

### Gauge-invariant energy-momentum tensor for massive QED

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For massive QED with a gauge-fixing term a candidate for the energy-momentum tensor is presented. Both cases of scalar and spinor matter fields are treated. The energy-momentum tensor is invariant under the restricted gauge transformations which exist in that model. This property guarantees that the unphysical scalar photons do not contribute to the energy-momentum densities. The difference between the translational generators and the energy-momentum observables is pointed out.

The indefiniteness of the space-time metric  $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$  makes it impossible to realize a canonically quantized free Hermitian vector field with four independent degrees of freedom  $A_\mu$  on a positive-metric Hilbert space in a Lorentz-covariant way so that the space-time translation generators have their spectra confined to the forward light cone.<sup>1</sup> This means that vector-field models in general contain unphysical degrees of freedom. An exception is Proca's model<sup>2,1</sup> in which no ghost states are present due to the constraint  $\partial \cdot A = 0$ . Yet this model has problems<sup>3</sup> when the vector field is coupled to charged matter and, in addition, the vector field's propagator is singular in the zero-mass limit<sup>4</sup> which makes it cumbersome to recover<sup>5</sup> QED in that limit. These problems have been resolved<sup>3,6-8</sup> by the inclusion of a so-called "gauge-fixing term"<sup>9</sup> into the massive vector field's Lagrangian. (A coherent textbook treatment of perturbative massive QED with a gauge-fixing term is presented in Ref. 4.) Such models with a gauge-fixing term contain unphysical spin-zero quanta which, however, can be eliminated from the physical asymptotic states through defining the physical subspace  $\mathcal{H}$  by the weak Lorentz condition  $\partial \cdot A^{(+)}\Phi = 0, \forall \Phi \in \mathcal{H}$ .<sup>4</sup> In contrast with zero-mass QED (Refs. 10 and 11) no zero-norm states are left in  $\mathcal{H}$  and all three helicity states of the spin-one boson are physical. The models, however, fit into the general frame of indefinite-metric quantum field theory as formulated by Wightman and Garding.<sup>12</sup> In this framework not all Hermitian operators are physical observables. As observables only those operators on the full Hilbert space can be used which leave invariant the physical subspace and whose restriction to  $\mathcal{H}$  is Hermitian on  $\mathcal{H}$ .<sup>12</sup> It is the aim of this paper to present an energy-momentum tensor for massive QED which is observable in this sense.

The class of models we have in mind is generated by Lagrangians of the type  $\mathcal{L} \equiv \mathcal{L}_A + \mathcal{L}_M$  with  $(a^2 > 0, \kappa^2 > 0)$

$$\mathcal{L}_A \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{a^2}{2}(\partial \cdot A)^2 + \frac{\kappa^2}{2}A_\mu A^\mu, \tag{1}$$

$$F_{\mu\nu} \equiv A_{\nu/\mu} - A_{\mu/\nu},$$

and  $A_\mu$  being a Hermitian vector field on Minkowski

space  $\mathbb{M}^{1,3}$ . The matter part  $\mathcal{L}_M$  contains the matter fields and their coupling to  $A_\mu$  but shall not depend on the vector field's derivatives. Scalar QED chooses a non-Hermitian field  $\phi$  and, with  $U: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ ,

$$\mathcal{L}_M \equiv [(\partial^\mu - ieA^\mu)\phi]^*[(\partial_\mu - ieA_\mu)\phi] - U(\phi^*\phi). \tag{2}$$

Spinor QED is characterized by

$$\mathcal{L}_M \equiv \frac{i}{2}\bar{\psi}\gamma^\mu[(\partial_\mu - ieA_\mu) - (\overleftarrow{\partial}_\mu + ieA_\mu)]\psi - m\bar{\psi}\psi. \tag{3}$$

The vector field equation reads, with  $j^\mu \equiv -\partial \mathcal{L}_M / \partial A_\mu$ ,

$$\partial_\nu F^{\nu\mu} + a^2 \partial \cdot A^{|\mu} + \kappa^2 A^\mu = j^\mu. \tag{4}$$

Introducing  $\tilde{\mathcal{D}}^\mu \equiv \partial^\mu - ieA^\mu$  and  $\overleftarrow{\tilde{\mathcal{D}}}^\mu \equiv \overleftarrow{\partial}^\mu + ieA^\mu$  the current is given, respectively, by  $j^\mu = -e\bar{\psi}\gamma^\mu\psi$  and  $j^\mu = -ie\phi^*(\tilde{\mathcal{D}}^\mu - \overleftarrow{\tilde{\mathcal{D}}}^\mu)\phi$ .

