Gauge Potts model with generalized action: A Monte Carlo analysis

H. Fanchiotti, C. A. García Canal, and S. J. Sciutto
Laboratorio de Física Teórica, Departamento de Física, Universidad Nacional de La Plata, C.C. 67-1900 La Plata, Argentina
(Received 27 November 1984)

Results of a Monte Carlo calculation on the q-state gauge Potts model in d dimensions with a generalized action involving planar 1×1, plaquette, and 2×1, fenêtre, loop interactions are reported. For $d=3$ and $q=2$, first- and second-order phase transitions are detected. The phase diagram for $q=3$ presents only first-order phase transitions. For $d=2$, a comparison with analytical results is made. Here also, the behavior of the numerical simulation in the vicinity of a second-order transition is analyzed.

I. INTRODUCTION

Recently, there has been an increasing interest in the study of different generalizations of standard lattice gauge theories. In particular, the inclusion of larger interaction loops in the action seems to be very appealing because it could probably improve the continuum limit of the model without destroying universality. In order to investigate the effects of these generalizations in simple gauge models, we have taken a q-state gauge Potts model, P_q, whose fundamental properties are well known.[1] Then we have considered the most simple addition to the standard plaquette action: An interaction term including a 2×1 loop usually called fenêtre.

We report here the most interesting results coming from a Monte Carlo analysis of the plaquette plus fenêtre gauge Potts model. We have chosen to present the particular cases $q=2$, 3, 4, 5 for $d=2$ dimensions and $q=2$, 3, for $d=3$.

There exists a previous study of the $P_2\equiv Z_2$ model with both plaquette and fenêtre interactions in four dimensions,[2] and with only fenêtre action[3] for $d=3$ and 4. We have reobtained these results for checking purposes.

In Sec. II we present the model under consideration, the magnitudes used in the calculations, and the results obtained. Section III contains some concluding remarks.

II. MODEL AND RESULTS

The generalized q-state Potts model here considered is defined through the action

$$S=\beta_1 S_1+\beta_2 S_2 \tag{1}$$

with

$$S_1=\sum_{\text{plaq}}\delta_{U_p,1},\quad S_2=\sum_{\text{fen}}{}'\delta_{U_f,1}\ , \tag{2}$$

where plaq denotes a primitive square, or plaquette, fen stands for a rectangular plaquette of 2×1 links, or fenêtre, and $U_p=U_1U_2U_3U_4$ and $U_f=U_1U_2U_3U_4U_5U_6$ are the ordered products of link variables around plaquettes and fenêtres, respectively. Each link variable can take the value $\exp(i2k\pi/q)$, $k=0,1,2,\ldots,q-1$; $q\geq 2$. δ is the standard Kronecker symbol and β_1 and β_2 are the corresponding coupling parameters. The sums indicated in (2) run over all plaquettes and fenêtres of an N^d hypercubical lattice with periodic boundary conditions.

The magnitudes used in our Monte Carlo calculations are the average plaquette action

$$E_1=\frac{\langle S_1\rangle}{N_p}\ , \tag{3}$$

N_p being the number of plaquettes on the lattice, the average fenêtre action

$$E_2=\frac{\langle S_2\rangle}{N_p}\ , \tag{4}$$

and the plaquette and fenêtre contributions to the specific heat, given by

$$C_1=\frac{|\beta_1|}{N_p}(\langle {S_1}^2\rangle-\langle S_1\rangle^2) \tag{5}$$

and

$$C_2=\frac{|\beta_2|}{N_p}(\langle {S_2}^2\rangle-\langle S_2\rangle^2)\ , \tag{6}$$

respectively. Notice that $0\leq E_1\leq 1$ and $0\leq E_2\leq 2$.

We have used in the computations the "heat bath" algorithm,[4] and we have analyzed several lattice sizes in order to control eventual size dependences. When periodic boundary conditions are used and the region $\beta_2<0$ is explored, the number of sites in each direction is required to be a multiple of 4. It is not sufficient to take an even number of sites[2] to ensure the boundary conditions because an antiferromagneticlike configuration presents a link periodicity 4 (see, for example, the figure in footnote 1 of Ref. 2).

