Polarization effects in production and decay of Z^0

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In the standard model, the polarization of Z^0, singly produced by arbitrarily polarized incident electron and positron beams, is obtained in a manifestly covariant way. It is shown that the vector polarization of Z^0 depends on the longitudinal polarization of the incident beams while the tensor polarization of Z^0 depends on both the transverse polarization and longitudinal polarization of the incident beams. We also obtain the polarization effects on the forward-backward asymmetry, the longitudinal asymmetry, and the final polarization of an outgoing fermion in the decay process $Z^0 \to f\bar{f}$.

I. INTRODUCTION

The discovery of the intermediate vector bosons[1,2] $W^{\pm}$ and Z^0 at the CERN $p\bar{p}$ collider renders bright prospects for physics to be studied by two future machines: the Stanford Linear Collider (SLC) and the CERN LEP. Many Z^0's may be seen in the future and hence an investigation of reactions related to Z^0 is very promising. The standard model of Glashow, Weinberg, and Salam[3] is more or less confirmed except for the higher-order corrections and the nature of the symmetry breaking. Nevertheless, many gauge models whose low-energy effects are very similar to those of the standard model can exist. Of course, these different gauge models can predict different phenomena at some level. For this purpose, we concentrate in this paper on the polarization effects in e^-e^+ reactions. Typical examples are the polarization of Z^0, the decay distribution of Z^0, and τ polarization from Z^0 decay.

It is also known[4] that the polarization of an incident electron beam is important in the production and decay process of Z^0 because they change the Z^0 polarization. Experimentally the processes we are considering in this paper are feasible. Polarized high-energy electron beams are available; the e^-e^+ collision experiment with a longitudinally polarized electron beam at the SLC will be done in the near future, and the experiment with transversely polarized e^-e^+ beams at LEP might be possible.

The purpose of this paper is to obtain the vector polarization and rank-2 tensor polarization of Z^0 in a manifestly covariant way through the density-matrix formalism. This formalism makes it easy for the case of arbitrary polarization of incident beams. The vector polarization depends only on the longitudinal polarization of incident beams. On the other hand, the tensor polarization of Z^0 depends on the transverse polarization as well as longitudinal polarization of incident beams in the standard model. Once Z^0 is produced in e^-e^+ annihilation, it decays into various channels. In this case, the density matrix of Z^0 obtained in the production process is useful to discuss the various decay processes of Z^0's. The decay $Z^0 \to f\bar{f}$ (two fermions) is simple enough to discuss the polarization of a final-state fermion. We show that it depends on the polarization of Z^0.

The method discussed in this paper can be used in various other processes involving spin-1 particles. It goes without saying that the polarization effects discussed in this paper can distinguish various gauge theory models.

II. POLARIZATION OF VECTOR BOSON

The pure-state density matrix for the spin-1 state described by the Proca vector $\epsilon^{\mu}(K)$ with a spin state K becomes[5,6]

$$\epsilon^{\mu}(K)\epsilon^{\nu}(K)^{*} = \tfrac{1}{3}I^{\mu\nu} - \frac{iK}{2M}\varepsilon^{\mu\nu\lambda\tau}p_{\lambda}\eta_{\tau} + (\tfrac{1}{3} - \tfrac{1}{2}\,|\,K\,|\,)\eta^{\mu\nu} . \tag{1}$$

Here M and p_{λ} denote the mass and momentum of the spin-1 particle. η^{μ} is the polarization four-vector which is related to the polarization three-vector $\mathbf{s}$ by the rest-to-laboratory Lorentz transformation,

$$\eta^{i} = s^{i} + \frac{p^{i}(\mathbf{p}\cdot\mathbf{s})}{M(E+M)} , \tag{2a}$$

$$\eta^{0} = \mathbf{p}\cdot\mathbf{s}/M , \tag{2b}$$

$$I^{\mu\nu} = -g^{\mu\nu} + p^{\mu}p^{\nu}/M^{2} , \tag{3}$$

$$\eta^{\mu\nu} = 3\eta^{\mu}\eta^{\nu} - I^{\mu\nu} . \tag{4}$$

The mixed-state density matrix can be obtained by introducing the probability weight factor $W(K)$ to Eq. (1) as[7]

$$\rho^{\mu\nu} = \sum_{K} W(K)\epsilon^{\mu}(K)\epsilon^{\nu}(K)^{*} = \tfrac{1}{3}I^{\mu\nu} - \frac{i}{2M}\varepsilon^{\mu\nu\lambda\tau}p_{\lambda}P_{\tau} - \tfrac{1}{2}Q^{\mu\nu} . \tag{5}$$

P_{τ} and $Q^{\mu\nu}$ are called the covariant vector and tensor polarization of the spin-1 particle, respectively.

