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Radiation from initially static vacuum structures

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An alternate scheme of quantizing extended vacuum structures is further developed to include the case of scalar multiplets. Approximate static bubble and multibubble states are presented for several scalar theories which possess a symmetry that creates energetically degenerate minima for their potentials. The first theory is the standard ψ^4 model, while the second theory is a model which exhibits spontaneous *CP* violation. The approximate bubble solution is consistent with selecting the particle spectrum associated with a constant vacuum expectation value for the scalar field. The spectrum of radiation associated with the collapse is calculated in lowest-order approximation and the average number of emitted particles is found to be proportional to the area of the bubble. Approximate multibubble solutions are presented and the energy of these configurations exhibits an attraction between the bubbles.

I. INTRODUCTION

Many of the cosmological ramifications of extended vacuum structures, which might be formed during a primordial phase transition early in the history of the Universe, have been examined. Such structures will be possible within the context of grand unified models whenever there is a discrete symmetry which causes the manifold of the invariance group to be multiply connected. The domain walls and strings which may be formed can be either stable or unstable depending upon the dynamics of the parent theory. The initial nucleation and subsequent growth of bubbles of true vacuum have been examined.[1] The formation and effects of domain walls have been discussed, and it has been shown that the mass-energy associated with stable domain walls renders them inconsistent with current ideas regarding cosmological evolution, and hence should be avoided.[2] The gravitational field associated with planar walls has been investigated,[3] and the gravitational and particle radiation from oscillating vacuum structures has been calculated.[4]

In previous work the collapse of a time-dependent vacuum bubble formed in a simple scalar theory was evaluated.[5] This process was modeled within the framework of an alternate technique for quantizing a theory in the presence of an extended vacuum structure.[6] This technique allows use of the particle spectrum associated with the choice of a constant vacuum expectation value, as long as all the constant vacua are induced by the discrete symmetry and are thus energetically degenerate. Such an approach is ideal for modeling the collapse of an unstable vacuum structure into the translationally invariant sector. In addition, the equation for determining the shape of the vacuum structure is considerably easier to solve in this approach.

This paper will treat the case of a vacuum bubble which is initially static. Consideration will be limited to the scalar sector of several theories in both one and three spatial dimensions, the first being the standard ψ^4 model, while the second will be a model which exhibits spontaneous *CP* violation. However, the technique of quantization will be extended to the case where there is more than one scalar field present in the theory. The difference between the expectation value of the Hamiltonian in the bubble state and the translationally invariant ground state to which it will collapse is evaluated in lowest order, and this yields the total energy of emitted particles. The spectrum of emitted particles is obtained from the matrix element of the bubble state and the one-particle translationally invariant state to which it can collapse. The square of this matrix element gives the relative spectrum of emitted particles. The absolute spectrum is obtained by assuming that it is proportional to the relative spectrum and then calculating the total energy of emitted particles. The coefficient of normalization is then obtained by matching to the energy calculated from the Hamiltonian. The values for the number of emitted particles and the Hamiltonian are found to be proportional to the surface area of the bubble.

II. GENERAL BUBBLE SOLUTIONS

Since previous work[6] was limited to theories of a single scalar field a brief extension to the case of scalar multiplets will be presented. However, only domain-wall formation will be considered. The gauge field sector of the theory, and hence the possibility of string formation, will be ignored except to assume that all Goldstone bosons disappear as the longitudinal components of massive Proca bosons.

