$ transition, it suffices to take both initial and final states with spin up: $|B_i\rangle = |\Sigma^-\uparrow\rangle$ and $|B_f\rangle = |n\uparrow\rangle$. In terms of creation and annihilation operators, these SU(6) wave functions are

$$|\Sigma^-\uparrow\rangle = \frac{1}{\sqrt{18}}[-2a_{d\uparrow}^{1\dagger}a_{s\downarrow}^{2\dagger}a_{d\uparrow}^{3\dagger} + a_{s\uparrow}^{1\dagger}a_{d\downarrow}^{2\dagger}a_{d\uparrow}^{3\dagger} + a_{d\uparrow}^{1\dagger}a_{d\downarrow}^{2\dagger}a_{s\uparrow}^{3\dagger} + (2\leftrightarrow 3) + (1\leftrightarrow 3)]\,|0\rangle , \tag{14}$$

and

$$|n\uparrow\rangle = -|\Sigma^-\uparrow(s\to u)\rangle . \tag{15}$$

Here we have suppressed reference to all other quantum numbers besides color, spin, and flavor, taking quarks in each baryon to be in their lowest-energy eigenstate.

The use of SU(6) wave functions, Eqs. (14) and (15), assumes that quarks of the initial- and final-state baryons are in their ground states. Some mass splitting is taken into account by using quark wave functions in Eq. (12) which depend upon the appropriate quark masses. Excited-state amplitudes could be included via configuration mixing[14] and gluon exchange, although we do not do so in this paper.

With Eqs. (12)–(15), we have all the tools necessary to evaluate Eq. (11) for $\Sigma^-$ $\beta$ decay. Expanding the exponential factor $\exp(-i\mathbf{q}\cdot\mathbf{y})$ of Eq. (11) in powers of $\mathbf{q}\cdot\mathbf{y}$, it is then straightforward to identify the functions $\mathscr{V}_i$ and $\mathscr{A}_i$ of the weak current $\langle B_f \,|\, \mathscr{J}(\mathbf{q}) \,|\, B_i\rangle$ as per Eqs. (6) and (7). These can be consistently defined in terms of various integrals over quark wave functions, including the appropriate SU(6) Clebsch-Gordan coefficients. Explicit examples of this construction will be presented in the next section.

## III. FORM-FACTOR CALCULATIONS

In the previous section, we established a general framework for calculating the weak currents associated with spin-$\frac{1}{2}\to$spin-$\frac{1}{2}$ baryon transitions. We now specialize to the MIT bag model[7] and expand the quark field operator, Eq. (12), in terms of bag quark and antiquark wave functions whose explicit forms we give in the Appendix [cf. Eqs. (A5a) and (A7)]. Tables I and II list the quark and gluon modes and corresponding energy eigenvalues that we shall use in the expansion of bag quark and gluon propagators. We shall ignore the slight differences in bag radii among the octet baryons used in the fit of Ref. 7 and adopt the average value, $R = 5.0\ \text{GeV}^{-1}$.

Strictly speaking, we must evaluate $\mathscr{V}_i$ and $\mathscr{A}_i$ in the Lorentz frame in which the initial- and final-state baryons

TABLE I. Quark bag modes and energy eigenvalues $\omega_{\text{flavor}}$ (in MeV) included in the vertex calculation. Standard bag-model parameters are used, with $R=5$ GeV$^{-1}$, $m_u=5$ MeV, $m_s=280$ MeV, and $\alpha_s=0.55$. All eigenvalues correspond to the lowest mode ($n=0$) in each channel $(j,l)$, with the exception of $^*(\frac{1}{2},0)$ which is first-excited ($n=1$).

| $J$ | $l$ | $\omega_u$ | $\omega_s$ |
|---|---|---|---|
| $\frac{1}{2}$ | 0 | 411 | 571 |
| $\frac{3}{2}$ | 1 | 643 | 779 |
| $\frac{1}{2}$ | 1 | 763 | 838 |
| $\frac{5}{2}$ | 2 | 867 | 989 |
| $\frac{3}{2}$ | 2 | 1025 | 1081 |
| $^*\frac{1}{2}$ | 0 | 1080 | 1144 |
| $\frac{7}{2}$ | 3 | 1088 | 1199 |

are moving in opposite directions with momentum $|\mathbf{q}|/2$. This is, however, problematic in the MIT bag model, which only describes baryons in their rest frame; no entirely rigorous formalism exits for boosting bags to the nonzero momentum. We shall circumvent (but do not solve) this problem by approximating the values of $\mathscr{V}_i(0)$ and $\mathscr{A}_i(0)$ with their values obtained assuming both the initial- and final-state baryons are at rest.

### A. Bare form factors

To zeroth order in $\alpha_s$, $\Sigma^-$ $\beta$ decay is the process portrayed in Fig. 1. Evaluating Eq. (11) with the SU(6) wave functions given by Eqs. (14) and (15), we obtain

$$\langle n\uparrow|\mathscr{J}_\lambda^{(0)}(\mathbf{q})|\Sigma^-\uparrow\rangle = -\sum_{mm'}\left(\tfrac{1}{3}\delta_{m\uparrow}\delta_{m'\uparrow}+\tfrac{2}{3}\delta_{m\downarrow}\delta_{m'\downarrow}\right) \times\int_{\text{bag}} d^3y\, e^{-i\mathbf{q}\cdot\mathbf{y}}[\bar{q}^{(u)}_{jlmn}(\mathbf{y})\gamma_\lambda(1-\gamma_5) \times q^{(s)}_{j'l'm'n'}(\mathbf{y})]\ , \quad (16)$$

where $j=j'=\frac{1}{2}$, $l=l'=0$, and $n=n'=0$, which results from taking quarks in the initial- and final-state baryon wave functions to be in their lowest energy eigenstate. The sum is over spin states $m$ ($m'$) of the final (initial)

TABLE II. Gluon bag modes and energy eigenvalues $\nu_{JN}^{(T)}R$ included in the vertex calculation. For given $(T,J)$ all eigenvalues correspond to the lowest mode ($N=0$), with the exception of $^*1$, which is first-excited ($N=1$).

| $J$ | $\nu_J^{(\text{TE})}R$ | $\nu_J^{(\text{TM})}R$ |
|---|---|---|
| 1 | 2.744 | 4.493 |
| 2 | 3.870 | 5.764 |
| 3 | 4.973 | 6.988 |
| $^*1$ | 6.117 | 7.725 |

quark participating in the decay. Superscripts on the quark wave functions $q(\mathbf{y})$ denote its flavor.