In particular, the spin-1 density matrix Eq. (5) in the rest frame can be written as[7,8]

$$\rho = \tfrac{1}{3}I + \tfrac{1}{2}\mathbf{P}\cdot\mathbf{S} + \tfrac{1}{4}Q_{ij}(S_iS_j + S_jS_i - \tfrac{4}{3}I\delta_{ij}) , \tag{6}$$

where I and $\mathbf{S}$ are the usual 3×3 spin-1 matrices. Then $\mathbf{P}$ measures the mean spin vector as

$$\mathbf{P}=\mathrm{Tr}(\mathbf{S}\rho) \quad (7)$$

and traceless Q_{ij} measures the mean rank-two spin tensor as

$$Q_{ij}=\mathrm{Tr}[(S_iS_j+S_jS_i-\tfrac{4}{3}I\delta_{ij})\rho] . \quad (8)$$

The purity of states described by the density matrix of Eq. (5) is measured by the parameter ξ defined by

$$\xi=\tfrac{3}{4}P^2+\tfrac{3}{8}Q_{ij}Q_{ij} \quad (9)$$

which ranges from zero for an isotropic state to unity for a pure state. Therefore, the problem of treating the polarization of a spin-1 particle is equivalent to finding out P^μ and $Q^{\mu\nu}$ (or $\mathbf{P}$ and Q_{ij} in its rest frame).

III. POLARIZATION OF Z^0 PRODUCED BY e^-e^+ COLLISION

For arbitrarily polarized incident electron and positron beams, the Z^0 polarization can be calculated. In the standard model, the relevant Lagrangian for our discussion is

$$L=e\sum_f Q_f\bar{\psi}_f\gamma_\mu\psi_f A^\mu +g_Z\sum_f \bar{\psi}_f\gamma_\mu[\epsilon_L^f(1-\gamma_5)+\epsilon_R^f(1+\gamma_5)]\psi_f Z^\mu , \quad (10)$$

where f indicates specific fermions, Q_f is the fermionic electric charge of fermion f, and g_Z, ϵ_L^f, and ϵ_R^f are defined in terms of Weinberg angle θ_W following the notation of Refs. 9 and 10 by

$$g_z=\frac{e}{2\sin\theta_W\cos\theta_W}=(\sqrt{2}G_FM^2)^{1/2} , \quad (11)$$

$$\epsilon_L^f=T_{3L}-Q^f\sin^2\theta_W , \quad (12)$$

$$\epsilon_R^f=T_{3R}-Q^f\sin^2\theta_W . \quad (13)$$

For the Z_0 production by electron and positron beams, T_{3L}, T_{3R}, and Q^f are $-\frac{1}{2}$, 0, and -1, respectively.

The wave vector of Z^0 can be obtained from Eq. (10) as

$$\begin{aligned}\epsilon^\mu&=g_Z\bar{v}(k_2)\gamma_\mu[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)]u(k_1)\sum_{\text{spin}}Z^{\nu *}Z^\mu\\&=g_Z\bar{v}(k_2)\gamma_\mu[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)]\\&\quad\times u(k_1)\left[-g^{\mu\nu}+\frac{p^\mu p^\nu}{M}\right] ,\end{aligned} \quad (14)$$

where ϵ^μ is not normalized but Z^μ is a normalized pure state, and k_1, k_2, and p are momenta of e^-, e^+, and Z^0, respectively.

The covariant density matrix $\rho^{\mu\nu}$ which is normalized as $g_{\alpha\beta}\rho^{\alpha\beta}=-1$ becomes

$$\rho^{\mu\nu}=-\epsilon^\mu\epsilon^{\nu *}/g_{\alpha\beta}\epsilon^\alpha\epsilon^{\beta *} . \quad (15)$$

From Eqs. (14) and (15), one can obtain the explicit value for $\rho^{\mu\nu}$ in the form of Eq. (5) and the vector and tensor polarizations become

$$P^\mu=\frac{\Delta^\mu}{M}\frac{\epsilon_R{}^2(1+\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)-\epsilon_L{}^2(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1+\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)}{\epsilon_R{}^2(1+\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+\epsilon_L{}^2(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1+\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)} , \quad (16)$$

$$\begin{aligned}Q^{\mu\nu}=&\frac{1}{M^2}\Delta^\mu\Delta^\nu-\tfrac{1}{3}I^{\mu\nu}+2\epsilon_L\epsilon_R\frac{s_{1T}^\mu s_{2T}^\nu+s_{1T}^\nu s_{2T}^\mu+\mathbf{s}_{1T}\cdot\mathbf{s}_{2T}(I^{\mu\nu}-\Delta^\mu\Delta^\nu/M^2)}{\epsilon_R{}^2(1+\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+\epsilon_L{}^2(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1+\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)}\\&+\frac{4\epsilon_L\epsilon_R}{M^2}\frac{g^{\mu\nu}k_1\cdot s_{2T}k_2\cdot s_{1T}-k_1\cdot s_{2T}(k_2^\mu s_{1T}^\nu+k_2^\nu s_{1T}^\mu)-k_2\cdot s_{1T}(k_1^\mu s_{2T}+k_1^\mu s_{2T}^\nu)}{\epsilon_R{}^2(1+\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_2)+\epsilon_L{}^2(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1+\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)} ,\end{aligned} \quad (17)$$

where $\hat{\mathbf{k}}_i$ is the unit vector in the $\mathbf{k}_i$ direction, $\Delta^\mu=k_1^\mu-k_2^\mu$, and s_{iT} has only space components $\mathbf{s}_{iT}$ which is transverse to $\mathbf{k}_i$. From Eqs. (16) and (17), one can see that the vector polarization P^μ depends on the longitudinal polarization of the incident beam while the tensor polarization $Q^{\mu\nu}$ depends on the transverse polarization of the incident beam.