Consideration will be limited to those theories whose Higgs potentials $V(\psi)$ possess a discrete symmetry which creates a set of distinct and degenerate minima in field space. There are then different sets of solutions to the equation

$$\frac{\partial V}{\partial \psi_i}=0 , \tag{1}$$

where ψ_i is the ith scalar field. The jth possible solution to (1) for the ith field ψ_i will be labeled v_{ij}. The index i runs over all fields while the range of j is dictated by the form of the potential. The symmetry dictates that

$$V(\psi_i=v_{ij})=V(\psi_i=v_{ik}) , \tag{2}$$

for all j and k, and that the mass matrix

$$M_{ij}=\frac{1}{2}\left.\frac{\partial^2 V}{\partial\psi_i\partial\psi_j}\right|_{\psi_i=v_{ik}} \tag{3}$$

has identical eigenvalues regardless of which set of solutions to (1) is used. It then follows that domain walls are possible and describe regions of space where the vacuum expectation value of the field interpolates between two of the values available as solutions to (1). The form of these domain walls in the tree approximation is given by a classical solution to the equations of motion of the theory. Unfortunately, only a limited number of such solutions are known analytically. The picture is further complicated by Derrick's theorem,[7] which predicts all purely scalar solutions in higher than one spatial dimension will be unstable and will thus collapse into the translationally invariant sector.

It is possible to bypass some of these problems by using an alternate form of quantization in the presence of an extended vacuum structure which does not require the knowledge of an exact solution to the equations of motion. This approach employs interaction-picture fields whose masses are given by the eigenvalues of the mass matrix (3). The same unitary matrix U which diagonalizes the mass matrix is next applied to each possible set of solutions to (1). To be specific, if

$$\widetilde{M}_{ij}=U_{ik}M_{kl}U_{lj}^{\dagger} , \tag{4}$$

where $\widetilde{M}$ is a diagonal matrix, then the diagonalized solutions to (1) are defined as

$$\widetilde{v}_{ik}=U_{ij}v_{jk} . \tag{5}$$

A discontinuous function of space is then constructed by partitioning all space into sections and assigning one of the possible sets of (5) to each section. As an example, consider the spherically symmetric function

$$\widetilde{v}_i(x)=\widetilde{v}_{ij}\theta(r-a)+\widetilde{v}_{ik}\theta(a-r) , \tag{6}$$

where θ is the standard step function. This assigns the jth value outside a ball of radius a and the kth value inside the ball. Obviously, more complex structures are possible, the only rule being that each section of space is associated with only one of the possible sets of unitarily transformed solutions to (1).

It is consistent to assume that the interaction-picture fields exist as the weak limit of the interpolating fields,

$$\operatorname*{w-lim}_{t\to t_\pm} U_{ij}\psi_j=\phi_i . \tag{7}$$

The incident extended vacuum structure consistent with (6) and (7) is realized as a coherent state in n spatial dimensions,

$$|f\rangle=\exp\left[i\int d^n x(\phi_j\dot{f}_j-\pi_j f_j)\right]|0\rangle , \tag{8}$$

where $|0\rangle$ is the vacuum cyclic with respect to the algebra of the interaction-picture fields and π_j is the momentum canonically conjugate to ϕ_j. The functions f_j appearing in (8) are solutions to the equation

$$(\Box+m_j{}^2)f_j(x)=m_j{}^2\widetilde{v}_j(x) \tag{9}$$

(no summation on j), where m_j is the mass of the jth interaction-picture field and $\widetilde{v}_j$ is the function constructed from the values of (5), of which relation (6) is an example. It is straightforward to show that

$$\phi_{ic}(x)=\lim_{\hbar=0}\langle f,t_+|\psi_i(x)|f,t_-\rangle \tag{10}$$

is a classical solution to the original equation of motion such that $U_{ij}\phi_{jc}$ interpolates between the constant values chosen for $\widetilde{v}_i$. The approximate forms for bubble and multibubble extended structures are obtained by solving Eq. (9) with an appropriate choice for $\widetilde{v}_i(x)$. This is easily seen from the fact that

$$\langle f,t|\phi_i(x)|f,t\rangle=f_i(x) . \tag{11}$$

The simplest case is that of a one-dimensional system. While not of physical significance, such a system offers insights because of the mathematical simplifications obtained. The case of the kink has been analyzed elsewhere.[8] The general form of the static double kink or bubble in one dimension is obtained by choosing

