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Cabibbo-angle-favored $D \to PP$ and $D \to VP$ decays: A dispersion approach

A. N. Kamal
Theoretical Physics Institute, Department of Physics, University of Alberta, Edmonton, Alberta T6G 2J1 Canada
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We have studied $D \to PP$ and $D \to VP$ decays in a dispersion approach. We have explicitly included the flavor-annihilation process and investigated the circumstances where its contribution could become large. Data point to a substantial flavor-annihilation term in all these decays. We also estimate the branching ratios for $D^0 \to \bar{K}^0\eta$, $D^0 \to \bar{K}^0\eta'$, $D^0 \to \bar{K}^0\phi$, and $D^0 \to \bar{K}^0\omega$.

I. INTRODUCTION

Extensive data on two-body Cabibbo-angle-favored decays now exist[1-3] for the channels $D \to PP$ and $D \to VP$. The data confirm that $D^0 \to \bar{K}^0\pi^0$ is not color suppressed and that there is evidence of perhaps a larger degree of color suppression in the modes $D^0 \to \bar{K}^0\rho^0$ and $D^0 \to \bar{K}^{*0}\pi^0$. The relevant ratios are[1-3]

$$\begin{aligned} R_{00}^{K\pi} &\equiv \Gamma(D^0 \to \bar{K}^0\pi^0)/\Gamma(D^0 \to K^-\pi^+) \\ &= 0.35 \pm 0.07 \pm 0.07 , \\ R_{0+}^{K\pi} &\equiv \Gamma(D^0 \to K^-\pi^+)/\Gamma(D^+ \to \bar{K}^0\pi^+) \\ &= 3.7 \pm 1.0 \pm 0.08 ; \end{aligned} \tag{1}$$

$$\begin{aligned} R_{00}^{K\rho} &\equiv \Gamma(D^0 \to \bar{K}^0\rho^0)/\Gamma(D^0 \to K^-\rho^+) \\ &= 0.09 \pm 0.03 \pm 0.02 , \\ R_{0+}^{K\rho} &\equiv \Gamma(D^0 \to K^-\rho^+)/\Gamma(D^+ \to \bar{K}^0\rho^+) \\ &= 2.67 \pm 0.92 \pm 0.64 ; \end{aligned} \tag{2}$$

$$\begin{aligned} R_{00}^{K^*\pi} &\equiv \Gamma(D^0 \to \bar{K}^{*0}\pi^0)/\Gamma(D^0 \to K^{*-}\pi^+) \\ &= 0.30 \pm 0.14 \pm 0.08 , \\ R_{0+}^{K^*\pi} &\equiv \Gamma(D^0 \to K^{*-}\pi^+)/\Gamma(D^+ \to \bar{K}^{*0}\pi^+) \\ &= 5.45 \pm 3.61 \pm 3.25 . \end{aligned} \tag{3}$$

In arriving at the above numbers we have used $\tau_{D^+}/\tau_{D^0} = 2.5 \pm 0.6$ (statistical only). The use of[4] $\tau_{D^+}/\tau_{D^0} = 2.3^{+0.5+0.1}_{-0.4-0.1}$ makes very little difference to our analysis. A model-independent analysis[5] of the $D \to K\pi$ data has shown that (i) complex amplitudes are needed to fit the data and (ii) nonspectator processes play an important role. With the latter point in mind the problem of $D \to K\pi$ decay was investigated[6] in a vector-pole model constrained by current algebra. A near fit to the data was obtained with real amplitudes. After unitarization of the amplitudes through final-state interactions the authors of Ref. 6 obtained a fit to the data. In contrast with the quality of $D \to K\pi$ data[1,3] which necessitate[5] the use of complex amplitudes, the errors in the $D \to VP$ data[2,3] are large enough that a real-amplitude analysis suffices. The data, as we shall see later, already point to a significant flavor-annihilation contribution, as do the $D \to K\pi$ data.

In the present paper we have studied the two-body Cabibbo-angle-favored $D \to PP$ and $D \to VP$ decays from a dispersion viewpoint, including the flavor-annihilation (W-exchange process) which is usually argued to be helicity suppressed. We investigate the role of these channels and the circumstances under which the annihilation channel could become important. The reader is referred to earlier investigations, similar in spirit, by Fakirov and Stech,[7] Milosevic, Tadić, and Trampetić,[8] and Rückl.[9]

Section II of this paper deals with $D \to PP$ decays. In Sec. II A, dealing with $D \to K\pi$ decays, it is pointed out that if the hadronic matrix element of the divergence of the weak current satisfies a once-subtracted dispersion relation then the flavor-annihilation amplitude could become large. This, then, lifts color suppression of $D^0 \to \bar{K}^0\pi^0$, giving a near fit to the data with real amplitudes. Sections II B and II C deal with $D^0 \to \bar{K}^0\eta$ and $D^0 \to \bar{K}^0\eta'$ decays, respectively. In Sec. II D we unitarize $D \to K\pi$ decay amplitudes through final-state interactions and obtain a fit to the data. The prescription for unitarization used here is the simplest one can think of.

$D \to VP$ decays are discussed in Sec. III. In Sec. III A we study $D \to K\rho$ decays and argue that a significant flavor-annihilation contribution is needed. We show that a naive application of PCAC (partial conservation of axial-vector current) in the region $q^2 \approx m_D{}^2$ leads to a suppression of the flavor-annihilation terms by factors of order $m_K{}^2/m_D{}^2$. PCAC, however, is sure to fail in the annihilation region ($q^2 \approx m_D{}^2$) since one would need to extrapolate the amplitude through a region known to be populated by resonances. We show that PCAC requires a fine-tuning among the parameters, which include the parameters entering the decay $Q_1(1270) \to K\rho$. We argue that at $q^2 \approx m_D{}^2$ this fine-tuning may not occur. If so, the flavor-annihilation terms could become larger. With one parameter describing the flavor-annihilation contribution, we fit $D \to K\rho$ data. In Sec. III B we study $D \to K^*\pi$ decays. We show that these data can also be understood using an annihilation term of the same size as that needed to understand $D \to K\rho$ decays. In Secs. III C and III D we study $D^0 \to \bar{K}^0\phi$ and $D^0 \to \bar{K}^0\omega$ decays, respectively.

$D \to VP$ decays have also been studied in the past by several authors.[7,9,10] In all these studies $R_{00}^{K\rho}$ and $R_{00}^{K^*\pi}$ of (1) were found to be strongly suppressed. The analysis of the present work is more detailed and, in particular, includes the flavor-annihilation contribution. We conclude with a discussion of our results in Sec. IV.

II. $D\to PP$ DECAYS

A. $D\to K\pi$

The hard-gluon-corrected Hamiltonian for the Cabibbo-angle-favored charm decay is

$$H_W=\frac{G_F}{\sqrt{2}}\cos^2\theta_C[\tfrac{1}{2}(C_+ + C_-)(\bar{u}d)(\bar{s}c) + \tfrac{1}{2}(C_+ - C_-)(\bar{u}c)(\bar{s}d)] , \quad (4)$$

where $(\bar{u}d)$, etc., represent the left-handed hadronic current, θ_C is the Cabibbo angle, and C_+ and C_- are taken to be[5]

$$C_+=0.69,\quad C_-=2.09,\quad C_+{}^2C_-=1 . \quad (5)$$

Sandwiching the Hamiltonian (4) between the initial and the final states and linking up the quark lines in all color-singlet combinations, one obtains the decay amplitudes in the factorization approximation (details are provided for $D^0\to K^-\pi^+$ channel only),

$$\begin{aligned}A(D^0\to K^-\pi^+)&=C_1\langle\pi^+|(\bar{u}d)|0\rangle\langle K^-|(\bar{s}c)|D^0\rangle\\&\quad+C_2\langle\pi^+K^-|(\bar{s}d)|0\rangle\langle 0|(\bar{u}c)|D^0\rangle , \quad (6)\end{aligned}$$

where

$$C_1=\tfrac{1}{3}(2C_+ + C_-),\quad C_2=\tfrac{1}{3}(2C_+ - C_-) . \quad (7)$$

The first term in (6) is the usual spectator (and color-suppressed spectator) term while the second term is the flavor-annihilation term.

To proceed further we use

$$\begin{aligned}\langle 0|(\bar{u}c)|D^0\rangle&=i\sqrt{2}f_D p_D^\mu ,\\ \langle\pi^+|(\bar{u}d)|0\rangle&=-i\sqrt{2}f_\pi p_\pi^\mu ,\end{aligned} \quad (8)$$

and

$$\begin{aligned}\langle P_i|V_j^\mu|P_k\rangle&=if_{ijk}[(p_k+p_i)^\mu f_+(q^2)\\&\quad+(p_k-p_i)^\mu f_-(q^2)] , \quad (9)\end{aligned}$$

where i,j,k are the SU(4) indices and $q^\mu=(p_k-p_i)^\mu$. $f_+(0)=1$ in the SU(4) limit.