If incident beams are longitudinally polarized, we have s_{1T}, $s_{2T}=0$, and if incident beams are transversally polarized, we have $\hat{\mathbf{k}}_2\cdot\mathbf{s}_1$, $\hat{\mathbf{k}}_2\cdot\mathbf{s}_2=0$ in Eqs. (16) and (17). For unpolarized incident electron and positron beams, i.e., $s_1=0$ and $s_2=0$, we obtain

$$P_0^\mu=\frac{1}{M}\Delta^\mu\frac{\epsilon_R{}^2-\epsilon_L{}^2}{\epsilon_R{}^2+\epsilon_L{}^2} , \quad (18)$$

$$Q_0^{\mu\nu}=\frac{1}{M^2}\Delta^\mu\Delta^\nu-\tfrac{1}{3}I^{\mu\nu} . \quad (19)$$

In the center-of-mass frame of e^- and e^+, which is also the rest frame of Z^0, the last term of Eq. (17) does not appear and P^μ is $(0,\mathbf{P})$, Δ is $(0,M\hat{\mathbf{k}}_1)$, and $Q^{\mu\nu}$ has only Q^{ij} components. Therefore, in the c.m. frame of e^-e^+

$$\mathbf{P}=\hat{\mathbf{k}}_1 P , \quad (20a)$$

where

$$P_0=\frac{\epsilon_R{}^2-\epsilon_L{}^2}{\epsilon_R{}^2+\epsilon_L{}^2} , \quad (20b)$$

$$P\equiv\frac{P_0(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+(\hat{\mathbf{k}}_1\cdot\mathbf{s}_1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)}{(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+P_0(\hat{\mathbf{k}}_1\cdot\mathbf{s}_1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)} , \quad (20c)$$

and

$$Q^{ij}=Q_0^{ij}+\tilde{Q}^{ij} , \quad (21a)$$

where Q_0^{ij} and $\tilde{Q}^{ij}$ are polarization-independent and polarization-dependent parts of Q^{ij}, respectively,

$$Q_0^{ij}=\hat{\mathbf{k}}_1^i\hat{\mathbf{k}}_1^j-\tfrac{1}{3}\delta^{ij}\ , \tag{21b}$$

$$\tilde{Q}^{ij}=\frac{2\epsilon_R\epsilon_L}{\epsilon_R{}^2+\epsilon_L{}^2}\frac{s_{1T}^i s_{2T}^j+s_{1T}^j s_{2T}^i+\mathbf{s}_{1T}\cdot\mathbf{s}_{2T}(\hat{\mathbf{k}}_1^i\hat{\mathbf{k}}_1^j-\delta^{ij})}{(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+P_0(\hat{\mathbf{k}}_1\cdot\mathbf{s}_1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)}\ . \tag{21c}$$

The parameter ξ defined in Eq. (9) becomes

$$\xi=\tfrac{1}{4}+\tfrac{3}{4}P^2+\tfrac{3}{8}\tilde{Q}^2\ , \tag{22}$$

where

$$\tilde{Q}=(\tilde{Q}^{ij}\tilde{Q}^{ij})^{1/2}$$
$$=\frac{2\sqrt{2}\epsilon_R\epsilon_L}{\epsilon_R{}^2+\epsilon_L{}^2}\frac{|\mathbf{s}_{1T}||\mathbf{s}_{2T}|}{(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+P_0(\hat{\mathbf{k}}_1\cdot\mathbf{s}_1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)}\ . \tag{23}$$

From Eq. (22), one can see that Z^0 is not in a pure state in general.

If the incident positron beam is not polarized as in the SLC machine, the $\mathbf{s}_2$ terms can be neglected and one obtains

$$P=\frac{P_0+\hat{\mathbf{k}}_1\cdot\mathbf{s}_1}{1+P_0\hat{\mathbf{k}}_1\cdot\mathbf{s}_1}\ , \tag{24}$$

$$Q^{ij}=\hat{\mathbf{k}}_1^i\hat{\mathbf{k}}_2^j-\tfrac{1}{3}\delta^{ij}\ . \tag{21d}$$

Equation (24) was obtained by Prescott.[4] From Eq. (20c), one can see that the vector polarization becomes maximum in magnitude when one of the incident beams is completely longitudinally polarized.

IV. DECAY DISTRIBUTION OF Z^0

Once Z^0's are produced, they decay into various decay modes. To show how the Z^0 density matrix obtained in the previous section can be used, consider in particular the case that Z^0 decays into two fermions (fermion and antifermion). The fermions can be ν, e, μ, τ or quarks such as u, d, s, c, b, t. In the standard model, the transition amplitude for the decay becomes

$$M=g_z\bar{u}(p_1)\gamma_\mu[\epsilon_L'(1-\gamma_5)+\epsilon_R'(1+\gamma_5)]v(p_2)\epsilon^\mu\ , \tag{25}$$

where p_1, p_2, and ϵ^μ are the momenta of two decay products and the Proca vector of the initial Z^0.

