Nonperturbative propagators in quantum chromodynamics

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Leading nonperturbative corrections to the quark and gluon propagators are derived following the assumption that the nonperturbative QCD vacuum can be described in terms of nonvanishing vacuum expectation values for the composite operators $[\bar{\psi}\psi]$ and $[G_{\mu\nu}{}^2]$. The nonperturbative quark propagator can be described in terms of a running gauge-dependent quark mass and a running normalization function. The nonperturbative gluon propagator can be described in terms of a running gauge parameter and a running normalization function.

I. INTRODUCTION

In this article we study quark and gluon propagation in the presence of quark and gluon condensates. The basic assumption underlying our analysis is that the nonperturbative QCD vacuum can be described in terms of nonvanishing vacuum expectation values (VEV's) of gauge-invariant composite operators such as the quark condensate $[\bar{\psi}\psi]$ and the gluon condensate $[G_{\mu\nu}{}^2]$. This hypothesis has been used extensively in successful attempts to account for nonperturbative phenomena. It is the key assumption in the so-called QCD sum rules[1] which have been used to study (among other subjects) charmonium decays, SU(3)-symmetry-breaking effects, and hadron wave functions.[2] Another application of the same assumption is a recent analysis[3] of the MIT bag model where it is found that the existence of quark and gluon condensates allows for the possibility of having a small strong coupling constant inside the bag.

Mueller[4] has recently shown that it is possible to define the nonperturbative condensates in a consistent way. Fukuda and Kazama[5] have demonstrated condensation of $[G_{\mu\nu}{}^2]$ by constructing the effective potential through the trace-anomaly equation. However, we feel that this latter derivation remains somewhat ambiguous since it is not based on a rigorous definition of the gluon condensate.

In the following analysis, the composite operators will appear in the operator-product expansion[6] (OPE) of the quark and gluon propagators, respectively. The operators in this expansion must carry vacuum quantum numbers, and from general grounds one may conclude that they must be gauge singlets. The operators in the OPE are furthermore characterized by their dimension. The contribution from an operator with a higher dimension falls off more rapidly with momentum than a lower-dimension operator.

Apart from the unit operator, the operators $[\bar{\psi}\psi]$ (dimension 3) and $[G_{\mu\nu}{}^2]$ (dimension 4) are the gauge-independent operators of lowest dimension in QCD. In the following we neglect higher-dimension operators. It should be mentioned that if higher-dimension operators acquire nonzero VEV's, their influence can be neglected only for large momenta. We also note that the OPE can be taken seriously only for hard momenta in the propagator to be expanded.

In Refs. 1 and 2 the following phenomenological values for the VEV's of $[\bar{\psi}\psi]$ and $[G_{\mu\nu}{}^2]$ are given:

$$\langle\Omega\,|\,[\bar{u}u]\,|\,\Omega\rangle=\langle\Omega\,|\,[\bar{d}d]\,|\,\Omega\rangle=1.3\langle\Omega\,|\,[\bar{s}s]\,|\,\Omega\rangle \simeq-(0.25\text{ GeV})^3\,,$$
$$\langle\Omega\,|\,[\bar{c}c]\,|\,\Omega\rangle=\langle\Omega\,|\,[\bar{b}b]\,|\,\Omega\rangle=\langle\Omega\,|\,[\bar{t}t]\,|\,\Omega\rangle\simeq 0\,, \tag{1}$$
$$\left\langle\Omega\left|\left[\frac{\alpha_s}{\pi}G_{\mu\nu}{}^2\right]\right|\Omega\right\rangle\simeq+0.012\text{ GeV}^4\,.$$

The authors of Ref. 1 use a value close to 1 for α_s, corresponding to $Q^2=0.2$ GeV and $\Lambda_{\text{QCD}}=0.1$ GeV.

The effect of the quark condensate on the quark propagator was analyzed in 1976 by Politzer[7] and the analysis was later revised by Pascual and de Rafael[8] who pointed out some numerical errors in Politzer's paper. Politzer's main interest was in the derivation of the so-called nonperturbative quark mass, obtained through the nonperturbative quark propagator. Here we revise the analysis in Refs. 7 and 8 and we extend it to include the gluon-condensate contribution to the quark propagator. The nonperturbative quark propagator can be described in terms of a running (momentum-dependent) normalization function and a nonperturbative quark mass. We derive these quantities and compare the results to those of Refs. 7 and 8.

