Proof of one-loop finiteness of type-I SO(32) superstring theory

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We prove, in the Lorentz-covariant formulation of the type-I superstring theory, that the parity-conserving one-loop amplitudes with arbitrary numbers of external gauge bosons are finite if the internal-symmetry group is SO(32).

In the past year a great amount of interest has been rekindled in supersymmetric string theories by the prospect of developing an infinity-free and anomaly-free theory of fundamental interactions including Yang-Mills and gravitational forces.[1] Of particular interest from some points of view is the SO(32) type-1 theory which includes both open and closed strings. Using a nonmanifestly Lorentz-invariant light-cone formulation, Green and Schwarz[2] have recently shown that the one-loop four-point function is finite. One might expect from this, using a heuristic duality argument, that all the higher-N-point functions are finite at the one-loop level. However, in the light-cone formulation, the finiteness of the four-point function appears to depend explicitly on the number of external particles.

In this article we show directly the finiteness of all the parity-conserving one-loop N-point functions. The vanishing of the one-loop two- and three-point functions follows in the Lorentz-covariant formulation from simple relations among Jacobi θ functions.

The type-1 superstring theory corresponds to the Ramond-Neveu-Schwarz model projected onto the even-G-parity bosonic sector and the Majorana-Weyl fermion sector.[3]

Up to internal-symmetry factors the one-loop N-point functions in D dimensions with external gauge bosons are given by expressions of the form

$$L=g^N d^D p\, \mathrm{Tr} P \prod_{i=1}^{N} [V(k_i,1)\Delta] \,. \tag{1}$$

With bosons circulating in the loop the vertex for gauge-boson emission is

$$\begin{aligned} V(k_i,\rho_i) &= V_B(k_i,\rho_i) \\ &= e^{ik_i\cdot Q(\rho_i)}[\zeta_i\cdot P(\rho_i)+k_i\cdot H(\rho_i)\zeta_i\cdot H(\rho_i)] \;, \end{aligned} \tag{2}$$

and the string propagator is

$$\begin{aligned} \Delta=\Delta_B &= \frac{1}{L_0-1}(1+\Omega)\left[\frac{1+G}{2}\right] \\ &= \int_0^1 \frac{dx}{x} x^{L_0-1}(1+\Omega)\left[\frac{1+G}{2}\right] . \end{aligned} \tag{3}$$

Here Ω is the twist operator given (up to a gauge transformation) for an orthogonal-group internal symmetry by

$$\Omega=-e^{i\pi(L_0-p_0{}^2/2)} \,, \tag{4}$$

and G is the G-parity operator

$$G=e^{2\pi i(L_0-p_0{}^2)/2)} \,. \tag{5}$$

P in Eq. (1) is the Brink-Olive projection operator onto positive-norm states. The amplitude is given by a D-dimensional loop momentum integration times a trace over excited states of the circulating string. With fermions circulating, the amplitude takes the same form (except for a fermionic loop factor of -1) with the substitutions

$$V(k_i,\rho_i)=V_F(k_i,\rho_i)=e^{ik_i\cdot Q(\rho_i)}\zeta_i\cdot\Gamma(\rho_i) \,, \tag{6}$$

$$\Delta=\Delta_F=\frac{1}{F_0}(1+\Omega)\frac{(1+\Gamma^{11})}{2} \,. \tag{7}$$

In the following we will not treat the parity-violating term Γ^{11} in the propagator. Possible divergences due to these terms cannot cancel divergences in the parity-conserving terms. The parity-conserving loop is also given in an alternate (F_1) formulation by

$$V_F(k_i,\rho)=e^{ik_i\cdot Q(\rho)}[\zeta_i\cdot P(\rho)-\tfrac{1}{2}k_i\cdot\Gamma(\rho)\zeta_i\cdot\Gamma(\rho)] \,, \tag{8}$$

$$\Delta_F=\frac{1}{L_0}(1+\Omega) \,, \tag{9}$$

providing one supplies an extra factor of $\frac{1}{4}$ on the right-hand side of Eq. (1). With fermionic states circulating, the trace in Eq. (1) is, of course, also taken over the Dirac matrices.

Loops with a positive even number of twists in Eq. (1) are finite except on the poles of intermediate graviton states and need not be considered here. Loops with an odd number of twists are equivalent to the Mobius loop with a single twist if the integration range of the propagator variables is appropriately adjusted as discussed below.