Two dimensions

In this case one can expect at least a second-order phase transition for $\beta_1=0$. This is due to the fact that a simple extension of an argument by Turban[5] allows us to show that the two-dimensional model under consideration is equivalent to a two-dimensional spin Potts model in an

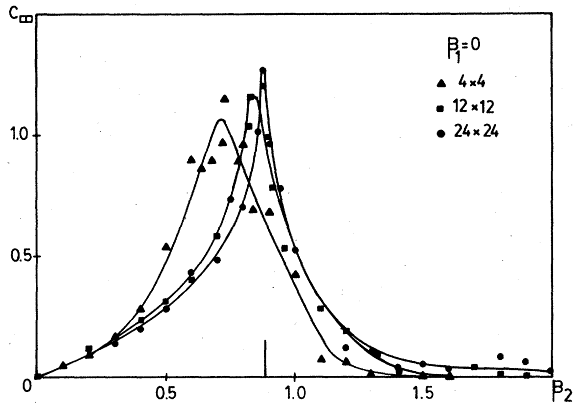

FIG. 1. Specific heat C_2 versus parameter β_2 for the two-dimensional P_2 model with $\beta_1=0$, for different lattice sizes. No error bars are presented for simplicity. The lines traced should be regarded as qualitative guides.

external field. In fact, the parameter β_1 plays the role of the magnetic field and β_2 works as the coupling constant between neighbor spins. The exact solution of the spin P_2 model with $\beta_1=0$ gives a second-order phase transition at $\beta_2=\pm 0.881$ where the $\pm$ sign appears because for $\beta_1=0$ the model presents a symmetry[2] $\beta_2 \rightarrow -\beta_2$. For $q>2$ there are no exact solutions for the spin model. However, it can be analytically shown[6] that phase transitions are present only in the case $\beta_1=0$, being of second order for $q<4$ and of first order otherwise. Notice that the phase diagrams of this model are sections of high-dimensional phase diagrams of conveniently generalized models,[6] and for that reason, it may happen that isolated first-order phase transitions show up.

It is very interesting to study Monte Carlo results for the gauge model with action (1) in order to investigate the behavior of these kinds of models in the vicinity of a second-order phase transition. For that reason we have analyzed the behavior of the specific heat C_2 in the $q=2$ case for different values of β_1 and particularly for $\beta_1=0$. In Fig. 1 we have plotted our results for $\beta_1=0$ and for three different lattice sizes. We can certainly speak of a second-order phase transition due to the presence of a neat

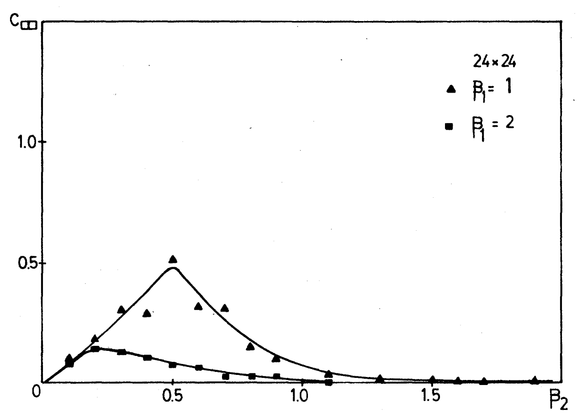

FIG. 2. The same as Fig. 1, but for two nonzero values of β_1. Observe the vanishing tendency of C_2 with increasing β_1.

TABLE I. Transition data for the two-dimensional model considered. The computations were made using lattices of size 32^2.

q	β_c	Order of transition
3	1.07 ± 0.13	Second
4	1.18 ± 0.13	Second
5	1.26 ± 0.21	First

sharpening of the curves and the corresponding displacement of their maximum when N, the number of lattice points considered, grows. Notice also that the larger the lattice size, the smaller the distance between the maximum of the curve and the exact transition point. It is important to contrast this behavior with the case $\beta_1 \neq 0$. To do this we present in Fig. 2 the graphs of C_2 versus β_2 for $\beta_1=1$ and $\beta_1=2$. It is clear that the maximum value of the specific heat C_2 decreases when β_1 grows. For larger values of β_1 the C_2 values become vanishingly smaller.