Expanding (16) in powers of $q^i$, we may easily obtain explicit expressions for $\mathscr{V}_i^{(0)}$ and $\mathscr{A}_i^{(0)}$ which we now illustrate for the case of $\mathscr{A}_S^{(0)}$ and $\mathscr{A}_0^{(0)}$. Inserting quark wave functions [Eq. (A5)] for the $q^i$-independent portion of the axial-vector weak current $A^i$, we find

$$\chi_\uparrow^\dagger\sigma^i\chi_\uparrow\mathscr{A}_S^{(0)}(0) = -\sum_{mm'}\left(\tfrac{1}{3}\delta_{m\uparrow}\delta_{m'\uparrow}+\tfrac{2}{3}\delta_{m\downarrow}\delta_{m'\downarrow}\right)\chi_m^\dagger\sigma^i\chi_{m'} \times\int_0^R r^2dr\, N_uN_s\left(W_+^uW_+^sj_0^uj_0^s-\tfrac{1}{3}W_-^uW_-^sj_1^uj_1^s\right)\ , \quad (17)$$

where $R$ is the bag radius and $j_{0,1}^f=j_{0,1}(x^fr/R)$ are $f$-flavor-dependent spherical Bessel functions. We have also used a simplifying notation where

$$N_{u,s}=N^{u,s}_{j=\frac{1}{2},l=0}\ ,\quad x^{u,s}=x^{u,s}_{j=\frac{1}{2},l=0,n=0}\ ,$$

and

$$W_\pm^{u,s}=W_\pm^{u,s}(j=\tfrac{1}{2},l=0,n=0)$$

[cf. Appendix, Eqs. (A4)–(A5)]. Similar steps for the temporal component of the axial-vector current give $\mathscr{A}_0$:

$$\chi_\uparrow^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\chi_\uparrow\mathscr{A}_0^{(0)}(0) = -\sum_{mm'}\left(\tfrac{1}{3}\delta_{m\uparrow}\delta_{m'\uparrow}+\tfrac{2}{3}\delta_{m'\downarrow}\delta_{m\downarrow}\right)\chi_m^\dagger(\mathbf{q}\cdot\boldsymbol{\sigma})\chi_{m'} \times\tfrac{1}{3}\int_0^R r^2dr\left[\frac{r}{R}\right]N_uN_s\left(W_+^uW_-^sj_0^uj_1^s - W_-^uW_+^sj_1^uj_0^s\right)\ . \quad (18)$$

Note that this second-class coefficient vanishes for equal quark masses, $m_u=m_s$. With the values $\mathscr{V}_i^{(0)}$ and $\mathscr{A}_i^{(0)}$ in hand, we invert Eqs. (8a)–(8f) to determine the bare form factors, $f_i^{(0)}$ and $g_i^{(0)}$, $i=1,2,3$. The numerical results are presented in Table III.

As alluded to in Sec. II, we see that $f_3^{(0)}$ and $g_3^{(0)}$ are not negligible and contribute significantly to the second-class form factors [cf. Eq. (10)]. Also we observe that $g_2^{(0)}$ is small and negative; this contradicts the estimate of Ref. 6 that $g_2$ is of order $\Delta m_q/\omega_q$, which for $\Delta S=1$ transitions is near unity. This estimate of $g_2$ neglected contributions from $g_1$ and $g_3$, and the factor multiplying $\Delta m_q/\omega_q$ was found to be numerically quite small.

From Table III, we may also compute the ratio,

$$(g_2^{(0)}/g_1^{(0)})=-0.17\ . \quad (19)$$

This is somewhat smaller than and opposite in sign to the value obtained by Donoghue and Holstein,[15] who find $g_2/g_1=0.27$, employing the nonrelativistic quark model; the discrepancy with our value Eq. (19) is due to contributions from $g_3$, which are neglected in their computations. Had we set $g_3^{(0)}$ to zero in Eq. (8d) and used our values of $\mathscr{A}_S^{(0)}=0.237$ and $\mathscr{A}_0^{(0)}=-0.0053$ in Eqs. (8d) and (8e) to

TABLE III. Form factors for $\Sigma^- \to n$ evaluated at $q^2 \cong (\Delta m)^2 = (0.258\ \text{GeV}/c^2)^2$. Listed are the bare form factors, $(f_i^{(0)}, g_i^{(0)})$, order-$\alpha_s$ $(\equiv g_s^2/4\pi)$ QCD vertex corrections $(f_i^{(1)}, g_i^{(1)})$ and the renormalized form factors calculated using Eqs. (31) and (32).

| | $f_1$ | $f_2$ | $f_3$ | $g_1$ | $g_2$ | $g_3$ |
|---|---|---|---|---|---|---|
| Bare | $-1.015$ | 1.637 | 0.246 | 0.232 | $-0.040$ | $-0.558$ |
| Vertex correction | $-0.514$ | 0.269 | 0.024 | $-0.015$ | $-0.047$ | $-0.491$ |
| Renormalized form factors | $-0.924$ | 1.906 | 0.270 | 0.250 | $-0.087$ | $-1.049$ |

determine $g_1^{(0)}$ and $g_2^{(0)}$, we would have found $g_2^{(0)}/g_1^{(0)} = 0.21$. We conclude that the inclusion of $g_3$ (or, equivalently, $\mathscr{A}_T$) is essential for obtaining the correct sign and magnitude of $g_2/g_1$. This point has recently been emphasized by Kubodera, Kohyama, Oikawa, and Kim.[11]

### B. QCD corrections to form factors

The rigorous treatment of gluon exchanges between quarks confined to a bag is not a straightforward problem. There are ambiguities associated with the double counting of gluon degrees of freedom and quark self-energies. The former is ameliorated somewhat by the fact that, by confining gluons to a bag, gluonic energies are bounded from below by a minimal zero-point energy. This presumably minimizes double counting of soft-gluon effects which, according to traditional bag-model wisdom, are solely responsible for the formation of the bag. Effects due to soft gluons are then implicitly incorporated in the quark-bag wave functions.

The ambiguities introduced by using physical (that is, renormalized) quark masses may be removed by eliminating all quark self-energy diagrams. However, the common practice is to adopt the so-called minimal prescription,[7] which instructs us to neglect all quark self-energy graphs except those self-energy diagrams which, together with exchange graphs (Fig. 2), will provide for zero radial color flux at the surface of the bag. This eliminates well-known infrared divergences introduced by considering exchange graphs alone, which in the Coulomb gauge are associated with the $s$-wave component of the bag Coulomb propagator $D_{00}$.[16]

This prescription is often applied as an approximation in calculating decay diagrams, such as those shown in Figs. 2–4. In particular, in the Coulomb gauge, it treats the bubble (Fig. 3) and vertex (Fig. 4) diagrams as self-energy graphs, keeping only Coulomb gluon ($D_{00}$) contributions. This is the approach used, for instance, by Ushio and Konashi[17] in their calculation of $g_1/f_1$ of hyperon semileptonic decay. These authors find that the contributions to form factors due to Coulomb gluons in Figs. 2–4 tend to cancel, leaving as the dominant correction to $g_1/f_1$ the static transverse gluon-exchange graphs of Fig. 2.