It is convenient to replace the loop momentum integral in Eq. (1) by a trace over canonical zeroth-mode oscillators using the limiting procedure of Ref. 4. Then the bosonic loop takes the form

$$L_B=g^N \int_0^1 d\Omega\, \mathrm{Tr} w^{L_0-1} P(1+\Omega)\left[\frac{1+G}{2}\right] \times \prod_{i=1}^{N} V_B(k_i,\rho_i,a,a^\dagger) \,, \tag{10}$$

where

$$d\Omega=\frac{dw}{w}\prod_{i=2}^{N}\frac{d\rho_i}{\rho_i} . \tag{11}$$

The arguments $a,a^\dagger$ in Eq. (10) refer collectively to the boson oscillators in Q_μ and P_μ and to the fermion oscillators in H_μ. The string coordinate $Q_\mu(\rho,a,a^\dagger)$ and conjugate momentum $P_\mu(\rho,a,a^\dagger)$ are

$$Q_\mu(\rho,a,a^\dagger)=\sum_{n=0}^{\infty}\left[\frac{\Gamma(n+2\epsilon)}{n!}\right]^{1/2}(a_\mu^n\rho^{-n-\epsilon}+a_\mu^{n\dagger}\rho^{n+\epsilon}) , \tag{12}$$

$$P_\mu(\rho,a,a^\dagger)=i\rho\frac{d}{d\rho}Q_\mu(\rho,a,a^\dagger) . \tag{13}$$

After evaluating the trace in Eq. (10) one takes the limit $\epsilon\to 0$ using the relation

$$\lim_{\epsilon\to 0}(4\pi\epsilon)^{-D/2}\exp\left[-\left[\sum_i k_i\right]^2\Big/4\epsilon\right]=\delta^D\left[\sum k_i\right] . \tag{14}$$

Using the results in Appendix C of Ref. 4, the four terms in Eq. (10) can be written

$$L_B=\frac{g^N}{2}d\Omega\{(\mathrm{Tr}\lambda^0)[F_{\mathrm{NS}}(w)T_{\mathrm{NS}}(\rho_i,w)+F_{\mathrm{NS}}(we^{2\pi i})T_{\mathrm{NS}}(\rho_i,we^{2\pi i})] -F_{\mathrm{NS}}(-w)T_{\mathrm{NS}}(\rho_i,-w)-F_{\mathrm{NS}}(-we^{2\pi i})T_{\mathrm{NS}}(\rho_i,-we^{2\pi i})\} . \tag{15}$$

The projected Neveu-Schwarz partition function is

$$F_{\mathrm{NS}}(w)=\frac{w^{1/2}}{w}\prod_{n=1}^{\infty}\left[\frac{1+w^{n-1/2}}{1-w^n}\right]^{D-2} . \tag{16}$$

We have taken the liberty of including in this a factor of $1/w$, which is more properly a part of $d\Omega$. This gives a differential $d\Omega$, which is then the same for boson and fermion loops. We assume the effect of the physical-state projection operator is such as to maintain the form of Eq. (16) in the Mobius as well as in the planar loop. We have also defined

$$T_{\mathrm{NS}}(\rho_i,w)=(-\epsilon\ln|w|)^{-D}\left\langle 0\left|\prod_{i=1}^{N}V_B(\rho_i,d(w),\bar d(w))\right|0\right\rangle \tag{17}$$

with

$$d^n(w)=\frac{a^n}{1\pm w^{n-J}}+a'^{n\dagger} , \tag{18}$$

$$\bar d^{\,n}(w)=a^{n\dagger}+\frac{a'^n w^{n-J}}{1\pm w^{n-J}} , \tag{19}$$

the plus and minus signs referring, respectively, to the fermionic and bosonic oscillators which have, respectively, $J=-\frac{1}{2}$ and $-\epsilon$. The zeroth modes in Eq. (17) are understood to depend only on the absolute values of ρ_i and w.