In Table I we have summarized the results of a similar study, now for $q=3$, 4, and 5. As indicated there, a single phase transition for $\beta_1=0$, $\beta_2>0$ was found in each case.

Three dimensions

For this case we present results corresponding to the models P_2 and P_3.

The P_2 model has first-order phase transitions at the points[3] $\beta_1=0$, $\beta_2=\pm(0.46\pm 0.04)$ and a second-order one at[7] $\beta_1=1.400\pm 0.025$, $\beta_2=0$. The complete phase diagram, obtained by using the E_2 thermal cycle method,[4] is shown in Fig. 3. These cycles, in the vicinity of the dashed line of this figure, do not show definite hysteresis phenomena as to ensure first-order transitions. However,

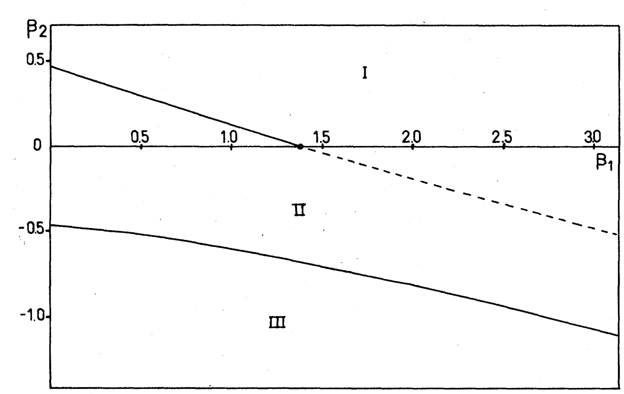

FIG. 3. Phase diagram for P_2, $d=3$. Solid lines represent first-order phase transitions. The $\beta_1\beta_2$ plane is divided in three different regions corresponding to a "ferromagnetic" (I), "paramagnetic" (II), and "antiferromagnetic" (III) phase.

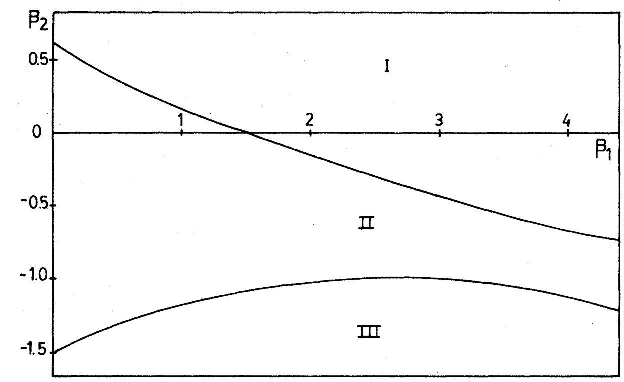

FIG. 4. The same as Fig. 3, but for P_3 model where only first-order phase transitions were detected.

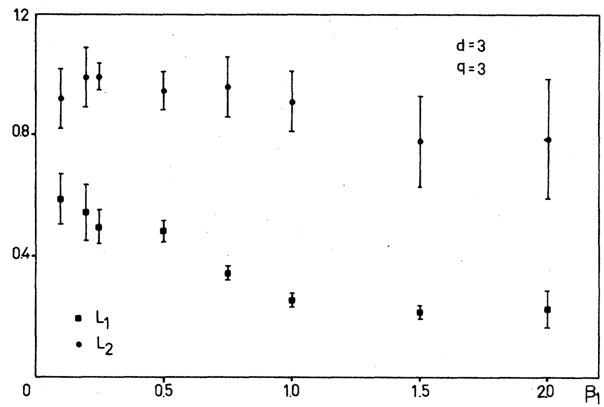

FIG. 6. Same as Fig. 5, but for $q=3$.

a clear signal for transitions exists in that region. On the other hand, due to computer-time restrictions, it is not possible for us to do an exhaustive analysis of the specific-heat behavior in order to confirm the second-order character of these transitions. It is worth noticing that if this would be the case the point $\beta_1=1.4$, $\beta_2=0$ would be a tricritical one.

The corresponding phase diagram for the P_3 model is presented in Fig. 4. It was obtained in a similar way as above and only first-order phase transitions were detected.