If the decay products are unpolarized, the absolute square of the amplitude, after the spin states of decay products are summed and $\epsilon^\mu\epsilon^{\nu *}$ is replaced by the density matrix $\rho^{\mu\nu}$ of Eq. (5), becomes

$$\sum|M|^2=\frac{2g_z{}^2}{3m^2}[(\epsilon_R'^2+\epsilon_L'^2)(M^2-m^2)+6m^2\epsilon_R'\epsilon_L'$$
$$+3(\epsilon_L'^2-\epsilon_R'^2)Mp_1\cdot P$$
$$+3(\epsilon_R'^2-\epsilon_L'^2)Q^{\mu\nu}p_{1\mu}p_{1\nu}]\ , \tag{26}$$

where m is the mass of decay products.

If the values given in Eqs. (16) and (17) are used, and one chooses the vector $\mathbf{k}_1$, $\mathbf{p}_1$, $\mathbf{s}_{1T}$, and $\mathbf{s}_{2T}$ according to[10,11]

$$\hat{\mathbf{k}}_1=-\hat{\mathbf{k}}_1=(0,0,1)\ , \tag{27a}$$

$$\hat{\mathbf{p}}_1=-\hat{\mathbf{p}}_2=(\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta)\ , \tag{27b}$$

$$\mathbf{s}_{1T}=|\mathbf{s}_{1T}|(\cos\phi_1,\sin\phi_1,0)\ , \tag{28a}$$

$$\mathbf{s}_{2T}=|\mathbf{s}_{2T}|(\cos\phi_2,\sin\phi_2,0)\ , \tag{28b}$$

one obtains the angular distribution for one fermion of Z^0 decay products:

$$\frac{d\Gamma}{d\Omega}=\frac{m^2}{16M\pi^2}\left[1-\frac{4m^2}{M^2}\right]^{1/2}\sum|M|^2$$
$$=\frac{g_z{}^2M}{32\pi^2}\left[1-\frac{4m^2}{M^2}\right]^{1/2}(\epsilon_R'^2+\epsilon_L'^2)$$
$$\times\left[R(m^2)+2\cos\theta\left[1-\frac{4m^2}{M^2}\right]^{1/2}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]P+\frac{1}{\sqrt{2}}\sin^2\theta\cos(2\phi-\phi_1-\phi_2)\left[1-\frac{4m^2}{M^2}\right]\tilde{Q}\right]\ , \tag{29}$$

where $R(m^2)$ is defined as

$$R(m^2)=1+\left[1-\frac{4m^2}{M^2}\right]\cos^2\theta+8\frac{m^2}{M^2}\left[\frac{\epsilon_R'\epsilon_L'}{\epsilon_R'^2+\epsilon_L'^2}\right]\ . \tag{30}$$

The charge asymmetry is defined by

$$A_{\rm ch}(\theta,\phi)=\frac{(d\Gamma/d\Omega)_f-(d\Gamma/d\Omega)_{\bar f}}{(d\Gamma/d\Omega)_f+(d\Gamma/d\Omega)_{\bar f}}\ , \tag{31}$$

where $(d\Gamma/d\Omega)_{\bar f}$ implies the decay distribution when the antifermion is going out in (θ,ϕ) direction instead of the fermion, and $(d\Gamma/d\Omega)_f$ is defined similarly. This is the same as the forward-backward asymmetry $A_{\rm FB}$ of the outgoing fermion defined by

$$A_{\rm FB}(\theta,\phi)=\frac{\frac{d\Gamma}{d\Omega}(\theta,\phi)-\frac{d\Gamma}{d\Omega}(\pi-\theta,\pi+\phi)}{\frac{d\Gamma}{d\Omega}(\theta,\phi)+\frac{d\Gamma}{d\Omega}(\pi-\theta,\pi+\phi)}\ , \tag{32}$$

where $d\Gamma/d\Omega$ is for the fermion f. From Eqs. (29) and (31), one obtains at the Z^0 pole

$$A_{\rm ch}(\theta,\phi)=A_{\rm FB}(\theta,\phi)$$
$$=\frac{2\cos\theta\left[1-\frac{4m^2}{M^2}\right]^{1/2}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]P}{R(m^2)+\frac{1}{\sqrt{2}}\sin^2\theta\cos(2\phi-\phi_1-\phi_2)\widetilde{Q}}\ . \tag{33}$$

In particular, for small m^2/M^2, $R(m^2)$ becomes $1+\cos^2\theta$. If incident particles are longitudinally polarized, Eq. (33) becomes

$$A_{\rm ch}(\theta,\phi)=\frac{2\cos\theta}{1+\cos^2\theta}\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}P \tag{34}$$

which is given in Ref. 4.