Similarly, the parity-conserving fermion loop with N external gauge bosons is

$$L_F=-\frac{g^N}{2}d\Omega[(\mathrm{Tr}\lambda^0)F_R(w)T_R(\rho_i,w) -F_R(-w)T_R(\rho_i,-w)] , \tag{20}$$

where the projected partition function in the Ramond sector is

$$F_R(w)=2^{(D-2)/2}\prod_{n=1}^{\infty}\left[\frac{1+w^n}{1-w^n}\right]^{D-2} \tag{21}$$

and

$$T_R(\rho_i,w)=(-\epsilon\ln|w|)^{-D}\left\langle 0\left|\prod_{i=1}^{N}V_F(\rho_i,d(w),\bar d(w))\right|0\right\rangle . \tag{22}$$

The vertices V_F of Eq. (8) appear in Eq. (22) with their fermionic and bosonic oscillators transformed by Eqs. (18) and (19) with, effectively, $J=0$ and $-\epsilon$, respectively. The vacuum expectation value in Eq. (22) is defined to include a normalized Dirac trace:

$$\langle 0|f(\gamma_\mu)|0\rangle\equiv 2^{-D/2}\mathrm{tr}f(\gamma_\mu) .$$

The vacuum expectation values in Eqs. (17) and (22) are completely determined by the elementary correlations of pairs of conformal fields. The automorphisms of Jacobi θ functions relate these to similar correlations with transformed variables:

$$\rho_i'=e^{2\pi i\ln\rho_i/\ln w} , \tag{23}$$

$$w'=e^{4\pi^2/\ln w} , \tag{24}$$

$$\rho_i''=(\rho_i')^{1/2} , \tag{25}$$

$$w''=(w')^{1/4} . \tag{26}$$

More specifically, with

$$\tau=\frac{\ln w}{2\pi i} , \tag{27}$$

one finds

$$\langle 0|Q_\mu(\rho_i,d,\bar d)Q_\nu(\rho_j,d,\bar d)|0\rangle =\langle 0|\hat Q_\mu(\rho_i',d',\bar d')\hat Q_\nu(\rho_j',d',\bar d')|0\rangle +g_{\mu\nu}\left[-\frac{1}{\epsilon^2\ln w}-\frac{1}{2\epsilon}-\frac{\pi i\tau}{6}-\ln\tau\right] , \tag{28a}$$

$$\langle 0|P_\mu(\rho_i,d,\bar{d})Q_\nu(\rho_j,d,\bar{d})|0\rangle = \tau^{-1}\langle 0|\hat{P}_\mu(\rho_i',d',\bar{d}')\hat{Q}_\nu(\rho_j',d',\bar{d}')|0\rangle , \quad (28b)$$

$$\langle 0|P_\mu(\rho_i,d,\bar{d})P_\nu(\rho_j,d,\bar{d})|0\rangle = \tau^{-2}\langle 0|\hat{P}_\mu(\rho_i',d',\bar{d}')\hat{P}_\nu(\rho_j',d',\bar{d}')|0\rangle . \quad (28c)$$

Equations (28b) and (28c) follow a simple differentiation of Eq. (28a). Here we have defined

$$\hat{Q}_\mu(x,d,\bar{d}) = \Gamma(2\epsilon)^{1/2}(a_\mu^0 x^{-\epsilon} + a_\mu^{0\dagger}x^{\epsilon}) + \sum_{n=1}^{\infty}\left[\frac{\Gamma(n+2\epsilon)}{n!}\right]^{1/2} \times (d_\mu^n x^{-n-\epsilon} + \bar{d}_\mu^n x^{n+\epsilon}) , \quad (29)$$

$$\hat{P}_\mu(x,d,\bar{d}) = ix\frac{d}{dx}\hat{Q}_\mu(x,d,\bar{d}) , \quad (30)$$

i.e., in $\hat{Q}_\mu$, by definition, the zeroth mode remains untransformed by Eqs. (18) and (19). The prime on d and $\bar{d}$ indicates that the argument w in Eqs. (18) and (19) is replaced by w'

$$d' \equiv d(w'), \quad \bar{d}' \equiv \bar{d}(w') . \quad (31)$$

The Neveu-Schwarz field H_μ satisfies

$$\langle 0|H_\mu(\rho_i,d,\bar{d})H_\nu(\rho_j,d,\bar{d})|0\rangle = \tau^{-1}\langle 0|H_\mu(\rho_i',d',\bar{d}')H_\nu(\rho_j',d',\bar{d}')|0\rangle . \quad (32)$$

The nonsingular, ρ-independent terms in the large parentheses of Eq. (28a) do not contribute to the loop amplitude with massless external particles since the $Q_\mu Q_v$ correlations only occur in the combination

$$\sum_{i<j} k_{i_\mu} k_{j_\nu} \langle 0|Q_\mu(\rho_i,d,\bar{d})Q_\nu(\rho_j,d,\bar{d})|0\rangle \quad (33)$$

and the total four-momentum is conserved. The singular terms in the large parentheses only affect the coefficient of the momentum-conserving δ function.