However this treatment, in its neglect of transverse gluon-bubble graphs and vertex corrections, also removes the desired QCD corrections to the form factors. The point is that, for our choice of baryon states [Eqs. (14) and (15)] in which individual quarks have well-defined energies, the intermediate quark states of Figs. 2–4 may be *off-mass shell* since the $W$ boson removes energy from the decaying baryon. Thus, only on-mass-shell intermediate quark states should be discarded, which in Figs. 2 and 3 correspond to baryon-wave-function and quark-mass renormalization, respectively. (An even more comprehensive treatment would include configuration mixing[14] for the initial- and final-state baryons, including amplitudes for quarks to be in excited states.)

Here we consider only order-$\alpha_s$ vertex corrections to form factors due to time-dependent transverse gluon exchange (Fig. 4), thus extending the work of Halprin, Lee, and Sorba[6] to $\Delta S = 1$ transitions. A more complete calculation would include contributions from one-gluon-exchange and off-mass-shell bubble graphs (Figs. 2 and 3, respectively) as well as from graphs involving Coulomb gluons.

The order-$\alpha_s(\equiv g_s^2/4\pi)$ vertex graph of Fig. 4 corrects the baryon weak current as given by

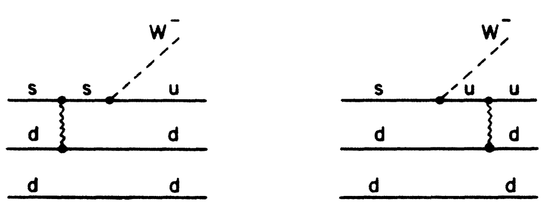


FIG. 2. Exchange diagrams contributing to $\Sigma^-$ $\beta$ decay.

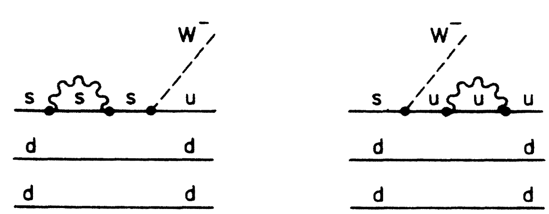


FIG. 3. Bubble diagrams contributing to $\Sigma^-$ $\beta$ decay.

$$2\pi\delta(E_\Sigma - E_n - \Delta m)\langle n\uparrow | \mathscr{J}_\lambda^{(1)}(\mathbf{q}) | \Sigma^-\uparrow\rangle$$
$$= \int d^4x\, d^4y\, d^4z\, e^{iq\cdot y} \sum_{cc'c''} \langle n\uparrow | :\bar\psi_u^c(z) i g_s \gamma_\alpha \lambda^a_{cc'} [iS_F^u(z,y)]\, \gamma_\lambda(1-\gamma_5)[iS_F^s(y,x)] i g_s \gamma_\beta \lambda^b_{c'c''} \psi_s^{c''}(x) : [iD^{\alpha\beta}_{ab}(z,x)] \,|\, \Sigma^-\uparrow\rangle\ , \tag{20}$$

where $\psi_f^c$ are $f$-flavored, $c$-colored quark-field operators (12) and the propagators $D^{\alpha\beta}_{ab}$ and $S_F^f$ are given in the Appendix [cf. Eqs. (A9), (A10), (A21), and (A22)]. The color matrices $\lambda^a$ are the familiar Gell-Mann SU(3) matrices. The extraction of $\mathscr{V}_i^{(1)}$ and $\mathscr{A}_i^{(1)}$ proceeds as in the QCD-uncorrected case; the only complications encountered here are additional factors due to (1) energy denominators of quark and gluon propagators, (2) quark wave function overlaps at the quark-quark-gluon vertices, and (3) a color trace, providing an overall factor of $\mathrm{Tr}(\lambda^a\lambda^a)/3 = \frac{16}{3}$. For example, the vertex correction to $\mathscr{A}_0$, namely, $\mathscr{A}_0^{(1)}$, is given by

$$\chi_\uparrow^\dagger(\sigma\cdot\mathbf{q})\chi_\uparrow \mathscr{A}_0^{(1)} = -\tfrac{16}{3} g_s{}^2 \sum_{JMTN} \sum_{jln} \sum_{j'l'n'} \sum_{\substack{mm'\\ m_1m_2}} \left(\tfrac{1}{3}\delta_{m_1\uparrow}\delta_{m_2\uparrow} + \tfrac{2}{3}\delta_{m_1\downarrow}\delta_{m_2\downarrow}\right)\left[2\nu_{JN}^{(T)}(\nu_{JN}^{(T)} - \omega^u_{j_1l_1n_1} + \omega^u_{jln})(\nu_{JN}^{(T)} - \omega^s_{j_2l_2n_2} + \omega^s_{j'l'n'})\right]^{-1}$$
$$\times J_u(j_1l_1m_1n_1, jlmn; JMTN)\,\mathbf{q}\cdot\mathbf{K}_{us}(jlmn, j'l'm'n')$$
$$\times J_s^\dagger(j'l'm'n', j_2l_2m_2n_2; JMTN)\ , \tag{21a}$$

where the matrices $J_f$ are quark-quark overlaps with the gluon wave functions $\mathbf{G}_{JMN}^{(T)}$ [Eq. (A17)],

$$J_f(jlmn, j'l'm'n'; JMTN) \equiv \int d^3x\, \bar q^{\,f}_{jlmn}(\mathbf{x})\gamma\cdot\mathbf{G}^{(T)}_{JMN}(\mathbf{x}) q^f_{j'l'm'n'}(\mathbf{x}) \tag{21b}$$

and

$$\mathbf{K}_{ff'}(jlmn, j'l'm'n') \equiv -i \int d^3x\, \mathbf{x}\bar q^{\,f}_{jlmn}(\mathbf{x})\gamma_0\gamma_5 q^{f'}_{j'l'm'n'}(\mathbf{x})\ , \tag{21c}$$

with external quark legs evaluated in the lowest-energy eigenmode ($j_1 = j_2 = \frac{1}{2}$, $l_1 = l_2 = 0$, and $n_1 = n_2 = 0$). $J_f^\dagger$ is related to $J_f$ via

$$J_f^\dagger(jlmn, j'l'm'n'; JMTN) = J_f^*(j'l'm'n'; jlmn; JMTN)\ . \tag{22}$$

Equations (21) specify contributions to $\mathscr{A}_0^{(1)}$ when the intermediate quarks are in positive-energy bag eigenstates. Similar expressions may be found for contributions from quark-antiquark, antiquark-quark, and antiquark-antiquark intermediate states.