We evaluate the nonperturbative corrections to the gluon propagator in a general covariant gauge. The corrected propagator can be described in terms of a running normalization function and a running gauge parameter.

The ghost propagator is not influenced by $[\bar{\psi}\psi]$ or $[G_{\mu\nu}{}^2]$. In the following our analysis will be done at the tree level and all the calculations are done in Euclidean space.

II. THE NONPERTURBATIVE QUARK PROPAGATOR

The operator-product expansion of the inverse quark propagator in the presence of nonvanishing VEV's for the operators $[\bar{\psi}\psi]$ and $[G_{\mu\nu}{}^2]$ is

$${}^{ab}_{\alpha\beta}S_{np}{}^{-1}\approx{}^{ab}_{\alpha\beta}C^{\mathbf{1}}(p)\mathbf{1}+{}^{ab}_{\alpha\beta}C^{[\bar{\psi}\psi]}(p)\langle\Omega\,|\,[\bar{\psi}\psi]\,|\,\Omega\rangle +{}^{ab}_{\alpha\beta}C^{[G_{\mu\nu}{}^2]}(p)\langle\Omega\,|\,[G_{\mu\nu}{}^2]\,|\,\Omega\rangle\,. \tag{2}$$

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The coefficients carry the spinor and color quantum numbers as indicated by the indices α,β (spinor) and a,b (quark color). The coefficients are calculated perturbatively. Here they will be calculated to first order in α_s. ${}^{ab}_{\alpha\beta}C^{\mathbf{1}}(p)$, the coefficient associated with the unit operator, is equal to the perturbative inverse quark propagator. In Euclidean space we hence have

$$ {}^{ab}_{\alpha\beta}C^{\mathbf{1}}(p)=i\delta^{ab}(\not p+m)\ . \tag{3}$$

In order to calculate ${}^{ab}_{\alpha\beta}C^{[\bar\psi\psi]}(p)$, we will follow the standard recipe in the literature.[9] The coefficient is obtained by equating a $(2+n)$-point, one-particle irreducible Green's function—where two of the external legs have hard momentum and the remaining n external legs are assigned zero momentum—with the coefficient times an n-point Green's function with an insertion of the operator under study at zero momentum. The number n corresponds to the number of elementary fields contained in the composite operator.

In our present case, we chose to study the quark-antiquark, one-gluon-exchange, four-point function. The operator-product expansion of the four-point function is illustrated in Fig. 1. It reads

$$P^{cd}_{\gamma\delta}\,{}^{abcd}_{\alpha\beta\gamma\delta}\Gamma^{(2+2)}(p,p,0,0)\approx{}^{ab}_{\alpha\beta}C^{[\bar\psi\psi]}(p)\times P^{cd}_{\gamma\delta}\,{}^{cd}_{\gamma\delta}\Gamma^{(2)}_{[\bar\psi\psi(0)]}(0)\ . \tag{4}$$

In this equation, the one-particle irreducible Green's function ${}^{abcd}_{\alpha\beta\gamma\delta}\Gamma^{(2+2)}(p,p,0,0)$ is the diagram on the left-hand side in Fig. 1. The role of the contraction operator $P^{cd}_{\gamma\delta}$ is to ensure that the collective quantum numbers of the soft legs are those of the vacuum. It also effectuates the necessary summation over color and spinor indices. In this case $P^{cd}_{\gamma\delta}$ is simply equal to $\delta^{cd}\delta_{\gamma\delta}$. Note that $[\bar\psi\psi]$ is defined as $[\bar\psi^a_\alpha\psi^a_\alpha]$.