Equations (28) and (32) imply that the vacuum expectation value of Eq. (17) is simply related to another vacuum expectation value in which each $Q_\mu(\rho,d,\bar{d})$ is replaced by $\hat{Q}_\mu(\rho_i',d',\bar{d}')$, each $P_\mu(\rho_i,d,d)$ is replaced by $\tau^{-1}\hat{P}_\mu(\rho_i',d',\bar{d}')$, and each $H_\mu(\rho_i,d,d)$ is replaced by $\tau^{-1/2}H_\mu(\rho_i',d',\bar{d}')$.

More specifically, we define parallel to Eq. (17),

$$\hat{T}_{\rm NS}(\rho_i',w') \equiv (4\pi\epsilon)^{-D/2}\left\langle 0\left|\prod_{i=1}^{N}\hat{V}_B(\rho_i',d',\bar{d}')\right|0\right\rangle , \quad (33a)$$

$$\hat{T}_R(\rho_i',w') \equiv (4\pi\epsilon)^{-D/2}\left\langle 0\left|\prod_{i=1}^{N}\hat{V}_F(\rho_i',d',\bar{d}')\right|0\right\rangle , \quad (33b)$$

where each Q_μ,P_μ in V_B or V_F has been replaced by $\hat{Q}_\mu,\hat{P}_\mu$, to form $\hat{V}_B$ or $\hat{V}_F$. Then

$$T_{\rm NS}(\rho_i,w) = \left[\frac{\tau}{i}\right]^{-D/2}\tau^{-N}\hat{T}_{\rm NS}(\rho_i',w') . \quad (34)$$

Under the same transformation the Neveu-Schwarz partition function satisfies

$$F_{\rm NS}(w) = \left[\frac{\tau}{i}\right]^{D/2-1}F_{\rm NS}(w') \quad (35)$$

and the differential transforms as

$$d\Omega = \tau^{N+1}d\Omega' , \quad (36)$$

where $d\Omega'$ is given by Eq. (11) in terms of primed variables. Putting Eqs. (34), (35), and (36) together implies for the first term in Eq. (15)

$$d\Omega F_{\rm NS}(w)T_{\rm NS}(\rho_i,w) = i\,d\Omega' F_{\rm NS}(w')\hat{T}_{\rm NS}(\rho_i',w') . \quad (37)$$

One can write similar relations for the correlations occurring in the Mobius loop and G-parity-projected loops. The results are that, in the Mobius loop, the Neveu-Schwarz field correlation transforms into the G-parity-projected NS correlation and vice versa. Similarly the Planar-Ramond correlation transforms into the G-parity-projected planar Neveu-Schwarz correlation. That is, if

$$\langle 0|H_\mu(\rho_i,d,\bar{d})H_\nu(\rho_j,d,\bar{d})|0\rangle = g_{\mu\nu}\chi(\rho_j/\rho_i,w) \quad (38)$$

and

$$\left\langle 0\left|\frac{\Gamma_\mu(\rho_i,d,\bar{d})}{i\sqrt{2}}\frac{\Gamma_\nu(\rho_j,d,\bar{d})}{i\sqrt{2}}\right|0\right\rangle = g_{\mu\nu}\chi_0(\rho_j/\rho_i,w) , \quad (39)$$

then

$$\chi(x,w) = \tau^{-1}\chi(x',w') , \quad (40a)$$

$$\chi(x,we^{2\pi i}) = \tau^{-1}\chi_0(x',w') , \quad (40b)$$

$$\chi_0(x,w) = \tau^{-1}\chi(x',w'e^{2\pi i}) , \quad (40c)$$

$$\chi(x,-w) = (2\tau)^{-1}\chi(x'',-w''e^{2\pi i}) , \quad (40d)$$

$$\chi(x,-we^{2\pi i}) = (2\tau)^{-1}\chi(x'',-w'') , \quad (40e)$$

$$\chi_0(x,-w) = (2\tau)^{-1}\chi_0(x'',-w'') . \quad (40f)$$

The Mobius loop involves correlations of fields with negative w:

$$\langle 0|Q_\mu(\rho_i,d(-w),\bar{d}(-w)),Q_\nu(\rho_j,d(-w),\bar{d}(-w))|0\rangle = \langle 0|\hat{Q}_\mu(\rho_i'',d(-w''),\bar{d}(-w''))\hat{Q}_\nu(\rho_j'',d(-w''),\bar{d}(-w''))|0\rangle + g_{\mu\nu}\left[-\frac{1}{\epsilon^2\ln w}-\frac{1}{2\epsilon}-\frac{\pi i\tau}{6}-\ln 2\tau\right] . \quad (41)$$