Sometimes the forward-backward asymmetry $A_{\rm FB}$ is defined by integrating out the solid angle to give

$$A_{\rm FB}=\frac{\int_0^{2\pi}d\phi\int_0^1\frac{d\Gamma}{d\Omega}d\cos\theta-\int_0^{2\pi}d\phi\int_{-1}^0\frac{d\Gamma}{d\Omega}d\cos\theta}{\int_0^{2\pi}d\phi\int_0^1\frac{d\Gamma}{d\Omega}d\cos\theta+\int_0^{2\pi}d\phi\int_{-1}^0\frac{d\Gamma}{d\Omega}d\cos\theta}\ , \tag{35}$$

$$=\frac{3}{4}\frac{\left[1-\frac{4m^2}{M^2}\right]^{1/2}}{1-\frac{m^2}{M^2}+6\frac{m^2}{M^2}\frac{\epsilon_R'\epsilon_L'}{\epsilon_R'^2+\epsilon_L'^2}}\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}P\ . \tag{36}$$

In the SLC, the longitudinal polarization of the incident electron beam can be reversed without difficulty, and hence, it is useful to consider the longitudinal asymmetry defined by

$$A_L(\theta)=\frac{\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=+)-\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=-)}{\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=+)+\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=-)}\ , \tag{37}$$

where s_{1T} and s_{2T} are assumed to be zero. Since the differential cross section is of the form

$$\frac{d\sigma}{d\Omega}\propto[\epsilon_R{}^2(1+\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)+\epsilon_L{}^2(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1)(1+\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)]\frac{d\Gamma}{d\Omega}\ , \tag{38}$$

one obtains explicitly

$$A_L(\theta)=\frac{\widetilde{P}+\frac{2\cos\theta}{R(m^2)}\left[1-\frac{4m^2}{M^2}\right]^{1/2}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]}{1+\frac{2\cos\theta}{R(m^2)}\left[1-\frac{4m^2}{M^2}\right]^{1/2}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2-\epsilon_L'^2}\right]\widetilde{P}}\ , \tag{39}$$

where $\widetilde{P}$ is defined by

$$\widetilde{P}=\frac{P_0-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2}{1-P_0\hat{\mathbf{k}}_2\cdot\mathbf{s}_2}\ . \tag{40}$$

If the incident positron beam is unpolarized and the mass of the outgoing fermion is small, i.e., $m^2\ll M^2$, one obtains[4]

$$A_L(\theta)=\frac{P_0+\frac{2\cos\theta}{1+\cos^2\theta}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]}{1+\frac{2\cos\theta}{1+\cos^2\theta}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]P_0}\ . \tag{41}$$

The longitudinal polarization is sometimes defined by integrating out the solid angle to give

$$A_L=\frac{1}{P_1^{\parallel}}\frac{\int d\Omega\left[\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=P_1^{\parallel})-\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=-P_1^{\parallel})\right]}{\int d\Omega\left[\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=P_1^{\parallel})+\frac{d\sigma}{d\Omega}(\theta,\hat{\mathbf{k}}_1\cdot\mathbf{s}_1=-P_1^{\parallel})\right]} \tag{42}$$

and it becomes

$$A_L=\widetilde{P}=\frac{P_0-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2}{1-P_0\hat{\mathbf{k}}_2\cdot\mathbf{s}_2}\ . \tag{43}$$

For the unpolarized incident beam, one obtains

$$A_L=P_0=\frac{\epsilon_R{}^2-\epsilon_L{}^2}{\epsilon_R{}^2+\epsilon_L{}^2} \tag{44}$$

which is small. But since $\widetilde{P}$ can be increased with the negative helicity of a positron beam, $A_L(\theta)$ and A_L can be enhanced if polarized positron beams are available.

If muon pairs are produced by the collision of a polarized electron beam and an unpolarized positron beam at the Z^0 resonance, as expected from SLC and LEP, and if electron-muon universality is assumed, one obtains, from Eqs. (36) and (44),

$$A_{\rm FB}=\tfrac{3}{4}(A_L)^2\ . \tag{45}$$

The transverse polarization asymmetry is defined by

$$A_\perp(\theta,\phi)=\frac{1}{P_1^\perp P_2^\perp}\left[\frac{\frac{d\sigma}{d\Omega}\left[|\mathbf{s}_{1T}|=P_1^\perp,|\mathbf{s}_{2T}|=P_2^\perp,\phi_1=-\phi_2=\frac{\pi}{2}\right]}{\frac{d\sigma}{d\Omega}(\text{unpolarized})}-1\right] \tag{46}$$

and from Eq. (29) one obtains at the Z^0 peak

$$A_\perp(\theta,\phi)=\frac{\left[1-\frac{4m^2}{M^2}\right]\sin^2\theta\cos2\phi\left[\frac{2\epsilon_R\epsilon_L}{\epsilon_R{}^2+\epsilon_L{}^2}\right]}{R(m^2)+2P_0\cos\theta\left[1-\frac{4m^2}{M^2}\right]^{1/2}\left[\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]}\,. \tag{47}$$

This asymmetry is of great interest at LEP. The azimuthal distribution of the outgoing fermions produced by the transversely polarized electron-positron collision becomes

$$\langle\cos2\phi\rangle=\frac{\int\cos2\phi(d\sigma/d\Omega)d\Omega}{\int(d\sigma/d\Omega)d\Omega}=\frac{\frac{1}{4}P_1^\perp P_2^\perp\left[1-\frac{4m^2}{M^2}\right]\frac{2\epsilon_R\epsilon_L}{\epsilon_R{}^2+\epsilon_L{}^2}}{1-\frac{m^2}{M^2}+\frac{6m^2}{M^2}\left[\frac{\epsilon_R'\epsilon_L'}{\epsilon_R'^2+\epsilon_L'^2}\right]} \tag{48}$$

and $\langle\sin2\phi\rangle$ is related with the γ-Z^0 interference term which is relatively small at the Z^0 peak.