$$\widetilde{v}_i(x)=\widetilde{v}_{ij}\theta(a-x)+\widetilde{v}_{ik}\theta(x-a)\theta(b-x)+\widetilde{v}_{il}\theta(x-b) , \tag{12}$$

where it is assumed that $a<b$. Equation (9) gives

$$\begin{aligned} f_i(x)=&\theta(a-x)[\widetilde{v}_{ij}-\tfrac{1}{2}(\widetilde{v}_{ij}-\widetilde{v}_{ik})e^{-m_i(a-x)}-\tfrac{1}{2}(\widetilde{v}_{ik}-\widetilde{v}_{il})e^{-m_i(b-x)}] \\ &+\theta(x-a)\theta(b-x)[\widetilde{v}_{ik}+\tfrac{1}{2}(\widetilde{v}_{ij}-\widetilde{v}_{ik})e^{-m_i(x-a)}-\tfrac{1}{2}(\widetilde{v}_{ik}-\widetilde{v}_{il})e^{-m_i(b-x)}] \\ &+\theta(x-b)[\widetilde{v}_{il}+\tfrac{1}{2}(\widetilde{v}_{ij}-\widetilde{v}_{ik})e^{-m_i(x-a)}+\tfrac{1}{2}(\widetilde{v}_{ik}-\widetilde{v}_{il})e^{-m_i(x-b)}] . \end{aligned} \tag{13}$$

The generalization of (13) to the N-kink solution is straightforward.

The static solution of (9) consistent with the choice of (6) for three spatial dimensions is

$$f_i(r)=\theta(r-a)\left[\widetilde{v}_{ij}-\frac{c_i}{r}(\widetilde{v}_{ij}-\widetilde{v}_{ik})e^{-m_i(r-a)}\right]+\theta(a-r)\left[\widetilde{v}_{ik}+\frac{d_i}{r}(\widetilde{v}_{ij}-\widetilde{v}_{ik})\sinh(m_i r)\right] , \tag{14}$$

where

$$c_i = a - \frac{1}{2m_i}(1+m_i a)(1-e^{-2m_i a}), \quad d_i = \frac{1}{m_i}(1+m_i a)e^{-m_i a} . \tag{15}$$

A double-bubble solution is also possible. For simplicity each bubble will be assumed to have the same radius a. One bubble will be placed at position $\mathbf{r}_1$, the other at $\mathbf{r}_2$, and it will be assumed that the radius a is such that the two bubbles do not overlap. The form of $\tilde{v}_i(x)$ will be

$$\tilde{v}_i(x) = \tilde{v}_{ij}\theta(a - |\mathbf{x}-\mathbf{r}_1|) + \tilde{v}_{ik}\theta(a - |\mathbf{x}-\mathbf{r}_2|) + \tilde{v}_{il}\theta(|\mathbf{x}-\mathbf{r}_1| - a)\theta(|\mathbf{x}-\mathbf{r}_2| - a) , \tag{16}$$

so that

$$\begin{aligned} f_i(x) = {} & \theta(a - |\mathbf{x}-\mathbf{r}_1|)\left[\tilde{v}_{ij} + \frac{d_i}{|\mathbf{x}-\mathbf{r}_1|}(\tilde{v}_{il}-\tilde{v}_{ij})\sinh(m_i|\mathbf{x}-\mathbf{r}_1|) - \frac{c_i}{|\mathbf{x}-\mathbf{r}_2|}(\tilde{v}_{il}-\tilde{v}_{ik})e^{-m_i(|\mathbf{x}-\mathbf{r}_2|-a)}\right] \\ & + \theta(a - |\mathbf{x}-\mathbf{r}_2|)\left[\tilde{v}_{ik} + \frac{d_i}{|\mathbf{x}-\mathbf{r}_2|}(\tilde{v}_{il}-\tilde{v}_{ik})\sinh(m_i|\mathbf{x}-\mathbf{r}_2|) - \frac{c_i}{|\mathbf{x}-\mathbf{r}_1|}(\tilde{v}_{il}-\tilde{v}_{ij})e^{-m_i(|\mathbf{x}-\mathbf{r}_1|-a)}\right] \\ & + \theta(|\mathbf{x}-\mathbf{r}_1| - a)\theta(|\mathbf{x}-\mathbf{r}_2| - a)\left[\tilde{v}_{il} - \frac{c_i}{|\mathbf{x}-\mathbf{r}_1|}(\tilde{v}_{il}-\tilde{v}_{ij})e^{-m_i(|\mathbf{x}-\mathbf{r}_1|-a)}\right. \\ & \left. - \frac{c_i}{|\mathbf{x}-\mathbf{r}_2|}(\tilde{v}_{il}-\tilde{v}_{ik})e^{-m_i(|\mathbf{x}-\mathbf{r}_2|-a)}\right] . \end{aligned} \tag{17}$$