The tensorial fields are quantized via canonical equal-time commutation relations, the nonvanishing ones of which read

$$[\pi^\mu, A_\nu]_{\text{ET}} = -i\delta^\mu_\nu \delta^s, \quad [\pi, \phi]_{\text{ET}} = -i\delta^s, \tag{5}$$

$$[\pi^*, \phi^*]_{\text{ET}} = -i\delta^s$$

with the canonical momenta

$$\pi^\mu \equiv \frac{\partial \mathcal{L}}{\partial A_{\mu/0}} = F^{\mu 0} - a^2 \eta^{\mu 0} \partial \cdot A, \quad \pi \equiv \frac{\partial \mathcal{L}}{\partial \phi_{/0}} = \phi^* \tilde{\mathcal{D}}^0, \tag{6}$$

$$\pi^* \equiv \frac{\partial \mathcal{L}}{\partial \phi^*_{/0}} = \tilde{\mathcal{D}}^0 \phi.$$

Anticommutator quantization rules are imposed on the spinor field:

$$\{\psi, \bar{\psi}\}_{\text{ET}} = \gamma^0 \delta^s, \quad \{\psi, \psi\}_{\text{ET}} = \{\bar{\psi}, \bar{\psi}\}_{\text{ET}} = 0. \tag{7}$$

Choosing  $a = 0$  in  $\mathcal{L}_A$  gives Proca's model which we exclude in order to be able to employ the canonical formalism without having to care for the constraint which the zero component of the vector field equation constitutes for  $a = 0$ .<sup>13</sup> In addition, we exclude from the outset the zero-mass case  $\kappa = 0$  of ordinary QED with a gauge-fixing term since we need factors  $\kappa^{-2}$  well defined for our treatment.

Current conservation  $\partial \cdot j = 0$  renders trivial the dynamics of  $\partial \cdot A$ , the negative-metric field,<sup>14</sup> which is constrained to vanish in Proca's model.  $\partial \cdot A$  solves the free Klein-Gordon equation with mass parameter  $\kappa^2 a^{-2}$ . [We have excluded  $a^2 < 0$  in order to have  $\partial \cdot A$  as a positive (mass)<sup>2</sup> field.]

At the quantum level formally, and at the  $c$ -number level rigorously, the ghost field  $\partial \cdot A$  can be extracted from the fields  $A_\mu$  and  $\psi$  (denoting either spinor or scalar matter fields). Define the fields<sup>3</sup>  $V_\mu, \psi_V$  by  $A_\mu \equiv V_\mu - a^2 \kappa^{-2} \partial_\mu \partial \cdot A$ ,  $\psi \equiv \exp(-iea^2 \kappa^{-2} \partial \cdot A) \psi_V$ . From the equations of motion for  $A_\mu, \psi$  it follows that  $\partial \cdot V = 0$  holds and that  $V_\mu, \psi_V$  solve Proca's field equations.<sup>3</sup> The equal-time commutators which result for  $V_\mu, \psi_V$  from the imposed canonical quantization rules coincide with the ones derived from Proca's Lagrangian.<sup>13</sup> Thus  $V_\mu$  and  $\psi_V$  are insensitive to  $a^2$  and identical with Proca's fields. Note also  $[\partial \cdot A(x), V_\mu(y)] = 0 = [\partial \cdot A(x), \psi_V(y)]$  for arbitrary  $(x, y)$ .<sup>3</sup> Thus the field  $\partial \cdot A$  is decoupled completely from  $V_\mu, \psi_V$  and the Hilbert space of the model factorizes into a tensor product of a space  $\mathcal{H}_V$  carrying  $V_\mu, \psi_V$  and a space  $\mathcal{H}_{\partial \cdot A}$  carrying  $\partial \cdot A$ :  $\mathcal{H} = \mathcal{H}_V \otimes \mathcal{H}_{\partial \cdot A}$ .