So far, we have considered the asymmetries in the $e^-e^+\to f\bar{f}$ process on the Z^0 pole, but for completeness in treating the process off the Z^0 resonance as well, single photon exchange must be considered. From Eq. (10) the transition amplitude for the process $e^-e^+\to\gamma,\ Z\to f\bar{f}$ becomes

$$M=\frac{ie^2Q_eQ_f}{s}\bar{v}(k_2s_2)\gamma^\mu u(k_1s_1)\bar{u}(p_1s_1')\gamma_\mu v(p_2s_2')$$
$$+\frac{ig_z{}^2}{s-M^2+iM\Gamma}\bar{v}(k_2s_2)\gamma_\mu[\epsilon_L(1-\gamma_5)+\epsilon_R(1+\gamma_5)]u(k_1s_1)\bar{u}(p_1s_1')\gamma^\mu[\epsilon_L'(1-\gamma_5)+\epsilon_R'(1+\gamma_5)]v(p_2s_2')\,. \tag{49}$$

This amplitude is applicable to all final fermions except $f=e$ or ν. After the spin sum of the final fermions, the absolute square of the amplitude becomes

$$|M|^2=\frac{N_c}{16m^2m_f{}^2}\left\{e^4Q_f{}^2\left[(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)\left[1+\cos^2\theta+\frac{4}{s}m_f{}^2\sin^2\theta\right]+|\mathbf{s}_{1T}||\mathbf{s}_{2T}|\left[1-\frac{4}{s}m_f{}^2\right]\sin^2\theta\cos(2\phi-\phi_1-\phi_2)\right]\right.$$
$$-\frac{2e^2Q_fg_z{}^2s(s-M^2)}{(s-M^2)^2+M^2\Gamma^2}\left\{(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)\left[(\epsilon_R+\epsilon_L)(\epsilon_R'+\epsilon_L')\left[1+\cos^2\theta+\frac{4}{s}m_f{}^2\sin^2\theta\right]\right.\right.$$
$$\left.+2\cos\theta\left[1-\frac{4}{s}m_f{}^2\right]^{1/2}(\epsilon_R-\epsilon_L)(\epsilon_R'-\epsilon_L')\right]$$
$$+(\hat{\mathbf{k}}_1\cdot\mathbf{s}_1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)\left[(\epsilon_R-\epsilon_L)(\epsilon_R'+\epsilon_L')\left[1+\cos^2\theta+\frac{4}{s}m_f{}^2\sin^2\theta\right]\right.$$
$$\left.+2\cos\theta\left[1-\frac{4}{s}m_f{}^2\right]^{1/2}(\epsilon_R+\epsilon_L)(\epsilon_R'-\epsilon_L')\right]$$
$$\left.+|\mathbf{s}_{1T}||\mathbf{s}_{2T}|\left[1-\frac{4}{s}m_f{}^2\right](\epsilon_R+\epsilon_L)(\epsilon_R'+\epsilon_L')\sin^2\theta\cos(2\phi-\phi_1-\phi_2)\right\}$$
$$-\frac{2e^2Q_fg_z{}^2sM\Gamma}{(s-M^2)^2+M^2\Gamma^2}|\mathbf{s}_{1T}||\mathbf{s}_{2T}|\left[1-\frac{4}{s}m_f{}^2\right](\epsilon_R-\epsilon_L)(\epsilon_R'+\epsilon_L')\sin^2\theta\sin(2\phi-\phi_1-\phi_1)$$
$$+\frac{4g_z{}^4s^2}{(s-M^2)^2+M^2\Gamma^2}\left\{(1-\hat{\mathbf{k}}_1\cdot\mathbf{s}_1\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)\left[(\epsilon_R{}^2+\epsilon_L{}^2)(\epsilon_R'^2+\epsilon_L'^2)\left[1+\cos^2\theta-\frac{4}{s}m_f{}^2\cos^2\theta\right]\right.\right.$$
$$+2\cos\theta(\epsilon_R{}^2-\epsilon_L{}^2)(\epsilon_R'^2-\epsilon_L'^2)\left[1-\frac{4}{s}m_f{}^2\right]^{1/2}$$

$$+\frac{8}{s}m_f^2(\epsilon_R^2+\epsilon_L^2)\epsilon'_R\epsilon'_L\Bigg]$$

$$+(\hat{\mathbf{k}}_1\cdot\mathbf{s}_1-\hat{\mathbf{k}}_2\cdot\mathbf{s}_2)\Bigg[(\epsilon_R^2-\epsilon_L^2)(\epsilon'^2_R+\epsilon'^2_L)\left[1+\cos^2\theta-\frac{4}{s}m_f^2\cos^2\theta\right]$$

$$+2\cos\theta(\epsilon_R^2+\epsilon_L^2)(\epsilon'^2_R-\epsilon'^2_L)\left[1-\frac{4}{s}m_f^2\right]^{1/2}$$

$$+\frac{8}{s}m_f^2(\epsilon_R^2-\epsilon_L^2)\epsilon'_R\epsilon'_L\Bigg]$$

$$+2\,|\,\mathbf{s}_{1T}|\,|\mathbf{s}_{2T}\,|\left[1-\frac{4}{s}m_f^2\right]\epsilon_R\epsilon_L(\epsilon'^2_R+\epsilon'^2_L)\sin^2\theta\cos(2\phi-\phi_1-\phi_2)\Bigg\}\Bigg], \quad (50)$$

where the number N_c takes into account the color degree of freedom. Schiller[12] has obtained a special case of Eq. (50) when the final-fermion masses are neglected. Hollik[13] has considered the e^-e^+ annihilation with polarized beams in multiboson-exchange models and Eq. (50) is contained in his result as a special case.