The t channel does not contribute to the left-hand side in Fig. 1 since it is identical to zero when the two lower legs are assigned zero momentum. The fact that only the s channel enters the calculation implies that only condensates with the same flavor as the quark with hard momentum (the propagating quark) contribute. The diagram on the left-hand side of Fig. 1 is gauge dependent due to the gauge dependence in the gluon propagator. We evaluate this diagram in a general covariant gauge. When contracted in spinor and color space, the diagram on the left-hand side in Fig. 1 yields

$$P^{cd}_{\gamma\delta}\,{}^{abcd}_{\alpha\beta\gamma\delta}\Gamma^{(2+2)}(p,p,0,0)=\tfrac{4}{3}(3+\alpha_G)g^2\delta^{ab}\delta_{\alpha\beta}\frac{1}{p^2}\ . \tag{5}$$

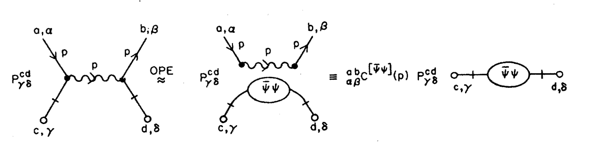

FIG. 1. Graphical illustration of Eq. (4). The bars on the lower legs indicate that the corresponding propagator is assigned zero momentum. The upper diagram in the middle part of the equation is defined to be the graphical representation of the coefficient ${}^{ab}_{\alpha\beta}C^{[\bar\psi\psi]}(p)$.

An insertion of $[\bar\psi\psi]$, at zero momentum in the inverse quark propagator, $\Gamma^{(2)}(0)$, is obtained by differentiation with respect to minus the quark mass:

$$\Gamma^{(2)}_{[\bar\psi\psi(0)]}(0)=\frac{\partial}{\partial(-m)}\Gamma^{(2)}(0)\ . \tag{6}$$

After contraction with $P^{cd}_{\gamma\delta}$ we have

$$P^{cd}_{\gamma\delta}\,{}^{cd}_{\gamma\delta}\Gamma^{(2)}_{[\bar\psi\psi(0)]}(0)=12i\ . \tag{7}$$

Combining (4), (5), and (7) we now obtain

$${}^{ab}_{\alpha\beta}C^{[\bar\psi\psi]}(p)=\frac{(3+\alpha_G)}{9}ig^2\delta^{ab}\delta_{\alpha\beta}\frac{1}{p^2}\ . \tag{8}$$

In order to obtain ${}^{ab}_{\alpha\beta}C^{[G_{\mu\nu}{}^2]}$ in (2), we study the expansion of a six-point function with two hard quark legs and four soft gluon legs. The OPE expansion of this diagram is illustrated in Fig. 2, and it reads

$$P^{ABCD}_{\mu\nu\rho\sigma}\,{}^{ab,ABCD}_{\alpha\beta,\mu\nu\rho\sigma}\Gamma^{(2+4)}(p,p,0,0,0,0)\approx{}^{ab}_{\alpha\beta}C^{[G_{\mu\nu}{}^2]}(p)P^{ABCD}_{\mu\nu\rho\sigma}\,{}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)}_{[G_{\mu\nu}{}^2(0)]}\ . \tag{9}$$

The contraction operator, $P^{ABCD}_{\mu\nu\rho\sigma}$, plays the same role here as in the previous case. It ensures that the collective quantum numbers of the soft gluon legs are those of vacuum and it effectuates the summation over gluon color and space-time indices compatible with that requirement. $P^{ABCD}_{\mu\nu\rho\sigma}$ is identical to the contraction operator in the four-gluon vertex:

$$\begin{aligned}P^{ABCD}_{\mu\nu\rho\sigma}&=f^{ABE}f^{CDE}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\nu\rho}\delta_{\mu\sigma})\\&\quad+f^{CBE}f^{ADE}(\delta_{\mu\rho}\delta_{\nu\sigma}-\delta_{\nu\mu}\delta_{\rho\sigma})\\&\quad+f^{DBE}f^{CAE}(\delta_{\mu\nu}\delta_{\rho\sigma}-\delta_{\nu\rho}\delta_{\mu\sigma})\ .\end{aligned} \tag{10}$$

$\Gamma^{(2+4)}$ is a one-particle irreducible six-point function and $\Gamma^{(4)}_{[G_{\mu\nu}{}^2(0)]}$ is the gluon four-point vertex with an insertion of $[G_{\mu\nu}{}^2]$ at zero momentum. Such an insertion in a one-particle irreducible Green's function is obtained through the following formula:[10]

$$\Gamma_{[G_{\mu\nu}{}^2(0)]}=-4\left[-\frac{g}{2}\frac{\partial}{\partial g}+\alpha_G\frac{\partial}{\partial\alpha_G}+\frac{n_g}{2}\right]\Gamma\ . \tag{11}$$