Table III summarizes our results of vertex corrections to form factors, $f_i^{(1)}$ and $g_i^{(1)}$, $i=1,2,3$. These include contributions from all combinations of all intermediate quark/antiquark and gluon modes listed in Tables I and II, providing an effective energy cutoff of $\Lambda \cong 1$ GeV. This proved sufficient to attain convergence to within 10% of $f_i^{(1)}$ and $g_i^{(1)}$, $i=2,3$, which are not expected to diverge with $\Lambda$.

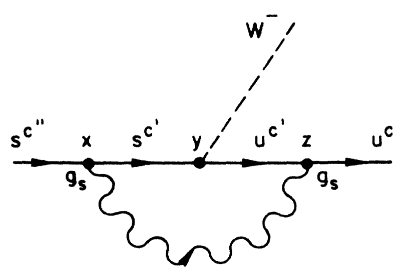


FIG. 4. Vertex graph contributing to $\Sigma^- \beta$ decay.

### C. Renormalization

To obtain physically meaningful results, the form factors must also be renormalized. As in quantum electrodynamics, vertex corrections introduce ultraviolet divergences which must be eliminated to produce a finite, well-defined theory. Here, to order $\alpha_s$, we expect contributions to $f_1$ and $g_1$ which depend logarithmically on the high-energy cutoff,[6] as has been demonstrated for the bag quark self-energy.[16] We did not observe this weak divergence in our numerical calculations, although this may be due to possible nondivergent dependences on $\Lambda$ (e.g., power-law dependences: $\Lambda^{-p}$, $p > 0$).

However, prescriptions of renormalization elucidated for perturbative quantum chromodynamics (PQCD) are not directly applicable to the MIT bag model, which distinguishes implicit soft gluons responsible for producing confinement from hard gluons explicitly incorporated in the model. While this separation provides for the phenomenological success of the MIT bag model in describing a number of hadronic properties (mass spectra, magnetic moments, axial couplings, etc.), it does lead to ambiguities in extracting physical results from loop diagrams involving hard gluons. Confinement of quarks in a bag produces distortions of quark wave functions and quark/gluon propagators, leading to corrections (even for the bare vertex) to form factors which are *implicitly* of order $\alpha_s$. A consistent renormalization of strong-interaction effects should take these implicit dependences on $\alpha_s$ into account.

Our approach shall be to mimic PQCD by assuming that all strong-interaction-induced contributions to the form factors may be treated perturbatively. These induced contributions to the bare from factors, naively of zeroth order in $\alpha_s$, are given by the deviations of $f_i^{(0)}$ and $g_i^{(0)}$ from their free-quark values. In the limit, strong interactions are turned off (or, in our calculation, as the bag radius is sent to infinity), $f_i^{(0)}$ and $g_i^{(0)}$, $i=2,3$, which are absent in the bare weak vertex, all vanish and $f_1^{(0)}$ and $g_1^{(0)}$ assume their SU(6) values: for $\Sigma^-\to n$, these are given by $-1$ and $\frac{1}{3}$, respectively. Indicating implicit strong-interaction corrections by $\Delta f_i^{(0)}$ and $\Delta g_i^{(0)}$, we have

$$f_1^{(0)}=-1+\Delta f_1^{(0)},\quad g_1^{(0)}=\tfrac{1}{3}+\Delta g_1^{(0)}\,,$$
$$f_2^{(0)}=\Delta f_2^{(0)},\quad g_2^{(0)}=\Delta g_2^{(0)}\,, \tag{23}$$
$$f_3^{(0)}=\Delta f_3^{(0)},\quad g_3^{(0)}=\Delta g_3^{(0)}\,.$$

Hence, to lowest order in $\alpha_s$, $\Delta f_i^{(0)}$ and $\Delta g_i^{(0)}$ are of the same order as $f_i^{(1)}$ and $g_i^{(1)}$.

The remainder of the renormalization procedure proceeds in the usual manner. We introduce two constants $Z_V$ and $Z_A$ which, respectively, renormalize the vector and axial-vector currents, viz.,

$$V_\mu^{\rm ren}=Z_VV_\mu=Z_V(V_\mu^{(0)}+V_\mu^{(1)}+\cdots)\,, \tag{24a}$$

$$A_\mu^{\rm ren}=Z_AA_\mu=Z_A(A_\mu^{(0)}+A_\mu^{(1)}+\cdots)\,, \tag{24b}$$

where, here and below, the ellipsis denote order-$\alpha_s^2$ contributions. A condition on $V_\mu^{\rm ren}$ determining $Z_V$ is obtained from the Cabibbo hypothesis[4] which states that, in the limit of perfect SU(3) symmetry, $V_\mu^{\rm ren}$ is a member of an octet of conserved vector currents which includes the isovector portion of the electromagnetic current. This relates the electric charge of the proton ( + 1) to the normalization of $V_\mu^{\rm ren}(\Sigma^-\to n)$ at zero momentum transfer, giving

$$V_\mu^{\rm ren}(\Sigma^-\to n)\,|_{q=0,m_s=m_u,m_{\Sigma^-}=m_n}$$
$$=-\bar u(n)\gamma_\mu u(\Sigma^-)\,. \tag{25}$$

At the same renormalization point, $q_\mu$ vanishes and so $A_\mu^{\rm ren}$ must be pure axial-vector, resulting in the condition,

$$A_\mu^{\rm ren}(\Sigma^-\to n)\,|_{q=0,m_s=m_u,m_{\Sigma^-}=m_n}$$
$$=C_A(\Sigma^-\to n)\bar u(n)\gamma_\mu\gamma_5u(\Sigma^-)\,, \tag{26}$$

where $C_A(\Sigma^-\to n)$ is a numerical constant depending on the values of $m_s$ $(=m_u)$ and $m_{\Sigma^-}$ $(=m_n)$ in the SU(3)-symmetry limit. If we realize this limit as $m_s=m_u=5$ MeV and $m_{\Sigma^-}=m_n=939$ MeV, then the Cabibbo theory together with SU(6) symmetry relates $C_A(\Sigma^-n)$ to the experimentally measured axial-vector form factor $C_A(n\to p)$ of neutron $\beta$ decay:[3]

$$C_A(\Sigma^-\to n)=\tfrac{1}{5}C_A(n\to p)=0.251\pm0.001\,. \tag{27}$$

Here again we shall isolate the strong-interaction corrections to $C_A$ as $\Delta C_A=C_A-\frac{1}{3}$.