Since the differential cross section becomes

$$\frac{d\sigma}{d\Omega}=\frac{m^2m_f^2}{4\pi^2 s}\left[1-\frac{4m_f^2}{s}\right]^{1/2}|M|^2\,, \quad (51)$$

one can obtain various asymmetries from these equations. In particular, at the Z^0 peak, i.e., $S=M^2$, the magnitude of the last term in Eq. (50) becomes about 10^2 times larger than the rest terms. Therefore, if one considers only the last term in Eq. (50), one can obtain the asymmetries given in Eqs. (33), (39), and (47).

It is also noted that, if incident electron and positron beams are unpolarized and final-fermion masses are small ($m_f^2 \ll s$), one can obtain the so-called master formula for the spin-averaged cross section given in Ref. 14 from Eqs. (50) and (51). The contribution of one photon exchange besides Z^0 exchange can be obtained from these equations.

V. FINAL-LEPTON POLARIZATION

Since the outgoing fermions in the process, $e^-e^+\to Z^0\to f\bar{f}$, may be polarized and a direct measurement of τ appears to be possible,[15] the calculation of the final-fermion polarization is very useful to check e-μ-τ universality and other gauge-model predictions.

In the standard model, the transition amplitude for the process $Z^0\to f\bar{f}$, Eq. (25), can be used to obtain the final state of f

$$u^f=g_z\sum_{\text{spin}}u(p_1)\bar{u}(p_1)\gamma_\mu[\epsilon'_L(1-\gamma_5)+\epsilon'_R(1+\gamma_5)]\times\sum_{\text{spin}}v(p_2)\bar{v}(p_2)\epsilon^\mu$$

$$=g_z\frac{m+\not{p}_1}{2m}\gamma_\mu[\epsilon'_L(1-\gamma_5)+\epsilon'_R(1+\gamma_5)]\frac{-m+\not{p}_2}{2m}\epsilon^\mu\,. \quad (52)$$

The density matrix of the outgoing fermion becomes

$$\rho=u^f\bar{u}^f/\mathrm{Tr}(u^f\bar{u}^f)\,, \quad (53)$$

where $\epsilon^\mu\epsilon^{\nu *}$ is replaced by the density matrix $\rho^{\mu\nu}$ of Eq. (5) or Eq. (15). Then the fermion polarization four-vector η^μ can be obtained,

$$\eta^\mu=\mathrm{Tr}(\gamma^\mu\gamma_5\rho)\,. \quad (54)$$

Using the method given in Refs. 10 and 16, the longitudinal polarization and transverse polarization of the outgoing fermion become

$$\hat{\mathbf{p}}_1\cdot\mathbf{s}'=\frac{\left[1-\frac{4m^2}{M^2}\right]^{1/2}\left[\frac{\epsilon'^2_R-\epsilon'^2_L}{\epsilon'^2_R+\epsilon'^2_L}\right]R+2\cos\theta\left[1+\frac{4m^2}{M^2}\frac{\epsilon'_R\epsilon'_L}{\epsilon'^2_r+\epsilon'^2_L}\right]P}{\left[1-\frac{4m^2}{M^2}\right]R+\frac{4m^2}{M^2}\frac{(\epsilon'_R+\epsilon'_L)^2}{\epsilon'^2_R+\epsilon'^2_L}+2\cos\theta\left[1-\frac{4m^2}{M^2}\right]^{1/2}\frac{\epsilon'^2_L-\epsilon'^2_L}{\epsilon'^2_R+\epsilon'^2_L}P} \quad (55)$$

and

$$\mathbf{s}'_T=\frac{2m}{M}\left\{\left[\frac{2\epsilon'_R\epsilon'_L}{\epsilon_R'^2+\epsilon_L'^2}P+\left[1-\frac{4m^2}{M^2}\right]^{1/2}\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\cos\theta\right]\hat{\mathbf{p}}_1\times(\hat{\mathbf{k}}_1\times\hat{\mathbf{p}}_1)\right.$$

$$\left.-\frac{1}{\sqrt{2}}\left[1-\frac{4m^2}{M^2}\right]^{1/2}\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\tilde{Q}\hat{\mathbf{p}}_1\times[\hat{p}_1\times(\mathbf{s}_{1T}\cdot\mathbf{s}_{2T}\hat{\mathbf{k}}_1\cos\theta+\hat{\mathbf{p}}_1\cdot\mathbf{s}_{2T}\mathbf{s}_{1T}+\hat{\mathbf{p}}_1\cdot\mathbf{s}_{1T}\mathbf{s}_{2T})]\right\}$$

$$\times\left[\left[1-\frac{4m^2}{M^2}\right]R+\frac{4m^2}{M^2}\frac{(\epsilon'_R+\epsilon'_L)^2}{(\epsilon_R'^2+\epsilon_L'^2)}+2\cos\theta\left[1-\frac{4m^2}{M^2}\right]^{1/2}\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}P\right]^{-1}, \tag{56}$$

where R is defined by

$$R=1+\cos^2\theta+\frac{1}{\sqrt{2}}\tilde{Q}\sin^2\theta\cos(2\phi-\phi_1-\phi_2)\ . \tag{57}$$