Here, n_g is the number of external gluon legs and α_G is the gauge parameters. In our case $n_g=4$. With

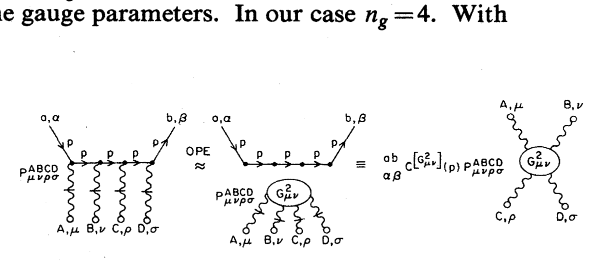

FIG. 2. Graphical representation of Eq. (9). Notations are the same as in Fig. 1.

$$ {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)} = -g^2 P^{ABCD}_{\mu\nu\rho\sigma} \, , \tag{12} $$

we hence get

$$ {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)}_{[G_{\mu\nu}{}^2(0)]} = 4g^2 P^{ABCD}_{\mu\nu\rho\sigma} \tag{13} $$

and

$$ P^{ABCD}_{\mu\nu\rho\sigma}\, {}^{ABCD}_{\mu\nu\rho\sigma}\Gamma^{(4)}_{[G_{\mu\nu}{}^2(0)]}(0) = 31104 g^2 \, . \tag{14} $$

The left-hand side of (9) is evaluated to be

$$ P^{ABCD}_{\mu\nu\rho\sigma}\, {}^{ab,ABCD}_{\alpha\beta,\mu\nu\rho\sigma}\Gamma^{(2+4)}(p,p,0,0,0,0) = 36 i g^4 \delta^{ab} \frac{1}{(p^2+m^2)^3}(2p^2\not{p} + 3mp^2 + 3m^2\not{p} + 4m^3)_{\alpha\beta} \, . \tag{15} $$

From (9), (14), and (15) we now get

$$ {}^{ab}_{\alpha\beta}C^{[G_{\mu\nu}{}^2]}(p) \approx \frac{i}{864} g^2 \delta^{ab} \frac{1}{(p^2+m^2)^3}(2p^2\not{p} + 3mp^2 + 3m^2\not{p} + 4m^3)_{\alpha\beta} \, . \tag{16} $$

We are now in a position to obtain the nonperturbative inverse quark propagator. From (2), (3), (8), and (16) we get

$$ \begin{aligned} {}^{ab}_{\alpha\beta}S^{-1}_{np} \approx{} & i\delta^{ab}(\not{p}+m)_{\alpha\beta} + \frac{3+\alpha_G}{9} i g^2 \delta^{ab}\delta_{\alpha\beta}\frac{1}{p^2}\langle \Omega \,|\, [\bar\psi\psi] \,|\, \Omega \rangle \\ & + \frac{1}{864} i g^2 \delta^{ab} \frac{1}{(p^2+m^2)^3}(2p^2\not{p} + 3mp^2 + 3m^2\not{p} + 4m^3)_{\alpha\beta} \langle \Omega \,|\, [G_{\mu\nu}{}^2] \,|\, \Omega \rangle \, . \end{aligned} \tag{17} $$

In Fig. 3 we give a diagrammatic interpretation of this equation. The nonperturbative inverse propagator can be expressed in terms of a running normalization function, $N_q(p^2)$, and a running mass, $M(p^2)$, in the following way:

$$ {}^{ab}_{\alpha\beta}S^{-1}_{np} = i\delta^{ab}\frac{1}{N_q(p^2)}[\not{p} + M(p^2)]_{\alpha\beta} \, . \tag{18} $$

This corresponds to Politzer's definition of the nonperturbative mass.[7] We have

$$ N_q(p^2) = \left[1 + \frac{1}{864} g^2 \frac{1}{(p^2+m^2)^3}(2p^2 + 3m^2)\langle \Omega \,|\, [G_{\mu\nu}{}^2] \,|\, \Omega \rangle \right]^{-1} \tag{19} $$

and

$$ M(p^2) = \left[m + \frac{3+\alpha_G}{9} g^2 \frac{1}{p^2}\langle \Omega \,|\, [\bar\psi\psi] \,|\, \Omega \rangle + \frac{1}{864} g^2 \frac{1}{(p^2+m^2)^3}(3mp^2 + 4m^3)\langle \Omega \,|\, [G_{\mu\nu}{}^2] \,|\, \Omega \rangle \right] N_q(p^2) \, . \tag{20} $$