Accordingly, the translation generators  $P^\mu$  must decompose into a sum of two commuting and therefore separately conserved contributions  $P^\mu = P_V^\mu + P_{\partial \cdot A}^\mu$ , each generating translations on the respective space.

The commutator

$$[\partial \cdot A(x), \partial \cdot A(y)] = -\frac{\kappa^2}{a^4} i \Delta(x-y; \kappa^2 a^{-2}) \quad (8)$$

differs from the one of a canonical Klein-Gordon field by the factor  $(-\kappa^2 a^{-4}) [k^\mu \equiv (\omega(\mathbf{k}), \mathbf{k})]$ :

$$i \Delta(x; m^2) \equiv \int \frac{d^s k}{2\omega(\mathbf{k})} (2\pi)^{-s} (e^{-ikx} - e^{ikx}).$$

From this commutator  $P_{\partial \cdot A}^\mu$  can be read off immediately:

$$P_{\partial \cdot A}^\mu = -\frac{a^4}{\kappa^2} \int d^s x \left[ (\partial \cdot A)'^\mu (\partial \cdot A)'^0 - \eta^{\mu 0} \frac{1}{2} \left[ (\partial \cdot A)_{/\rho} (\partial \cdot A)'^\rho - \frac{\kappa^2}{a^2} (\partial \cdot A)^2 \right] \right]. \quad (9)$$

If the field  $\partial \cdot A$  is realized so that its positive-frequency part acts as an annihilation operator the spectra of  $P_{\partial \cdot A}^\mu$  are, due to the minus sign in the commutator function of  $\partial \cdot A$ , in the forward light cone, yet the scalar product, which is defined implicitly by  $\partial \cdot A = (\partial \cdot A)^*$ , is indefinite. In the other way, to realize  $\partial \cdot A$ , its negative-frequency part acts as a creation operator. In this case the spectra of  $P_{\partial \cdot A}^\mu$  are in the backward light cone, but the scalar product is positive definite. Both realizations seem to exclude a physical interpretation of the scalar photons connected with  $\partial \cdot A$  and since these quanta do not interact they indeed escape detection by any measurement process whose dynamics is determined by the model.

There are two equivalent ways to deal with this situation. The usual one<sup>1,4</sup> is to restrict the physically realizable states to the subspace without spin-zero photons  $\mathcal{H} = \mathcal{H}_V \otimes \Omega_{\partial \cdot A}$  with  $\Omega_{\partial \cdot A}$  being the vacuum in  $\mathcal{H}_{\partial \cdot A}$ .

The observables are then given by the operators which leave invariant  $\mathcal{H}$  and whose restriction to  $\mathcal{H}$  is Hermitian. On  $\mathcal{H}$  they have the form  $B = B_V \otimes \Omega_{\partial \cdot A}(\Omega_{\partial \cdot A}, \cdot)$ . In case of the positive-metric realization of  $\partial \cdot A$ , one may, however, equally well allow arbitrary states from  $\mathcal{H}$  and restrict the observables to the form  $B = B_V \otimes \mathbb{1}_{\partial \cdot A}$  with  $B_V$  being Hermitian on  $\mathcal{H}_V$ . By this procedure the states on  $\mathcal{H}$ , i.e., the density operators, are grouped into equivalence classes of physically indistinguishable states. Any two density operators on  $\mathcal{H}$  are equivalent if for all  $B = B_V \otimes \mathbb{1}_{\partial \cdot A}$  holds  $\text{Tr}(W_1 B) = \text{Tr}(W_2 B)$ . The usual treatment then simply chooses the representative  $W_V \otimes \Omega_{\partial \cdot A}(\Omega_{\partial \cdot A}, \cdot)$  from each class.

In both ways of keeping the scalar photons unobservable it is trivial to identify the energy-momentum observables. They are given by  $P_V^\mu$ , since these generate the translations for the observables. It is with the construction of the energy-momentum tensor  $T^{\mu\nu}$  that the second approach proves of advantage.