If the final outgoing fermion is a τ lepton and its mass m is neglected (compared with M), the longitudinal polarization of τ becomes

$$\hat{\mathbf{p}}_1\cdot\mathbf{s}_\tau=\frac{\left[\dfrac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]R+2\cos\theta P}{R+2\cos\theta\dfrac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}P} \tag{58}$$

and the transverse polarization becomes negligible. The longitudinal τ polarization depends on the longitudinal polarization of incident electron beams through P in Eq. (20) and also depends on the transverse polarization of incident beams through R which depends on $\tilde{Q}$. This is the same as the τ longitudinal polarization asymmetry $A_{\tau\text{pol}}$ on Z^0 resonance defined by

$$A_{\tau\text{pol}}(\theta,\phi)=\frac{\dfrac{d\sigma}{d\Omega}(e^+e^-\to\tau^+\tau_L^-)-\dfrac{d\sigma}{d\Omega}(e^+e^-\to\tau^+\tau_R^-)}{\dfrac{d\sigma}{d\Omega}(e^+e^-\to\tau^+\tau_L^-)+\dfrac{d\sigma}{d\Omega}(e^+e^-\to\tau^+\tau_R^-)}\ . \tag{59}$$

The contribution of the photon exchange to $A_{\tau\text{pol}}$ can be calculated, but it is small at the Z^0 peak.

If the incident electron beams are unpolarized, Eq. (58) becomes

$$\hat{\mathbf{p}}_1\cdot\mathbf{s}_\tau=\frac{\left[\dfrac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_R'^2}\right]+\dfrac{2\cos\theta}{1+\cos^2\theta}P_0}{1+\dfrac{2\cos\theta}{1+\cos^2\theta}\left[\dfrac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]P_0}\ . \tag{60}$$

One can see that this is the same as $A_L(\theta)$ of Eq. (39) if e-μ-τ universality is assumed. This asymmetry is of great interest at the SLC and LEP.

If one integrates out the solid angles in the numerator and denominator of Eq. (55) or Eq. (59), one obtains

$$\hat{\mathbf{p}}_1\cdot\mathbf{s}_\tau=\left[1-\frac{4m^2}{M^2}\right]^{1/2}\times\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\Bigg/\left[1-\frac{m^2}{M^2}+6\frac{m^2}{M^2}\frac{\epsilon'_R\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\right]\approx\frac{\epsilon_R'^2-\epsilon_L'^2}{\epsilon_R'^2+\epsilon_L'^2}\quad\text{for } m\ll M\ . \tag{61}$$

In general the outgoing fermions have longitudinal as well as transverse polarizations. Equation (56) implies that the transverse polarization of one of the outgoing fermions is negligible for a small fermion mass if all the other particle polarizations are summed, but it is not so for a large fermion mass (compared with Z^0 mass).

VI. DISCUSSION

We have discussed in the density-matrix formalism the Z^0 polarization produced by the e^+e^- collision. The result has been used for its decay processes into two fermions, in particular. The method described here can also be applied to various others cases involving spin-1 particles, e.g., production and decay processes of W. We have shown that, once Z^0's are produced, they will have a definite polarization which is described by the density matrix. Various decay processes of Z^0 can be described through this density matrix and sometimes one does not need to go into the tedious work of considering the production and decay processes simultaneously.

In particular, we have applied our result to the $e^-e^+\to f\bar{f}$ process in the standard electroweak model of Glashow, Weinberg, and Salam.[3] Our consideration has been limited to the tree level, but the final fermion mass is included explicitly. The result obtained here may be useful for future discovery of new heavy quarks or new heavy leptons.[17]

Our method can be applied to multiboson-exchange models as well.[9,18] In particular, if another neutral heavy boson $Z^{0\prime}$ exists, as predicted by the left-right symmetry model of Barger, Ma, and Whisnant,[9] one can get the similar formula for A_{FB}, A_L, and $A_\perp$ in terms of ϵ'_R and ϵ'_L, where ϵ' is the $Z^{0\prime}$ coupling defined similarly as the Z^0 coupling. Specific polarization effects and asymmetries for supersymmetric particle production in e^-e^+ collision with polarized beams have been considered in

Ref. 19 and our method can be applied there as well.

It is very important to find new particles and new interactions which are not contained in the standard model. But also the standard electroweak theory allows a systematic calculation of radiative correction even though the choice of the renormalization parameters is not, at present, unique beyond the tree level. The radiative corrections to various asymmetries in the $e^-e^+ \to \mu^-\mu^+$ process have been considered in Refs. 20–22 up to one-loop level in the standard model. The resulting magnitude from these radiative corrections is very small. Since we expect many Z^0's to be produced in SLC and LEP in the future, precise experimental measurements of various asymmetries at the Z^0 resonance and of the weak-boson masses would lead to a test of the standard model at the one-loop level.

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