Having secured two conditions on the renormalized currents, it is now a straightforward matter to use Eqs. (24) to determine $Z_V$ and $Z_A$, and hence the renormalized form factors. For this we need our results for the unrenormalized form factors, $f_i$ and $g_i$, $i=1,2,3$, as well as the values of $f_1$ and $g_1$ at the SU(3)-symmetric renormalization point,

TABLE IV. Form factors $\tilde f_1$ and $\tilde g_1$ for $\Sigma^-\to n$ at the SU(3)-symmetric renormalization point: $m_s=m_u=5$ MeV, $m_{\Sigma^-}=m_n=939$ MeV.

| | $\tilde f_1$ | $\tilde g_1$ |
|---|---|---|
| Bare | $-1.0$ | 0.218 |
| Vertex correction | $-0.606$ | $-0.001$ |

$$\tilde f_1=f_1\,|_{q^2=0,m_s=m_u,m_{\Sigma^-}=m_n}\,, \tag{28a}$$

$$\tilde g_1=g_1\,|_{q^2=0,m_s=m_u,m_{\Sigma^-}=m_n}\,. \tag{28b}$$

The latter are listed in Table IV.

Applying now Eqs. (24)–(26), we find

$$-1=Z_V(-1+\Delta\tilde f_1^{(0)}+\tilde f_1^{(1)}+\cdots)\,, \tag{29a}$$

$$\tfrac{1}{3}+\Delta C_A=Z_A(\tfrac{1}{3}+\Delta\tilde g_1^{(0)}+\tilde g_1^{(1)}+\cdots)\,, \tag{29b}$$

where $\Delta\tilde f_1^{(0)}$, etc., are defined analogously to Eq. (23). Equations (29) then determine $Z_V$ and $Z_A$ to first order in $\alpha_s$ and strong-interaction effects:

$$Z_V=1+\Delta\tilde f_1^{(0)}+\tilde f_1^{(1)}+\cdots\,, \tag{30a}$$

$$Z_A=1+3\Delta C_A-3\Delta\tilde g_1^{(0)}-3\tilde g_1^{(1)}+\cdots\,. \tag{30b}$$

Inserting into Eqs. (24), we see that to first order in $\alpha_s$ and confinement effects, the form factors $f_i$ and $g_i$, $i=2,3$ are unrenormalized:

$$f_i^{\rm ren}=f_i^{(0)}+f_i^{(1)}+\cdots\,, \tag{31a}$$

$$g_i^{\rm ren}=g_i^{(0)}+g_i^{(1)}+\cdots,\quad i=2,3\,. \tag{31b}$$

Only $f_1$ and $g_1$ experience nontrivial rescalings, as given by

$$f_1^{\rm ren}=Z_Vf_1$$
$$=-1+(\Delta f_1^{(0)}-\Delta\tilde f_1^{(0)})+(f_1^{(1)}-\tilde f_1^{(1)})+\cdots\,, \tag{32a}$$

$$g_1^{\rm ren}=Z_Ag_1$$
$$=\tfrac{1}{3}+\Delta C_A+(\Delta g_1^{(0)}-\Delta\tilde g_1^{(0)})+(g_1^{(1)}-\tilde g_1^{(1)})+\cdots\,. \tag{32b}$$

Numerical results for $f_i^{\rm ren}$ and $g_i^{\rm ren}$ are presented in Table III.

## IV. DISCUSSION

In Table III all listed form factors have been calculated at $q_\mu^2=(\Delta m)^2$. Cabibbo-theory-predicted form-factor values, however, are usually presented at $q_\mu^2$ equal to zero. To factor out $q_\mu^2$ dependence one usually adopts a dipole formula[3] which is a function of both $q_\mu^2$ and a mass parameter that must be fit to experiment for each form factor separately. Using our calculated values of $f_1$,

$f_2$, and $g_1$ (here and henceforth, renormalized form factors are understood) we can extract their values at $q_\mu{}^2=0$ by dividing out the dipole correction factor, viz.,

$$f_1(0)=f_1(\Delta m)\left[1-2\left[\frac{\Delta m}{M_v}\right]^2\right]=-0.80\ ,$$
$$f_2(0)=f_2(\Delta m)\left[1-2\left[\frac{\Delta m}{M_v}\right]^2\right]=1.64\ , \qquad (33)$$
$$g_1(0)=g_1(\Delta m)\left[1-2\left[\frac{\Delta m}{M_a}\right]^2\right]=0.23\ ,$$

where we used $M_v=0.98$ GeV and $M_a=1.25$ GeV as found in Ref. 3. These values for $f_1$, $f_2$, and $g_1$ differ by approximately 20% from their Cabibbo-theory-predicted values:

$$f_1(0)\,|_{\text{Cab}}=-1.0\ ,$$
$$f_2(0)\,|_{\text{Cab}}=-\mu_p-2\mu_n=2.03\ , \qquad (34)$$
$$g_1(0)\,|_{\text{Cab}}=-(F-D)=0.28\ ,$$

where $\mu_p$ and $\mu_n$ are the anomalous magnetic moments of the proton and neutron, respectively, and $D$ ($F$) is the SU(3) (anti)symmetric axial-vector coupling.

The reasonable agreement found between (33) and (34) provides confidence in our calculation of the other less well-known form factors $f_3$, $g_2$, and $g_3$. Examining Table III we see that the second-class form factor $f_3$ is comparable in magnitude to $g_1$.[12] However, as mentioned before, a suppression factor of $(m_e/m_{fi})^2$ makes $f_3$ irrelevant for the process $\Sigma^-\to ne\bar{\nu}_e$, although it may be significant in transition amplitudes involving final-state muons since the amplitude suppression factor $(m_\mu/m_{fi})^2$ is $(200)^2$ times larger than in $\Sigma^-\to ne\bar{\nu}_e$. In contrast to $f_3$, $g_2$ is relatively small, approximately $-0.1$, and thus does not support the estimate of Ref. 6 that $g_2(\Delta S=1)\simeq-1.0$.

A value for $g_2$ of this magnitude will not have a significant effect on the electron asymmetry $\alpha_e$ of semileptonic decay, which measures the correlation of the spin of the decaying baryon with the momentum of the final-state electron. The dependence of the electron symmetry $\alpha_e$ on $f_1$, $f_2$, and $g_2$ to first order in $\Delta m/m_{fi}$, is given by[18]

$$\alpha_e=(f_1{}^2+3g_1{}^2)^{-1}\left[2g_1(f_1-g_1)-\frac{2}{3}\frac{\Delta m}{m_{fi}}[(f_1+g_1)(f_1+f_2+g_1+g_2)-6g_1g_2]\right]$$
$$+(f_1{}^2+3g_1{}^2)^{-2}\left[4\frac{\Delta m}{m_{fi}}g_1g_2\right]+O\left[\left[\frac{\Delta m}{m_{fi}}\right]^2\right]. \qquad (35)$$

We see that $g_2$, already of order $\Delta m$, appears in this expression suppressed by an additional kinematical factor of $\Delta m/m_{fi}$. Roughly speaking, this gives a contribution to $\alpha_e$ of $(\Delta m/m_{fi})|g_2|\cong0.01$, which is negligible compared to current experimental error, $\Delta\alpha_e=\pm0.14$.