Starting from the canonical tensor<sup>4</sup> we shall construct a candidate  $T^{\mu\nu}$  by employing the property of "gauge invariance"<sup>7</sup> of observables. The "gauge transformations"<sup>3,7,15,16</sup> of massive QED are mappings of the type  $A_\mu \mapsto A_\mu + \Lambda_{/\mu}$ ,  $\psi \mapsto \exp(ie\Lambda)\psi$  with  $(a^2 \square + \kappa^2)\Lambda = 0$  and  $\Lambda: \mathbb{M}^{1,s} \rightarrow \mathbb{R}$ . They leave the (anti)commutator algebra, the field equations and the fields  $V_\mu, \psi_V$  invariant, while  $\partial \cdot A$  transforms as  $\partial \cdot A \mapsto \partial \cdot A - (\kappa^2/a^2)\Lambda$ . Therefore the observables  $B = B_V \otimes \mathbb{1}_{\partial \cdot A}$  are identical to those field functionals  $\mathcal{F}(A_\mu, \psi, \bar{\psi})$  which are gauge invariant, i.e., functionals of the form  $\mathcal{F}(V_\mu, \psi_V, \bar{\psi}_V)$ . From this it follows, since  $V_\mu, \psi_V$  are independent of  $a^2$ , that all observable physics of massive QED is insensitive to that parameter.

After this summary of the general structure of massive QED we shall now describe the construction of  $T^{\mu\nu}$  and prove its main properties which make it a reasonable candidate for the energy-momentum tensor of the model.

The canonical tensor reads  $K^{\mu\nu} = K_A^{\mu\nu} + K_M^{\mu\nu}$  with

$$K_A^{\mu\nu} = (F^{\rho\mu} - a^2 \eta^{\rho\mu} \partial \cdot A) A_\rho{}^{/\nu} - \eta^{\mu\nu} \mathcal{L}_A$$

and

$$K_M^{\mu\nu} = \phi^* \mathcal{D}^{\bar{\mu}} \phi^{/\nu} + \phi^{*/\nu} \mathcal{D}^{\bar{\mu}} \phi - \eta^{\mu\nu} \mathcal{L}_M \quad (10)$$

or

$$K_M^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma^\mu (\bar{\partial}^\nu - \bar{\partial}^\nu) \psi,$$

respectively.

In the spinorial case, use has been made of the matter field equation which implies  $\mathcal{L}_M = 0$  for solutions. Now, by construction,  $\partial_\mu K^{\mu\nu} = 0$  holds, but  $K^{\mu\nu} \neq K^{\nu\mu}$ . As to be expected,  $K^{\mu\nu}$  is gauge variant. By introducing the canonical variables into  $K^{\mu\nu}$  it is straightforward to verify that  $P^\nu \equiv \int d^s x K^{0\nu}$  indeed generates the translations of the fields:

$$i[P^\mu, A_\rho] = A_\rho{}^{/\mu}, \quad i[P^\mu, \psi] = \psi^{/\mu}. \quad (11)$$

Let us see what can be achieved by following Minkowski's modification of  $K^{\mu\nu}$ , designed for the case  $\kappa^2 = 0$ ,  $a^2 = 0$ .<sup>4</sup> It consists of the definition  $\Theta^{\mu\nu} \equiv K^{\mu\nu} - \partial_\rho (F^{\rho\mu} A^\nu)$  which implies  $\partial_\mu \Theta^{\mu\nu} = 0$ , and leaves

the global conserved quantities unchanged. For solutions of the field equations,  $\theta^{\mu\nu}$  can be written as  $\theta^{\mu\nu} = \theta_A^{\mu\nu} + \theta_M^{\mu\nu}$  with

$$\theta_A^{\mu\nu} \equiv F^{\rho\mu} F^\nu{}_\rho + \frac{1}{4} \eta^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + \kappa^2 (A^\mu A^\nu - \frac{1}{2} \eta^{\mu\nu} A_\rho A^\rho) + a^2 [(\partial \cdot A)^\mu A^\nu - (\partial \cdot A) A^{\mu\nu} + \frac{1}{2} \eta^{\mu\nu} (\partial \cdot A)^2] \quad (12)$$