In order to find the energy available from the collapse it is necessary to calculate the expectation value of the Hamiltonian in the bubble state. Care must be taken since this will be, in general, a divergent quantity even at the tree level. However, the physically meaningful quantity is the difference between this value and the one obtained using the translationally invariant ground state, and this will be finite at the tree level. The value to be computed is

$$E = \langle f | \hat{H} | f \rangle - \langle v | \hat{H} | v \rangle , \tag{18}$$

where $|v\rangle$ is the translationally invariant ground state to which the bubble will collapse. The state $|v\rangle$ may also be realized as a coherent state

$$|v\rangle = \exp\left[-i\int d^n x(v_i \pi_i)\right]|0\rangle , \tag{19}$$

where v_i is the value to which f_i will collapse. At the lowest level it follows that

$$E = H[f] - H[v] . \tag{20}$$

It is possible to expand (20) around the function $\tilde{v}_i(x)$ in a functional power series in $g_i(x)$. It is not difficult to show that the lowest-order term in $g_i(x)$ is given by

$$\begin{aligned} & E \approx \sum_i \int d^n x[-\tfrac{1}{2}m_i^2\tilde{v}_i(x)g_i(x)] , \\ & g_i(x) = f_i(x) - \tilde{v}_i(x) . \end{aligned} \tag{21}$$

It is to be remembered that (20) does not contain all the tree-graph contributions since the choice of free fields will create terms in the evolution operator which are linear and quadratic in the interaction-picture fields. However, in the next section it will be shown that for the case of the kink in one dimension, where the exact tree-level energy may be computed, expression (20) is an excellent approximation.

Using the forms (8) and (18), along with a plane-wave decomposition of the interaction-picture fields, gives

$$\langle v | f \rangle = \exp\left[-\tfrac{1}{2}\int d^n k\, \omega_k^{(i)} |g_i(k)|^2\right] , \tag{22}$$

where

$$\begin{aligned} & \omega_k^{(i)} = (k^2 + m_i^2)^{1/2} , \\ & g_i(k) = \int \frac{d^n x}{(2\pi)^{n/2}}[f_i(x) - v_i]e^{i\mathbf{k}\cdot\mathbf{x}} . \end{aligned} \tag{23}$$

The spectrum of emitted particles is obtained by performing the following two steps. First, the matrix element between the bubble state and a particle state in the translationally invariant sector is found. It follows that

$$\langle \mathbf{k}^{(i)}, v | f \rangle = (\omega_k^{(i)})^{1/2} g_i(k) \langle v | f \rangle . \tag{24}$$

The relative probability of emission, P_k^i, is then obtained by squaring the matrix element, so that

$$P_k^i = \omega_k^{(i)} |g_i(k)|^2 . \tag{25}$$

The spectrum of the bubble collapse is then proportional to (25), so that

$$N_i(k) = C\omega_k^{(i)} |g_i(k)|^2 . \tag{26}$$

The coefficient C is next determined by matching the energy of the emitted radiation obtained from (26) to the energy (20) released in the collapse. This gives

$$C = E\left[\sum_i \int d^n k (\omega_k^{(i)})^2 |g_i(k)|^2\right]^{-1} . \tag{27}$$