In evaluating the matrix elements in (6) one encounters the hadronic matrix element of the divergence of the vector current

$$\begin{aligned}q_\mu(P_i|V_j^\mu|P_k)&=if_{ijk}[(m_k{}^2-m_i{}^2)f_+(q^2)+q^2f_-(q^2)]\\&\equiv if_{ijk}f_0(q^2) . \quad (10)\end{aligned}$$

The scalar form factor $f_0(q^2)$ is normalized such that

$$f_0(0)=(m_k{}^2-m_i{}^2)f_+(0) . \quad (11)$$

$f_0(q^2)$ appearing in the first term in (6) gets contribution from a 0^+ state with flavor content $\bar{s}c$, i.e., F_s, while the second term gets contribution from a 0^+ state with flavor content $\bar{s}d$, i.e., κ (Ref. 11).

The final result for the $D\to K\pi$ decay amplitudes, up to an overall constant is[8]

$$\begin{aligned}A(D^0\to K^-\pi^+)&=[C_1f_\pi f_0^{F_s}(m_\pi{}^2)-C_2f_Df_0^\kappa(m_D{}^2)] ,\\ A(D^0\to\bar{K}^0\pi^0)&=\frac{C_2}{\sqrt{2}}[f_Kf_0^{D_s}(m_K{}^2)+f_Df_0^\kappa(m_D{}^2)] ,\\ A(D^+\to\bar{K}^0\pi^+)&=[C_1f_\pi f_0^{F_s}(m_\pi{}^2)+C_2f_Kf_0^{D_s}(m_K{}^2)] .\end{aligned} \quad (12)$$

Note that the $\Delta I=1$ isospin sum rule

$$A(D^0\to K^-\pi^+)+\sqrt{2}A(D^0\to\bar{K}^0\pi^0)=A(D^+\to\bar{K}^0\pi^+) \quad (13)$$

is identically satisfied.

The naive-spectator-model results are obtained from (12) in the limit $f_\pi=f_K$, $f_0^{F_s}(m_\pi{}^2)=f_0^{D_s}(m_K{}^2)$, and the neglect of terms proportional to $f_0^\kappa(m_D{}^2)$, the flavor-annihilation channel contribution.

Let us next assume that $f_0(q^2)$ satisfies an unsubtracted dispersion relation, which would require $f_-(q^2)$ to decay faster than $1/q^2$ asymptotically, then

$$f_0^{F_s}(q^2)=\frac{f_+(0)(m_D{}^2-m_K{}^2)}{1-q^2/m_{F_s}{}^2} \quad (14)$$

and

$$f_0^{F_s}(m_\pi{}^2)\approx f_+(0)(m_D{}^2-m_K{}^2) . \quad (15)$$

Similarly,

$$f_0^{D_s}(m_K{}^2)=\frac{f_+(0)(m_D{}^2-m_\pi{}^2)}{1-m_K{}^2/m_{D_s}{}^2}\approx f_+(0)(m_D{}^2-m_\pi{}^2) \quad (16)$$

and

$$\begin{aligned}f_0^\kappa(m_D{}^2)&=\frac{f_+(0)(m_K{}^2-m_\pi{}^2)}{1-m_D{}^2/m_\kappa{}^2}\\&\simeq-1.1f_+(0)(m_K{}^2-m_\pi{}^2)\end{aligned} \quad (17)$$

with[11] $m_\kappa=1.35$ GeV.

The factor $(m_K{}^2-m_\pi{}^2)$ in (17) signals helicity suppression. Clearly $f_0^\kappa(m_D{}^2)$ is helicity suppressed relative to $f_0^{F_s}(m_\pi{}^2)$ or $f_0^{D_s}(m_K{}^2)$. Hence, if we assume an unsubtracted dispersion relation for $f_0(q^2)$, and further assume that it is dominated by a single-particle pole, then helicity suppression is not lifted and the nonspectator processes are not important.

It is clear from (10) that in an exact SU(4) limit, $f_-(q^2)=0$, helicity suppression will always operate. However, SU(4) symmetry is broken and since $f_0^\kappa(q^2)$ is required at $q^2=m_D{}^2$, it is quite possible that $f_+(m_D{}^2)$ and $f_-(m_D{}^2)$ are comparable.[12] Since $f_-(q^2)$ appears multiplied by $m_D{}^2$ in $f_0^\kappa(q^2)$, it is also quite possible that $m_D^2 f_-(m_D^2)$ would dominate over $(m_K^2 - m_\pi^2) f_+(m_D^2)$. We have just shown, however, that as long as $f_0(q^2)$ satisfies an unsubtracted dispersion relation helicity suppression is unlikely to be lifted.

Let us assume, next, that $f_-(q^2)$ decays no faster than $1/q^2$ asymptotically. $f_0(q^2)$ then satisfies a subtracted dispersion relation. Let us assume further, that $f_0(q^2)$ satisfies a once-subtracted dispersion relation such that

$$f_0^{F_s}(q^2) = f_+(0)(m_D^2 - m_K^2) + \frac{\lambda^{F_s} q^2}{q^2 - m_{F_s}^2} \tag{18}$$

and

$$f_0^{F_s}(m_\pi^2) \simeq f_+(0)(m_D^2 - m_K^2) \tag{19}$$

unless, of course, λ^{F_s} is unusually large. Notice that (19) has not changed from the unsubtracted value (15).

Similarly,

$$f_0^{D_s}(q^2) = f_+(0)(m_D^2 - m_\pi^2) + \frac{\lambda^{D_s} q^2}{q^2 - m_{D_s}^2} \tag{20}$$

and

$$f_0^{D_s}(m_K^2) \approx f_+(0)(m_D^2 - m_\pi^2) \tag{21}$$

which is also the same as in (16).

However,

$$f_0^{\kappa}(q^2) = f_+(0)(m_K^2 - m_\pi^2) + \frac{\lambda q^2}{q^2 - m_\kappa^2} \tag{22}$$

and

$$f_0^{\kappa}(m_D^2) = f_+(0)(m_K^2 - m_\pi^2) + \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2} . \tag{23}$$

If we look for solutions with the condition [λ stands for any λ in (18), (20), or (22)]

$$m_K^2 \ll \frac{|\lambda|}{f_+(0)} \ll \frac{m_D^2 m_{D_s}^2}{m_K^2} \tag{24}$$

then (19) and (21) will remain unaltered; however, for (23) we will obtain

$$f_0^{\kappa}(m_D^2) \approx \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2} . \tag{25}$$

It is now possible to lift the helicity suppression of the annihilation process. The decay amplitudes are then

$$A(D^0 \to K^- \pi^+) = \left[C_1 f_\pi f_+(0)(m_D^2 - m_K^2) - C_2 f_D \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2} \right],$$

$$A(D^0 \to \bar{K}^0 \pi^0) = \frac{C_2}{\sqrt{2}} \left[f_K f_+(0)(m_D^2 - m_\pi^2) + f_D \frac{\lambda m_D^2}{m_D^2 - m_\kappa^2} \right], \tag{26}$$

$$A(D^+ \to \bar{K}^0 \pi^+) = [C_1 f_\pi f_+(0)(m_D^2 - m_K^2) + C_2 f_K f_+(0)(m_D^2 - m_\pi^2)] .$$

Note that in the limit $f_\pi = f_K$ and $m_D^2 - m_\pi^2 \approx m_D^2 - m_K^2 \simeq m_D^2$ and the neglect of flavor-annihilation terms, signaled by the parameter λ, the usual color suppression occurs,

$$\Gamma(D^0 \to \bar{K}^0 \pi^0)/\Gamma(D^0 \to K^- \pi^+) \approx \tfrac{1}{2}(C_2/C_1)^2 = \tfrac{1}{50} .$$

However, $\lambda > 0$ raises both $A(D^0 \to \bar{K}^0 \pi^0)$ and $A(D^0 \to \bar{K} \pi^+)$. The percentage rise in $A(D^0 \to \bar{K}^0 \pi^0)$ is larger since the two terms appear with equal weights. In contrast, the annihilation term in $A(D^0 \to K^- \pi^+)$ is proportional to C_2 while the spectator term is proportional to C_1. Since $C_2/C_1 \approx -\frac{1}{5}$, the percentage rise in $A(D^0 \to K^- \pi^+)$ is smaller.

In Table I we have compiled the ratios $R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$ as functions of f_D/f_π and $\lambda/f_+(0)$. Clearly, though one does not expect to fit both the ratios $R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$ with real amplitudes, one comes close to fitting the data. The important point being that helicity suppression of the flavor-annihilation process is lifted. In turn, this lifts the color suppression of $D^0 \to \bar{K}^0 \pi^0$ decay.