and  $\theta_M^{\mu\nu} \equiv K_M^{\mu\nu} - j^\mu A^\nu$ . For our scalar and spinor paradigms  $\theta_M^{\mu\nu}$  reads

$$\theta_M^{\mu\nu} = \phi^* (\tilde{\mathcal{D}}^\mu \tilde{\mathcal{D}}^\nu + \tilde{\mathcal{D}}^\nu \tilde{\mathcal{D}}^\mu) \phi - \eta^{\mu\nu} \mathcal{L}_M, \quad (13)$$

$$\theta_M^{\mu\nu} = \frac{i}{2} \bar{\psi} \gamma^\mu (\tilde{\mathcal{D}}^\nu - \tilde{\mathcal{D}}^\nu) \psi. \quad (14)$$

The electric current and  $\theta_M^{\mu\nu}$  are gauge invariant, but  $\theta_A^{\mu\nu}$  is not. In addition  $\theta^{\mu\nu}$  is not symmetric and thus Minkowski's procedure does not do the job. If we replace now in  $\theta_A^{\mu\nu}$  the field  $A_\mu$  by its gauge-invariant part,  $V_\mu \equiv A_\mu + a^2 \kappa^{-2} (\partial \cdot A)_{,\mu}$ , we obtain the gauge-invariant tensor

$$T_A^{\mu\nu} \equiv \theta_A^{\mu\nu}(V) = F^{\rho\mu} F^\nu{}_\rho + \frac{1}{4} \eta^{\mu\nu} F_{\rho\sigma} F^{\rho\sigma} + \kappa^2 (V^\mu V^\nu - \frac{1}{2} \eta^{\mu\nu} V_\rho V^\rho). \quad (15)$$

Since  $V_\mu$  solves the vector field equation, conservation of  $\tilde{\theta}^{\mu\nu} \equiv T_A^{\mu\nu} + \theta_M^{\mu\nu}$  follows:  $\partial_\mu \tilde{\theta}^{\mu\nu} = 0$ . The vector field's contribution  $T_A^{\mu\nu}$  enjoys the following properties: (i)  $T_A^{\mu\nu}(A_\rho + \Lambda_{,\rho}) = T_A^{\mu\nu}(A_\rho)$  (gauge invariance); (ii)  $T_A^{\mu\nu} = T_A^{\nu\mu}$  (symmetry); (iii)  $T_A^{00} \geq 0$  (positivity). In the case of scalar QED the matter part  $\theta_M^{\mu\nu}$  is also gauge invariant, symmetric, and  $\theta_M^{00} \geq 0$  holds. In the case of spinor QED the matter part  $\theta_M^{\mu\nu}$  again is gauge invariant, but it is neither symmetric nor is  $\theta_M^{00}$  positive.

Before going on to symmetrize  $\tilde{\theta}^{\mu\nu}$  in the spinorial case, we shall compare the global conserved quantities connected with  $K^{\mu\nu}$  or equivalently  $\theta^{\mu\nu}$  on one side and  $\tilde{\theta}^{\mu\nu}$  on the other. A simple computation shows that the following equations hold for both cases of QED:

$$\theta^{\mu\nu} - \tilde{\theta}^{\mu\nu} = \theta_A^{\mu\nu} - T_A^{\mu\nu}$$

and

$$\theta_A^{\mu\nu} = T_A^{\mu\nu} - a^2 \partial_\rho [\eta^{\nu\rho} (\partial \cdot A) A^\mu - \eta^{\nu\mu} (\partial \cdot A) A^\rho] - a^4 \kappa^{-2} T_{\partial \cdot A}^{\mu\nu} \quad (16)$$

with

$$T_{\partial \cdot A}^{\mu\nu} \equiv (\partial \cdot A)^\mu (\partial \cdot A)^\nu - \frac{1}{2} \eta^{\mu\nu} [(\partial \cdot A)_{,\rho} (\partial \cdot A)^\rho - \kappa^2 a^{-2} (\partial \cdot A)^2].$$