Since the twist operator of Eq. (4) involves no zeroth modes, the d's of negative argument in Eq. (41) and elsewhere are defined so that the zeroth mode depends only on the absolute value of the argument. By differentiating Eq. (41) with respect to $\ln\rho_i$ and/or $\ln\rho_j$ we find analogs of Eqs. (28b) and (28c) with primes replaced by double primes on the right-hand side and τ replaced by 2τ.

Putting these together with Eqs. (40a)–(40f) implies, in addition to Eq. (34), that

$$T_{\rm NS}(\rho_i,we^{2\pi i})=\left[\frac{\tau}{i}\right]^{-D/2}\tau^{-N}\hat{T}_R(\rho_i',w')\ , \tag{42a}$$

$$T_R(\rho_i,w)=\left[\frac{\tau}{i}\right]^{-D/2}\tau^{-N}\hat{T}_{\rm NS}(\rho_i',w'e^{2\pi i})\ , \tag{42b}$$

$$T_{\rm NS}(\rho_i,-w)=\left[\frac{\tau}{i}\right]^{-D/2}(2\tau)^{-N}\hat{T}_{\rm NS}(\rho_i'',-w''e^{2\pi i})\ , \tag{42c}$$

$$T_{\rm NS}(\rho_i,-we^{2\pi i})=\left[\frac{\tau}{i}\right]^{-D/2}(2\tau)^{-N}\hat{T}_{\rm NS}(\rho_i'',-w'')\ , \tag{42d}$$

$$T_R(\rho_i,-w)=\left[\frac{\tau}{i}\right]^{-D/2}(2\tau)^{-N}\hat{T}_R(\rho_i'',-w'')\ . \tag{42e}$$

Note that the $(\tau/i)^{-D/2}$ factor is independent of twists or G-parity projections since it arises from the zeroth mode of the bosonic oscillators. Elsewhere the appearance of $-w$ in the Mobius loop correlations leads to extra factors of 2. In addition to Eq. (35), the Neveu-Schwarz and Ramond partition functions satisfy

$$F_{\rm NS}(we^{2\pi i})=-\left[\frac{\tau}{i}\right]^{D/2-1}F_R(w')\ , \tag{43a}$$

$$F_R(w)=-\left[\frac{\tau}{i}\right]^{D/2-1}F_{\rm NS}(w'e^{2\pi i})\ , \tag{43b}$$

$$F_{\rm NS}(-w)=\left[\frac{2\tau}{i}\right]^{D/2-1}F_{\rm NS}(-w''e^{2\pi i})\ , \tag{43c}$$

$$F_{\rm NS}(-we^{2\pi i})=\left[\frac{2\tau}{i}\right]^{D/2-1}F_{\rm NS}(w'')\ , \tag{43d}$$

$$F_R(-w)=\left[\frac{2\tau}{i}\right]^{D/2-1}F_R(-w'')\ . \tag{43e}$$

Extended to include double-primed variables, Eq. (36) reads

$$d\Omega=\tau^{N+1}d\Omega'=(2\tau)^{N+1}d\Omega''\ . \tag{44}$$

Inclusion of Mobius loops with multiple twists merely fills in the integration range of the ρ'' variables to match that of the ρ' variables. Thus, when we substitute Eqs. (34), (35), (42a)–(42e), (43a)–(43e), and (44) into the sum of Eqs. (15) and (20), we may drop all primes and double primes. Then labeling the ρ_i' variables by z_i,

$$\begin{aligned}L&=L_B+L_F\\&=i\frac{g^N}{2}d\Omega\{\,({\rm Tr}\lambda^0)[F_{\rm NS}(w)\hat{T}_{\rm NS}(z_i,w)+F_{\rm NS}(we^{2\pi i})\hat{T}_{\rm NS}(z_i,we^{2\pi i})-F_R(w)\hat{T}_R(z_i,w)]\\&\qquad-2^{D/2}[F_{\rm NS}(-w)\hat{T}_{\rm NS}(z_i,-w)+F_{\rm NS}(-we^{2\pi i})\hat{T}_{\rm NS}(z_i,-we^{2\pi i})-F_R(-w)\hat{T}_R(z_i,-w)]\}\ .\end{aligned} \tag{45}$$