To be more quantitative, we shall evaluate Eq. (35) for the $\Sigma^-\to n$ transition using renormalized form factors given in Table III, leaving $g_1(0)/f_1(0)$ as a free parameter to fit experiment. The calculated values of $\alpha_e$ vs $g_1(0)/f_1(0)$ are shown in Fig. 5 for the $\Sigma^-\to n$ transition. The curves in Fig. 5 also display the dependence of $\alpha_e$ on $g_2(\Delta m)/f_1(0)$, where $g_2(\Delta m)/f_1(0)=0.11$ is our calculated value from Table III and Eq. (33) while $g_2(\Delta m)/f_1(0)=0.0$ is the usual assumed value used in data analysis and phenomenological fits[2,3] to the Cabibbo model. From the plot, we find for $g_2(\Delta m)/f_1(0)=0.11$ an extracted value of $g_1(0)/f_1(0)=-0.29\pm0.08$, while for $g_2(\Delta m)/f_1(0)=0.0$ we find that $g_1(0)/f_1(0)$ is $-0.30\pm0.08$. The latter differs slightly from the experimentally extracted value (2) of Ref. 1 which uses the Cabibbo value (34) for $f_2$. These values are to be compared with the Cabibbo value $g_1(0)/f_1(0)=-0.28\pm0.02$, and our calculated value, $g_1(0)/f_1(0)_{\text{calc}}=-0.29$ as determined from (33). The message is now clear: Including $g_2$ in the calculation of $\alpha_e$ produces a negligible shift in $g_1/f_1$ (from $-0.30$ to $-0.29$), remaining in accord with

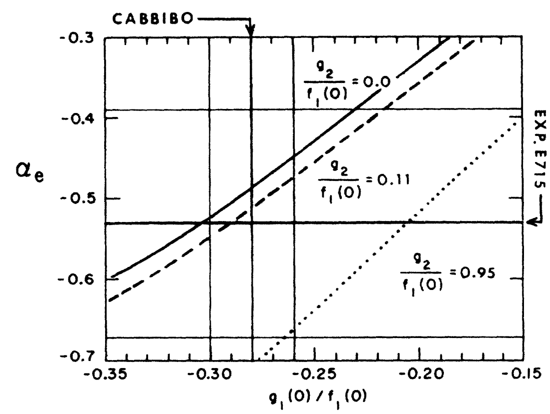


FIG. 5. Electron asymmetry $\alpha_e$ vs the axial-vector-to-vector form-factor ratio $g_1/f_1$ for several values of the weak-electric dipole form factor $g_2(\Delta m)/f_1(0)$: (1) $g_2/f_1(0)=0$ (solid line), (2) our calculated value, $g_2/f_1(0)=0.11$ (dashed line), and (3) $g_2/f_1(0)=0.95$ (dotted line), obtained via the Goldberger-Treiman estimate of $g_3$ [Eq. (36)]. Horizontal lines indicate the Fermilab E715 measurement (Ref. 1) of $\alpha_e$ with error bars [Eq. (1)], the vertical lines a recent fit (Ref. 2) with error bars of $g_1/f_1$ for baryon semileptonic decays based on the Cabibbo model [Eq. (3)].

the Cabibbo prediction and well within the experimental error bars. Therefore, the effects on $g_1/f_1$ due to the second-class form-factor $g_2$ are too small for early detection in electron asymmetry measurements.

Recalling the important dependence of $g_2$ on $g_3$ (cf. Sec. II) it is useful to compare our value of $g_2$ with that obtained by an independent estimate of $g_3$. The form factor $g_3$ can be related to $g_1$ using PCAC (partial conservation of axial-vector current) and the Goldberger-Treiman (GT) relation suitably modified for $\Delta S=1$ transitions.[19] This identity relies on the assumptions that the dominant contribution to $\Sigma^-\to n$ decay is mediated by the exchange of a $K^-$ meson and that PCAC remains a good approximation for $\Delta S=1$ transitions. Following the usual GT procedure we arrive at the estimate,[20]

$$g_3(q^2)\simeq-\frac{(M_\Sigma+M_n)^2 g_1(q^2)}{M_K{}^2-(M_\Sigma-M_n)^2} . \tag{36}$$

Using our value of $g_1$ from Table III and Eq. (36), we find $g_3$ takes the value of $-6.4$, which is significantly larger than our calculated value for $g_3$ of $-1.0$. This leads via Eq. (10) to a new value of $g_2(\Delta m)/f_1(0)$ of 0.95. Recalculating $\alpha_e$ (see Fig. 5) using this value of $g_2$ results in an extracted value for $g_1(0)/f_1(0)$ of $-0.21\pm0.06$, which is apperciably different from the Cabibbo-theory-predicted value ($-0.28$), as well as our calculated value ($-0.29$). We offer this as possible evidence of the unsuitability of using PCAC (by way of the GT relation) in the estimation of $g_3$ for $\Delta S=1$ transitions.

## ACKNOWLEDGMENTS

We thank J. Lach for discussions concerning the Fermilab E715 experiment. Two of us (R.J.O. and C.R.W.) would like to thank the Division of Educational Programs (DEP) and the High Energy Physics (HEP) Division at Argonne National Laboratory for their support and encouragement during the preparation of the manuscript. This research was also supported in part by the National Science Foundation and the U.S. Department of Energy.

## APPENDIX: MIT-BAG-MODEL WAVE FUNCTIONS AND PROPAGATORS

In this appendix, we review formulas for MIT-bag-model[7,16] quark and gluon wave functions and propagators which are needed for the evaluation of the weak currents $V_\lambda(\mathbf{q})$ and $A_\lambda(\mathbf{q})$ in Sec. III. In particular, we present expressions for excited spin $j>\frac{1}{2}$ quark-bag wave functions which have not, to our knowledge, appeared previously in the literature. In the following, we use the notation $x=(t,\mathbf{r})$ for the components of the position four-vector $x$.