Since  $\partial \cdot A$  is freely propagating with mass parameter  $\kappa^2 a^{-2}$ , the tensor  $T_{\partial \cdot A}^{\mu\nu}$  has a vanishing divergence:  $\partial_\mu T_{\partial \cdot A}^{\mu\nu} = 0$ . This term produces the separately conserved, scalar-photon contribution to  $P^\nu = \int d^3x \theta^{0\nu}$  with eigenvalues in the backward light cone. The term proportional to  $a^2$  again has a zero divergence, but it leads to vanishing global conserved quantities. Therefore we have verified explicitly the decomposition of the translation generators into a conserved observable part and a conserved nonobservable one:

$$P^\nu = \int d^3x \tilde{\theta}^{0\nu} - a^4 \kappa^{-2} \int d^3x T_{\partial \cdot A}^{0\nu} = P_V^\nu + P_{\partial \cdot A}^\nu. \quad (17)$$

Indeed  $P_V^\nu$  commutes with  $\partial \cdot A$  and  $P_{\partial \cdot A}^\nu$  with  $V_\mu$  and  $\psi_V$ . Thus the (gauge-invariant) energy-momentum observables are given by  $P_V^\nu \equiv \int d^3x \tilde{\theta}^{0\nu}$ .

Let us return to the search for a symmetric energy-momentum tensor for spinor QED. Following Belinfante's<sup>16</sup> construction we have to check whether the antisymmetric part of  $\tilde{\theta}^{\mu\nu}$  is a divergence. This, indeed, is the case:

$$\tilde{\theta}_M^{\mu\nu} - \tilde{\theta}_M^{\nu\mu} = i \partial_\rho H^{\rho[\mu\nu]} \quad (18)$$

with

$$H^{\rho[\mu\nu]} \equiv \frac{\partial \mathcal{L}}{\partial \psi_{/\rho}} \Sigma^{[\mu\nu]} \psi - \bar{\psi} \Sigma^{[\mu\nu]} \frac{\partial \mathcal{L}}{\partial \bar{\psi}_{/\rho}}$$

and  $\Sigma^{[\mu\nu]} \equiv (i/4)[\gamma^\mu, \gamma^\nu]$ . Now the way is open for defining a symmetric energy-momentum tensor:

$$T^{\mu\nu} \equiv \tilde{\theta}^{\mu\nu} - \frac{i}{2} \partial_\rho (H^{\rho[\mu\nu]} + H^{\mu[\nu\rho]} + H^{\nu[\rho\mu]}). \quad (19)$$

A further simplification can be achieved since  $\partial_\rho H^{\mu[\nu\rho]}$  is antisymmetric in  $(\mu, \nu)$ . This reduces  $T^{\mu\nu}$  to the form  $T^{\mu\nu} = \frac{1}{2} (\tilde{\theta}^{\mu\nu} + \tilde{\theta}^{\nu\mu}) = T_A^{\mu\nu} + \frac{1}{2} (\theta_M^{\mu\nu} + \theta_M^{\nu\mu})$  which is valid for both cases of QED.

$T^{\mu\nu}$  obeys the following crucial properties by construction: (i)  $\partial_\mu T^{\mu\nu} = 0$ ; (ii)  $[T^{\mu\nu}(x), \partial \cdot A(y)] = 0$ ; (iii)  $T^{\mu\nu} = T^{\nu\mu}$ ; (iv)  $\int d^3x T^{0\nu} = P_V^\nu$ ; (v)  $\dot{P}_V^\nu = 0$ ; (vi) for scalar QED only:  $T^{00} \geq 0$ .

It is because of these properties that we consider  $T^{\mu\nu}$  an energy-momentum tensor candidate. The zero commutator (ii) follows from our construction which made sure that  $T^{\mu\nu}$  can be expressed in terms of the fields  $V_\mu, \psi_V$  exclusively, and it signals the factorization  $T^{\mu\nu} = T_V^{\mu\nu} \otimes 1_{\partial \cdot A}$  which means that  $T^{\mu\nu}$  is indeed an observable.

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