Once C is known the total number of particles emitted by the collapsing bubble is given by

$$N_i = \int d^n k\, N_i(k) . \tag{28}$$

III. SPECIFIC MODELS

The first model to be examined is the standard single scalar field theory described by

$$V(\psi) = -\tfrac{1}{2}\alpha^2\psi^2 + \tfrac{1}{4}\lambda\psi^4 , \tag{29}$$

which has the discrete symmetry $\psi \to -\psi$. The two constant solutions to (1) are

$$v \equiv v_{11} = \left[\frac{\alpha^2}{\lambda}\right]^{1/2}, \quad v_{12} = -v , \tag{30}$$

while the mass of the interaction-picture field is

$$m^2 = 2\alpha^2 . \tag{31}$$

Using (28) and (24) gives the general form

$$E = \int d^n x [-\alpha^2 \tilde{v}(x) g(x) + \lambda \tilde{v}(x) g^3(x) + \tfrac{1}{4}\lambda g^4(x)] , \tag{32}$$

where

$$g(x) = f(x) - \tilde{v}(x) . \tag{33}$$

Before proceeding to the examination of bubble solutions it is instructive to examine the value of (32) in the case of the approximate kink, which is given by the one-dimensional form

$$f(x) = \theta(x-a) v (1 - e^{-m(x-a)}) - \theta(a-x) v (1 - e^{-m(a-x)}) . \tag{34}$$

For this case

$$E = \frac{35}{96}\frac{\alpha^3}{\lambda} . \tag{35}$$

The exact tree-level solution to the equation of motion is

$$\phi_c(x) = v \tanh \tfrac{1}{2} m (x-a) , \tag{36}$$

which gives the tree-level energy of this configuration

$$E_{\text{tree}} = \frac{1}{3}\frac{\alpha^3}{\lambda} . \tag{37}$$

Expression (35) is only slightly larger than (37), so that (33) is a very good approximation of (36). Form (32) is also well approximated by the first term of the Hamiltonian which is linear in g, whose general form is given by (21). The value of this term for the model (29) is

$$E \approx -\alpha^2 \int dx\, \tilde{v}(x) g(x) . \tag{38}$$

Using (34) gives

$$E \approx \frac{1}{2}\frac{\alpha^3}{\lambda} . \tag{39}$$

The approximation (38) has the advantage of yielding integrable expressions for the three-dimensional case, while the terms cubic and quartic in g are not easily integrated.

Inserting form (13) and (32) gives E for the one-dimensional bubble

$$E = \frac{\alpha^3}{\lambda}\left(\tfrac{35}{48} - \tfrac{15}{8} e^{-mL} + \tfrac{19}{16} e^{-2mL} + \tfrac{3}{8} mL\, e^{-3mL} - \tfrac{1}{24} e^{-3mL}\right) , \quad L = (b-a) . \tag{40}$$

For large separation (40) reduces to twice the energy (35) of a single kink. Expression (40) exhibits an attractive force between the kinks, which drops off as the exponential of the bubble size.

The spectrum of the one-dimensional bubble collapse is obtained from

$$g(k) = \int \frac{dx}{\sqrt{2\pi}} [f(x) - v] e^{ikx} = 2 v m^2 \sin(kL) [\sqrt{2\pi}\, k^2 (k^2 + m^2)]^{-1} , \tag{41}$$

so that, from (25) and (26),

$$N(k) = 2 C v^2 m^4 \sin^2(kL) [\pi k^2 (k^2 + m^2)^{3/2}]^{-1} , \tag{42}$$

with

$$C = E\left[\frac{4\alpha^4 L}{\lambda} - \frac{2\alpha^4}{m\lambda}(1 - e^{-2mL})\right]^{-1} , \tag{43}$$

where E is given by (40).