Confine quarks of mass $m_q$ to move freely inside a spherical cavity of radius $R$. Then inside the bag, quark eigenfunctions of energy $\omega$ satisfy the free Dirac equation,

$$(i\boldsymbol{\gamma}\cdot\boldsymbol{\nabla}+\gamma^0\omega-m_q)q(\mathbf{r})=0,\ \ r<R , \tag{A1a}$$

subject to the boundary condition,

$$(1+i\hat{\mathbf{r}}\cdot\boldsymbol{\gamma})q(\mathbf{r})\,|_{r=R}=0 , \tag{A1b}$$

which ensures that the radial quark current vanishes at the bag surface. As is familiar, the Hamiltonian operator defined by (A1a) commutes with the total angular momentum operator, $\mathbf{J}=\mathbf{L}+\boldsymbol{\sigma}/2$, where $\mathbf{L}=-i\mathbf{r}\times\boldsymbol{\nabla}$, and so states of definite energy are also characterized by the quantum numbers $(j,m)$. While orbital angular momentum $\mathbf{L}$ is not conserved, it still proves useful to expand the upper and lower components of $q(\mathbf{r})$ in terms of the spin-$\frac{1}{2}$ spherical harmonics $\phi_{jlm}(\hat{\mathbf{r}})$, eigenstates of $J^2$, $L^2$, and $J_3$, given by

$$\phi_{jlm}(\hat{\mathbf{r}})=\langle\hat{\mathbf{r}}\,|\,jm;l\tfrac{1}{2}\rangle$$
$$=\sum_{l_3 s_3} Y_{ll_3}(\hat{\mathbf{r}})\chi_{s_3}\langle ll_3\tfrac{1}{2}s_3\,|\,jm;l\tfrac{1}{2}\rangle . \tag{A2}$$

Here $Y_{ll_3}(\hat{\mathbf{r}})$ are the familiar spherical harmonics while $\chi_{s_3}$ are two-component Pauli spinors. Clearly $l=j\pm\frac{1}{2}$ are the only allowed values of orbital momentum and in the following we use the notation $\bar{l}=j\mp\frac{1}{2}$ whenever $l=j\pm\frac{1}{2}$. From (A2) we see that $\phi_{jlm}$ and $\phi_{j\bar{l}m}$ are of opposite parity and are related by a pseudoscalar operator,

$$\boldsymbol{\sigma}\cdot\hat{\mathbf{r}}\phi_{jlm}(\hat{\mathbf{r}})=-\phi_{j\bar{l}m}(\hat{\mathbf{r}}) . \tag{A3}$$

Employing an expansion in spin harmonics, positive energy ($\omega>0$) eigenfunctions of Eqs. (A1) are easily obtained. Let $\omega(x)=(x^2/R^2+m_q{}^2)^{1/2}$. Then the energy eigenvalues $\omega_{jln}=\omega(x_{jln})$, $n=0,1,2,\ldots$, are determined by the roots $x_{jln}$ of the transcendental equation,

$$j_l(x)=(\bar{l}-l)\left[\frac{\omega(x)-m_q}{\omega(x)+m_q}\right]^{1/2} j_{\bar{l}}(x) . \tag{A4}$$

The corresponding eigenfunctions are

$$q_{jlmn}(\mathbf{r})=N_{jl}(x_{jln})\begin{bmatrix} iW_+(jln)j_l\left[x_{jln}\dfrac{r}{R}\right]\phi_{jlm}(\hat{\mathbf{r}}) \\ (\bar{l}-l)W_-(jln)j_{\bar{l}}\left[x_{jln}\dfrac{r}{R}\right]\phi_{j\bar{l}m}(\hat{\mathbf{r}}) \end{bmatrix} , \tag{A5a}$$

where

$$W_\pm(jln)\equiv\left[\frac{\omega_{jln}\pm m_q}{\omega_{jln}}\right]^{1/2} \tag{A5b}$$

and

$$N_{jl}{}^{-2}(x)=\frac{R^3 j_l(x)}{\omega(x)[\omega(x)-m_q]}\times\left[2\omega(x)\left[\omega(x)-(\bar{l}-l)\frac{(j+\frac{1}{2})}{R}\right]+\frac{m_q}{R}\right] \tag{A5c}$$

is the normalization factor. For $j=\frac{1}{2}$, Eqs. (A5) reproduce the lowest-mode eigenfunctions of Ref. 7.

Negative-energy eigenfunctions may be derived in a similar fashion, or by simply applying charge conjugation to the positive-energy eigenfunctions given above. For this, we observe that if $q(\mathbf{r})$ solves Eqs. (A1), then $i\gamma_2 q^*(\mathbf{r})$ does so also with $\omega \to -\omega$. As this operation also flips $J_3$, we define, up to an irrelevant phase, the charge-conjugate negative-energy eigenstates via

$$\tilde{q}_{jlmn}(\mathbf{r}) \equiv e^{i\phi} i\gamma_2 q^*_{jl-mn}(\mathbf{r}) . \tag{A6}$$

Explicitly, our particular choice of phase produces

$$\tilde{q}_{jlmn}(\mathbf{r}) = N_{jl}(x_{jln}) \begin{bmatrix} iW_-(jln) j_{\bar{l}}\left[x_{jln}\frac{r}{R}\right]\phi_{j\bar{l}n}(\hat{\mathbf{r}}) \\ (\bar{l}-l)W_+(jln) j_{l}\left[x_{jln}\frac{r}{R}\right]\phi_{jlm}(\hat{\mathbf{r}}) \end{bmatrix} . \tag{A7}$$

The bag quark propagator $S_F(x,x')$ satisfying

$$(i\gamma^\mu \partial_\mu - m_q) S_F(x,x') = \delta^4(x-x') , \tag{A8a}$$

$$(1 + i\hat{\mathbf{r}}\cdot\gamma) S_F(x,x')\,|_{r=R} = 0 , \tag{A8b}$$

is constructed in the usual way by Fourier transforming to frequency space,

$$S_F(x,x') = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{-i\omega(t-t')} S_F(\mathbf{r},\mathbf{r}';\omega) \tag{A9}$$

and expanding the kernel $S_F(\mathbf{r},\mathbf{r}';\omega)$ in energy eigenstates (A5a) and (A7). Imposing Feynman boundary conditions, the result is

$$S_F(\mathbf{r},\mathbf{r}';\omega) = \sum_{jlmn} \frac{q_{jlmn}(\mathbf{r})\bar{q}_{jlmn}(\mathbf{r}')}{\omega - \omega_{jln} + i\epsilon} + \frac{\tilde{q}_{jlmn}(\mathbf{r})\bar{\tilde{q}}_{jlmn}(\mathbf{r}')}{\omega + \omega_{jln} - i\epsilon} . \tag{A10}$$

Strictly speaking, (A9) defines $S_F(\mathbf{r},\mathbf{r}';\omega)$ only for $\mathrm{Im}\omega > 0$. However, analytic continuation to $\mathrm{Im}\omega < 0$ is possible by virtue of Schwarz reflection,

$$S_F(\mathbf{r},\mathbf{r}';\omega^*) = \gamma^0 S_F^\dagger(\mathbf{r}',\mathbf{r};\omega)\gamma^0 \tag{A11}$$

which is a consequence of Eqs. (A1) extended to complex $\omega$. But the expression (A10) for $S_F(\mathbf{r},\mathbf{r}';\omega)$ satisfies (A11) as it stands, and thus is valid for all (complex) $\omega$.