Inverting (18) we obtain the nonperturbative quark propagator

$$ {}^{ab}_{\alpha\beta}S_{np} = N_q(p^2) \left[\frac{-i\delta^{ab}}{\not{p} + M(p^2)} \right]_{\alpha\beta} \, . \tag{21} $$

If we let both m and $\langle \Omega \,|\, [G_{\mu\nu}{}^2] \,|\, \Omega \rangle$ be zero, we see that this expression agrees with that of Ref. 8. We note that for large p^2 the mass is identical to Politzer's mass (up to a constant factor, see Ref. 8). For low momenta, where the gluon condensate is important, our mass goes like p^2, differing drastically from Politzer's mass which goes like $1/p^2$ for all momenta. However we want to stress that for low momenta the nonperturbative quark mass might get large contributions from higher-dimension operators. The only region where we can feel confident about our results is in the asymptotic region where our result coincides with that of Politzer.

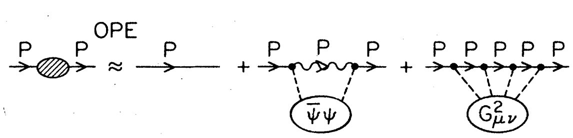

FIG. 3. Graphical interpretation of Eq. (17).

III. THE NONPERTURBATIVE GLUON PROPAGATOR

The OPE for the inverse gluon propagator is

$$ \begin{aligned} (D^{\mu\nu,AB}_{np})^{-1} \approx{} & {}^{AB}_{\mu\nu}C^{1}(p)\mathbb{1} + {}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)\langle \Omega \,|\, [\bar\psi\psi] \,|\, \Omega \rangle \\ & + {}^{AB}_{\mu\nu}C^{[G_{\mu\nu}{}^2]}(p)\langle \Omega \,|\, [G_{\mu\nu}{}^2] \,|\, \Omega \rangle \, . \end{aligned} \tag{22} $$

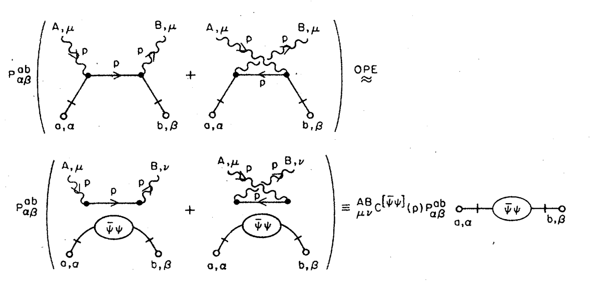

FIG. 4. Graphical representation of Eq. (24). Notations are the same as in Fig. 1.

We evaluate this expression in a general covariant gauge where we have the perturbative inverse propagator:

$$ {}^{AB}_{\mu\nu}C^{1}(p)=\delta^{AB}p^{2}\left[\delta_{\mu\nu}-\left[1-\frac{1}{\alpha_G}\right]\frac{p_\mu p_\nu}{p^2}\right] . \qquad (23)$$

Here α_G is the gauge parameter and the notation is such that $\alpha_G=1$ corresponds to the Feynman gauge. The coefficient ${}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)$ is obtained from the following equation, graphically represented in Fig. 4:

$$P^{ab}_{\alpha\beta}\,{}^{AB,ab}_{\mu\nu,\alpha\beta}\Gamma^{(2+2)}(p,p,0,0) \approx {}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)P^{ab}_{\alpha\beta}\,{}^{ab}_{\alpha\beta}\Gamma^{(2)}_{[\bar\psi\psi(0)]}(0) . \qquad (24)$$

Here $\Gamma^{(2+2)}$ is the sum of the two diagrams on the left-hand side in Fig. 4. The contraction operator $P^{ab}_{\alpha\beta}$ is the same as the one we used in the derivation of the quark-condensate contributions to the quark propagator. After contraction with $P^{ab}_{\alpha\beta}$ we have

$$P^{ab}_{\alpha\beta}\,{}^{AB,ab}_{\mu\nu,\alpha\beta}\Gamma^{(2+2)}(p,p,0,0)=-12ig^2\delta^{AB}\delta_{\mu\nu}\frac{m}{p^2+m^2} . \qquad (25)$$