TABLE I. $R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$ without final-state interactions.

f_D/f_π	$\lambda/f_+(0)$ (GeV^2)	R_{00}	R_{0+}
1	5	0.15	4.96
1	6	0.17	5.78
1.1	5	0.16	5.36
1.1	6	0.18	6.30
1.2	5	0.17	5.78
1.2	6	0.19	6.84
1.3	4	0.16	5.12
1.3	5	0.18	6.21
1.4	4	0.16	5.44
1.4	5	0.19	6.66

B. $D^0 \to \bar{K}^0 \eta$

We assume that η is a pure SU(3) octet, i.e.,

$$\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) = P_8 ,$$

in SU(4). In factorization approximation

$$A(D^0 \to \bar{K}^0 \eta) = C_2[\langle \bar{K}^0 | (\bar{s}d) | 0 \rangle \langle \eta | (\bar{u}c) D^0 \rangle + \langle \bar{K}^0 \eta | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle] . \tag{27}$$

Relating $\langle \eta | (\bar{u}c) | D^0 \rangle$ to $\langle \pi^0 | (\bar{u}c) | D^0 \rangle$ through the SU(4) rotation (9), one obtains, up to an overall constant,

$$A(D^0 \to \bar{K}^0 \eta) = \frac{C_2}{\sqrt{6}}[f_K f_0^{D_s}(m_K^2) - 3 f_D f_0^{\kappa}(m_D^2)] . \tag{28}$$

It is worth noting that the flavor-annihilation term is larger than in $D^0\to\bar{K}^0\pi^0$ by a factor of $\sqrt{3}$. This is due to the fact that η has a $s\bar{s}$ component while π^0 does not.

However, the flavor-annihilation term may not be significant for two reasons. First, one expects that it is harder to excite a $s\bar{s}$ pair from the vacuum[13] and second, the κ does not appear to couple to $\bar{K}\eta$ channel.[14] If the latter statement is taken seriously then one does not expect the κ structure in $f_0^{\kappa}(q^2)$ appearing in (28). $f_0^{\kappa}(q^2)$ will be essentially structure-free and approximated by its value at $q^2=0$,

$$f_0^{\kappa}(q^2)\approx f_0^{\kappa}(0)=f_+(0)(m_K{}^2-m_\eta{}^2)\ . \tag{29}$$

Notice that, because of the closeness of the K mass to the η mass, the mass suppression is rather strong.

This argument will apply regardless of whether $s\bar{s}$ pair is excited or not so long as the κ does not couple to $\bar{K}^0\eta$. *If the arguments made here apply* then flavor annihilation will not contribute significantly to $D^0\to\bar{K}^0\eta$ and one will obtain

$$A(D^0\to\bar{K}^0\eta)\simeq\frac{C_2}{\sqrt{6}}f_Kf_+(0)(m_D{}^2-m_\eta{}^2)\ . \tag{30}$$

Since $A(D^+\to\bar{K}^0\pi^+)$ does not depend on the flavor-annihilation amplitude either, one can calculate

$$B(D^0\to\bar{K}^0\eta)/B(D^+\to\bar{K}^0\pi^+)=0.7\times10^{-2}\ , \tag{31}$$

where we have used $\tau_{D^+}/\tau_{D^0}=2.5$ and $f_K/f_\pi=1.2$. In the naive spectator model one expects the ratio of (31) to be 2.2×10^{-2}. The difference is due to $f_\pi\neq f_K$ and the inclusion of the pseudoscalar masses.

On the other hand, if the flavor-annihilation term in (28) is *not* suppressed, and κ couples to $\bar{K}^0\eta$ through SU(3), then the ratio in (31) could be larger by as much as 2 orders of magnitude. A measurement of $B(D^0\to\bar{K}\eta)$ will be quite a sensitive test of nonspectator contribution to $D^0\to\bar{K}^0\eta$. In Table II we have tabulated $B(D^0\to\bar{K}^0\eta)/B(D^+\to\bar{K}^0\pi^+)$ as a function of f_D/f_π and $\lambda/f_+(0)$. In anticipation of the results of Sec. II D we have allowed $\lambda/f_+(0)$ to be of order 10 GeV^2. We notice from this table that $B(D^0\to\bar{K}^0\eta)\sim1-2\,\%$ signals a large flavor-annihilation contribution. The reader is reminded that $B(D^+\to\bar{K}^0\pi^+)$ is 2–3 %.

TABLE II. $B(D^0\to\bar{K}^0\eta)/B(D^+\to\bar{K}^0\pi^+)$ as a function of f_D/f_π and $\lambda/f_+(0)$. Larger values of $\lambda/f_+(0)$ are used in anticipation of the results of Sec. II D.

f_D/f_π	$\lambda/f_+(0)$ (GeV^2)	$\dfrac{B(D^0\to\bar{K}^0\eta)}{B(D^+\to\bar{K}^0\pi^+)}$
1	6	0.10
	8	0.20
	10	0.33
	12	0.51
1.2	6	0.15
	8	0.30
	10	0.51
	12	0.76
1.4	6	0.22
	8	0.43
	10	0.71
	12	1.06

C. $D^0\to\bar{K}^0\eta'$

We assume η' to be an SU(3) singlet,

$$\eta'=\frac{1}{\sqrt{3}}(u\bar{u}+d\bar{d}+s\bar{s})=\tfrac{1}{2}(\sqrt{3}P_0+P_{15})\ ,$$

in SU(4). We obtain, in a fashion analogous to the analysis of the last section,

$$\begin{aligned}A(D^0\to\bar{K}^0\eta')&=C_2[\langle\bar{K}^0\,|\,(\bar{s}d)\,|\,0\rangle\langle\eta'\,|\,(\bar{u}c)\,|\,D^0\rangle\\&\quad+\langle\bar{K}^0\eta'\,|\,(\bar{s}d)\,|\,0\rangle\langle0\,|\,(\bar{u}c)\,|\,D^0\rangle]\\&=\frac{C_2}{\sqrt{3}}f_Kf_+(0)(m_D{}^2-m_{\eta'}{}^2)\ .\end{aligned} \tag{32}$$

The flavor-annihilation term vanishes due to the vanishing of the SU(4) structure function f_{ijk}, as one would expect from the appearance of $d\bar{d}$ and $s\bar{s}$ with equal weights in the η'. However, the same reasons which allow us to argue away the flavor-annihilation terms in $D^0\to\bar{K}^0\eta$ conspire to resurrect the flavor-annihilation term in $D^0\to\bar{K}^0\eta'$. For example, if a $s\bar{s}$ pair could not be excited from the vacuum with the same probability as the $d\bar{d}$ pair then the cancellation of the flavor-annihilation term would not be complete.

In the approximation that the $s\bar{s}$ pair is *not* excited from the vacuum and the κ *does not couple* to the closed channel $\bar{K}^0\eta'$, such that $f_0^{\kappa}(q^2)\approx f_0^{\kappa}(0)$, one obtains

$$\begin{aligned}A(D^0\to\bar{K}^0\eta')=\frac{C_2}{\sqrt{3}}[&f_Kf_+(0)(m_D{}^2-m_{\eta'}{}^2)\\&+f_Df_+(0)(m_{\eta'}{}^2-m_K{}^2)]\ .\end{aligned} \tag{33}$$

This leads to (with $f_K=f_D$)

$$B(D^0\to\bar{K}^0\eta')/B(D^+\to\bar{K}^0\pi^+)=0.93\times10^{-2}\ . \tag{34}$$

On the other hand, *if flavor annihilation is absent in* $A(D^0\to\bar{K}^0\eta')$, that is, the amplitude is given by (32), then one obtains (using $f_D\approx f_K$)

$$B(D^0\to\bar{K}^0\eta')/B(D^+\to\bar{K}^0\pi^+)=0.58\times10^{-2}\ . \tag{35}$$

Thus this ratio is not a very sensitive test of the presence of an annihilation term, unless it can be measured very accurately.

D. Final-state interactions

Rescattering in the final state endows the weak decay amplitudes with phases. A number of authors[15] have studied the problem of final-state interactions in $D\to K\pi$ decays.