For a large bubble (42) simplifies to

$$\lim_{L \to \infty} N(k) \sim \frac{35}{24\pi}\frac{\alpha^6}{\lambda(\alpha L)}\frac{\sin^2(kL)}{k^2(k^2 + 2\alpha^2)^{3/2}} . \tag{44}$$

The total number of particles emitted during the collapse of a large bubble is

$$N = \int dk\, N(k) \sim \frac{35\sqrt{2}}{96}\frac{\alpha^2}{\lambda} . \tag{45}$$

It should be noted that the main contribution to the integral of (45) comes from the small interval $(-1/L, 1/L)$ in k space. Not surprisingly the average energy of an emitted particle is

$$E_{\text{avg}} = \frac{E}{N} \sim m . \tag{46}$$

For a small bubble (42) becomes

$$N(k) \sim \frac{4\alpha^8}{\pi\lambda^2}\frac{\sin^2(kL)}{k^2(k^2 + m^2)^{3/2}} , \tag{47}$$

so that

$$N \sim \frac{\sqrt{2}\,\alpha^5 L}{\lambda^2} . \tag{48}$$

Result (45) shows that, for large L, the number of particles emitted by and the energy of a one-dimensional bubble are independent of L, so that both can be considered to be proportional to the area of the bubble.

It is interesting to examine the energy of the one-dimensional double-bubble solution. This is given by solving (9) with the choice

$$\tilde{v}(x) = v\theta(a-x) - v\theta(x-a)\theta(b-x) + v\theta(x-b)\theta(c-x) - v\theta(x-c)\theta(d-x) + v\theta(x-d) . \tag{49}$$

The size of each bubble will be fixed to be L, so that

$$L = b - a = d - c, \quad L > 0 , \tag{50}$$

while the bubble separation will be S, so that

$$S = c - a = d - b, \quad S > L . \tag{51}$$

Using the approximation (38) for the Hamiltonian gives

$$E \approx \frac{2\alpha^3}{\lambda}\{1-e^{-mL}-e^{-mS}[\cosh(mL)-1]\} . \quad (52)$$

This clearly exhibits the attraction between the two bubbles, which grows exponentially with the size of the bubbles. For large L and larger S the expression (52) reduces to four times the energy of a single kink in the approximation (39).

The same analysis can be extended to the three-dimensional bubble given by the general form (14) for the specific model of (29). Unfortunately, the form (14) generates intractable integrals for the terms cubic and quartic in $g(x)$. As a result, approximation (38) will be employed. This gives

$$E \approx \frac{2\pi m}{\lambda}[(ma)^2-1+(1+ma)^2 e^{-2ma}] , \quad (53)$$

which is clearly proportional to the area of the bubble. Repeating the previous steps yields

$$N(k)=\frac{8}{\pi}Cv^2 m^4 k^{-6}(k^2+m^2)^{-3/2} \times[\sin(ka)-ka\cos(ka)]^2 . \quad (54)$$

Matching the two forms (53) and (54) gives, for ma large,

$$N(k)\approx \frac{108}{\pi^3}\frac{m^5}{\lambda a}k^{-6}(k^2+m^2)^{-3/2} \times[\sin(ka)-ka\cos(ka)]^2 . \quad (55)$$

Result (55) gives

$$N=\int d^3k\, N(k)\approx \frac{2\pi}{\lambda}(ma)^2 , \quad (56)$$

which again gives the result that the average particle created by the collapse of a large bubble has the energy $\sim m$.