We have studied also the latent heats L_1, plaquette contribution, and L_2, fenêtre contribution, along the transition between phases I and II (see Figs. 3 and 4) in the cases presented above. The results are summarized in Figs. 5 and 6, respectively. They were obtained from direct measurements on the E_1 and E_2 thermal cycles and should be regarded as rough estimations. The P_2 (tri-)critical point can be seen in Fig. 5.

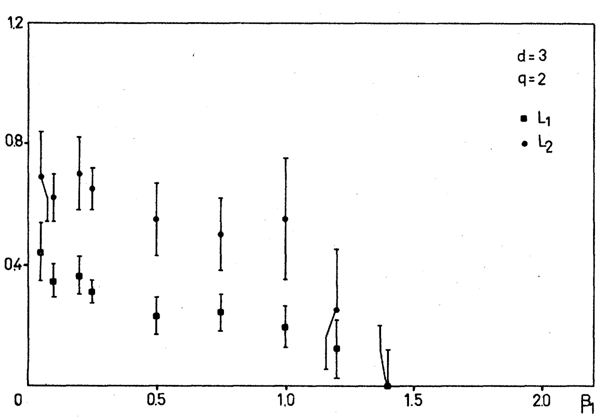

FIG. 5. Latent heat corresponding to I—II transition in Fig. 3 versus β_1 for the P_2 model in three dimensions. The squares and circles represent the plaquette and fenêtre contributions, respectively.

III. CONCLUSIONS AND REMARKS

The results presented in this paper complete, in a certain way, those of Edgar[3] and Bhanot *et al.*,[2] because two- and three-dimensional lattices as well as several values of q were considered.

The two-dimensional model is nontrivial. Therefore, a generalization of the theorem[8] which states that Wilson actions do not produce phase transitions in two dimensions is not possible. However, it would be very interesting to search for other models with a richer phase structure in two dimensions. Work is in progress in that direction.

Only first-order phase transitions were detected for the P_2 model in four dimensions.[2] Due to this fact we discarded the study of the four-dimensional model for $q>2$ because no qualitative different phase transitions were to be found. For the same reason, we did not consider $q>3$ in three dimensions.

Finally, we want to make some remarks concerning the numerical work. Much care is to be taken with Monte Carlo calculations on models with competitive interactions. The presence of metastable states makes difficult the identification of phase transitions in the thermal cycles. In the present model, these problems appear when $\beta_1>0$ and $\beta_2<0$, especially for large values of $|\beta_1|$ and $|\beta_2|$. For instance, the $P_2(d=2)$ thermal cycles showed hysteresis effects for $\beta_1\neq 0$, marking the existence of two, and sometimes more, phase transitions for a single value of β_1. Of course, no discontinuities in both E_1 and E_2 were detected when long runs with mixed initial configurations were used to evaluate the specific-heat behavior. These metastable states can also be interpreted as a remainder of the three-dimensional phase structure, in a similar way as it is done in Ref. 9.

ACKNOWLEDGMENTS

We are indebted to L. N. Epele for very useful remarks and suggestions. This work was partially supported by Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Comisión de Investigaciones Científicas de la Provincia de Buenos Aires (CIC PBA), and Subsecretaría de Ciencia y Técnica (SUBCYT), Argentina.

[1]See, for example, C. Camarata, L. N. Epele, H. Fanchiotti, and C. A. García Canal, Nucl. Phys. **B235** [FS11], 299 (1984), and references therein.

[2]G. Bhanot, J. M. Drouffe, A. Schiller, and I. O. Stamatescu, Phys. Lett. **125B**, 67 (1983).

[3]R. C. Edgar, Nucl. Phys. **B200** [FS4], 345 (1982).

[4]M. Creutz, L. Jacobs, and C. Rebbi, Phys. Rep. **95**, 201 (1983).

[5]L. Turban, J. Phys. A **17**, 419 (1984).

[6]F. Y. Wu, Rev. Mod. Phys. **54**, 235 (1982).

[7]H. Fanchiotti, S. J. Sciutto, and H. Vucetich, Z. Phys. B **55**, 57 (1984).

[8]L. P. Kadanoff, Rev. Mod. Phys. **49**, 267 (1977).

[9]M. Baig, Phys. Rev. Lett. **54**, 167 (1985).