Let us introduce the decay amplitudes A_1 and A_3 for decays into $I=\frac{1}{2}$ and $\frac{3}{2}$ states and their phases δ_1 and δ_3 as

$$A(D^0\to K^-\pi^+)=\frac{1}{\sqrt{3}}(A_3e^{i\delta_3}-\sqrt{2}A_1e^{i\delta_1}),$$
$$A(D^0\to \bar{K}^0\pi^0)=\frac{1}{\sqrt{3}}(\sqrt{2}A_3e^{i\delta_3}+A_1e^{i\delta_1}), \quad (36)$$
$$A(D^+\to \bar{K}^0\pi^+)=\sqrt{3}A_3e^{i\delta_3}.$$

If the scattering in the final state is elastic then the phase of the weak decay amplitude is the scattering phase shift in the relevant two-body channel. The effect is to generate a complex amplitude ($i=1,3$)

$$A_i(s)e^{i\delta_i}(s)=A_i(s_0)\exp\left[\frac{(s-s_0)}{\pi}\int\frac{\delta_i(s')ds'}{(s'-s_0)(s'-s+i\epsilon)}\right] \quad (37)$$

through the solution of the Muskhelishvili-Omnes integral equation.[16] s_0 is a normalization point and, in our case, we eventually set $s=m_D{}^2$.

If one parametrizes the partial-wave scattering amplitude in the N/D form,[17] then the Muskhelishvili-Omnes function, the exponential in (37), is proportional to $D^{-1}(s)$. A real decay amplitude $A_i^{(0)}(s)$ may then be unitarized by final-state interactions through

$$A_i(s)e^{i\delta_i(s)}=A_i^{(0)}(s)\frac{D_i(s_0)}{D_i(s)}. \quad (38)$$

We choose s_0, the normalization point, to be the πK threshold $s_0=(m_K+m_\pi)^2$, such that the phase vanishes at this point, as it indeed should, and $A_i(s_0)=A_i^{(0)}(s_0)$.

We begin by switching off the final-state interactions and evaluate $A_i^{(i)}(s)$ using (12) and (36) with $\delta_i=0$. We obtain (eventually we set $s=m_D{}^2$)

$$A_1^{(0)}(s)=-(\tfrac{3}{2})^{1/2}[\tfrac{2}{3}C_1f_\pi f_0^{F_s}(s,m_\pi{}^2)-\tfrac{1}{3}C_2f_Kf_0^{D_s}(s,m_K{}^2)-C_2f_Df_0^{\kappa}(s,m_\kappa{}^2)],$$
$$A_3^{(0)}(s)=\frac{1}{\sqrt{3}}[C_1f_\pi f_0^{F_s}(s,m_\pi{}^2)+C_2f_Kf_0^{D_s}(s,m_K{}^2)], \quad (39)$$

where

$$f_0^{F_s}(s,m_\pi{}^2)=f_+(0)(s-m_K{}^2),$$
$$f_0^{D_s}(s,m_K{}^2)=f_+(0)(s-m_\pi{}^2), \quad (40)$$
$$f_0^{\kappa}(s,m_\kappa{}^2)=\frac{\lambda s}{s-m_\kappa{}^2}.$$

We make a simplifying assumption for $D_3(s)$. We assume that there is very little rescattering in the nonresonant $I=\frac{3}{2}$ channel. Thus $\delta_3(s)=0$ and $D_3(s)=1$. $I=\frac{1}{2}$, 0^+ channel, on the other hand, resonates.[14] The *simplest* way to unitarize $A_1^{(0)}(s)$ would be by the prescription (38) where $D_1(s)$ is chosen to have a resonance structure. The unitarized form of (39) is then

$$A_1(s)e^{i\delta_1(s)}=\frac{A_1^{(0)}(s)(s_0-m_\kappa{}^2)}{(s-m_\kappa{}^2+i\gamma k)}, \quad (41)$$

$$A_3(s)e^{i\delta_3(s)}\approx A_3^{(0)}(s), \quad (42)$$

where γ is the reduced width of the κ and k, the three-momentum in the πK center of mass. We have chosen the subtraction point to be the πK threshold, $s_0=(m_K+m_\pi)^2$, and

$$D(s)=(s-m_\kappa{}^2+i\gamma k). \quad (43)$$

$R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$ are then calculated by using

$$R_{00}=\frac{1+2r^2+2\sqrt{2}r\cos\delta}{2+r^2-2\sqrt{2}r\cos\delta}, \quad (44)$$

$$R_{0+}=\frac{1}{9}\left[1+\frac{2}{r^2}-\frac{2\sqrt{2}}{r}\cos\delta\right], \quad (45)$$

where $\delta=\delta_1-\delta_3=\delta_1$ (in our case) and $r=A_3/A_1$.

The results are summarized in Table III. We choose rather a broad κ, $\gamma=1.4$ GeV, to generate $\delta_1=144°$ at $s=m_D{}^2$. m_κ was chosen to be[11] 1.35 GeV. We get a fit to the data with $\lambda/f_+(0)$ in the vicinity of 10 GeV^2, well within the limits of (24).

A word about the effect of final-state interaction of $D^0\to\bar{K}^0\eta$ and $D^0\to\bar{K}^0\eta'$ is in order. Since these decays involve only one isospin amplitude and thus only one overall phase, their rates are unaffected by final-state interactions. Further, since $D\to\bar{K}^0\pi^+$ also depends on a single isospin amplitude, the ratios $B(D^0\to\bar{K}^0\eta)/B(D^+\to\bar{K}^0\pi^+)$ and $B(D^0\to\bar{K}^0\eta')/B(D^+\to\bar{K}^0\pi^+)$ as also $B(D^0\to\bar{K}^0\eta)/B(D^0\to\bar{K}^0\eta')$ are insensitive to the details of final-state interactions.

III. $D\to VP$ DECAYS

A. $D\to K\rho$

Using the Hamiltonian (4) in the factorization approximation one can write the decay amplitude for $D^0\to K^-\rho^+$

TABLE III. $R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$ without final-state interactions. Parameters used: $m_\kappa=1.35$ GeV, $\gamma=1.4$ GeV ($\delta_1=144°$).

f_D/f_π	$\lambda/f_+(0)$ (GeV^2)	R_{00}	R_{0+}
1.0	7	0.18	3.38
	8	0.19	3.85
	9	0.20	4.34
	10	0.21	4.86
1.2	7	0.19	4.04
	8	0.21	4.65
	9	0.22	5.30
	10	0.23	6.00
1.4	7	0.21	4.75
	8	0.23	5.53
	9	0.24	6.36
	10	0.25	7.25

mode as

$$A(D^0 \to K^- \rho^+) = C_1 \langle \rho^+ | (\bar{u}d) | \rangle \langle K^- | (\bar{s}c) | D^0 \rangle + C_2 \langle \rho^+ K^- | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle , \quad (46)$$

where C_1 and C_2 are defined in (7).

We next use the following ingredients:

$$\langle 0 | V_i^\mu | V_j \rangle = \delta_{ij} g_V \epsilon_\mu , \quad (47)$$

where V_i^μ is the vector current and V_j a member of the vector SU(4) 15-plet. i and j are SU(4) indices. g_V is related to f_ρ, the leptonic decay constant of the ρ meson, by $g_V = m_\rho^2 / f_\rho$ with $f_\rho^2/4\pi = 2.0$. The matrix element of a vector current between two pseudoscalar states is given by (9). Using (9) and (47) the first term in (46), the spectator term, is reduced to

$$\langle \rho^+ | (\bar{u}d) | 0 \rangle \langle K^- | (\bar{s}c) | D^0 \rangle = 2\sqrt{2} \epsilon \cdot k g_V f_+^{F^*}(m_\rho^2) . \quad (48)$$

k is the kaon four-momentum. $f_+(q^2)$ gets contribution from a vector particle with flavor content $\bar{s}c$, F^*, and may be written in a single-particle-saturated form as $[f_+(0) = 1$ in SU(4) limit$]$

$$f_+^{F^*}(q^2) = 1/(1 - q^2/m_{F^*}^2) . \quad (49)$$

$f_-(q^2)$ in (9) does not contribute to the process due to the gauge condition $\epsilon \cdot q = 0$.

The flavor-annihilation form in (46) is needed at $q^2 = m_D^2$. An extrapolation of the matrix element through PCAC will almost certainly fail since the energy region 1–2 GeV is populated by resonances. First, we demonstrate the consequences of using a naive PCAC extrapolation to $q^2 = m_D^2$.