In passing, we note that our expression for $S_F(x,x')$ is equivalent, with suitable modifications for a nonzero quark mass $m_q$, to that derived by Hansson and Jaffe.[16] By employing a multiple-reflection expansion, these authors are able to isolate the free-quark contribution to $S_F(x,x')$ and thereby demonstrate that the ultraviolet divergences of the bag model are identical to the free (unconfined) theory, and thus can be similarly eliminated by renormalization. To achieve this, however, it is necessary to include contributions from higher-spin modes $j > \frac{1}{2}$. This justifies the inclusion of $j = \frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$ modes in the calculations of this paper, despite the fact that such modes violate the quadratic boundary condition found in earlier versions of the MIT bag model.[7]

Next we construct gluon wave functions and propagators. Working in the Coulomb gauge, the gluon energy eigenmodes $\mathbf{G}(\mathbf{r})$ are transverse,

$$\nabla\cdot\mathbf{G}(\mathbf{r}) = 0 , \tag{A12}$$

everywhere in the bag and satisfy the Helmholtz wave equation,

$$(\nabla^2 + \nu^2)\mathbf{G}(\mathbf{r}) = 0 , \tag{A13}$$

with bag boundary conditions,

$$\hat{\mathbf{r}}\cdot\mathbf{G}(\mathbf{r})\,|_{r=R} = 0 , \tag{A14a}$$

$$\hat{\mathbf{r}}\times[\nabla\times\mathbf{G}(\mathbf{r})]\,|_{r=R} = 0 . \tag{A14b}$$

The solution of Eqs. (A13) and (A14) is characterized by quantum numbers $(J,M)$ of $J^2$ and $J_3$, where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ as in the quark case except that $\mathbf{S}$ is now the spin operator for spin-1 representations. Invariance under orbital rotations generated by $\mathbf{L}$ is spoiled by the boundary conditions (A14). Thus an expansion of $\mathbf{G}(\mathbf{r})$ in vector spherical harmonics,

$$\mathbf{Y}_{JLM}(\hat{\mathbf{r}}) \equiv \langle \hat{\mathbf{r}} \,|\, JM;L1\rangle = \sum_{L_3 S_3} Y_{LL_3}(\hat{\mathbf{r}})\hat{\mathbf{e}}_{S_3} \langle LL_3 1S_3 \,|\, JM;L1\rangle , \tag{A15}$$

$$\hat{\mathbf{e}}_\pm = \mp 2^{-1/2}(\hat{\mathbf{x}} \pm i\hat{\mathbf{y}}), \quad \hat{\mathbf{e}}_0 = \hat{\mathbf{z}} ,$$

will mix coefficients of different $L$, which for given $J$ assumes the values $J$, $J \pm 1$. The relevant linear combinations satisfying Eqs. (A13) and (A14) and transversality (A12) are of two types, transverse-electric (TE) and -magnetic (TM), with energies $\nu_{JN}^{(T)}$ ($T$=TE, TM, $N$=0,1,2,...) determined by the conditions

$$\frac{d}{dx}[x j_J(x)]\,|_{x=\nu_J^{(\mathrm{TE})}R} = 0 , \tag{A16a}$$

$$j_J(x)\,|_{x=\nu_J^{(\mathrm{TM})}R} = 0 . \tag{A16b}$$

The corresponding eigenfunctions are

$$\mathbf{G}_{JMN}^{(\mathrm{TE})}(\mathbf{r}) = A_J^{(\mathrm{TE})}(\nu_{JN}^{(\mathrm{TE})}R) j_J(\nu_{JN}^{(\mathrm{TE})} r)\mathbf{Y}_{JJM}(\hat{\mathbf{r}}) , \tag{A17a}$$

$$\mathbf{G}_{JMN}^{(\mathrm{TM})}(\mathbf{r}) = \frac{A_J^{(\mathrm{TM})}(\nu_{JN}^{(\mathrm{TM})}R)}{i\nu_{JN}^{(\mathrm{TM})}} \nabla\times[j_J(\nu_{JN}^{(\mathrm{TM})} r)\mathbf{Y}_{JJM}(\hat{\mathbf{r}})] , \tag{A17b}$$

where the normalization constants $A_J^{(T)}$ are chosen to secure unit normalization, i.e.,

$$\int_{\mathrm{bag}} d^3r\, |\mathbf{G}_{JMN}^{(T)}(\mathbf{r})|^2 = 1 . \tag{A18}$$

Using the eigenvalue conditions (A16), these are found to be

$$[A_J^{(\mathrm{TE})}(x)]^{-2} = \tfrac{1}{2}R^3 j_J^2(x)\left[1 - \frac{J(J+1)}{x^2}\right] , \tag{A19a}$$

$$[A_J^{(\mathrm{TM})}(x)]^{-2} = \tfrac{1}{2}R^3 j_{J+1}^2(x) . \tag{A19b}$$

For the evaluation of vertex corrections to the form factors we will specifically require the transverse gluon propagator, $D_{ij}(x,x')$ which in a dyadic notation solves the equations,

$$(-\partial_t{}^2+\nabla^2)D(x,x')=\delta^4(x-x')\ , \qquad \text{(A20a)}$$

$$\nabla\cdot D(x,x')=0\ , \qquad \text{(A20b)}$$

with boundary conditions at $r=R$ analogous to Eqs. (A14) and the usual Feynman boundary conditions on the temporal development of energy eigenmodes. From Eq. (A13), it is straightforward to show that

$$D(x,x')=\int_{-\infty}^{\infty}\frac{d\nu}{2\pi}e^{-i\nu(t-t')}D(\mathbf{r},\mathbf{r}';\nu) \qquad \text{(A21)}$$

solves Eqs. (A20), where

$$D(\mathbf{r},\mathbf{r}';\nu)=\sum_{T=\mathrm{TE,TM}}\sum_{JMN}\frac{\mathbf{G}^{(T)}_{JMN}(\mathbf{r})\mathbf{G}^{(T)*}_{JMN}(\mathbf{r}')}{\nu^2-(\nu^{(T)}_{JN})^2+i\epsilon}\ . \qquad \text{(A22)}$$

As a final point, we mention that in the Coulomb gauge the gluon propagator $D_{\mu\nu}$ also has a nonzero component $D_{00}$ which is the instantaneous Coulomb propagator. We shall not have occasion in this paper to utilize this object, whose explicit form in the bag has been given by Lee.[21]

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[9]In another common notation, one would identify our form factors as follows:

$$f_1=g_v,\ \ f_2=g_M,\ \ f_3=g_s\ ,$$

$$g_1=g_A,\ \ g_2=g_{II},\ \ g_3=g_p\ .$$

[10]See, for example, E. D. Commins, *Weak Interactions* (McGraw-Hill, New York, 1973), p. 95.
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