Combining with (7) and (23) we have

$$ {}^{AB}_{\mu\nu}C^{[\bar\psi\psi]}(p)\approx g^2\delta^{AB}\delta_{\mu\nu}\frac{m}{p^2+m^2} . \qquad (26)$$

In order to obtain the contribution from the gluon condensate, we study the following equation, graphically represented in Fig. 5:

$$P^{CDEF}_{\rho\sigma\tau\lambda}\,{}^{AB,CDEF}_{\mu\nu,\rho\sigma\tau\lambda}\Gamma^{(2+4)}(p,p,0,0,0,0) \approx {}^{AB}_{\mu\nu}C^{[G_{\mu\nu}{}^2]}(p)P^{CDEF}_{\rho\sigma\tau\lambda}\,{}^{CDEF}_{\rho\sigma\tau\lambda}\Gamma^{(4)}_{[G_{\mu\nu}{}^2(0)]}(0) . \qquad (27)$$

Here $\Gamma^{(2+4)}$ is the sum of the five diagrams on the left-hand side in Fig. 5. The left-hand side in (27) is evaluated[11] to be (in a general covariant gauge, Euclidean space)

$$P^{CDEF}_{\rho\sigma\tau\lambda}\,{}^{AB,CDEF}_{\mu\nu,\rho\sigma\tau\lambda}\Gamma^{(2+4)}(p,p,0,0,0,0)=\frac{27}{2}\delta^{AB}g^4\frac{1}{p^2}\left[\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)^2-31\frac{p_\mu p_\nu}{p^2}(1-\alpha_G) -\delta_{\mu\nu}(1-\alpha_G)^2+28\delta_{\mu\nu}(1-\alpha_G)-13\frac{p_\mu p_\nu}{p^2}+13\delta_{\mu\nu}\right] . \qquad (28)$$

From (14) and (23) we get

$$ {}^{AB}_{\mu\nu}C^{[G_{\mu\nu}{}^2]}(p)\approx\frac{1}{2304}\delta^{AB}g^2\frac{1}{p^6}\left[\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)^2-31\frac{p_\mu p_\nu}{p^2}(1-\alpha_G) -\delta_{\mu\nu}(1-\alpha_G)^2+28\delta_{\mu\nu}(1-\alpha_G)-13\frac{p_\mu p_\nu}{p^2}+13\delta_{\mu\nu}\right] . \qquad (29)$$

We now get the nonperturbative inverse gluon propagator from (22), (23), (26), and (29):

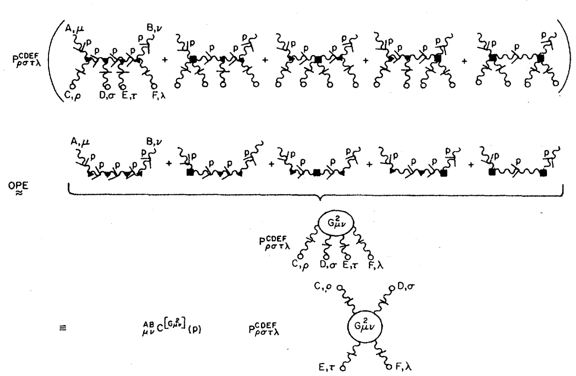

FIG. 5. Graphical representation of Eq. (27). Notations are the same as in Figs. 1 and 2. The small squares in the diagram represent four-gluon couplings and the triangles three-gluon couplings.

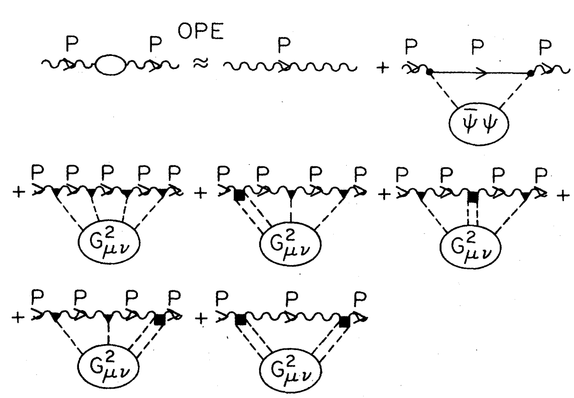

FIG. 6. Graphical interpretation of Eq. (30).