Another model examined in the literature[9] is one which exhibits spontaneous CP violation. The scalar sector consists of two charged scalar fields interacting through the potential

$$\begin{aligned} V(\psi,\psi_2)=&-\alpha^2(\psi_1^*\psi_1+\psi_2^*\psi_2)+\tfrac{1}{2}\lambda[(\psi_1^*\psi_1)^2+(\psi_2^*\psi_2)^2] \\ &+\tfrac{1}{4}B(\psi_1^*\psi_1+\psi_2^*\psi_2)(\psi_1^*\psi_2+\psi_2^*\psi_1) \\ &+\tfrac{1}{2}A[(\psi_1^*\psi_2)^2+(\psi_2^*\psi_1)^2] . \end{aligned} \quad (57)$$

Apart from the usual gauge invariance this potential possesses the discrete exchange symmetry

$$\psi_1\to\psi_2,\quad \psi_2\to\psi_1 , \quad (58)$$

which ensures the occurrence of degenerate minima. The potential is minimized by the solutions

$$\psi_1=ve^{i\theta/2},\quad \psi_2=ve^{-i\theta/2} ,$$

where

$$v^2=\alpha^2\left[\lambda-\frac{B^2}{8A}-A\right]^{-1},\quad \cos\theta=-\frac{B}{4A} . \quad (59)$$

The presence of a nonvanishing value for θ induces CP violation in the theory. The two degenerate minima are characterized by

$$\theta=\theta_1\equiv\left|\cos^{-1}\left[-\frac{B}{4A}\right]\right|,\quad \theta_2=-\theta , \quad (60)$$

which is a direct result of the symmetry (58). When the scalar fields are coupled to a set of spinor fields, in a way which manifestly breaks the symmetry (58), spinor particles or antiparticles are created preferentially during decay processes, the preference depending on the sign of θ. This model has been used to create a cosmology where the Universe has no preference for baryons or antibaryons, but can give rise to domains where either baryons or antibaryons predominate. Of course, there must be domain walls between the respective sectors, and the formation of unstable bubbles will be possible. That such a bubble could be stabilized against collapse by the presence of particles in its interior will not be considered here, although such a situation is clearly not out of the question. Instead, the collapse will be modeled by the techniques outlined in Sec. II.

The potential (57) is diagonalized by first rewriting it in terms of four Hermitian scalar fields which are related to the charged fields through a unitary transformation, explicitly given by[10]

$$\begin{aligned} \psi_1&=\tfrac{1}{2}e^{i\theta/2}[\phi_1+\phi_4+i(\phi_2+\phi_3)] , \\ \psi_2&=\tfrac{1}{2}e^{-i\theta/2}[\phi_1-\phi_4-i(\phi_2-\phi_3)] . \end{aligned} \quad (61)$$

The two minima of expression (59) are reproduced by the following two sets of expectation values for the new scalar fields:

$$\langle\phi_1\rangle=2v,\quad \langle\phi_2\rangle=\langle\phi_3\rangle=\langle\phi_4\rangle=0 , \quad (62)$$

or

$$\begin{aligned} &\langle\phi_1\rangle=2v\cos\theta , \\ &\langle\phi_2\rangle=-2v\sin\theta , \\ &\langle\phi_3\rangle=\langle\phi_4\rangle=0 , \end{aligned} \quad (63)$$

where the positive solution for θ is understood. Rewriting the potential in terms of the fields (61) and then shifting them by either of the two sets (62) or (63) removes all terms linear in the fields and yields a diagonal set of mass terms. The values of the masses are

$$\begin{aligned} &m_1^2=2\alpha^2 , \\ &m_2^2=4\alpha^2\left[A-\frac{B^2}{16A}\right]\left[\lambda-\frac{B^2}{8A}-A\right]^{-1} , \\ &m_3^2=0 , \\ &m_4^2=2\alpha^2(\lambda+A)\left[\lambda-\frac{B^2}{8A}-A\right]^{-1} . \end{aligned} \quad (64)$$

It is now possible to construct a bubble which separates two different sectors of the theory. The form for $\tilde{v}_i(x)$

will be constructed from the solutions (62) and (63), and will be chosen to be

$$\tilde{v}_{11}=2v,\ \ \tilde{v}_{12}=2v\cos\theta\ ,$$
$$\tilde{v}_{21}=0,\ \ \tilde{v}_{22}=-2v\sin\theta\ . \tag{65}$$