By using (8) and the PCAC relation

$$\partial^\mu A_\mu^{6-i7} = m_K^2 f_K P^{6-i7} \quad (50)$$

we obtain

$$\langle \rho^+ K^- | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle = 2\sqrt{2} (\epsilon \cdot k) \left[\frac{m_K^2}{m_D^2 - m_K^2} \right] f_K f_D g_{VPP} , \quad (51)$$

where we have introduced the $V_i \to P_j P_k$ coupling constant via the vertex

$$g_{ijk} = i f_{ijk} \epsilon \cdot (p_j - p_k) g_{VPP} . \quad (52)$$

If we use $g_{VPP}^2/4\pi \simeq 3$ from $\rho \to \pi\pi$ decay width, $f_K = f_D = 0.12$ GeV and $f_\rho^2/4\pi \simeq 2.0$ from $\rho \to e^+e^-$ decay width, we obtain the following ratio of the flavor-annihilation (51) to the spectator term (48) in $A(D^0 \to K^- \rho^+)$:

$$\frac{\text{flavor-annihilation}}{\text{spectator}} \simeq \left| \frac{C_2}{C_1} \frac{m_K^2}{m_D^2} \frac{f_K f_D g_{VPP}}{g_V} \right| \simeq 0.01 . \quad (53)$$

Note that $C_2/C_1 = -\frac{1}{5}$ has helped to pull this ratio down. Thus a naive extrapolation of the flavor-annihilation amplitude to $q^2 = m_D^2$ using PCAC leads to a highly suppressed flavor-annihilation amplitude. If this were indeed true then $D \to K\rho$ decays would occur by the spectator processes only and, as we shall confirm later, $D^0 \to \bar{K}^0 \rho^0$ would be more strongly suppressed than it is observed to be.

It is now known[5] (see Sec. II also) that in order to lift the color suppression of $D^0 \to \bar{K}^0 \pi^0$ mode one needs to enhance the flavor-annihilation amplitude. The same is true of $D^0 \to \bar{K}^0 \rho^0$ amplitude (as we shall see later) also. In the following we carry out a careful analysis of the flavor-annihilation term and study the conspiracy among the various parameters that is required to give the naive PCAC result.

Let us study $\langle K^-(k) | A^{6-i7} | \rho^-(p) \rangle$. The flavor-annihilation matrix element $\langle \rho^+ K^- | A_\mu^{6-i7} | 0 \rangle$ is obtained by the substitution, $p^\mu \to -p^\mu$.

In general,[18]

$$\langle P_i(k) | A_j^\mu | V_k(p) \rangle = f_{ijk} [\epsilon^\mu K_1(q^2) + \epsilon \cdot k (p+k)^\mu K_2(q^2) + \epsilon \cdot k (p-k)^\mu K_3(q^2)] . \quad (54)$$

In the flavor-annihilation term in (46) one needs to contract (54) with q^μ and evaluate it at $q^2 = m_D^2$. PCAC made the detailed knowledge of $K_i(q^2)$ in (54) unnecessary since the divergence of the axial-vector current was replaced by the pseudoscalar field (in this case, K meson). On the other hand one can evaluate $K_i(q^2)$ in the approximation that they are determined by 0^- (K meson) and 1^+ [$Q_1(1270)$] mesons. Note that $Q_1(1270)$ has about a 50% branching ratio[11] into $K\rho$ channel while $Q_2(1400)$ appears to decay almost entirely[11] into $K^*\pi$ channel. We assume after Das, Mathur, and Okubo[18] that $K_2(q^2)$ and $K_3(q^2)$ satisfy unsubtracted dispersion relations but $K_1(q^2)$ satisfies a once-subtracted dispersion relation. The subtraction point is chosen to be the soft-kaon limit where the matrix element of (54) is determined entirely by $K_1(q^2)$; the kinematic factors before $K_2(q^2)$ and $K_3(q^2)$ ensure that they do not contribute in the soft limit. Evaluating $K_i(q^2)$ much in the fashion of Ref. 18 we obtain

$$\begin{aligned} K_1(q^2) &= \frac{g_V}{f_K} + \frac{\sqrt{2} g_A G_S}{m_{Q_1}^2 - m_\rho^2} \frac{q^2 - m_\rho^2}{q^2 - m_{Q_1}^2} \\ K_2(q^2) &= \frac{1}{\sqrt{2}} \frac{g_A G_D}{q^2 - m_{Q_1}^2} , \\ K_3(q^2) &= \frac{2 f_K g_{VPP}}{q^2 - m_K^2} - \frac{\sqrt{2} g_A}{m_{Q_1}^2 (q^2 - m_{Q_1}^2)} \left[G_S + \frac{G_D}{2} (m_\rho^2 - m_K^2) \right] . \end{aligned} \quad (55)$$

Here g_A is defined analogously to g_V in (47) by

$$\langle 0 | A_i^\mu | A_j \rangle = \delta_{ij} g_A \epsilon^\mu . \quad (56)$$

G_S and G_D are the S- and D-wave decay parameters introduced in the $Q_1^0 \to K^- \rho^+$ decay amplitude,

$$A(Q_1^0 \to K^- \rho^+) = G_S \epsilon^{(Q)} \cdot \epsilon^{(\rho)} + G_D \epsilon^{(Q)} \cdot p \epsilon^{(\rho)} \cdot q , \quad (57)$$

where $\epsilon_\mu^{(Q)}$ and $\epsilon_\mu^{(\rho)}$ are the polarization vectors of Q_1 and ρ mesons and q and p their respective momenta. PCAC now requires

$$-K_1(q^2)+(m_\rho^2-m_K^2)K_2(q^2)+q^2K_3(q^2)=2f_K\frac{m_K^2}{q^2-m_K^2}g_{VPP}\ . \qquad (58)$$

Using $K_i(q^2)$ from (55) one finds that PCAC demands a conspiracy among the parameters introduced in (55) such that

$$2f_Kg_{VPP}-\frac{g_V}{f_K}-\frac{\sqrt{2}g_A}{m_{Q_1}^2}\left[G_S\frac{m_\rho^2}{m_{Q_1}^2-m_\rho^2}-\frac{G_D}{2}(m_\rho^2-m_K^2)\right]=0\ . \qquad (59)$$

The analogous relation for $A_1^+\to\rho^+\pi^0$ discussed in Ref. 18 is (note that our f_π and $g_{V,A}$ are related to F_π and $G_{V,A}$ of Ref. 18 through $\sqrt{2}f_\pi=F_\pi$ and $\sqrt{2}g_{V,A}=G_{V,A}$ and our G_S is opposite in sign to their G_S due to the metric used in Ref. 18)

$$2f_\pi g_{\rho\pi\pi}-\frac{g_V}{f_\pi}-\frac{\sqrt{2}g_A}{m_{A_1}^2}\left[G_S\frac{m_\rho^2}{m_{A_1}^2-m_\rho^2}-\frac{G_D}{2}(m_\rho^2-m_\pi^2)\right]=0\ . \qquad (60)$$

If G_S and G_D are related by

$$G_S\simeq\frac{G_D}{2}(m_{A_1}^2-m_\rho^2)\ . \qquad (61)$$

(This is equivalent to $\delta=0$ in Ref. 18.) Then PCAC requires

$$g_{\rho\pi\pi}=\frac{g_V}{2f_\pi^2}=\frac{m_\rho^2}{2f_\rho f_\pi^2}\ . \qquad (62)$$

If, in addition, we use $f_\rho=g_{\rho\pi\pi}$ for the correct normalization of the pion form factor, we recover the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin[19] (KSRF) relation

$$f_\rho^2=\frac{m_\rho^2}{2f_\pi^2} \qquad (63)$$

which is approximately correct. Note also that from what we know of the A_1 width[11] the calculation of Ref. 18 was not particularly successful.

Returning to our problem, note that the individual terms on the left-hand side of (58) are of order $2f_Kg_{VPP}$ while the right-hand side is of order $2f_Kg_{VPP}m_K^2/m_D^2$. Since $m_K^2/m_D^2\simeq0.07$, a fine-tuning among the parameters of (60) is required to give us the naive PCAC result at $q^2=m_D^2$. In addition, in the case of $Q_1\to K\rho$, the final-state three-momentum is only 45 MeV, the mass of Q_1 being very close to the $K\rho$ threshold. As a consequence the decay rate $\Gamma(Q_1\to K\rho)$ is insensitive to the size of G_D which could become rather large. It is conceivable that the fine-tuning required by (60) does not occur and that the left-hand side of (58) is comparable to the individual terms. In this scenario the flavor-annihilation amplitude could be considerably enhanced over its naive PCAC value.