$$(D_{np}^{\mu\nu,AB})^{-1} \approx \delta^{AB} p^2 \left[\delta_{\mu\nu} - \left[1 - \frac{1}{\alpha_G} \right] \frac{p_\mu p_\nu}{p^2} \right] + \delta^{AB} g^2 \delta_{\mu\nu} \sum_{q=d,u,s,c,b,t} \frac{m_q}{(p^2+m_q^2)} \langle \Omega | [\bar{q}q] | \Omega \rangle$$
$$+ \frac{1}{2304} \delta^{AB} g^2 \frac{1}{p^2} \left[\frac{p_\mu p_\nu}{p^2}(1-\alpha_G)^2 - 31 \frac{p_\mu p_\nu}{p^2}(1-\alpha_G) - \delta_{\mu\nu}(1-\alpha_G)^2 \right.$$
$$\left. + 28\delta_{\mu\nu}(1-\alpha_G) - 13\frac{p_\mu p_\nu}{p^2} + 13\delta_{\mu\nu} \right] \langle \Omega | [G_{\mu\nu}{}^2] | \Omega \rangle . \tag{30}$$

A diagrammatic representation of this equation is given in Fig. 6. In analogy with the inverse quark propagator, we write the inverse gluon propagator in the following form:

$$(D_{np}^{\mu\nu,AB})^{-1} = \frac{1}{N_g(p^2)} \delta^{AB} p^2 \left[\delta_{\mu\nu} - \left[1 - \frac{1}{A_G(p^2)} \right] \frac{p_\mu p_\nu}{p^2} \right] , \tag{31}$$

where we have introduced a running normalization function

$$N_g(p^2) = \left[1 + \sum_{q=d,u,s,c,b,t} \frac{g^2 m_q \langle \Omega | [\bar{q}q] | \Omega \rangle}{p^2(p^2+m_q^2)} + \frac{g^2 \langle \Omega | [G_{\mu\nu}{}^2] | \Omega \rangle}{2304 p^4} [13-(1-\alpha_G)^2 + 28(1-\alpha_G)] \right]^{-1} , \tag{32}$$

and a running gauge parameter, $A(p^2)$. The expression for the running gauge parameter is rather complicated in a general covariant gauge. The gauge in which the physics is most transparent is the Landau gauge where $\alpha_G = A_G(p^2) \equiv 0$. In this gauge all the nonperturbative effects are contained in the running normalization function and the expression for the nonperturbative gluon propagator reads

$$D_{np}^{\mu\nu,AB} = \left[1 + \sum_{q=d,u,s,c,b,t} \frac{g^2 m_q \langle \Omega | [\bar{q}q] | \Omega \rangle}{p^2(p^2+m_q^2)} + \frac{5g^2 \langle \Omega | [G_{\mu\nu}{}^2] | \Omega \rangle}{288 p^4} \right]^{-1} \times \frac{\delta^{AB}}{p^2} \left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] . \tag{33}$$

IV. DISCUSSION

Our main results are the expressions for the nonperturbative quark and gluon propagators, Eqs. (21) and (33). For low values of momenta, these expressions will have to be corrected by contributions from any operators of higher dimension than four that acquire nonzero VEV's, and from multiple insertions of $[\bar{\psi}\psi]$ and $[G_{\mu\nu}{}^2]$. We suggest (quite arbitrarily and probably very conservatively) that as long as the nonperturbative corrections that we have calculated here are one order of magnitude smaller than the regular propagator, further nonperturbative corrections can probably safely be neglected. With the phenomenological values of the VEV's of $[\bar{\psi}\psi]$ and $[G_{\mu\nu}{}^2]$ in (1), we thus estimate that the nonperturbative corrections that we have calculated here can be taken seriously for $(-p^2)^{1/2} \geq 0.60$ GeV in the case of the gluon propagator and $(-p^2)^{1/2} \geq 0.80$ GeV in the case of the quark propagator. One way to lower the limit of validity of the nonperturbative analysis is of course to calculate, or at least estimate, the contribution from higher-dimension operators and from multiple-operator insertions.

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