There are thus two functions, f_1 and f_2, necessary to model the domain wall, and these both take the general form (14) in three spatial dimensions. From form (21), to lowest order in g_1, and g_2 the energy of the bubble is

$$E\approx-\tfrac{1}{2}\int d^3x[m_1{}^2\tilde{v}_1(x)g_1(x)+m_2{}^2\tilde{v}_2(x)g_2(x)]\ ,$$
$$\lim_{a\to\infty}E\approx4\pi v^2a^2[m_1(1-\cos\theta)^2+m_2\sin^2\theta]\ . \tag{66}$$

From the form of the coherent state (8) and the fact that both f_1 and f_2 are present, it is apparent that the bubble collapse will couple directly to both type 1 and type 2 scalars. The spectra of radiation for the two types are, respectively,

$$N_1(k)=\frac{8}{\pi}Cv^2(1-\cos\theta)^2m_1{}^4k^{-6}(k^2+m_1{}^2)^{-3/2}[\sin(ka)-ka\cos(ka)]^2 \tag{67}$$

and

$$N_2(k)=\frac{8}{\pi}Cv^2\sin^2\theta m_2{}^4k^{-6}(k^2+m_2{}^2)^{-3/2}[\sin(ka)-ka\cos(ka)]^2\ , \tag{68}$$

where result (25) has been employed again. Matching the energy of the emitted particles predicted by (67) and (68) to the total energy of (66) gives the coefficient C. The final result, for a large bubble, is

$$N_1(k)\cong\left[\frac{6}{\pi}\right]^3\frac{m_1{}^4v^2(1-\cos\theta)^2}{k^6(k^2+m_1{}^2)^{3/2}}\frac{[m_1(1-\cos\theta)^2+m_2\sin^2\theta]}{[m_1{}^2(1-\cos\theta)^2+m_2{}^2\sin^2\theta]}[\sin(ka)-ka\cos(ka)]^2\ ,$$
$$N_2(k)\cong\left[\frac{6}{\pi}\right]^3\frac{m_2{}^4v^2\sin^2\theta}{k^6(k^2+m_2{}^2)^{3/2}}\frac{[m_1(1-\cos\theta)^2+m_2\sin^2\theta]}{[m_1{}^2(1-\cos\theta)^2+m_2{}^2\sin^2\theta]}[\sin(ka)-ka\cos(ka)]^2\ . \tag{69}$$

Integrating these expressions gives the total number of emitted particles of type 1 and type 2.

$$N_1\cong(4\pi a^2)m_1v^2(1-\cos\theta)^2\left[\frac{m_1(1-\cos\theta)^2+m_2\sin^2\theta}{m_1{}^2(1-\cos\theta)^2+m_2{}^2\sin^2\theta}\right],$$
$$N_2\cong(4\pi a^2)m_2v^2\sin^2\theta\left[\frac{m_1(1-\cos\theta)^2+m_2\sin^2\theta}{m_1{}^2(1-\cos\theta)^2+m_2{}^2\sin^2\theta}\right]. \tag{70}$$

Again, both results are proportional to the area of the bubble. The branching ratio of the bubble decay is given by

$$\frac{N_1}{N_2}=\frac{m_1}{m_2}\frac{(1-\cos\theta)}{(1+\cos\theta)}\ . \tag{71}$$

IV. CONCLUSIONS

The results presented in this paper have verified the oft-made assumption that the number of particles emitted in a bubble collapse is approximately the energy of the bubble divided by the mass of the particle emitted. However, the technique of this paper allows the calculation of particle production by more complicated vacuum structures, which may couple directly to more than one particle mode. In the *CP*-violating model examined in Sec. III this technique showed the branching ratio between type 1 and type 2 scalars in terms of the *CP*-violating parameter θ, the scalar masses, the coupling constants of the theory, and the size of the bubble. In addition, because the bubble is represented as a coherent state, this technique allowed a calculation of the momentum distribution of emitted particles. This technique is general enough to be applied to any model, non-Abelian or otherwise, which can support domain formation.

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