Using (8) and (54) we can write the flavor-annihilation term in (46) in the form

$$\langle\rho^+K^-|(\bar{s}d)|0\rangle\langle0|(\bar{u}c)|D^0\rangle=\sqrt{2}(\epsilon\cdot k)f_DK(m_D^2) \qquad (64)$$

where

$$K(m_D^2)=K_1(m_D^2)-(m_\rho^2-m_K^2)K_2(m_D^2)-m_D^2K_3(m_D^2)\ . \qquad (65)$$

In our calculations we use $K(m_D^2)$ as a free parameter. Finally combining (48) and (64) we get

$$A(D^0\to K^-\rho^+)=(\epsilon\cdot k)\sqrt{2}[2C_1g_Vf_+^{F^*}(m_\rho^2)+C_2f_DK(m_D^2)]\ . \qquad (66)$$

Next we evaluate $A(D^0\to\bar{K}^0\rho^0)$, which in the factorization approximation is written as

$$A(D^0\to\bar{K}^0\rho^0)=C_2[\langle\bar{K}^0|(\bar{s}d)|0\rangle\langle\rho^0|(\bar{u}c)|D^0\rangle+\langle\bar{K}^0\rho^0|(\bar{s}d)|0\rangle\langle0|(\bar{u}c)|D^0\rangle]\ . \qquad (67)$$

To reduce the first term, the spectator term, in (67) we use

$$\langle\bar{K}^0|(\bar{s}d)|0\rangle=-i\sqrt{2}f_KP_K^\mu\ . \qquad (68)$$

Further, the D meson being the lightest 0^- particle with flavor content $\bar{u}c$, we write

$$\partial^\mu A_\mu^{9+i10}=f_Dm_D^2P^{9+i10}\ . \qquad (69)$$

The VPP vertex $\rho^0\to D^0\bar{D}^0$ is written using (52). One finally obtains

$$\langle\bar{K}^0|(\bar{s}d)|0\rangle\langle\rho^0|(\bar{u}c)|D^0\rangle=2(\epsilon\cdot k)f_Df_Dg_{VPP}\left[\frac{m_D^2}{m_D^2-m_K^2}\right]\ . \qquad (70)$$

The flavor-annihilation term in (67) is $-1/\sqrt{2}$ times the flavor-annihilation term in $D^0\to K^-\rho^+$, by an SU(4) rotation. Finally one obtains

$$A(D^0\to\bar{K}^0\rho^0)=(\epsilon\cdot k)C_2\left[2f_Kf_Dg_{VPP}\frac{m_D^2}{m_D^2-m_K^2}-f_DK(m_D^2)\right]\ . \qquad (71)$$

By the same techniques one can also obtain $D^+\to\bar{K}^0\rho^+$ decay amplitude as

$$A(D^+\to\bar{K}^0\rho^+)=(\epsilon\cdot k)2\sqrt{2}\left[C_1 g_V f_+^{F^*}(m_\rho^2)+C_2 f_K f_D g_{VPP}\frac{m_D^2}{m_D^2-m_K^2}\right]. \qquad (72)$$

The spectator-model result is obtained by ignoring the flavor-annihilation term $K(m_D^2)$ in (66) and (71). One then gets[20]

$$A(D^0\to\bar{K}^0\rho^0)/A(D^0\to K^-\rho^+)=\frac{1}{\sqrt{2}}\left[\frac{C_2}{C_1}\right]\xi\ , \qquad (73)$$

where

$$\xi=f_K f_D g_{VPP}/[g_V f_+^{F^*}(m_\rho^2)]\ . \qquad (74)$$

If we use $g_{VPP}^2/4\pi\approx 3$, $g_V=m_\rho^2/f_\rho$, $f_\rho^2/4\pi\simeq 2$, and[21] $m_{F^*}=2.1$ GeV, we get $\xi=0.87$.

In Table IV we have listed $R_{00}^{K\rho}$ and $R_{0+}^{K\rho}$ as functions of the parameter $K(m_D^2)$ for two different values of f_D/f_K. From (71) it is clear that $|K(m_D^2)|\approx 2f_K g_{VPP}\approx 1.6$ GeV implies an annihilation term as large as the spectator term in $A(D^0\to\bar{K}^0\rho^0)$. From Table I we see that a fit to $K\rho$ data can be obtained with $K(m_D^2)\approx -2.0$ to -9.0 GeV. A larger value of f_D/f_K requires a smaller value of $K(m_D^2)$ since f_D scales $K(m_D^2)$ in $A(D^0\to K^-\rho^+)$ and $A(D^0\to\bar{K}^0\rho^0)$.

B. $D\to K^*\pi$

In the factorization approximation, as in (4), the decay amplitude for $D^0\to K^{*-}\pi^+$ is given by

$$A(D^0\to K^{*-}\pi^+)=C_1\langle\pi^+|(\bar{u}d)|0\rangle\langle K^{*-}|(\bar{s}c)|D^0\rangle+C_2\langle\pi^+K^{*-}|(\bar{s}d)|0\rangle\langle 0|(\bar{u}c)|D^0\rangle\ . \qquad (75)$$

The first term in (75), the spectator term, is manipulated by using (8) and PCAC for the $(\bar{s}c)$ axial-vector current (F meson is the lightest 0^- particle with flavor content $\bar{s}c$)

$$\partial^\mu A_\mu^{13+i14}=m_F^2 f_F P^{13+i14} \qquad (76)$$

together with the SU(4) VPP vertex defined in (52) for K^*-D-F vertex. We obtain [note that m_F does not appear in this expression since $m_F^2/(m_F^2-m_\pi^2)\approx 1$]

$$\langle\pi^+|(\bar{u}d)|0\rangle\langle K^{*-}|(\bar{s}c)|D^0\rangle=2\sqrt{2}(\epsilon\cdot k)f_\pi f_F g_{VPP}\ . \qquad (77)$$

The flavor-annihilation term in (75) can be handled the same way as in $D\to K\rho$ decays. We use (8) and (54) to obtain

$$\langle\pi^+K^{*-}|(\bar{s}d)|0\rangle\langle 0|(\bar{u}c)|D^0\rangle=\sqrt{2}(\epsilon\cdot k)f_D\hat{K}(m_D^2)\ , \qquad (78)$$

where $\hat{K}(m_D^2)$ is defined analogously to (65),

$$\hat{K}(m_D^2)=\hat{K}_1(m_D^2)-(m_{K^*}^2-m_\pi^2)\hat{K}_2(m_D^2)-m_D^2\hat{K}_3(m_D^2) \qquad (79)$$

with

$$\hat{K}_1(q^2)=\frac{g_V}{f_\pi}+\frac{\sqrt{2}g_A\hat{G}_S}{(m_{Q_2}^2-m_{K^*}^2)}\frac{(q^2-m_{K^*}^2)}{(q^2-m_{Q_2}^2)}\ ,$$

$$\hat{K}_2(q^2)=\frac{1}{\sqrt{2}}\frac{g_A\hat{G}_D}{(q^2-m_{Q_2}^2)}\ , \qquad (80)$$

$$\hat{K}_3(q^2)=\frac{2f_K g_{VPP}}{q^2-m_K^2}-\frac{\sqrt{2}g_A}{m_{Q_2}^2(q^2-m_{Q_2}^2)}\times\left[\hat{G}_S+\frac{\hat{G}_D}{2}(m_{K^*}^2-m_\pi^2)\right].$$

In (80) we are using $Q_2(1400)$ which appears to have a large branching ratio[11] ($\sim$100%) into the $K^*\pi$ channel instead of $Q_1(1270)$ which appears not to decay into this channel. $\hat{G}_S$ and $\hat{G}_D$ are the decay parameters for $Q_2^0\to K^{*-}\pi^+$, defined analogously to (57) for $Q_1^0\to K^-\rho^+$. There are a few other subtle differences between $K_i(m_D^2)$ of (55) and $\hat{K}_i(m_D^2)$ of (80). Notice, for example, the appearance of f_π in $\hat{K}_1(m_D^2)$ instead of f_K, the masses m_{Q_2} and m_{K^*} instead of m_{Q_1} and m_ρ, and $(m_{K^*}^2-m_\pi^2)$ instead of $(m_\rho^2-m_K^2)$. We expect that $\hat{K}(m_D^2)$, due to symmetry breaking, will be different from $K(m_D^2)$. We shall treat $\hat{K}(m_D^2)$ as a parameter, but, as will be shown later, a fit to $D\to K^*\pi$ data requires $\hat{K}(m_D^2)$ in the same range as $K(m_D^2)$.