We may now use Jacobi's famous "aequatio identica" relating, in effect, the Neveu-Schwarz and Ramond partition functions:

$$F_R(w)=F_{\rm NS}(w)+F_{\rm NS}(we^{2\pi i})\ . \tag{46}$$

The one-loop N-point function then becomes

$$\begin{aligned}L=\frac{i}{2}g^Nd\Omega(\,({\rm Tr}\lambda^0)\{F_{\rm NS}(w)[\hat{T}_{\rm NS}(z_i,w)-\hat{T}_R(z_i,w)]+F_{\rm NS}(we^{2\pi i})[\hat{T}_{\rm NS}(z_i,we^{2\pi i})-\hat{T}_R(z_i,w)]\}\\-2^{D/2}\{F_{\rm NS}(-w)[\hat{T}_{\rm NS}(z_i,-w)-\hat{T}_R(z_i,-w)]+F_{\rm NS}(-we^{2\pi i})[\hat{T}_{\rm NS}(z_i,-we^{2\pi i})-\hat{T}_R(z_i,-w)]\})\ .\end{aligned} \tag{47}$$

Since the potential divergence in Eq. (47) arises from the dw/w behavior of $d\Omega$, in order to prove finiteness one may discard terms in the curly brackets which vanish as $w\rightarrow0$. Note that $w\rightarrow0$ here is the point in the integration range where the original w of Eqs. (11)–(22) approaches 1. The necessary expansions are

$$F_{\rm NS}(w)=w^{-1/2}+8+O(w)\ , \tag{48}$$

$$\hat{T}_{\rm NS}(z_i,w)=\hat{T}_{\rm NS}(z_i,0)+w^{1/2}\hat{T}'_{\rm NS}(z_i,0)+O(w)\ , \tag{49}$$

$$\hat{T}_R(z_i,w)=\hat{T}_R(z_i,0)+O(w)\ . \tag{50}$$

We have then

$$\begin{aligned}L=ig^N\int d\Omega(\,[({\rm Tr}\lambda^0)-2^{D/2}]\\\times\{\,8[\hat{T}_{\rm NS}(z_i,0)-\hat{T}_R(z_i,0)]\\+\hat{T}'_{\rm NS}(z_i,0)\}+O(w))\ .\end{aligned} \tag{51}$$

Since $D=10$ in the superstring theory, all the N-point functions are finite for $\mathrm{Tr}\lambda^0=32$, i.e., for an SO(32) internal symmetry. From Eq. (47) it is straightforward to recover the results of the light-cone gauge calculations for small N. Since only the difference of the products of bosonic and fermionic vertices [Eq. (33)] appears in Eq. (47) there is no contribution from terms where only the $\zeta_i \cdot P_\mu$ pieces of V_B and V_F are taken. The vanishing of the two-point function follows then from the relation

$$F_{\mathrm{NS}}(w)[\chi^2(z,w)-\chi_0{}^2(z,w)]+(w \to we^{2\pi i})=0 . \quad (52)$$

Similarly, the vanishing of the one-loop three-point function follows from the relation

$$F_{\mathrm{NS}}(w)[\,\chi(z_2/z_1,w)\chi(z_3/z_2,w)\chi(z_3/z_1,w) - \chi_0(z_2/z_1,w)\chi_0(z_3/z_2,w)\chi_0(z_3/z_1,w)] + (w \to we^{2\pi i})=0 . \quad (53)$$

A similar relation among the x's also drastically simplifies the four-point loop. We note that the vanishing of the one-loop correction to the two- and three-point functions is independent of the internal-symmetry group.

The techniques used here also greatly simplify the analysis of one-loop graphs in the closed string models. Such results will appear in a later, more detailed article.

After completing this work we heard that some subset of our results has also been obtained by Frampton, Moxhay, and Ng[5] using the light-cone gauge.

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