Putting (77) and (78) in (75) we get

$$A(D^0\to K^{*-}\pi^+)=\sqrt{2}(\epsilon\cdot k)[2C_1 f_\pi f_F g_{VPP}+C_2 f_D\hat{K}(m_D^2)]\ . \qquad (81)$$

The amplitude for $D^0\to\bar{K}^{*0}\pi^0$ decay, in the factorization approximation, is

TABLE IV. $R_{00}^{K\rho}$ and $R_{0+}^{K\rho}$ as functions of $K(m_D^2)$ and f_D/f_K.

f_D/f_K	$K(m_D^2)$ (GeV)	$R_{00}^{K\rho}$	$R_{0+}^{K\rho}$
1.0	−2.0	0.036	1.80
	−3.0	0.051	2.08
	−4.0	0.066	2.39
	−5.0	0.081	2.71
	−6.0	0.095	3.06
	−7.0	0.110	3.42
	−8.0	0.122	3.81
	−9.0	0.135	4.22
1.2	−2.0	0.049	2.04
	−3.0	0.067	2.41
	−4.0	0.086	2.82
	−5.0	0.103	3.27
	−6.0	0.120	3.74
	−7.0	0.135	4.24

$$A(D^0 \to K^{*0}\pi^0) = C_2[\langle \bar{K}^{*0} | (\bar{s}d) | 0 \rangle \langle \pi^0 | (\bar{u}c) | D^0 \rangle + \langle \bar{K}^{*0}\pi^0 | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle] . \quad (82)$$

The second term, the flavor-annihilation term, is simply $-1/\sqrt{2}$ of the flavor-annihilation term in $A(D^0 \to K^{*-}\pi^+)$, Eq. (81). We exploit (8) and

$$\langle \bar{K}^{*0} | (\bar{s}d) | 0 \rangle = \sqrt{2} g_V \epsilon_\mu \quad (83)$$

to recast the spectator term in (82). The decay amplitude, then, is given by

$$A(D^0 \to \bar{K}^{*0}\pi^0) = (\epsilon \cdot k) C_2 [2 g_V f_+^{F^*}(m_{K^*}^2) - f_D \hat{K}(m_D^2)] . \quad (84)$$

Finally,

$$A(D^+ \to K^{*0}\pi^+) = C_1 \langle \pi^+ | (\bar{u}d) | 0 \rangle \langle \bar{K}^{*0} | (\bar{s}c) | D^0 \rangle + C_2 \langle \bar{K}^{*0} | (\bar{s}d) | 0 \rangle \langle \pi^+ | (\bar{u}c) | D^+ \rangle = 2\sqrt{2} (\epsilon \cdot k) [C_1 f_\pi f_F g_{VPP} + C_2 g_V f_+^{F^*}(m_{K^*}^2)] . \quad (85)$$

In the *absence* of flavor-annihilation terms in (81) and (84) one obtains

$$\frac{A(D^0 \to \bar{K}^{*0}\pi^0)}{A(D^0 \to K^{*-}\pi^+)} = \frac{1}{\sqrt{2}} \left[\frac{C_2}{C_1} \right] 1/\hat{\xi} , \quad (86)$$

where

$$\hat{\xi} = f_\pi f_F g_{VPP} / [g_V f_+^{F^*}(m_{K^*}^2)] . \quad (87)$$

Note that ξ of (74) and $\hat{\xi}$ of (87) would be equal in SU(4) limit. In Ref. 20 they are treated as equal.

In Table V we have compiled $R_{00}^{K^*\pi}$ and $R_{0+}^{K^*\pi}$ as functions of $\hat{K}(m_D^2)$ which we have varied in approximately the same range as $K(m_D^2)$ in Table IV. Note that since f_D scales the annihilation term $\hat{K}(m_D^2)$ in $A(D^0 \to K^{*-}\pi^+)$ and $A(D^0 \to \bar{K}^{*0}\pi^0)$, a larger value of f_F/f_D necessitates a larger value of $\hat{K}(m_D^2)$. We see from Table V that a fit to $D \to K^*\pi$ data can be obtained with the parameters in the same range as for $D \to K\rho$ data.

TABLE V. $R_{00}^{K^*\pi}$ and $R_{0+}^{K^*\pi}$ as functions of $\hat{K}(m_D^2)$ and f_F/f_D.

f_F/f_D	$\hat{K}(m_D^2)$ (GeV)	$R_{00}^{K^*\pi}$	$R_{0+}^{K^*\pi}$
1.0	−2.0	0.163	5.42
	−3.0	0.192	6.92
	−4.0	0.217	8.60
	−5.0	0.239	10.47
	−6.0	0.257	12.51
	−7.0	0.274	14.75
1.2	−2.0	0.124	3.95
	−3.0	0.150	4.90
	−4.0	0.174	5.94
	−5.0	0.195	7.09
	−6.0	0.214	8.33
	−7.0	0.230	9.68
	−8.0	0.245	11.13
	−9.0	0.259	12.68
	−10.0	0.271	14.33

C. $D \to \bar{K}^0\phi$

This decay proceeds only via the flavor-annihilation term [in absence of Okubo-Zweig-Iizuka-rule[22] violation). The decay amplitude is given by

$$A(D^0 \to \bar{K}^0\phi) = C_2 \langle \bar{K}^0\phi | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle , \quad (88)$$

ϕ $(=s\bar{s})$ in SU(4) is

$$\tfrac{1}{2} P_0 - \sqrt{2/3} P_8 + \frac{1}{2\sqrt{3}} P_{15} .$$

By a SU(4) rotation this amplitude can be related to the flavor-annihilation term in $A(D^0 \to K^-\rho^+)$ in (46). One finds, through the evaluation of the SU(4) structure function f_{ijk}, that the annihilation terms in $A(D^0 \to \bar{K}^0\phi)$ and $A(D^0 \to K^-\rho^+)$ have the same size and sign as one might naively expect by drawing the exchange diagram. One, then, gets

$$A(D^0 \to \bar{K}^0\phi) = \sqrt{2} (\epsilon \cdot k) C_2 f_D K(m_D^2) . \quad (89)$$

$K(m_D^2)$ in (89) is not exactly the same as in (64) and (65) due to SU(4) breaking. For example, m_ϕ^2 replaces m_ρ^2 in the expressions in (55). For the present we ignore these differences and allow $K(m_D^2)$ to vary in the same range as in $D \to K\rho$ problem.

We can compute $B(D^0 \to \bar{K}^0\phi)/B(D^+ \to \bar{K}^0\rho^+)$, a ratio that is not effected by the details of final-state interactions since both decays involve only one isospin amplitude. In Table VI we have compiled this ratio for $f_D = f_K = 0.12$ GeV and $\tau_{D^+}/\tau_{D^0} = 2.5$. Since[3] $B(D^+ \to \bar{K}^0\rho^+) = (14.1 \pm 4.1 \pm 2.7)\%$, our model calculation shows that $B(D^0 \to \bar{K}^0\phi)$ of the order of 1% is quite likely. Note that a branching ratio for $D^0 \to \bar{K}^0\phi$ of order 1% has been estimated in the past.[5,23]

D. $D^0 \to \bar{K}^0\omega$

This decay, like $D^0 \to \bar{K}^0\rho^0$, proceeds via the color-suppressed spectator process as well as the flavor-annihilation process. The decay amplitude is given by

$$A(D^0 \to \bar{K}^0\omega) = C_2[\langle \bar{K}^0 | (\bar{s}d) | 0 \rangle \langle \omega | (\bar{u}c) | D^0 \rangle + \langle \bar{K}^0\omega | (\bar{s}d) | 0 \rangle \langle 0 | (\bar{u}c) | D^0 \rangle] . \quad (90)$$

In SU(4), ω $[=(u\bar{u}+d\bar{d})/\sqrt{2}]$ is $1/\sqrt{2} P_0 + 1/\sqrt{3} P_8 + 1/\sqrt{6} P_{15}$. An evaluation of the SU(4) structure function shows that the spectator term in (90) has the same sign as the spectator term in $A(D^0 \to \bar{K}^0\rho^0)$; however, the flavor-annihilation terms have the opposite signs. This is intuitively expected since the $u\bar{u}$ content of ρ^0 and ω is the same but the $d\bar{d}$ content has the opposite sign. One obtains

TABLE VI. $B(D^0\to\bar K^0\phi)/B(D^+\to\bar K^0\rho^+)$ in column 2 and $B(D^0\to\bar K^0\omega)/B(D^+\to\bar K^0\rho^+)$ in column 3 as functions of $K(m_D^2)$. We use $f_D=f_K=0.12$ GeV.

$K(m_D^2)$ (GeV)	$\frac{B(D^0\to\bar K^0\phi)}{B(D^+\to\bar K^0\rho^+)}$	$\frac{B(D^0\to\bar K^0\omega)}{B(D^+\to\bar K^0\rho^+)}$
−2.0	0.0075	8.94×10^{-4}
−3.0	0.017	0.010
−4.0	0.030	0.030
−5.0	0.047	0.061
−6.0	0.067	0.101
−7.0	0.092	0.153
−8.0	0.120	0.214
−9.0	0.152	0.286

$$A(D^0\to\bar K^0\omega)=(\epsilon\cdot k)C_2\left[2f_Kf_Dg_{VPP}\frac{m_D^2}{m_D^2-m_K^2}+f_DK(m_D^2)\right]. \tag{91}$$

Since ω and ρ are almost degenerate in mass, $K(m_D^2)$ in (91) is almost the same as in $D^0\to\bar K^0\rho^0$ problem. For $K(m_D^2)\approx-1.6$ GeV this rate is very strongly suppressed. In Table VI we have tabulated $B(D^0\to\bar K^0\omega)/B(D^+\to\bar K^0\rho^+)$ for $f_D=f_K=0.12$ GeV. We note that for $-10\le K(m_D^2)\le-3.0$ GeV, $B(D^0\to\bar K^0\omega)>B(D^0\to\bar K^0\phi)$. Recalling that[3] $B(D^+\to\bar K^0\rho^+)=(14.1\pm4.1\pm2.7)\%$ we find that $B(D^0\to\bar K^0\omega)\approx3$–$4\%$ can be obtained with a value of $K(m_D^2)$ in the range required to fit $D\to K\rho$ and $D\to K^*\pi$ data. For the same values of $K(m_D^2)$, $B(D^0\to\bar K^0\phi)\approx1$–$2\%$ can be secured.

IV. CONCLUSIONS

In Sec. II we studied the Cabibbo-angle-favored $D\to PP$ decay. The $D\to K\pi$ decay amplitudes were written down in terms of the matrix elements of the hadronic weak currents in a factorization approximation. Single-particle-dominated dispersion relations were postulated for these hadronic matrix elements. The naive-spectator-model results are recovered from this analysis in the limits $f_\pi=f_K$ and $f_0^{F_s}(m_\pi^2)=f_0^{D_s}(m_K^2)$ and the neglect of the flavor-annihilation term proportional to $f_0^\kappa(m_D^2)$ in (12).

We showed that if $f_0^\kappa(q^2)$, which appears in the flavor-annihilation channel only, satisfies an unsubtracted dispersion relation then color suppression of $D^0\to\bar K^0\pi^0$ is not lifted. On the other hand, if $f_0^\kappa(q^2)$ satisfies a once-subtracted dispersion relation then it is possible to find a value of the new parameter, λ, introduced in (22) within the range required by (24), such that color suppression of $D^0\to\bar K^0\pi^0$ is lifted. Another way to see this result is that in the flavor-annihilation channel $f_+(q^2)$, introduced in (10), appears multiplied by a mass-suppression factor of $(m_K^2-m_\pi^2)$ while $f_-(q^2)$ appears with a large factor of m_D^2. Thus if $f_-(m_D^2)\approx f_+(m_D^2)$ then obviously $f_0^\kappa(m_D^2)$ will be large and the helicity suppression of the flavor-annihilation process will be lifted. This, in turn, lifts the color suppression of $D^0\to\bar K^0\pi^0$.

Experimentally, $f_-(q^2)$ is not accessible in $D\to\bar K l\nu$ due to the small charged-lepton mass. However, theoretical model calculations[8,11] favor $f_-(m_D^2)$ comparable to $f_+(m_D^2)$. Thus the conjecture that $m_D^2f_-(m_D^2)$ might dominate $(m_K^2-m_\pi^2)f_+(m_D^2)$ is very likely to be true.

We have also studied $D^0\to\bar K^0\eta$ and $D^0\to\bar K^0\eta'$ decays. Theoretically these are, surprisingly, not very clean channels. For example, in $D^0\to\bar K^0\eta$ a straightforward SU(4) rotation applied to $D\to K\pi$ amplitudes generates a large annihilation term. This is due to the additional $s\bar s$ content of η. However, SU(3) breaking makes the excitation of a $s\bar s$ pair from the vacuum less likely[13] than, say, a $d\bar d$ pair. This alone would reduce the annihilation term by a factor of 3. Further, since the κ does not appear[14] to couple to the $\bar K^0\eta$ channel, one expects $f_0^\kappa(q^2)$ not to have any structure. If so, then $f_0^\kappa(q^2)\approx f_0^\kappa(0)\approx(m_K^2-m_\eta^2)f_+(0)$, which is vanishingly small due to the closeness of K-meson and η-meson masses. This uncertainty in handling the annihilation term can give rise to an uncertainty of 2 orders of magnitude in the rate for $(D^0\to\bar K^0\eta)$. The ratio $B(D^0\to\bar K^0\eta)/B(D^+\to\bar K^0\pi^+)$ will test the presence of an annihilation term in $A(D^0\to\bar K^0\eta)$ since $A(D^+\to\bar K^0\pi^+)$ does not have an annihilation contribution. We have shown that with $\lambda/f_+(0)$ in the region of 10 GeV^2 one can generate $B(D^0\to\bar K^0\eta)\sim1$–$2\%$. If experiments would measure the branching ratio at this level, it would be an indication of a large flavor-annihilation contribution to $D^0\to\bar K^0\eta$.

Similar uncertainties apply to the flavor-annihilation term in $A(D^0\to\bar K^0\eta')$. However, in this channel one would be surprised if $B(D^0\to\bar K^0\eta')/B(D^+\to\bar K^0\pi^+)$ turned out very different from $\approx10^{-2}$.

Finally we unitarized $D\to K\pi$ decay amplitudes through final-state interactions and showed that it is possible to fit the data with the assumption of a broad κ meson on 0^+, $I=\frac12$ channel. The method of unitarization used here is the simplest one we can use (certainly not the last word on final-state interactions) and shows that once a mechanism for lifting color suppression is found, it is possible to fit $D\to K\pi$ data with final state interactions.

TABLE VII. $R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$ as functions of C_1/C_2. Final-state interactions included. $f_D/f_\pi=1.2$ The κ parameters are as in Table III.

C_1/C_2	$\lambda/f_+(0)$ (GeV^2)	R_{00}	R_{0+}
−5	7	0.19	4.04
	8	0.21	4.65
	9	0.22	5.30
	10	0.23	6.00
−4	5	0.20	4.30
	6	0.22	5.17
	7	0.24	6.13
−3	3	0.21	4.9
	4	0.24	6.4
−2	0.5	0.23	5.60
	0.6	0.23	5.96
	0.7	0.24	6.34

TABLE VIII. $R_{00}^{K\rho}$ and $R_{0+}^{K\rho}$ as functions of C_1/C_2 and $K(m_D{}^2)$. $f_D/f_K=1$ used.

C_1/C_2	$K(m_D{}^2)$ (GeV)	$R_{00}^{K\rho}$	$R_{0+}^{K\rho}$
−5	−2.0	0.036	1.80
	−4.0	0.066	2.39
	−6.0	0.095	3.06
	−8.0	0.122	3.81
	−10.0	0.146	4.65
−4	−2.0	0.052	2.10
	−4.0	0.091	2.94
	−6.0	0.126	3.93
	−8.0	0.157	5.05
−3	−2.0	0.082	2.74
	−4.0	0.133	4.14
	−6.0	0.175	5.84

The approach employed in this paper to $D\to K\pi$ decays is complementary to that used in Ref. 6 but couched in different, and hopefully more familiar, language. The approach of Ref. 6 was largely algebraic where current algebra was used to constrain the decay amplitudes. The approach adopted in this paper is analytic in nature where dispersion relations are invoked for the hadronic matrix elements. The particles are always kept on mass shell.

Finally, we could treat C_1/C_2 as a parameter. The values of C_+ and C_- used in this paper imply $C_1/C_2\simeq-5$. If C_-/C_+ is allowed to rise ("sextet dominance") then C_1/C_2 moves toward -1. The precise relationship between C_-/C_+ and C_1/C_2 is

$$\frac{C_-}{C_+}=2\frac{C_1/C_2-1}{C_1/C_2+1} . \qquad (92)$$

In Table VII we have listed $R_{00}^{K\pi}$ and $R_{0+}^{K\pi}$, computed with final-state interactions (the κ parameters as in Table III) and $f_D/f_\pi=1.2$, as functions of C_1/C_2. It is evident that the effect of lowering the magnitude of C_1/C_2 (equivalent to raising the ratio C_-/C_+) is to simulate the flavor-annihilation term, since less and less of it is needed (λ decreases) to fit the data.

In Sec. III $D\to K\rho$ and $D\to K^*\pi$ decays were analyzed with particular attention paid to the flavor-annihilation terms. As in the $D\to K\pi$ decay we find that substantial flavor-annihilation contribution is indicated in $D\to VP$ decays. Better statistics in future data will be of help in the theoretical analysis. The quality of $D\to VP$ data at present does not warrant the use of complex amplitudes and the inclusion of final-state interactions.

We have also demonstrated that the size of the annihilation term required to fit $D\to K\rho$ and $D\to K^*\pi$ data is consistent with $B(D^0\to\bar{K}^0\phi)\approx1-2\%$ and $B(D^0\to\bar{K}\omega)\approx3-4\%$.

Finally, as in the case of $D\to K\pi$ decays, we could allow the ratio C_-/C_+ to vary also. In Table VIII we show the effect of varying C_1/C_2 on $R_{00}^{K\rho}$ and $R_{0+}^{K\rho}$. As C_1/C_2 rises from -5 toward -1 data require less and less of the annihilation term, i.e., $K(m_D{}^2)$ decreases in magnitude. In this sense raising C_-/C_+ simulates the annihilation process. $D\to K^*\pi$ data show the same qualitative dependence on C_1/C_2.

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